PHYS2305-Week 2

S. Kunori (TTU)

04-Sep-2017

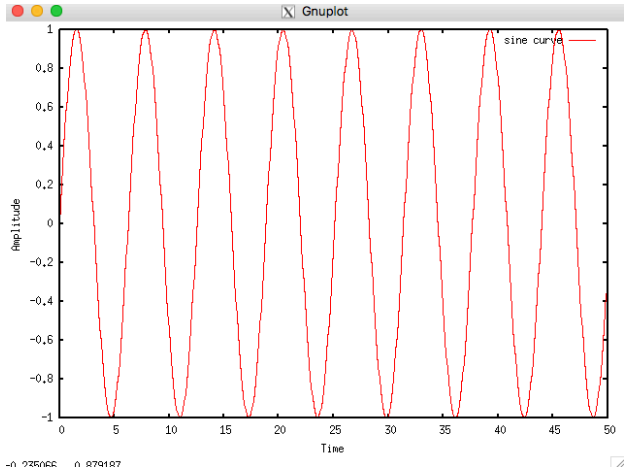
Previous Week

C++

Hello World!

Variabes: string, int, double
for-loop
If

function




```
#include <iostream>
#include <fstream>  // for output file
#include <math.h>   // for sin(), cos()

using namespace std; // to omit std on cout...

// my own function
double myfunction(double x) {
   double y=sin(x);
   return y;
}

int main()
{
   cout << "C++ code examples..."<<endl;

   // static sine curve...
   string fname="myOutfile.txt";   // define output file name
   ofstream outfile;               // open output file...
   outfile.open(fname.c_str());   // C++ lib on archer is old,
   // outfile.open(fname);    // new C++ lib use string.

   double xmin=0.0;
   double xmax=50.0;
   double dx=0.1;

   for (double x=xmin; x<xmax; x=x+dx) {
      double y=myfunction(x);
      outfile<<"   "<<x<<"   "<<y<<endl;
   }

   outfile.close();

   return 0;
}
```

This Week

Numerical equation solving

sin(x)+x = 2.0

Numerical Differentiation

d(sin(x))/dx

Numerical integration

Gaussian distribution

- three methods

Rectangle

Trapezoidal

Simpson's

Homework 1:

First submission: Mon 9/11 (required)

Second submission Mon 9/18 (for upgrade)

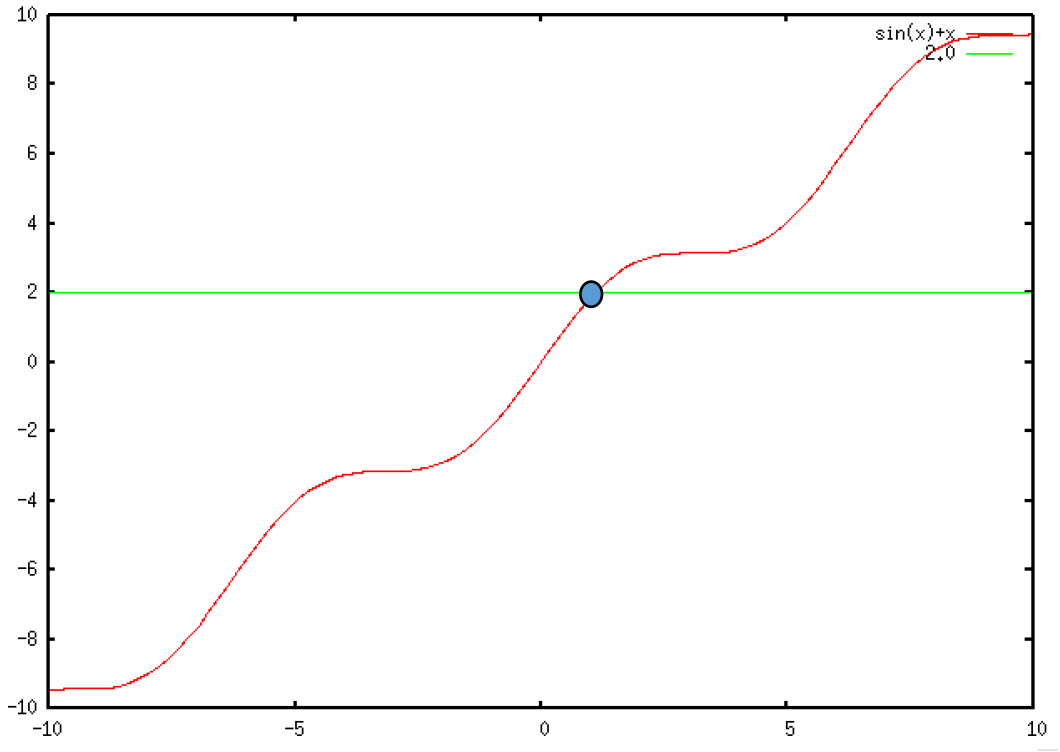
send plots and codes to shuichi.kunori@ttu.edu email subject: phys2305 homework-1

Numerical equation solving (1)

sin(x) + x = 2

check the shape of the function

gnuplot> plot sin(x)+x, 2.0



How to do?

Calculate y = sin(x)+x in small steps along x-axis and compare the y value with 2.0.

If y(x) > 2.0, the red curve has crossed the green curve. Take the x-value as a solution of the equation.

C++ code:

function: y=sin(x)+x

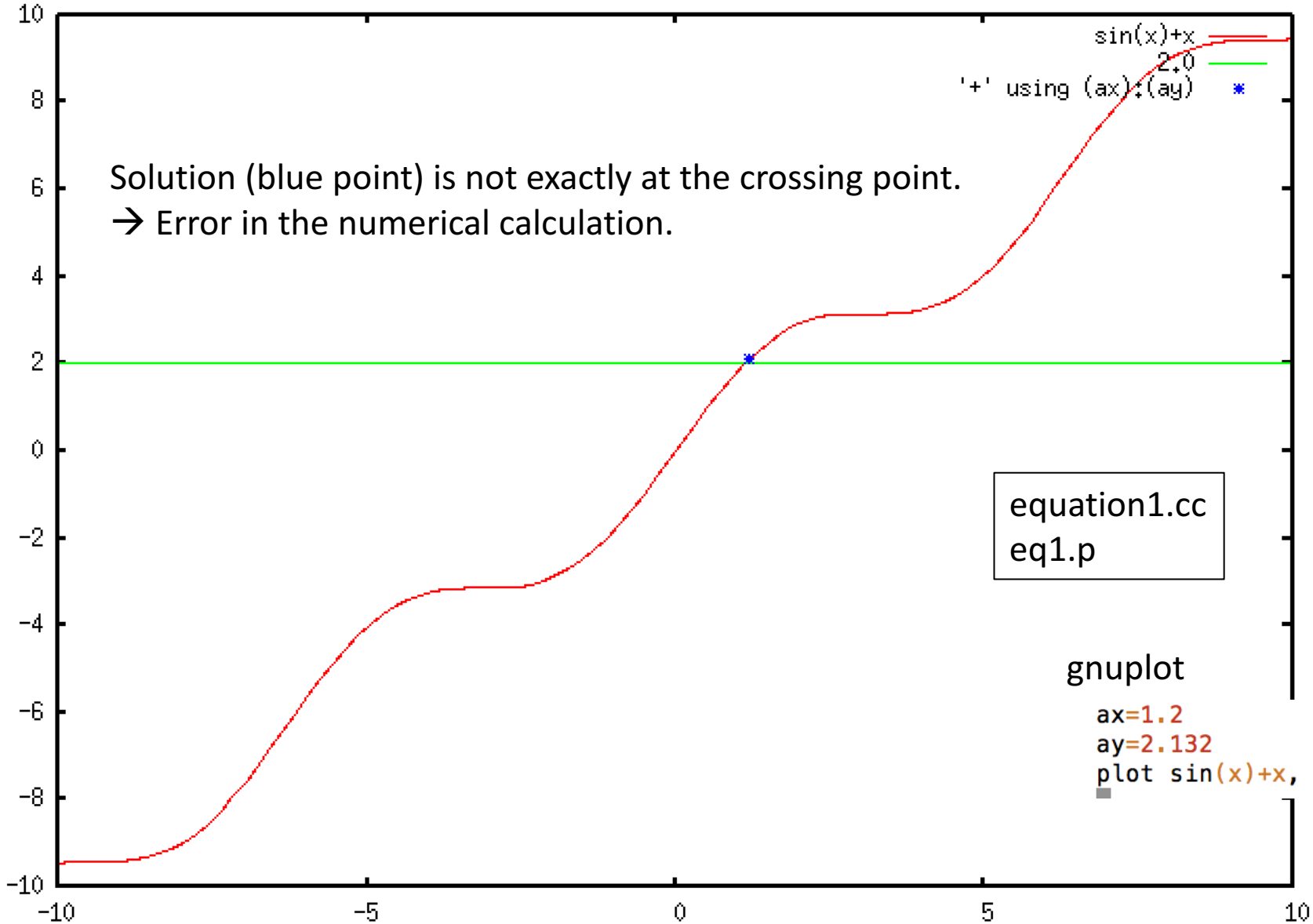
main:

for-loop over x

call function

compare y(x) vs 2.0

if y (red curve) becomes greater than 2.0, print out x and exit from the loop.



Solution (blue point) is not exactly at the crossing point.
→ Error in the numerical calculation.

equation1.cc
eq1.p

gnuplot

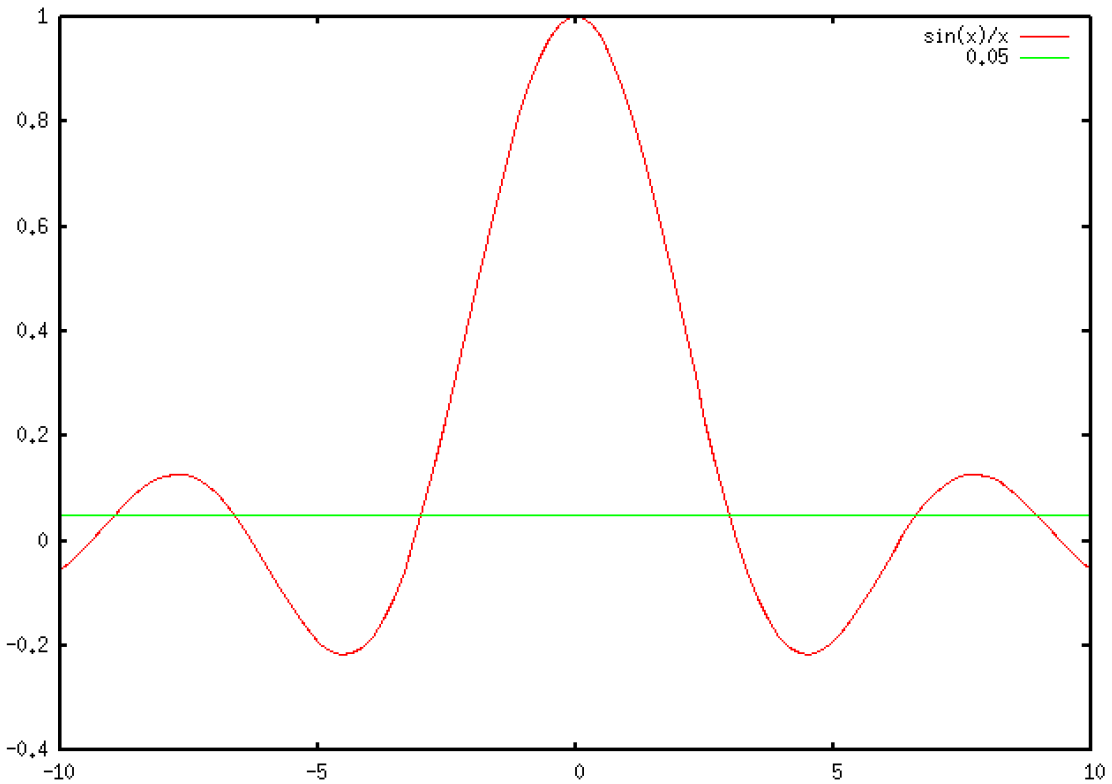
```
ax=1.2
ay=2.132
plot sin(x)+x, 2.0, '+' using (ax):(ay)
```

Numerical equation solving (2)

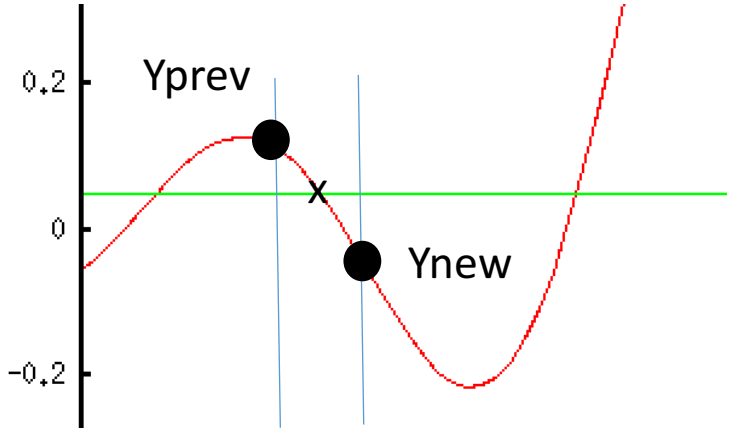
sin(x)/x = 0.05

Form of the function – six crossing points

gnuplot> plot sin(x)/x, 0.05



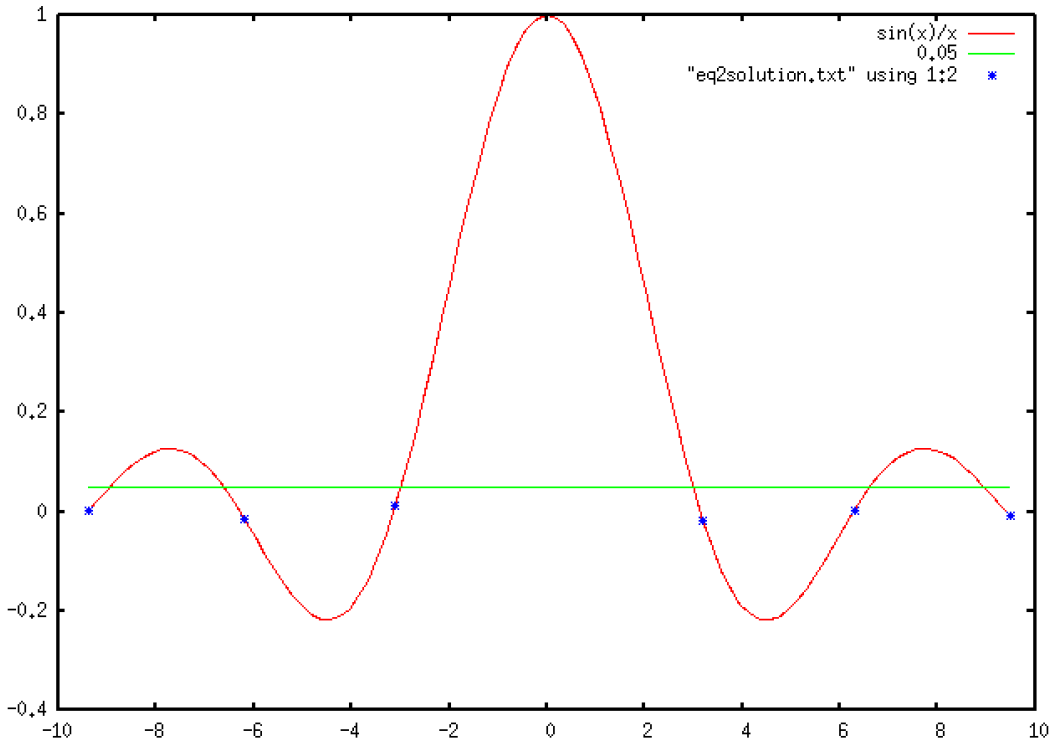
How to find the x values of six crossing points?



Yprod = (Yprev-0.05) * (Ynew-0.05),
Crossing point, if Yprod<0

C++ code:

function call in for-loop in main.



My first result with equation2bug.cc

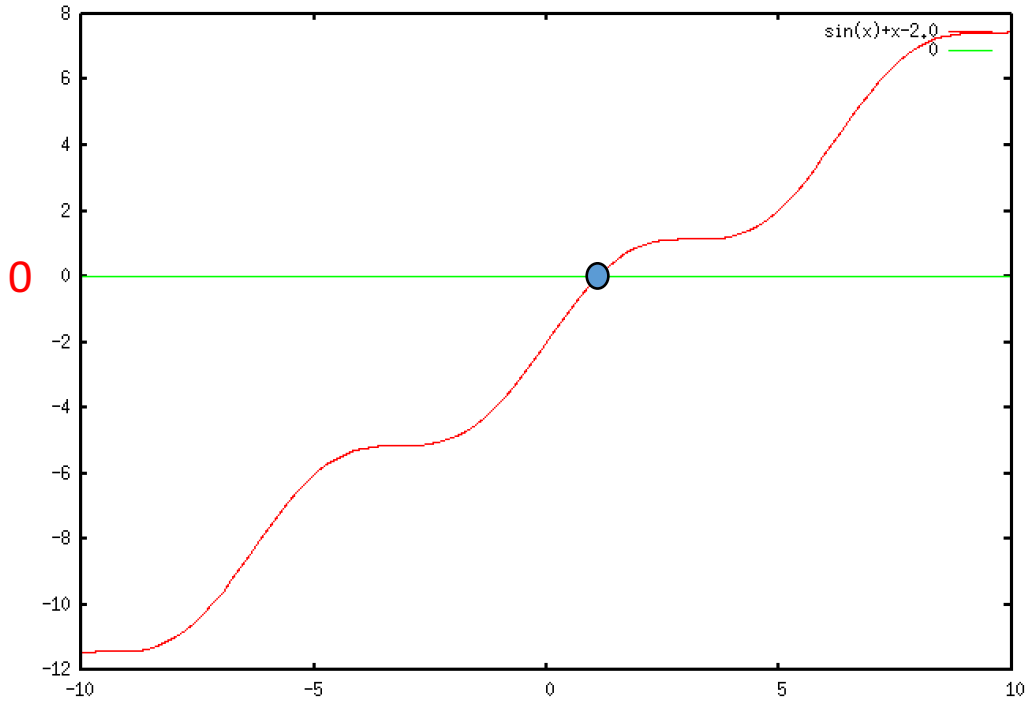
Solution points were off.
What was wrong in the code?

equation2bug.cc
eq2.p

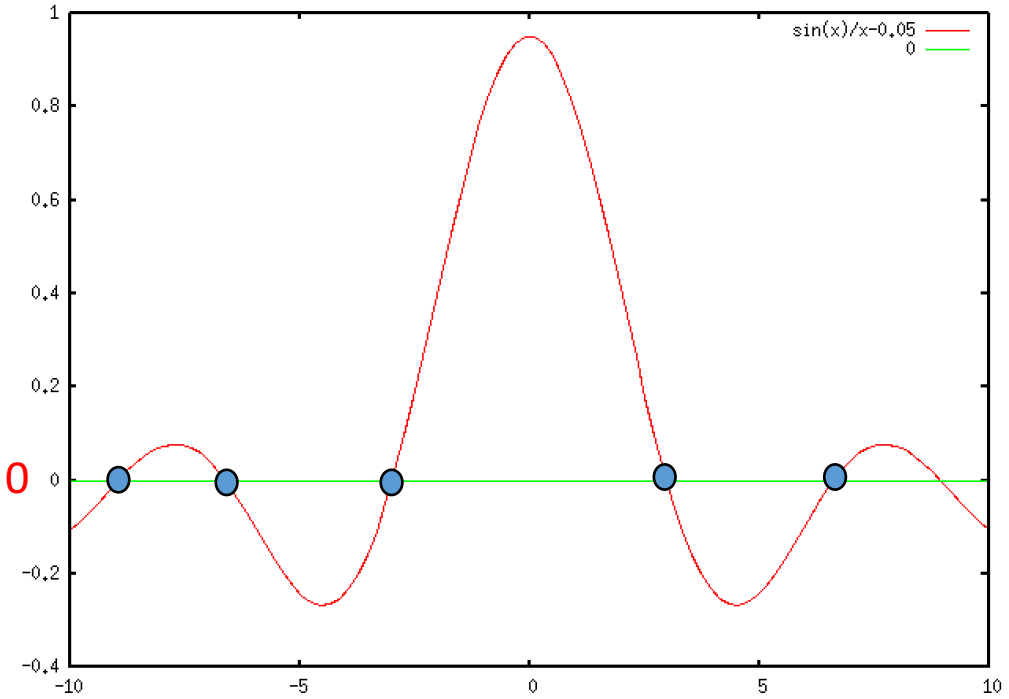
equation2fixed.cc
eq2.p

Numerical equation solving (3)

f(x) = sin(x) + x - 2.0 = 0



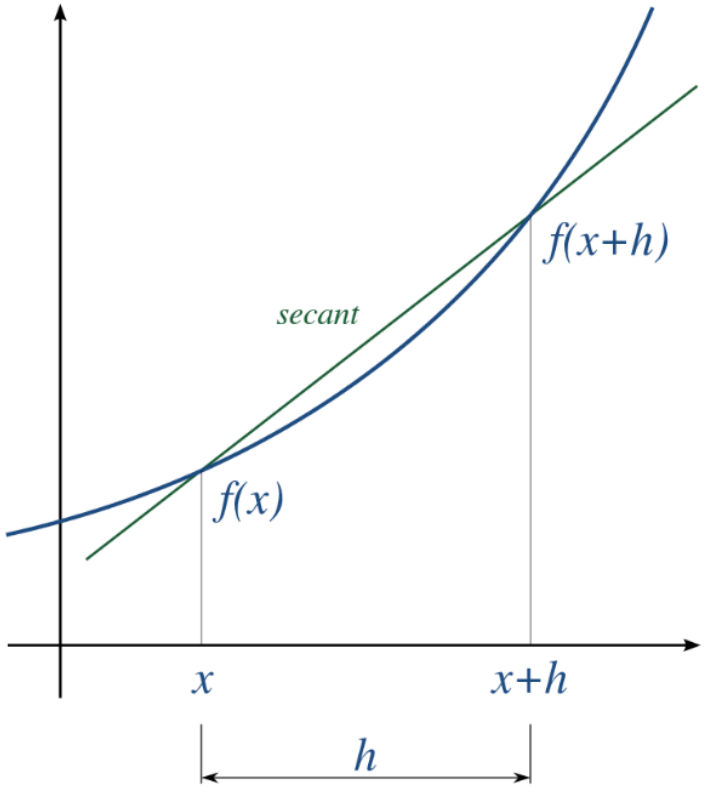
f(x) = sin(x)/x – 0.05 = 0



Use f(x) = 0 for a general equation solver.

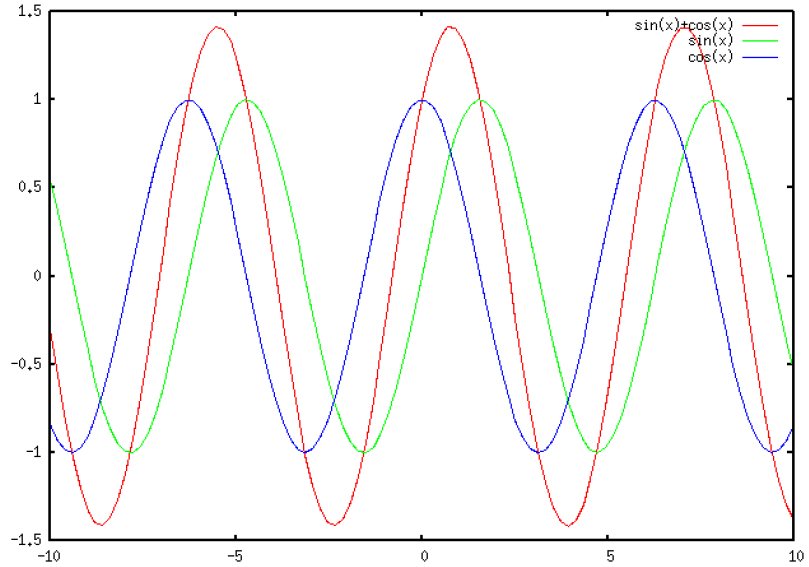
Numerical differentiation

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$



[sin(x)]'

[sin(x)+cos(x)]'



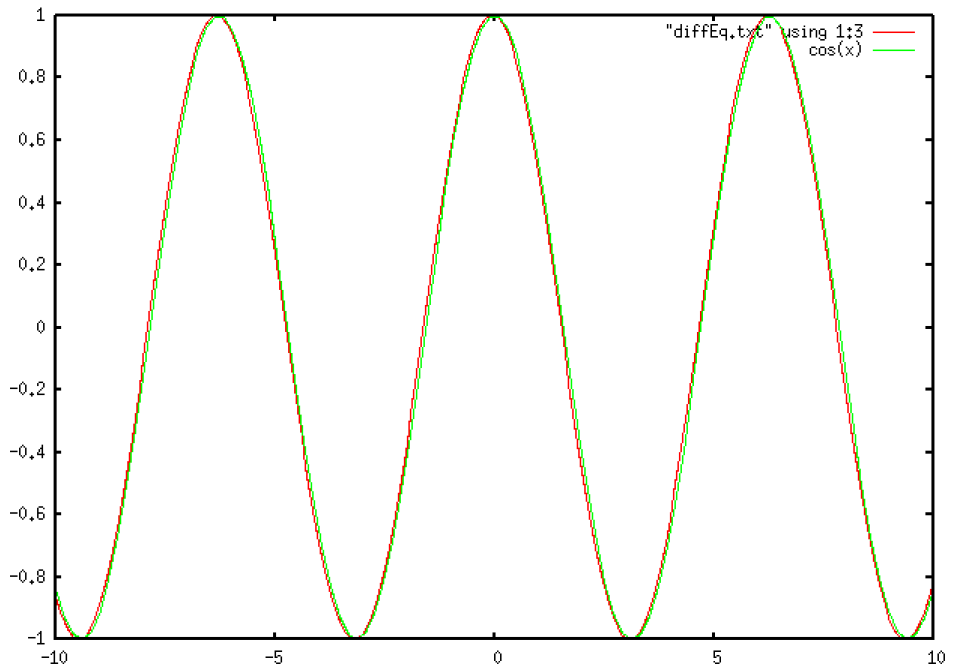
C++ code:

function sin(x) or sin(x)+cos(x)

main

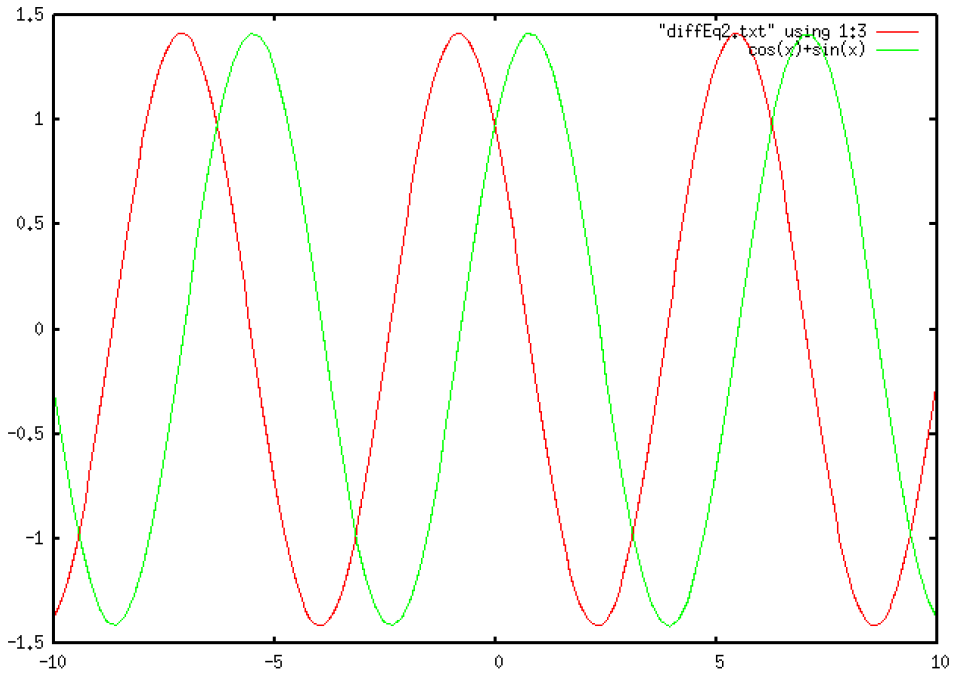
for-loop over x
function calls f(x) and f(x+dx)
dy=f(x+dx) – f(x)
write x, y, dy to output file

[sin(x)]’ with cos(x) superimposed



Two curves agree. Good!

[sin(x) + cos(x)]’ with cos(x)+sin(x) superimposed.

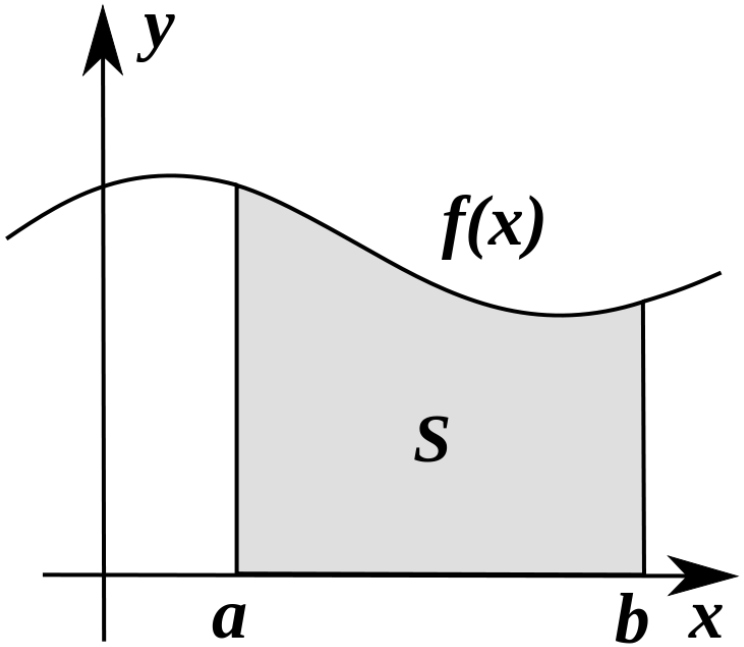


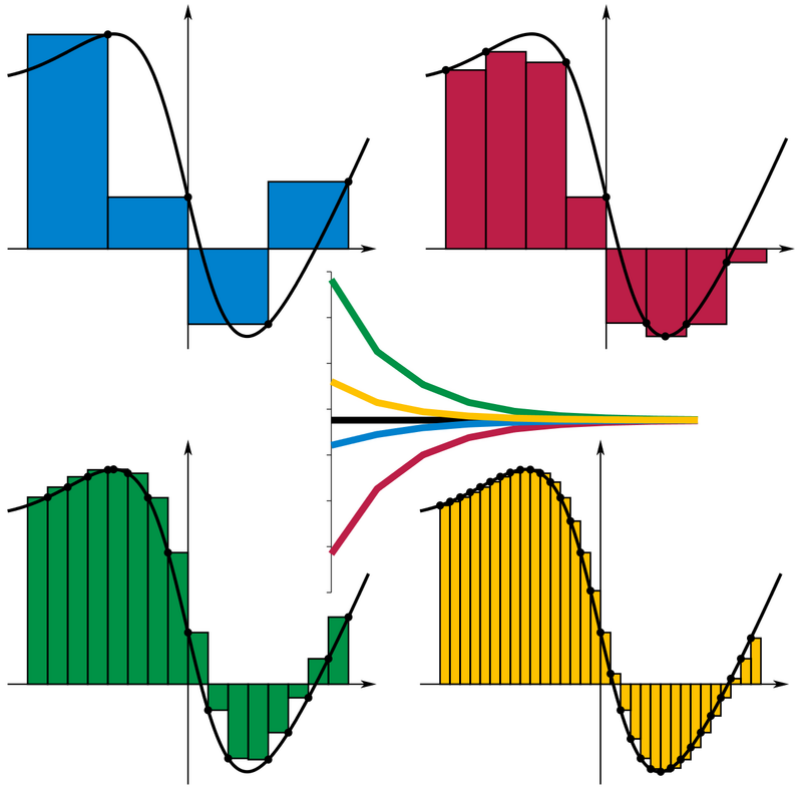
Two curves do not agree. Why?

Numerical Integration

$$\int_a^b f(x)\,dx$$

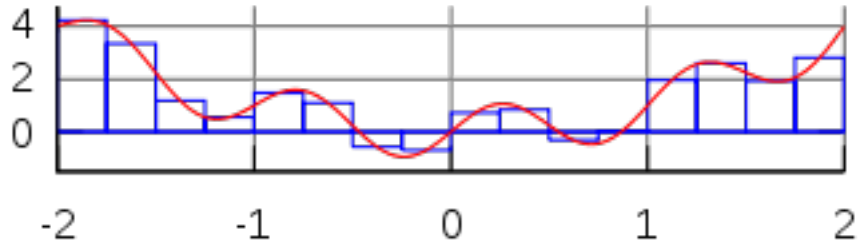
$$\sum f(x)\Delta x$$ (Riemann Sum)





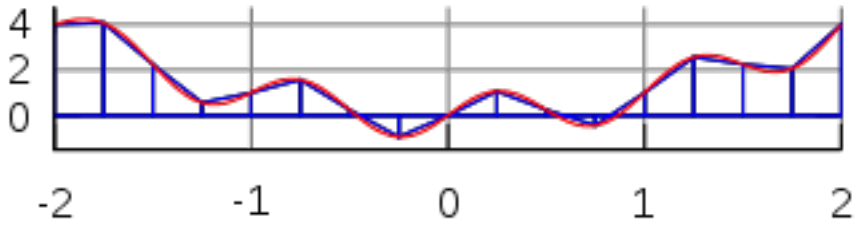
wikipedia

rules



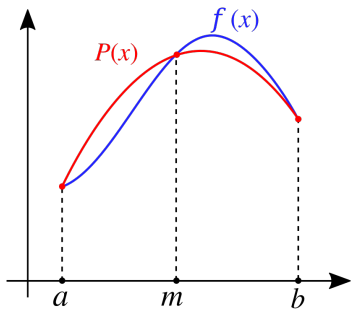
rectangle

$$\int_a^b f(x)\,dx \approx (b-a)\,f\left(\frac{a+b}{2}\right)$$



trapezoidal

$$\int_a^b f(x)\,dx \approx (b-a)\,\frac{f(a)+f(b)}{2}.$$



Simpson's

$$\int_a^b f(x)\,dx \approx \tfrac{b-a}{6}\left[f(a)+4f\left(\tfrac{a+b}{2}\right)+f(b)\right]$$

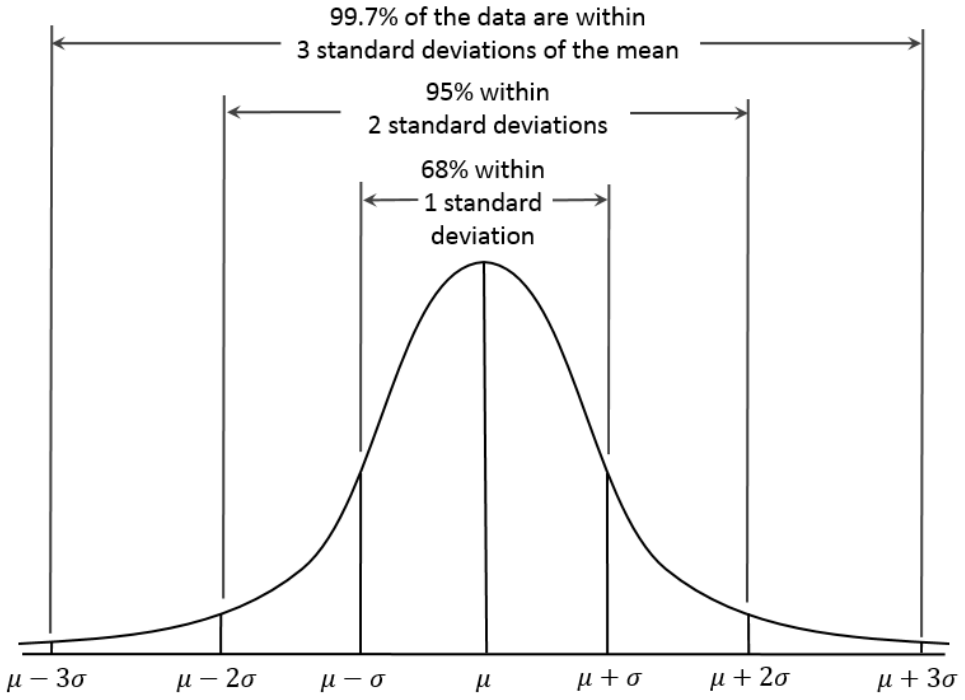
mid point

wikipedia

Integration of Gaussian distribution

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ: mean
σ: standard deviation



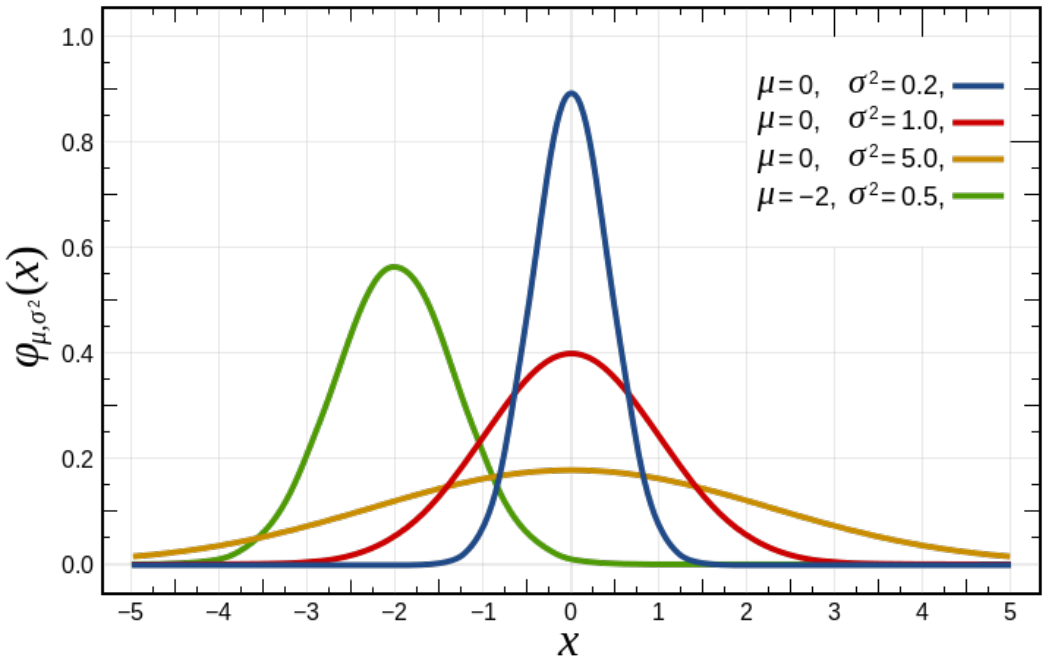

```
[kunori@archer week2]$ ./a.out 0.01 -2.0 2.0
 rectangular is called
 trapezoidal is called
 simpsons is called
 dx 0.01  xmin -2  xmax 2
result 1 0.954501
result 2 0.954498
result 3 0.9545

[kunori@archer week2]$ ./a.out 0.01 0.0 1.0
 rectangular is called
 trapezoidal is called
 simpsons is called
 dx 0.01  xmin 0  xmax 1
result 1 0.341346
result 2
result 3
```

Gaussian Distribution (Normal Distributions)

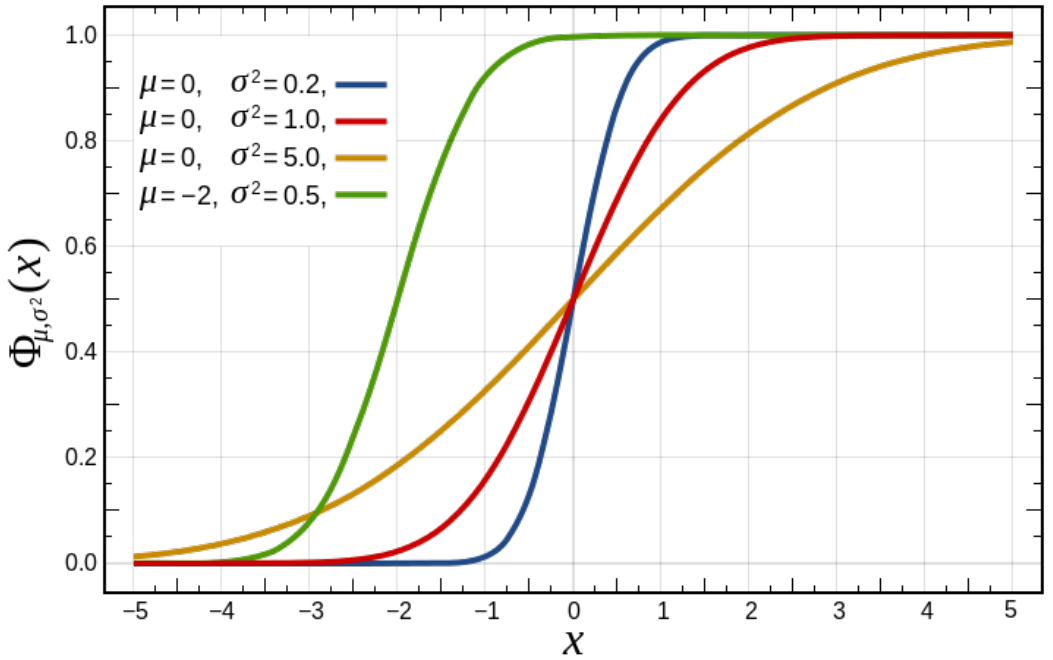
$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\sigma^2\pi}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ: mean
σ: standard deviation

Probability density function



Cumulative distribution function



wikipedia

Homework 1

First due: Mon. 9/11
Final due: Mon. 9/18 for upgrade
Email to shuichi.kunori@ttu.edu
Subject: phys2305 homework-1

C-course:

Solve the following equation: sin(x)*x=1

send 1) C++ code and a plot similar to one on the right (top). (with solution points)

Calculate differential of [sin(x)*x]

send 2) a plot similar to one on the right (bottom)

B-course:

Add two integration methods, trapezoidal and Simpson's to integralRectangle.cc from Dropbox

send 3) C++ code and values for range x=[0.0, 1.0]

A-course:

Make plots (probability density function, cumulative distribution) shown on page 14 by modifying integralRectangle.cc.

send 4) two plots