PHYS 3040: MODERN PHYSICS
Department of Physics and Astronomy
York University
Homework Set #3

Due on Wednesday November 6, 2024.
Please write solutions clearly and carefully.
Submit a single pdf file through eClass.

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1. Fermilab puts a muon into a uniform magnetic field $\vec{B} = B\hat{z}$ to measure the muon's magnetic moment, which is a way to search for new physics beyond the standard model. We can name the energy eigenvalues E_+ and E_-. Suppose a muon is in the state $\left|+\right\rangle_n$ at time $t = 0$, where $\hat{n} = \frac{1}{\sqrt{2}}(\hat{y} + \hat{z})$.

(a) Calculate an expression for the state of the muon at any time t, named $|\psi(t)\rangle$.

(b) Calculate the probability that a measurement of S_x will give $+\frac{\hbar}{2}$ at time t.

(c) Use your final expression from (b) to compute numerical values for the minimum and maximum probabilities as time passes.

(d) Calculate the expectation value of S_x at time t.

(e) Use your final expression from (d) to obtain simple algebraic answers for the minimum and maximum expectation values as time passes.

2. For a hospital MRI machine, the magnetic field has both a longitudinal component and a transverse component. The Hamiltonian matrix and its eigenvectors are

$$H = \frac{\hbar}{2}\begin{pmatrix} \omega\cos\theta & \omega\sin\theta \\ \omega\sin\theta & -\omega\cos\theta \end{pmatrix}, \qquad \left|+\right\rangle_B = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}, \qquad \left|-\right\rangle_B = \begin{pmatrix} \sin\frac{\theta}{2} \\ -\cos\frac{\theta}{2} \end{pmatrix}.$$

We know that H depends on time during normal operation, but for this homework question we will operate the machine with H held constant. Suppose the nucleus of a hydrogen atom in the patient is in the state $\left|-\right\rangle_z$ at time $t = 0$.

(a) Calculate the eigenvalues of H.

(b) Calculate an expression for the state of the nucleus at any time t, named $|\psi(t)\rangle$.

(c) Calculate the expectation value of S_x for the nucleus at time t.

(d) Suppose your MRI machine has $\theta = 45°$. Use your final expression from (c) to compute algebraic values for the minimum and maximum expectation values as time passes.