**Name – Siddhi Deshmane**

**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**ANS - OPTION B**

Since the probability is 0.26598552904870054

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**ANS –**

**A. False.** The statement is false because the mean age is 38 and the standard deviation is 6. The age range between 38 and 44 includes one standard deviation above the mean, which typically covers about 68% of the population in a normal distribution. Therefore, there would be more employees between 38 and 44 than older than 44.

**B. True**. Since the age distribution is normal, about 15.87% of the population (which is 100% - 84.13%, where 84.13% represents the cumulative percentage up to age 30) would be under 30. If there are 400 employees in total, 15.87% of 400 is approximately 63.5, which rounds down to 63. So, a training program for employees under 30 would be expected to attract about 63 employees, not 36.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**ANS -** Given that X1​ and X2are independent and identically distributed (iid) normal random variables, we can denote them as *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2)

This means both X1 and X2 follow a normal distribution with the same mean μ and variance σ2

Since X1​ follows a normal distribution,2X1will also follow a normal distribution.The mean of 2X1 will be 2 μ (since multiplying by 2 scales the mean) and the variance will be 4σ2 (since variance scales by the square of the multiplier).

Therefore, 2*X1* ~ *N*(2μ, 4σ2)

The sum of two independent normal random variables is also a normal random variable.

The mean of X1 + X2 will be μ + μ = 2μ (due to linearity of expectation for independent variables) and the variance will be σ2+ σ2=2 σ2 (since the variances add for independent variables).

Therefore, *X*1 + *X*2 ~ N(2μ,2 σ2)

In summary, the difference between 2X1 and ​ X1 + X2 lies in their variances and the resulting spread of values around the mean, despite both having the same mean due to the properties of independent and identically distributed normal random variables.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**ANS -** The values a and b are: 48.48 and 151.52

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

**ANS –**

A. Profit1 Rupee Range: (-39.59513791290732, 489.5951379129073) million

Profit2 Rupee Range: (-37.79351721720974, 667.7935172172097) million

B. 5th Percentile of Profit1: 2.9447603615511753 million

5th Percentile of Profit2: 18.926347148734862 million

C. Probability of Loss for Profit1: 0.0477903522728147

Probability of Loss for Profit2: 0.040059156863817086