Digital Signal Processing

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Abstract—**This manual provides a beginner level application of signal processing by filtering noise from an audio signal recorded using a mobile phone. A built-in Python module for the Butterworth low pass filter (LPF) is used for filtering out noise present in higher frequencies. Through this application, relevant concepts in DSP are explored.**

1 Software Installation

Run the following commands

```
sudo apt-get update
sudo apt-get install libffi-dev
sudo pip install pysoundfile
```

2 Digital Filter

Problem 1. Download the sound file in the link given below. http://tlc.iith.ac.in/img/sound//Sound_Noise.wav

Problem 2. Open the link given below. You will find a spectrogram. https://academo.org/demos/spectrum-analyzer

Problem 3. Upload the sound file that you downloaded in Problem 1 in the spectrogram in Problem 2 and play. Observe the spectrogram. What do you find?

Solution: There are a lot of yellow lines between 440 Hz to 5.1 KHz. These represent the synthesizer key tones. Also, the key strokes are audible along with background noise.

Problem 4. Write the python code for removal of out of band noise and execute the code.

Solution:

```
import soundfile as sf
from scipy import signal

#read .wav file
input_signal,fs = sf.read('
    Sound_Noise.wav')

#sampling frequency of Input
    signal
sampl_freq=fs

#order of the filter
order=4

#cutoff frquency 4kHz
cutoff_freq=4000.0

#digital frequency
Wn=2*cutoff_freq/sampl_freq

# b and a are numerator and
    denominator polynomials
    respectively
b, a = signal.butter(order,Wn, '
    low')

#filter the input signal with
    butterworth filter
output_signal = signal.filtfilt(b,
     a, input_signal)

#write the output signal into .wav
     file
sf.write('Sound_With_ReducedNoise.
    wav', output_signal, fs)
```

Problem 5. The output of the python script in Problem 4 is the audio file Sound_With_ReducedNoise.wav. Play the file

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in the spectrogram in Problem 2. What do you observe?

Solution: The key strokes as well as background noise is subdued in the audio. Also, the signal is blank for frequencies above 5.1 kHz. Answer the following questions by looking at the python code in Problem 4.

Problem 6. What is the sampling frequency of the input signal?

Solution: Sampling frequency(fs)=44.1kHZ.

Problem 7. What is type, order and cutoff-frequency of the above butterworth filter

Solution: The given butterworth filter is low pass with order=2 and cutoff-frequency=4kHz.

Problem 8. Modifying the code with different input parameters and to get the best possible output.

Problem 9. The command

```
output_signal = signal.
    filtfilt(b, a,
    input_signal)
```

in Problem 4 is executed through the following difference equation

$$\sum_{m=0}^{M} a(m)\, y(n-m) = \sum_{k=0}^{N} b(k)\, x(n-k) \tag{9.1}$$

where the input signal is $x(n)$ and the output signal is $y(n)$ with initial values all 0. Replace **signal.filtfilt** with your own routine and verify.

3 Z-transform

Problem 10. The Z-transform of $x(n)$ is defined as

$$X(z) = \mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \tag{10.1}$$

Show that

$$\mathcal{Z}\{x(n-1)\} = z^{-1}X(z) \tag{10.2}$$

and find

$$\mathcal{Z}\{x(n-k)\} \tag{10.3}$$

Problem 11. Find

$$H(z) = \frac{Y(z)}{X(z)} \tag{11.1}$$

from (9.1) assuming that the Z-transform is a linear operation.

Problem 12. Find the Z transform of

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases} \tag{12.1}$$

and show that the Z-transform of

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{12.2}$$

is

$$U(z) = \frac{1}{1 - z^{-1}}, \quad |z| > 1 \tag{12.3}$$

Problem 13. Show that

$$a^n u(n) \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1 - az^{-1}} \quad |z| > |a| \tag{13.1}$$

Problem 14. Show that $H(z)$ for b and a in Problem 4 can be expressed as

$$H(z) = \sum_k \frac{c_k}{1 - d_k z^{-1}} \tag{14.1}$$

using partial fractions. Find the values of c_k and d_k.

4 Impulse Response

Problem 15. Find an expression for $h(n)$ using $H(z)$ in Problem 14.1 and (13.1), given that

$$h(n) \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{15.1}$$

and there is a one to one relationship between $h(n)$ and $H(z)$. $h(n)$ is known as the *impulse response* of the system defined by (9.1).

Problem 16. Sketch $h(n)$. Is it bounded? Convergent?

Problem 17. The system with $h(n)$ is defined to be stable if

$$\sum_{n=-\infty}^{\infty} h(n) < \infty \tag{17.1}$$

Is the system defined by (9.1) stable for the impulse response in (15.1)?

Problem 18. Compute $h(n)$ using

$$\sum_{m=0}^{M} a(m)\, h(n-m) = \sum_{k=0}^{N} b(k)\, \delta(n-k) \tag{18.1}$$

This is the definition of $h(n)$.

Problem 19. Compute

$$y(n) = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x(k)h(n-k) \qquad (19.1)$$

where $x(k)$ is the **input_signal** in Problem 4. You will need to suitably truncate $h(n)$ calculated in Problem 15. Use $y(n)$ as **output_signal** in Problem 4. Comment. The operation in (19.1) is known as *convolution*.

Problem 20. Show that

$$y(n) = \sum_{n=-\infty}^{\infty} x(n-k)h(k) \qquad (20.1)$$

5 DFT and FFT

Problem 21. Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-\jmath 2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \qquad (21.1)$$

and $H(k)$ using $h(n)$.

Problem 22. Compute

$$Y(k) = X(k)H(k) \qquad (22.1)$$

Problem 23. Compute

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) \cdot e^{\jmath 2\pi kn/N}, \quad n = 0, 1, \ldots, N-1 \qquad (23.1)$$

Use $y(n)$ as **output_signal** in Problem 4.

Problem 24. Repeat the previous exercise by computing $X(k), H(k)$ and $y(n)$ through FFT and IFFT.