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RoadMap Based Robot Motion Planning

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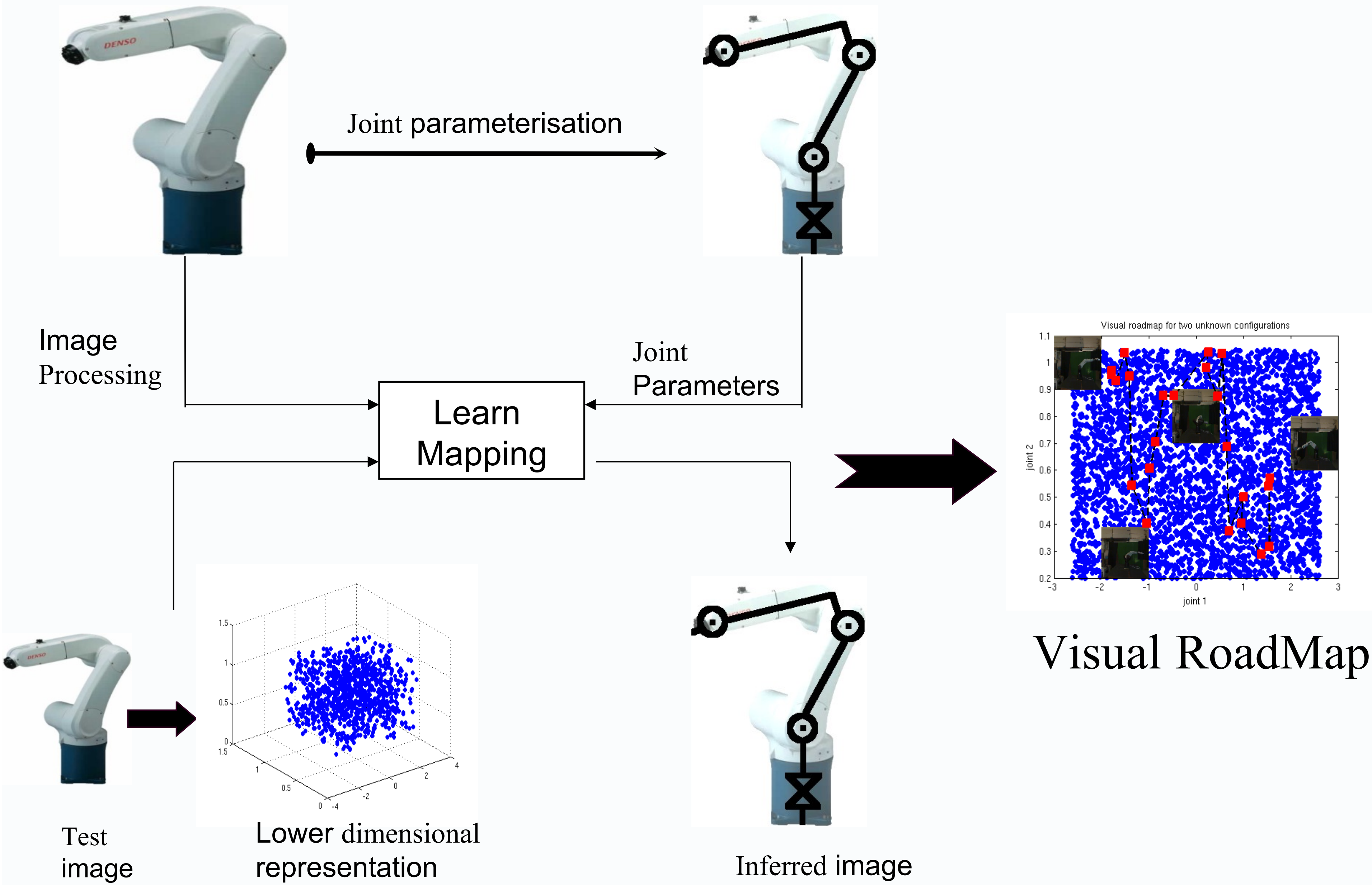
Introduction



Gaussian Processes

- GPs are distributions over functions – a set of random variables indexed by a continuous function *f (x)*.
- Inputs X={$x_1 x_2, x_3$……………} with corresponding function values f={f_1, f_2, f_3 …………}.
- Gaussian Process on functions

  - Any set of function variables has joint Gaussian distribution

$$P (f \mid X) \quad N (0 , K)$$

  - Covariance matrix comes from covariance function or Kernel

$$K_{ij} \quad k(x_i, x_j)$$

  -Covariance function determines the correlation between different points from the GP

$$k(x_i, x_j) \quad E(f(x_i) f(x_j))$$

$$k(x_i, x_j) \qquad {}^2 \exp \frac{\left| x_i \quad x_j \right|^2}{2l^2}$$

Gaussian Process Regression

- Assume functional relation between observation spaces,corrupted by noise(Gaussian distributed).

$$Y \quad f(X) \quad N(0, \quad {}^{2} I), \textit{ where } , f \quad GP(0, K)$$

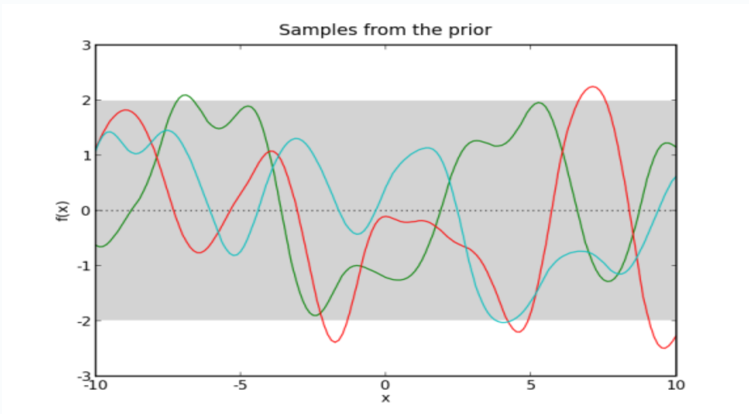
- Define joint distribution over the inferred observation space.For any new point x* ,corresponding y* can be obtained by maximizing the posterior distribution derived from the joint distribution.

$$\begin{matrix} y \\ y^{*} \end{matrix} \quad N \quad 0, \begin{matrix} K & K_{*}^{T} \\ K_{*} & K_{**} \end{matrix}$$

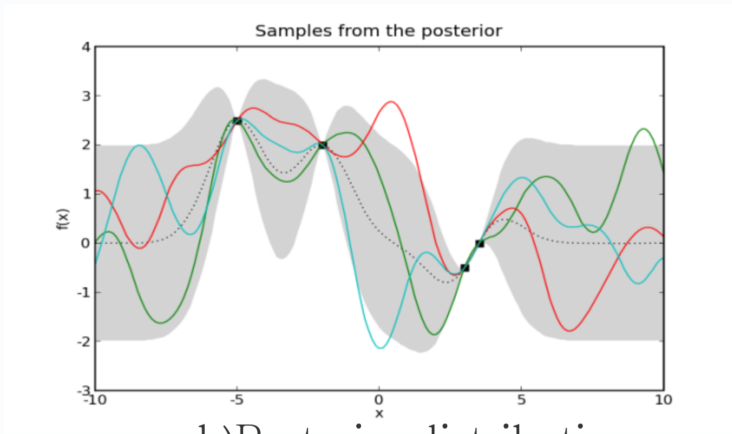
Joint distribution

Posterior distribution

$$y_{*} \mid y \quad N \quad K_{*} K^{\;1} y, K_{**} \quad K^{\;1} K_{*}^{T}$$



a)Prior distribution



b)Posterior distribution

Gaussian Process Latent Variable Model

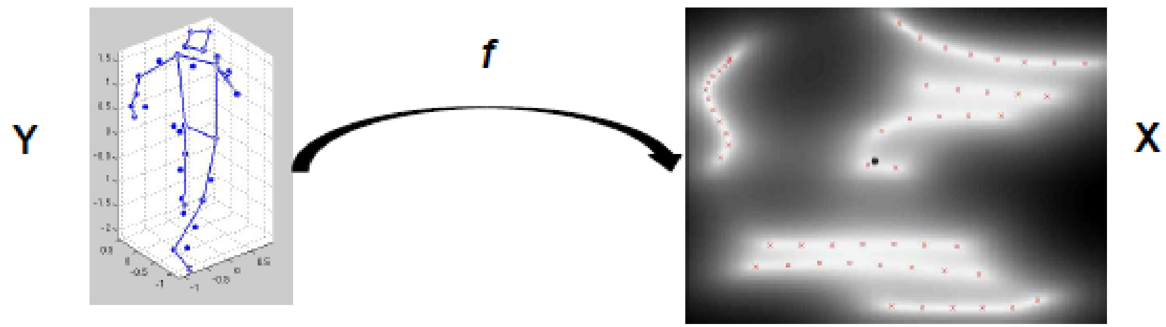
- Assume the observed data Y is related to the latent data X by a mapping f corrupted by noise

$$y_n \qquad f(x_n) \qquad , where \; , \qquad N(0, \quad {}^{1}I)$$

- A Gaussian Process prior is placed on f

$$P(Y \mid X, \quad) \qquad \prod_{n \; 1}^{N} P(y_n \mid x_n, \quad , f) P(f) df$$

- Optimise likelihood w.r.t. X using Scaled Conjugate Gradient method



Lawrence, N. D. *Probabilistic Non-Linear Principal Components Analysis with Gaussian Process Latent Variable Model*, JMLR 6:1783-1816, 2005

Combining the strengths of UMIST and
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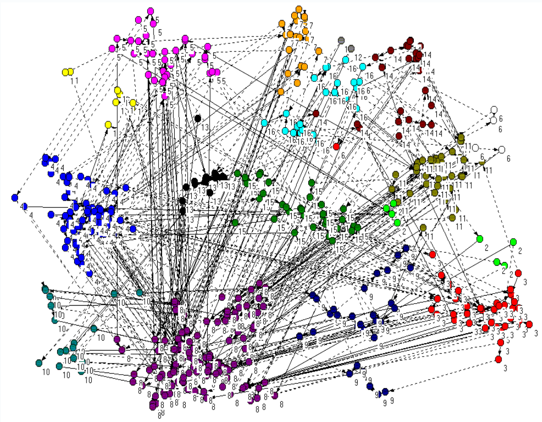
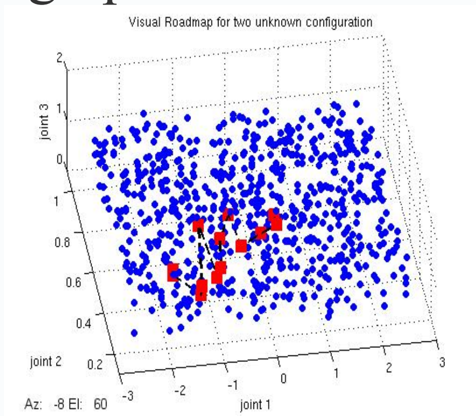
Visual RoadMap

- Proximity graph is constructed assuming all possible transitions in known configuration space.
- Similarity matrix for proximity graph is evaluated on basis of distance measure by using K-means clustering.
- Nearest neighbor for unknown configuration is found in known configuration space.
- A shortest path through known configuration space is reported by applying Dijakstra’s algorithm on proximity graph.



Proximity graph
(built using K–means clustering)



Visual Roadmap built
from proximity graph

Workflow

Random Projection
- Helps to reduce high dimensional image to lower dimension while preserving inter point distance

GPLVM
- Initialized using LLE(Locally Linear embedding)
- Optimized using SCG(Scaled Conjugate Gradient)

GP Regression
- Find a mapping Image space(Y)to joint angle space (Θ)

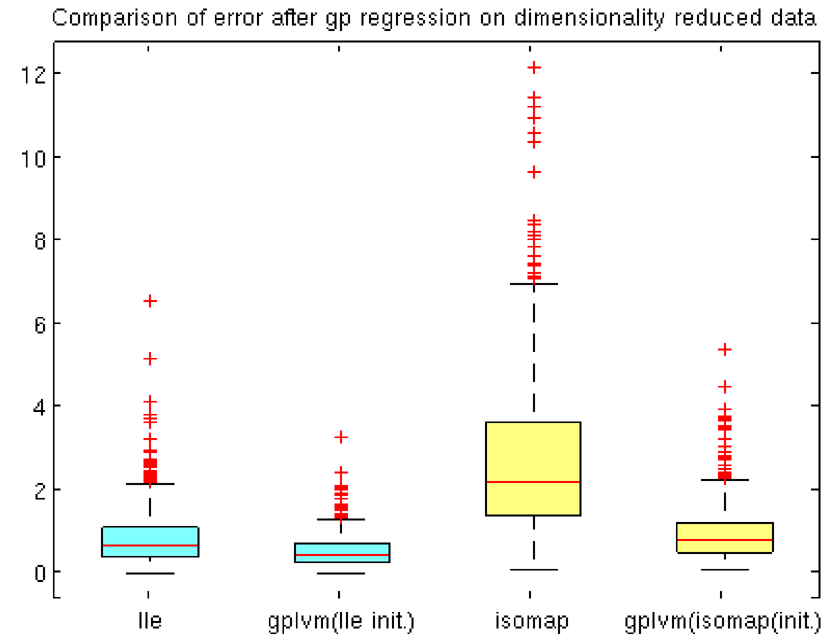
K Means
- Cluster n points into K clusters so that points within same cluster are closely placed

Visual Path Planning
- Generate a path between two given configurations through known configuration space via Dijakstra's algorithm.

Results

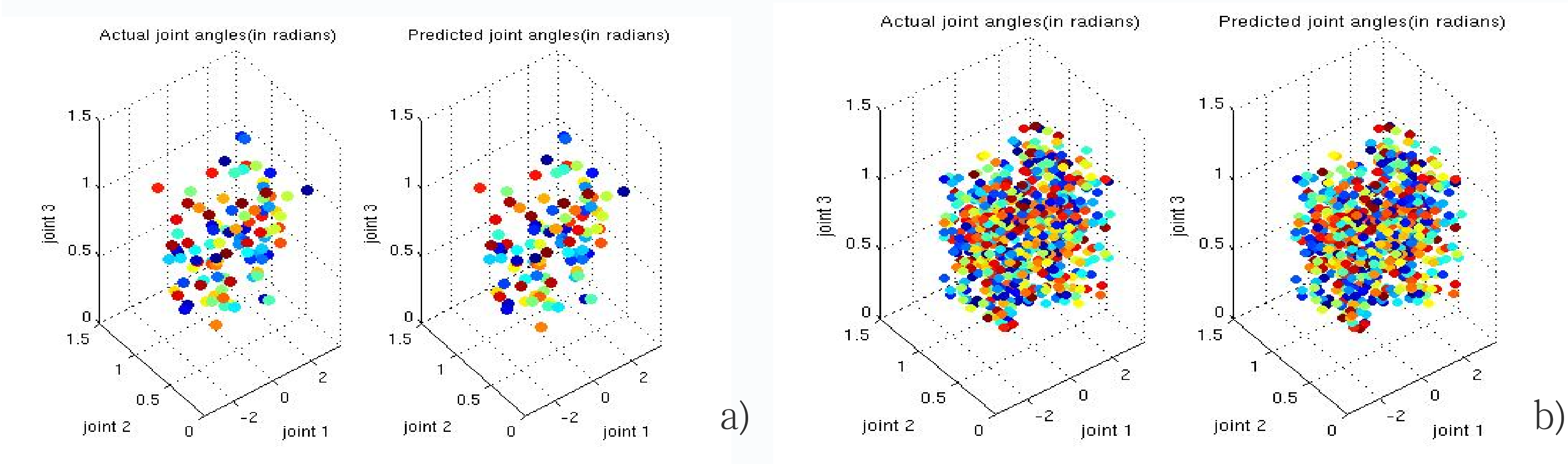
- Gaussian Process regression is applied on random projected data followed by dimensionality reduction using GPLVM,Isomap and LLE.
- GPLVM is generative dimensionality reduction method , requires initialization of latent space using other methods(GPLVM,Isomap).
- Statistical analysis of deviation from the ground truth is done for various techniques used.

Technique	Avg. error	Min error	Max error	Std. Dev.
LLE	0.855	0.030	6.507	0.66
GPLVM (LLE init)	0.558	0.013	3.247	0.38
Isomap	2.713	0.081	12.10	1.85
GPLVM (isomap init.)	0.968	0.084	5.378	0.66

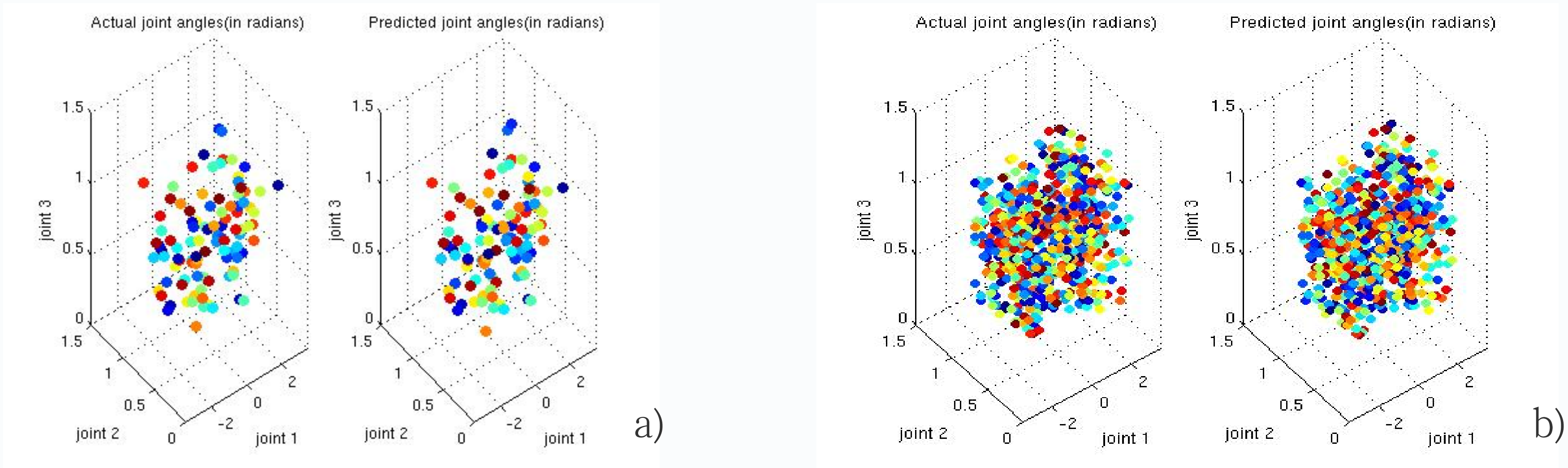


Comparison of error among dimensionality reduction techniques

Results

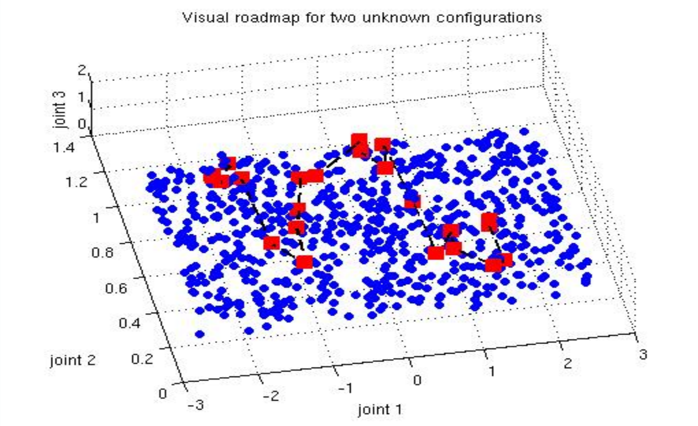


Comparison of GPLVM results with ground truth a)100 points b)675 points

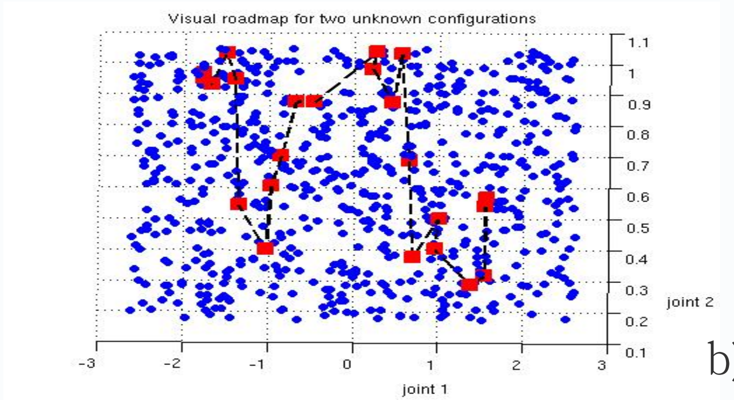


Comparison of LLE results with ground truth a)100 points b)675 points

Results



a)



b)

Visual Roadmap between two unknown configurations a)3D b) 2D

Image transitions over the roadmap

Conclusion

- Statistical analysis shows that GP Regression coupled with GPLVM (lle init.) gives superior results to other dimensionality reduction techniques which are favorably close to ground truth.
- Gaussian Methods are sensitive to parameters as no. of optimization iterations , no. of nearest neighbor in lle, and hence the result must be validated.

Future Work

- In future , we will like to reconstruct the manifold to extend it to generate an obstacle free path.
- Implement shared manifold models for direct multivalued regression.
- Dynamical shared manifold models can be used to consider usefulness of sequential data to improve manifold structure.
- To use unlabelled image data along with labeled data to improve upon the manifold structure for sparse datasets.

Acknowledgement

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