Numerical Analysis (10th ed)

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Chapter 9

Initial-Value Problems for Ordinary Differential Equations

▶ Chapter 9.1: Linear Algebra and Eigenvalues

▶ Chapter 9.2: Orthogonal Matrices and Similarity Transformations

▶ Chapter 9.3: The Power Method

▶ Chapter 9.4: Householder's Method

▶ Chapter 9.5: The QR Algorithm

▼ Chapter 9.6: Singular Value Decomposition*

In this section we consider the factorization of a general m x\002n matrix A into what is called a Singular Value Decomposition. We decompose the matrix A into $A = USV^t$ where U is an m x\002m orthogonal matrix, V is an n x\002n orthogonal matrix, and S is an m x n matrix whose only nonzero elements lie along the main diagonal.

Geometrically, the SVD maps spheres of the proper dimension in the domain into ellipsoids in the codomain.

SVD CONSTRUCTION:

1. Construct S in the factorization $A = USV^t$
   - Find the eigenvalues of the n x n symmetric matrix A^tA.
   - Order them from largest to smallest and denote as $s_1^2 \geq s_2^2 \geq \ldots \geq s_k^2 > s_{k+1}^2 = \ldots = s_n = 0$.
   - The diagonal entries of D are the square roots of singular values of $s_1, s_2, \ldots, s_n$.
2. Construct V in the factorization $A = USV^t$
   - Find the associated eigenvectors $v_1, v_2, \ldots, v_n$ for the eigenvalues of A^tA.
   - Normalize the eigenvectors $v_1, v_2, \ldots, v_n$ to obtain the columns of V.
3. Construct U in the factorization $A = USV^t$
   - Compute the first k columns of

- $$U : u_i = \frac{1}{s_i} A v_i \text{ for } i = 1, 2, \ldots, k.$$
- Use the Gram-Schmidt process to obtain additional columns .

ALTERNATE STEP 3 OF SVD CONSTRUCTION:

- Compute the m eigenvalues of $A^t A$.
- Find the set of m corresponding eigenvectors.
- Normalize these eigenvectors and make them the columns of U.

The SVD is often used in data compression and in finding least squared polynomials.