

# **A simple model of photovoltaic module electric characteristics**

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## **Keywords**

«Alternative energy», «Generation of electrical energy» «Photovoltaic», «Device modelling»

## **Abstract**

Power electronics is used in an increasing number of applications as connecting element between photovoltaic modules and electric loads. The design of power converters for these applications needs to consider hereby the electrical characteristics of solar cells. To support this, a behavioural model has been developed to characterise current, voltage and power of photovoltaic modules as function of solar cell temperature and solar radiation. It has the advantage to use only parameters that are available from data sheets plus a single fit parameter. Calculated diagrams are compared with diagrams of two photovoltaic module data sheets.

## **1. Introduction**

The electrical data of photovoltaic (pv) modules are influenced by solar radiation, solar cell temperature and load impedance. The knowledge of this characteristic is needed to design grid connected pv power plants such that both pv modules and inverter operate with maximum efficiency [1]. This knowledge is of special interest if one uses an inverter with a relative small input voltage range that hereby offers maximum conversion efficiency [2]. Also pv module integrated inverters [3] will operate with maximum efficiency if their DC input voltage range fits exactly with the target pv module voltage range by avoiding over dimensioning [4].

Several models of solar cell exist to calculate their electric characteristics [5, 6, 7, 8, 9, 10]. To improve the accuracy, model parameters such as “irradiance correction factor” and “Temperature coefficient of a series resistance” are discussed in [11]. All these models have the disadvantage that required parameters are not published in standard data sheets. Thus it is of interest to have a simpler model to calculate the electrical properties of photovoltaic modules with only those data that are available from data sheets. Such a model has been proposed in [12]. This model has, however, the disadvantage to calculate decreasing module voltages with increasing solar radiation for cell temperatures above about 60°C because of the definition of a “shading linear function”  $\gamma(t)$ . Therefore a new behavioural model has been developed to calculate the electrical characteristic of photovoltaic modules.

The model is presented in the next chapter. Two examples demonstrate the application of the model for both a pv model with crystalline solar cells as well as a thin-film pv module. Calculated V-I diagrams are compared with diagrams from datasheets. All model parameters are documented in the appendix.

## 2. Mathematical model

The proposed mathematical model of a pv module characteristics consists of three equations (1)...(3). Equations (1) and (2) are well known from [5]. Equation (3) is the new equation to calculate the pv module current as function of solar radiation “ $\alpha$ ”, solar cell temperature “ $T$ ” and module voltage “ $V$ ”. The pv module voltage is adjusted by the maximum power point (MPP) function of a photovoltaic power converter or inverter that adjusts the load impedance of pv modules. The pv module voltage can be adjusted between zero (short circuit) and the open module voltage.

To use this model it is required to know open module voltages “ $V_{\min}$ ” and “ $V_{\max}$ ” in two operation points with different solar radiation levels  $\alpha_{\min}$  (e.g. 200 W/m<sup>2</sup>) and  $\alpha_{\max}$  (1000 W/m<sup>2</sup>) at the same nominal solar cell temperature  $T_N$  (25°C). In addition to these four parameters, the model requires additional five parameters from a module data sheet that are the short circuit current  $I_{sc}$ , voltage  $V_{MPP}$  and current  $I_{MPP}$  in the maximum power point all at standard test conditions (STC) as well as the temperature coefficients of the short circuit current  $TC_I$  and open module voltage  $TC_V$ .

The parameter “ $b$ ” is the fit parameter of the model. It influences the V-I curve in the maximum power point. The value of “ $b$ ” can be found with a procedure that compares the calculated module voltage, current and power in the maximum power point with data from a datasheet that is discussed after equation (6) that defines the maximum power point.

$$\tau_i(T) = 1 + \frac{TC_i}{100\%} \cdot (T - T_N) \quad (1)$$

$$\tau_v(T) = TC_V \cdot (T - T_N) \quad (2)$$

$$I(\alpha, T, V) = \frac{\alpha}{1000 \frac{W}{m^2}} \cdot I_{sc} \cdot \tau_i(T) \cdot \left[ \frac{1 - e^{\left[ \frac{V}{b \left( 1 + \frac{V_{\max} - V_{\min}}{V_{\max}} \cdot \frac{\alpha - \alpha_{\max}}{\alpha_{\max} - \alpha_{\min}} \right) (V_{\max} + \tau_v(T))} \cdot \frac{1}{b} \right]}}{1 - e^{\frac{1}{b}}} \right] \quad (3)$$

The open circuit voltage of a module can be calculated from equation (3) using the condition

$$I(\alpha, T, V) = 0A.$$

$$V_{oc}(\alpha, T) = \left[ 1 + \frac{V_{\max} - V_{\min}}{V_{\max}} \cdot \frac{\alpha - \alpha_{\max}}{\alpha_{\max} - \alpha_{\min}} \right] \cdot [V_{\max} + \tau_v(T)] \quad (4)$$

Equation (5) is the definition of the pv module power that is required to calculate the maximum power point. The module voltage in the maximum power point can be calculated by using the condition  $\delta P / \delta V = 0$ . Equation (6) illustrates this condition derived from equations (3) and (5). Unfortunately, it is not possible to solve equation (6) analytically. Thus the author uses the “Find” function of the Mathcad software [13] to do this.

$$P(\alpha, T, V) = V \cdot I(\alpha, T, V) \quad (5)$$

$$0 = 1 - \left[ 1 + \frac{V_{MPP}}{b \cdot \left( 1 + \frac{V_{max} - V_{min}}{V_{max}} \cdot \frac{\alpha - \alpha_{max}}{\alpha_{max} - \alpha_{min}} \right) \cdot (V_{max} + \tau_v(T))} \right] \cdot e^{\left[ \frac{\frac{V_{MPP}}{b \cdot \left( 1 + \frac{V_{max} - V_{min}}{V_{max}} \cdot \frac{\alpha - \alpha_{max}}{\alpha_{max} - \alpha_{min}} \right) \cdot (V_{max} + \tau_v(T))} - 1}{b} \right]} \quad (6)$$

The author proposes the following procedure to determine a value of the fit variable “b”. Using a guess value e.g.  $b = 0.09$  one calculates  $V_{MPP}(1000 \text{ W/m}^2, 25^\circ\text{C})$  by solving equation (6). Secondly equations (3) and (5) are used to calculate  $I_{MPP}(1000 \text{ W/m}^2, 25^\circ\text{C}, V_{MPP})$  and  $P_{MPP}(1000 \text{ W/m}^2, 25^\circ\text{C}, V_{MPP})$ . These calculated values have to be compared with values from a module data sheet. Now the guess value of fit parameter  $b$  can be adjusted iteratively to minimise the error of  $P_{MPP}(1000 \text{ W/m}^2, 25^\circ\text{C})$ .

### 3. Application

#### 3.1 Photovoltaic module with crystalline solar cells

The model has been applied for a Sharp 180 W pv module with 48 mono-crystalline solar cells [14]. The datasheet offers two diagrams in Figures 1B and 2B that are helpful in determining the model parameters. The model fit parameter has been adjusted to  $b = 0.088$  as discussed above.

The comparison of calculated diagrams Figures 1 and 2 with diagrams from the datasheet illustrates a good correlation. Curiously the two diagrams from the datasheet show slightly different short circuit currents at  $600 \text{ W/m}^2$ . The short circuit current at  $600 \text{ W/m}^2$  in Figure 1B is with 5.2 A 4% higher than in Figure 2B. The proposed mathematical model considers a short circuit current that is proportional to the solar radiation as given in the datasheet in Figure 2B.

MPP voltage and open module voltage are depicted as function of both solar radiation and temperature in Figure 3 and Figure 4 respectively in 3-dimensional diagrams. Calculated solar cell and module voltages of individual operation points are collected in Table I. This collection indicates that the voltage of this pv module may vary by a ratio of 1:2 considering a temperature range of  $-25^\circ$  till  $+75^\circ\text{C}$  and a solar radiation of  $200 \dots 1000 \text{ W/m}^2$ . That is a conservative approach since a solar radiation of  $1000 \text{ W/m}^2$  will heat-up cold solar cells at  $-25^\circ\text{C}$  ambient that results in a voltage reduction at very low ambient temperatures. On the other side one has to add to the calculated data range additional product tolerances of  $\Delta P = \pm 5\%$  for this special module. Table I also includes data for a temperature of  $-10^\circ\text{C}$  for a comparison with [15].

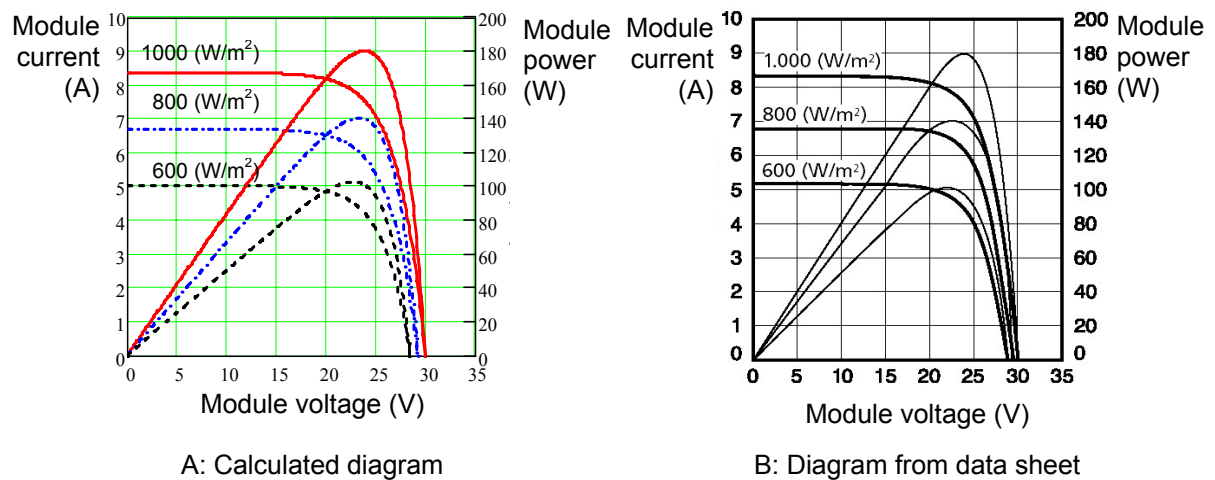
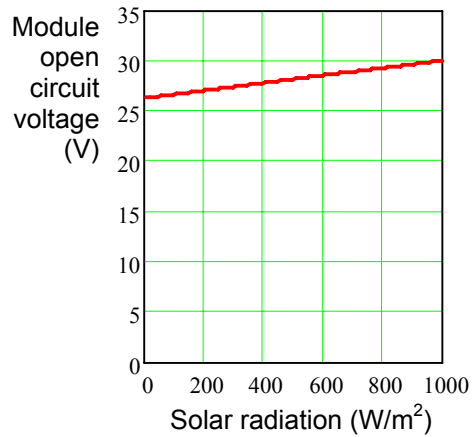
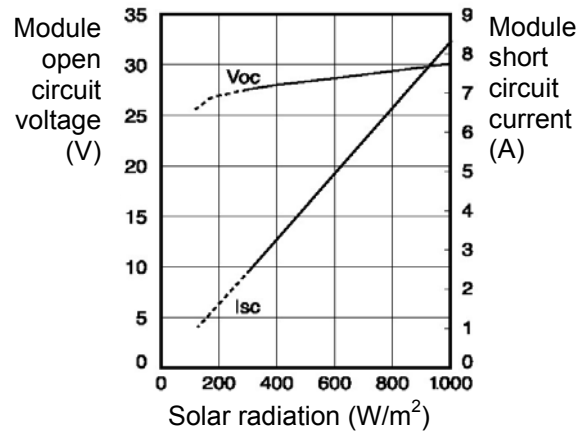


Figure 1: Comparison of calculated and data sheet diagrams ( $T = 25^\circ\text{C}$ )



A: Calculated diagram



B: Diagram from data sheet

Figure 2: Comparison of calculated and data sheet diagrams of the open circuit voltage at 25°C

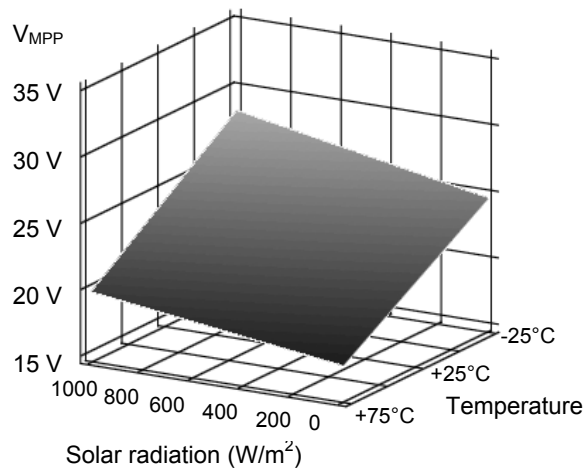


Figure 3: Calculated MPP module voltage

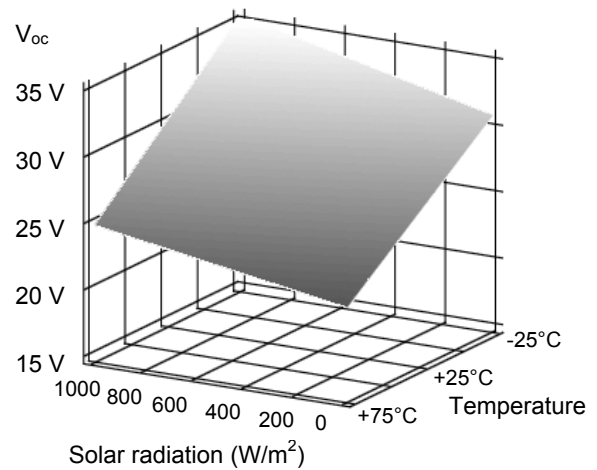


Figure 4: Calculated open module voltage

Table I: Calculated individual operation points of reference module [14]

Cell temperature	Solar radiation	Cell voltage		Module voltage	
		MPP point	open circuit	MPP point	open circuit
-25°C	1000 W/m <sup>2</sup>	585 mV	733 mV	28.1 V	35.2 V
-10°C	1000 W/m <sup>2</sup>	558 mV	700 mV	26.8 V	33.6 V
+25°C	1000 W/m <sup>2</sup>	498 mV	625 mV	23.9 V	30.0 V
+75°C	1000 W/m <sup>2</sup>	413 mV	517 mV	19.8 V	24.8 V
+75°C	200 W/m <sup>2</sup>	371 mV	465 mV	17.8 V	22.3 V

### 3.2 Thin film photovoltaic module

The model has been applied secondly for an 80 W Wuerth CIS-type thin-film pv module with 72 internal cells [16]. The model fit parameter has been adjusted to  $b = 0.095$ . The comparison of calculated diagrams in Figures 5 and diagrams from a datasheet in Figure 6 illustrates a good correlation.

A small difference is visible for the short circuit current in the diagrams at low radiation of 200 W/m<sup>2</sup>. The model calculates a short circuit current that is directly proportional to the solar radiation. The datasheet, however, shows a slightly higher short circuit current in this operation point.

The diagram from the datasheet in Figure 6 shows an open module voltage value of 41.0 V at 1000 W/m<sup>2</sup> and 55°C. This would result in a temperature coefficient of -0.150 V/K (4.5V/30K) that is more than documented in the datasheet. The model, on the other side, considers a temperature coefficient of -0.132 V/K from the datasheet. Thus the calculated open module voltage is 41.5 V at 1000 W/m<sup>2</sup> and 55°C in Figure 5.

Figure 7 depicts the MPP voltage and Figure 8 the open module voltage as function of both solar radiation and temperature. Calculated solar cell and pv module voltages of individual operation points are collected in Table II. This collection indicates that the voltage of this pv module may vary by a ratio of 1:1.9 considering a temperature range of -25° till +75°C and a solar radiation of 200...1000 W/m<sup>2</sup>. That is a little bit lower than of the Sharp module with crystalline solar cells and this is also indicated by the lower power temperature coefficient, TC<sub>P</sub> of this CIS-type module given in the appendix.

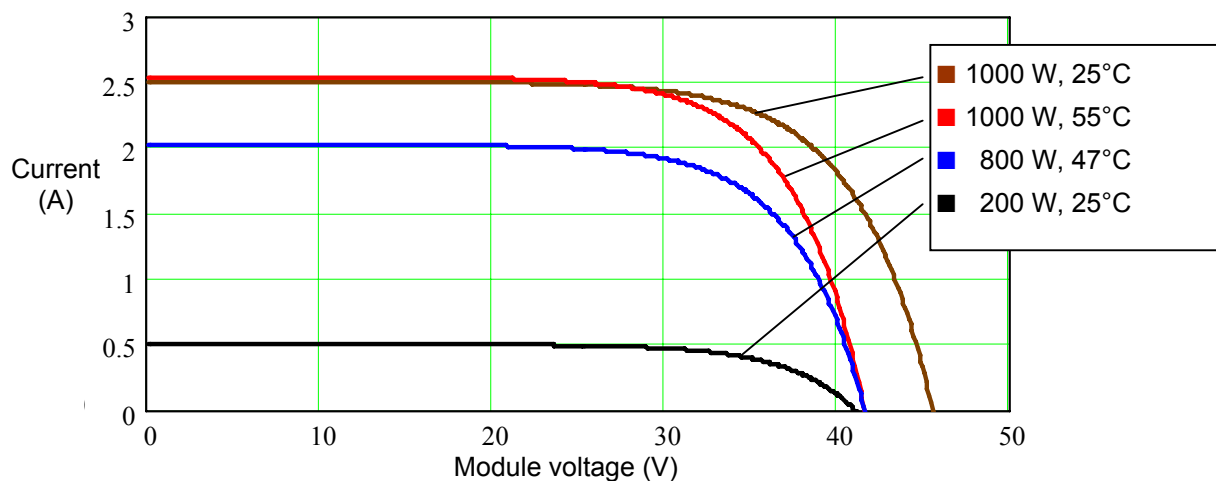


Figure 5: Calculated V-I diagram of a CIS thin-film module

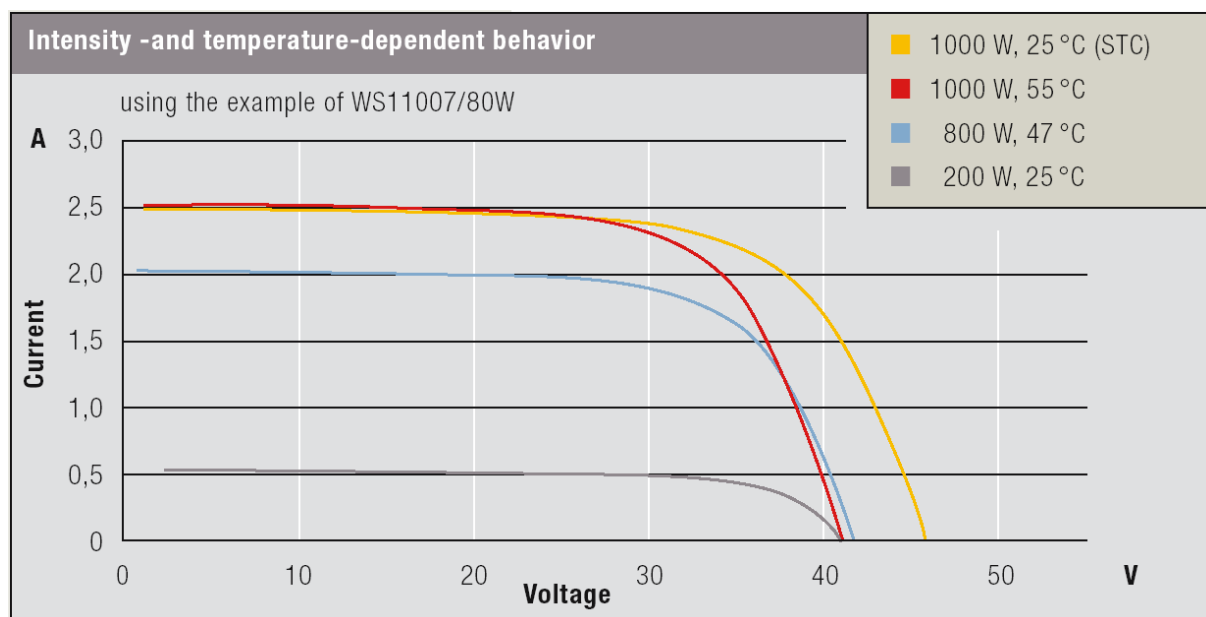


Figure 6 V-I diagram of a CIS thin-film module from datasheet [16]

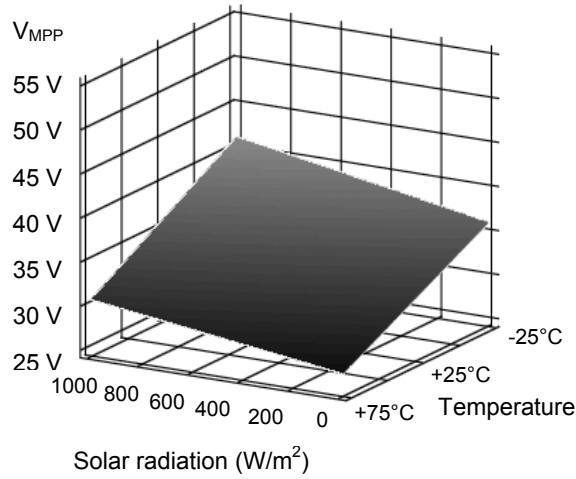


Figure 7: Calculated MPP module voltage

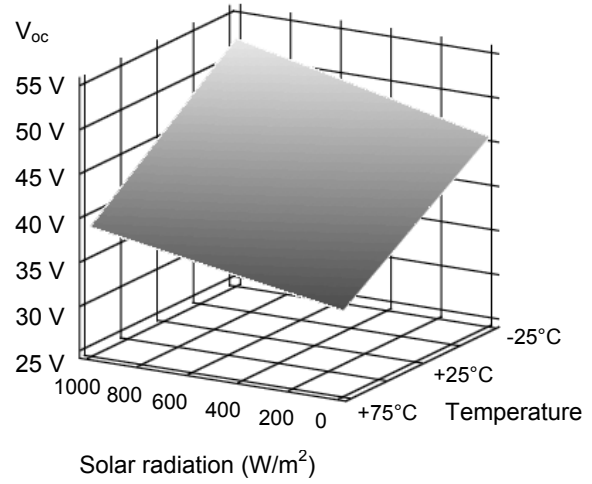


Figure 8: Calculated open module voltage

**Table II: Calculated individual operation points of reference module [16]**

Cell temperature	Solar radiation	Cell voltage		Module voltage	
		MPP point	open circuit	MPP point	open circuit
-25°C	1000 W/m <sup>2</sup>	571 mV	724 mV	41.1 V	52.1 V
-10°C	1000 W/m <sup>2</sup>	549 mV	696 mV	39.5 V	50.1 V
+25°C	1000 W/m <sup>2</sup>	499 mV	632 mV	35.9 V	45.5 V
+75°C	1000 W/m <sup>2</sup>	426 mV	540 mV	30.7 V	38.9 V
+75°C	200 W/m <sup>2</sup>	383 mV	488 mV	27.6 V	35.1 V

## 4. Conclusion

A mathematical model is proposed to calculate the electrical characteristic of a photovoltaic module as function of solar cell temperature and solar radiation. This behavioural model has the advantage to use only one unknown parameter that can be fitted by hand. All other parameters can be taken from photovoltaic module data sheets. Calculated diagrams show good correlation with diagrams from datasheets.

The author motivates to extend the diagrams in photovoltaic module datasheets in a systematically way. It is proposes to include measured V-I diagrams for two solar radiation levels of 200 W/m<sup>2</sup> and 1000 W/m<sup>2</sup> at three module temperatures of e.g. -25°C, +25°C and +75°C. Hence it is easier for constructors of photovoltaic power systems to validate the DC-bus voltage range as function of temperature and solar radiation. This would also allow a comparison of this and other models with measured data at different temperatures that is not possible with today's information in datasheets.

## 5. Appendix

### 5.1 Abbreviations

		Reference module data	
		[14]	[16]
b	model fit parameter	b = 0.088	0.095
CIS	Copper Indium Diselenid		
I <sub>MPP</sub>	Module current in the maximum power point and STC conditions	I <sub>MPP</sub> = 7.60 A	2.22 A
I <sub>sc</sub>	Short circuit current under standard test conditions	I <sub>sc</sub> = 8.37 A	2.50 A
MPP	Maximum Power Point		
P <sub>MPP</sub>	Module power in the maximum power point and STC conditions	P <sub>MPP</sub> = 180 W	80.0 W
pv	Photovoltaic		
STC	Standard Test Conditions: + 25°C, 1000 W/m <sup>2</sup> , air mass 1.5		
T	Temperature		
T <sub>N</sub>	Nominal Temperature according to standard test conditions	T <sub>N</sub> = 25°C	
TC <sub>i</sub>	Temperature coefficient of the module short circuit current	+0.053 %/K	+0.050 %/K
TC <sub>v</sub>	Temperature coefficient of the open module voltage	-0.104 V/K	-0.132 V/K
TC <sub>p</sub>	Temperature coefficient of the module nominal power	-0.485 %/K	-0.36 %/K
V	Voltage		
V <sub>min</sub>	Open module voltage at 25°C and low solar radiation ( $\alpha_{min}$ )	V <sub>min</sub> = 27.0 V	41.0 V
V <sub>max</sub>	Open module voltage at 25°C and high solar radiation ( $\alpha_{max}$ )	V <sub>max</sub> = 30.0 V	45.5 V
V <sub>MPP</sub>	Module voltage in the maximum power point and STC conditions	V <sub>MPP</sub> = 23.7 V	36.0 V
V <sub>oc</sub>	Voltage of an unload solar cell or module	V <sub>oc</sub> = 30.0 V	45.5 V
$\alpha$	Solar radiation, unit W/m <sup>2</sup>		
$\alpha_{min}$	Low solar radiation	$\alpha_{min}$ = 200 W/m <sup>2</sup>	
$\alpha_{max}$	High solar radiation	$\alpha_{max}$ = 1000 W/m <sup>2</sup>	

## 5.2 References

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