EX6

$f : [a,b] \longrightarrow [a,b]$

f croissante $\Rightarrow \exists\, \beta \in [a,b],\ f(\beta) = \beta$

Soit $A = \{ x \in [a,b],\ x \leqslant f(x) \}$

$f(a) \in [a,b] \Rightarrow a \leqslant f(a) \Rightarrow a \in A \Rightarrow A \neq \emptyset$

On pose $\beta = \operatorname{Sup} A$

$\beta = \operatorname{Sup}(A)$, $\beta \leqslant f(\beta)^{(1)}$, $(f \nearrow)$

$\Rightarrow f(\beta) \leqslant f(f(\beta))$

$\Rightarrow f(\beta) \in A \Rightarrow f(\beta) \leqslant \sup(A) = \beta \ (2)$

$\Rightarrow \boxed{f(\beta) = \beta}$ d'où le résultat

---

* f continue

on pose h(x

$\begin{cases} h(a) = f(a) - \\ h(b) = f(b) - \end{cases}$

Comme h(x) est

et change s

i.e $\exists\, \alpha \in [a,b]$ où

$\Rightarrow f(\alpha) - \alpha = 0 \Rightarrow$ f(α

d'où le résultat

$) = \beta$

$= A \Rightarrow A \neq \emptyset$

$) = \beta$ (2)

* f continue

on pose $h(x) = f(x) - x$

$$\begin{cases} h(a) = f(a) - a \geq 0 \\ h(b) = f(b) - b \leq 0 \end{cases}$$

Comme $h(x)$ est continue et change son signe

i.e $\exists \alpha \in [a, b]$ où $h(\alpha) = 0$

$$\Rightarrow f(\alpha) - \alpha = 0 \Rightarrow \boxed{f(\alpha) = \alpha}$$

d'où le résultat

EX5

$f : [0, 1] \longrightarrow [0, 1]$

$|f(x) - f(y)| \geq |x - y|$

$f = ??$

$\begin{cases} x = 0 \\ y = 1 \end{cases}$ $\quad |-3| = -(-3)$

$\Rightarrow |f(0) - f(1)| \geq 1$ (1)

$0 \leq f(1) \leq 1$

$0 \leq f(0) \leq 1 \quad \times(-1)$

$\Rightarrow -1 \leq -f(0) \leq 0$

$\Rightarrow -1 \leq f(1) - f(0) \leq 1$

$\Rightarrow 0 \leq |f(1) - f(0)| \leq 1$ (2)

Ex 7. $f : \mathbb{R}_+ \longrightarrow \mathbb{R}$

$$\lim_{n \to +\infty} \left[f(n+1) - f(n) \right] = \ell$$

$$\Leftrightarrow \forall \varepsilon > 0,\ \exists A > 0 ;\ \forall n > A$$

$$|f(n+1) - f(n) - \ell| < \varepsilon$$

$$\Leftrightarrow \ell - \varepsilon \leq f(n+1) - f(n) < \varepsilon + \ell$$

$$\Rightarrow \ k \text{ fois} \begin{cases} \ell - \varepsilon \leq f(n) - f(n-1) \leq \ell + \varepsilon \\ \ell - \varepsilon \leq f(n-1) - f(n-2) < \ell + \varepsilon \\ \quad \vdots \\ \ell - \varepsilon \leq f(n-k+1) - f(n-k) < \ell + \varepsilon \end{cases}$$

$k(\ell - \varepsilon) \leq$

$\Rightarrow k(\ell - \varepsilon) -$

$m \leq$

$\Rightarrow \frac{k(\ell - \varepsilon)}{n}$

$k \in \mathbb{N},\ k$

$E(n$

$\Rightarrow n -$

$x - A < E$

$\Rightarrow n - A$

$\longrightarrow \mathbb{R}$

$x)\big) = \ell$

$> 0 ;\ \forall x > A$

$x) - \ell| < \varepsilon$

$+1) - f(x) < \varepsilon + \ell$

$(x) - f(x-1) < \ell + \varepsilon$

$(x-1) - f(x-2) < \ell + \varepsilon$

$(x-k+1) - f(x-k) < \ell + \varepsilon$

$k(\ell-\varepsilon) \leq f(x) - f(x-k) \leq k(\ell+\varepsilon)$

$\Rightarrow k(\ell-\varepsilon) + f(x-k) \leq f(x) \leq k(\ell+\varepsilon) + f(x-k)$

$m \leq f \leq M$ (f borne

$\Rightarrow \dfrac{k(\ell-\varepsilon)}{x} + \dfrac{m}{x} \leq \dfrac{f(x)}{x} \leq \dfrac{k(\ell+\varepsilon)}{x} + \dfrac{M}{x}$

$k \in \mathbb{N},\ k = E(x - A + 1)$

$E(x) \leq x < E(x) + 1$

$\Rightarrow x - 1 \leq E(x) \leq x$

$x - A < E(x - A + 1) < x - A + 1$

$\Rightarrow x - A < k < x - A + 1$

$\Rightarrow \dfrac{(x-A)(\ell-\varepsilon)}{x} +$

$\xrightarrow[x \to +\infty]{}$

$\Rightarrow \left(1 - \dfrac{A}{x}\right)(\ell-\varepsilon)$

$\xrightarrow[x \to +\infty]{} \ell - \varepsilon \leq$

$\Rightarrow \left|\dfrac{f(x)}{x} - \ell\right.$

$- f(x-k) \le k(l+\varepsilon)$

$k) \le f(x) \le k(l+\varepsilon) + f(x-k)$

M (f bornée)

$\le \frac{f(x)}{x} \le \frac{k(l+\varepsilon)}{x} + \frac{M}{x}$

$E(x - A + 1)$

$x < E(x) + 1$

$E(x) \le x$

$< x - A + 1$

$A+1) < x - A + 1$

$$\Rightarrow \frac{(x-A)(l-\varepsilon)}{x} + \frac{m}{x} \le \frac{f(x)}{x} \le \frac{x-A+1}{x}(l+\varepsilon) + \frac{M}{x}$$

$x \to +\infty$

$$\Rightarrow \left(1 - \frac{A}{x}\right)(l-\varepsilon) + \frac{m}{x} \le \frac{f(x)}{x} \le \left(1 - \frac{A+1}{x}\right)(l+\varepsilon) + \frac{M}{x}$$

$x \to +\infty$

$$\Rightarrow l - \varepsilon \le \lim_{x \to +\infty} \frac{f(x)}{x} \le l+\varepsilon$$

$$\Rightarrow \left|\frac{f(x)}{x} - l\right| < \varepsilon \Rightarrow \boxed{\lim \frac{f(x)}{x} = l}$$

$$\boxed{|f(x) - l| < \varepsilon \Rightarrow l - \varepsilon < f(x) < l+\varepsilon}$$

Ex :

$$\lim_{x\to 0} \frac{\sqrt{x}\,\sin(\sqrt{x})}{x(x+2)}, \quad \sin(\sqrt{x}) \sim \sqrt{x}$$

$$= \lim_{x\to 0} \frac{\sqrt{x}\times\sqrt{x}}{x(x+2)} = \frac{\cancel{x}}{\cancel{x}(x+2)}$$

$$= \lim_{x\to 0} \frac{1}{x+2} = \boxed{\frac{1}{2}}$$

2) $$\lim_{x\to 0} \frac{(1-\cos(x))(1-e^{x})}{x^3+x}$$

$$1-\cos(x) \underset{0}{\sim} -\frac{x^2}{2}$$

$$1-e^{x} \underset{0}{\sim} -x$$

$$= \lim \frac{-\frac{x^2}{2}(-x)}{x^3+x} = \frac{\frac{x^3}{2}}{x^3+x}$$

3) lim

$$\frac{\frac{1}{2}\sqrt{}}{x^2\left(1+\frac{1}{x^2}\right)} = \frac{\frac{1}{2}}{1+\infty} = \boxed{0}$$

3) $\lim\limits_{x\to 0} \frac{x\log(1+x)}{tg^2(x)}$

$$= \frac{\ln(x+1)}{\ln(10)} \times \frac{x}{tg^2(x)}, \quad tg(x) \underset{0}{\sim} x$$

$$= \frac{\ln(x+1)}{\ln(10)} \times \frac{1}{x} = \lim_{x\to 0} \frac{1}{\ln(10)} \, \frac{\ln(x+1)}{x}$$

$$= \boxed{\frac{1}{\ln(10)}}$$

$$\lim_{x \to 0} \frac{\sin(x) - \sin(2x)}{x^2}$$

$$= \lim_{x \to 0} \frac{x - 2x}{x^2} = \lim_{x \to 0} \frac{-1}{x}$$

$$= \boxed{\begin{cases} +\infty ; & x < 0 \\ -\infty ; & x > 0 \end{cases}}$$

5) $\lim_{x \to 0} e^{x - \sin(x)} = e^{x - x} = e^{0} = \boxed{1}$

EX 10

1) $\lim\limits_{x \to +\infty} \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

$= \frac{\sqrt{x}}{\sqrt{x\left(1+\frac{\sqrt{x}}{x}\right)}}$

$= \frac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}$

$= \lim\limits_{x \to +\infty} \frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}} = \boxed{1}$

donc $\boxed{f \underset{+\infty}{\sim} g}$

2) $\lim\limits_{x \to \pi} \frac{x^{2n} - \pi^{2n}}{2n\pi^{2n-1}\, tg(x)} = \frac{f}{g}$

on pose $h = x - \pi \to h \to 0$

$= \frac{(h+\pi)^{2n} - \pi^{2n}}{2n\pi^{2n-1}\, tg(h+\pi)}$

$= \frac{(h+\pi)^{2n} - \pi^{2n}}{2n\pi^{2n-1}\, tg(h)}$

$= \frac{2n(h+\pi)^{2n-1}}{2n\pi^{2n-1}(1+tg^2 h)}$

$= \lim\limits_{h \to 0} \frac{\cancel{2n\pi^{2n-1}}\left(\frac{h}{\pi}+1\right)^{2n-1}}{\cancel{2n\pi^{2n-1}}(1+h^2)}$ car $tg(h) \underset{0}{\sim} h$

$= \lim\limits_{h \to 0} \frac{1}{1} = \boxed{1}$ $\boxed{f \underset{\pi}{\sim} g}$

$\boxed{\lim\limits_{x \to x_0} \frac{f}{g} = \lim\limits_{x \to x_0} \frac{f'}{g'}}$

EX 11 $f, g : \mathbb{R} \longrightarrow \mathbb{R}$

$\forall (x, y) \in \mathbb{R}^2,$

$(f(x) - f(y))(g(x) - g(y)) = 0$

f n'est pas constante

i.e $\exists (\beta_1, \beta_2) \in \mathbb{R}^2;$

$f(\beta_1) \neq f(\beta_2)$.

$\begin{cases} x = \beta_1 \\ y = \beta_2 \end{cases}$

$(\underbrace{f(\beta_1) - f(\beta_2)}_{\neq 0})(g(\beta_1) - g(\beta_2)) = 0$

$\Rightarrow g(\beta_1) = g(\beta_2)$ (1)

$\forall x \in \mathbb{R}$

$\begin{cases} x \\ y = \beta_1 \end{cases}$ $\begin{cases} x \\ y = \beta_2 \end{cases}$

$\begin{cases} (f(x) - f(\beta_1))(g(x) - g(\beta_1)) = 0 & (2) \\ (f(x) - f(\beta_2))(g(x) - g(\beta_2)) = 0 & (3) \end{cases}$

(2) - (3)

$(f(x) - f(\beta_1))(g(x) - g(\beta_1)) - (f(x) - f(\beta_2))(g(x) - g(\beta_1)) = 0$

$(f(x) - f(\beta_1))(g(x) - g(\beta_1))$

$f(x)g(x) - f(x)g(\beta_1) - f(\beta_1)g(x) + f(\beta_1)g(\beta_1)$

$\Rightarrow \cancel{f(x)g(x)} - \cancel{f(x)g(\beta_1)} - f(\beta_1)g(x) + f(\beta_1)g(\beta_1)$

$- \cancel{f(x)g(x)} + \cancel{f(x)g(\beta_1)} + f(\beta_2)g(x) - f(\beta_2)g(\beta_1) = 0$

$g(x)(f(\beta_2) - f(\beta_1)) + g(\beta_1)(f(\beta_1) - f(\beta_2)) = 0$

$(\underbrace{f(\beta_2) - f(\beta_1)}_{\neq 0})(g(x) - g(\beta_1)) = 0 \Rightarrow g(x) = g(\beta_1)$

donc g est constante

EX 19

1) $f : \mathbb{R}\setminus\{0,1\} \longrightarrow \mathbb{R}$

$$f(x) + f\left(\frac{-1}{x+1}\right) = 2 + 3x$$

on pose $h = \frac{-1}{x+1}$

$$h \circ h(x) = h(h(x)) = \frac{-1}{h(x)+1}$$

$$= \frac{-1}{\frac{-1}{x+1} + \frac{1}{1}} = \frac{1}{\frac{x}{x+1}} = \frac{-x-1}{x}$$

$$h \circ h \circ h(x) = h \circ h(h(x)) = \frac{-h(x)-1}{h(x)}$$

$$= \frac{\frac{1}{x+1} - 1}{\frac{-1}{x+1}} = \frac{\frac{-x}{x+1}}{\frac{-1}{x+1}} = \frac{-x}{-1} = \boxed{x}$$

$$\begin{cases} f(x) + f(h(x)) = 2 + 3x & (1) \\ f(h(x)) + f(h \circ h) = 2 + 3h(x) & (2) \\ f(h \circ h(x)) + f(x) = 2 + 3 h \circ h(x) & (3) \end{cases}$$

(1) - (2) + (3)

$$f(x) + f(x) = \cancel{2} + 3x - \cancel{2} - 3h(x) + 2 + 3 h \circ h$$

$$\boxed{f(x) = \frac{2 + 3x - 3h(x) - 3\frac{(x+1)}{x}}{2}}$$

2) $x f(x) + f(1-x) = x^3 + 1$

on pose $h(1-x)$

$$h \circ h(x) = 1 - h(x) = 1 - (1 - x) = \boxed{x}$$

$$\begin{cases} x f(x) + f(h(x)) = x^3 + 1 & (1) \\ h(x) f(h(x)) + f(x) = h^3(x) + 1 & (2) \end{cases}$$

$h(x)$ (1) - (2)

$$x h(x) f(x) + \cancel{h(x) f(h(x))} - \cancel{h(x) f(h(x))} - f(x)$$

$$= (x^3 + 1) h(x) - h^3(x) - 1$$

$$\Rightarrow f(x)(x h(x) - 1) = (x^3 + 1) h(x) - h^3(x) - 1$$

$$\Rightarrow \boxed{f(x) = \frac{(x^3 + 1) h(x) - h^3(x) - 1}{x h(x) - 1}}$$

3)

$$f(x+y^2) = f(x^2)+f(y)$$

$\begin{cases} x=0 \\ y=0 \end{cases}$

$f(0) = f(0)+f(0)$

$\Rightarrow f(0)-2f(0)=0 \Rightarrow \boxed{f(0)=0}$

$= -f(0)=0$

$\begin{cases} x \\ y=0 \end{cases}$

$\Rightarrow f(x) = f(x^2) \Rightarrow f(x^2) = f(x^4)$

$\times\ x=-y^2$

$f(0) = f(y^4)+f(y)=0$

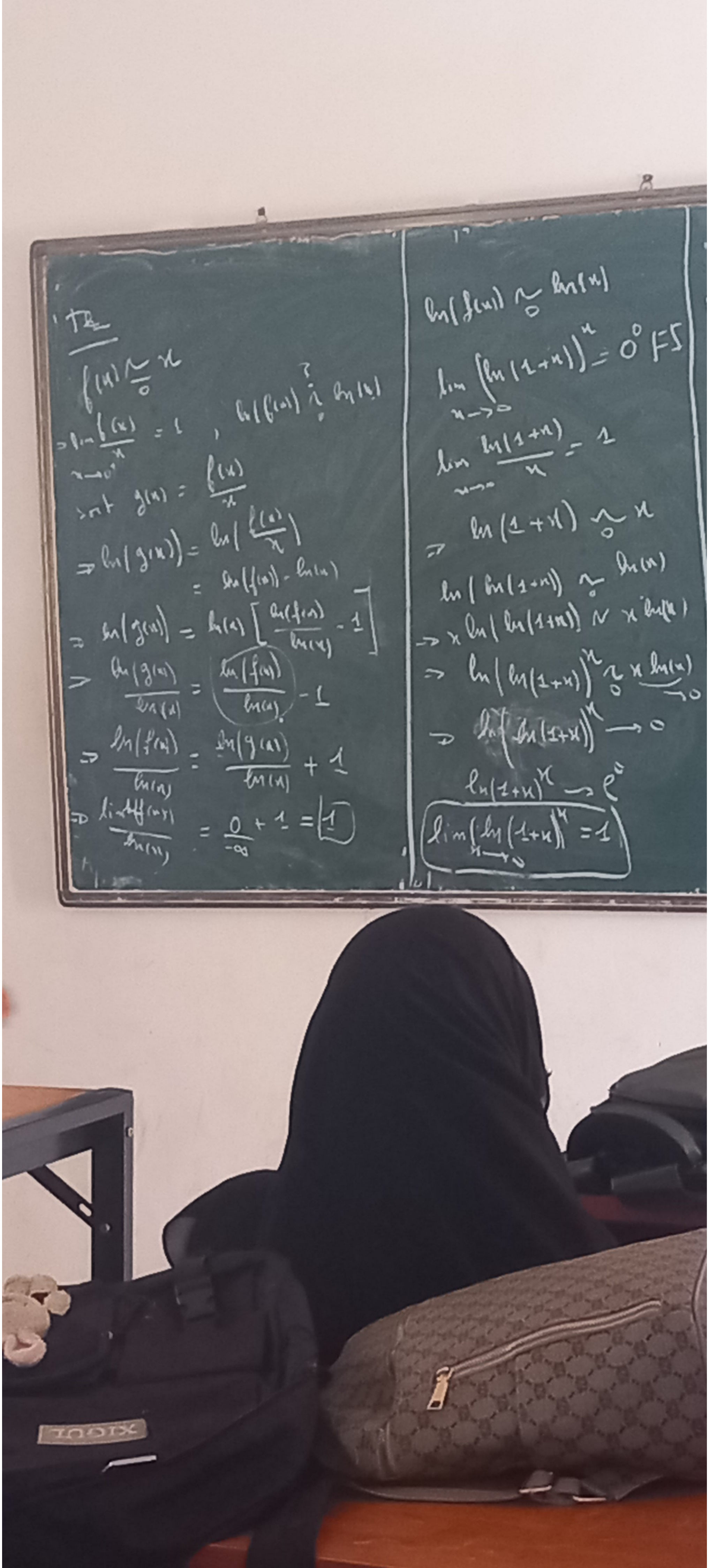
$\Rightarrow f(y^2)+f(y)=0$

$\Rightarrow f(y)+f(y)=0$

$\Rightarrow 2f(y)=0 \Rightarrow f(y)=0\ \forall y \in \mathbb{R}$

$\Rightarrow f=0$

Donc, f est nulle.



$f(x) \underset{0}{\sim} x$
$\Rightarrow \lim_{x \to 0^+} \frac{f(x)}{x} = 1$, $\ln(f(x)) \overset{?}{\underset{0}{\sim}} \ln(x)$
soit $g(x) = \frac{f(x)}{x}$
$\Rightarrow \ln(g(x)) = \ln\left(\frac{f(x)}{x}\right)$
$= \ln(f(x)) - \ln(x)$
$\Rightarrow \ln(g(x)) = \ln(x)\left[\frac{\ln(f(x))}{\ln(x)} - 1\right]$
$\Rightarrow \frac{\ln(g(x))}{\ln(x)} = \frac{\ln(f(x))}{\ln(x)} - 1$
$\Rightarrow \frac{\ln(f(x))}{\ln(x)} = \frac{\ln(g(x))}{\ln(x)} + 1$
$\Rightarrow \lim \frac{\ln(f(x))}{\ln(x)} = \frac{0}{-\infty} + 1 = \boxed{1}$
$\ln(f(x)) \underset{0}{\sim} \ln(x)$
$\lim_{x \to 0} (\ln(1+x))^x = 0^0$ FI
$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$
$\Rightarrow \ln(1+x) \underset{0}{\sim} x$
$\ln(\ln(1+x)) \underset{0}{\sim} \ln(x)$
$\Rightarrow x \ln(\ln(1+x)) \sim x \ln(x)$
$\Rightarrow \ln(\ln(1+x))^x \underset{0}{\sim} x \ln(x) \to 0$
$\Rightarrow \ln(\ln(1+x))^x \to 0$
$\ln(1+x)^x \to e^0$
$\boxed{\lim_{x \to 0} (\ln(1+x))^x = 1}$

$) \underset{0}{\sim} \ln(x)$

$(1+x))^{x} = 0^0$ FI

$\frac{(1+x)}{x} = 1$

$(1+x) \underset{0}{\sim} x$

$(1+x)) \underset{0}{\sim} \ln(x)$

$\ln(1+x)) \sim x \ln(x)$

$(\ln(1+x))^{x} \underset{0}{\sim} \underbrace{x \ln(x)}_{\to 0}$

$(\ln(1+x))^{x} \to 0$

$(1+x)^{x} \to 1$

$\boxed{\lim_{x \to 0} (1+x)^{x} = 1}$

Ex2 :

$f : \mathbb{R} \longrightarrow \mathbb{R}$ continue

$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = +\infty$

$\lim_{x \to -\infty} f(x) = -\infty$, $\forall A_2 < 0, \exists A_1 < 0, x \leq A_1 \Rightarrow f(x) \leq A_2$

$\lim_{x \to +\infty} f(x) = +\infty$, $\forall B_2 > 0, \exists B_1 > 0, x \geq B_1, f(x) \geq B_2$

donc, $f(A_1) \leq 0$ et $f(B_1) \geq 0$

$f(A_1) \times f(B_1) \leq 0$

d'après le théorème de valeur intermédiaire

$\exists c \in]A_1, B_1[\subset \mathbb{R}$ tq $\boxed{f(c) = 0}$

$f(x) \leq A_2$

$x \geq B_1, \; f(x) \geq B_2$

0

...médiaire

$) = 0$

2)
$$P_n = \sum_{k=0}^{2n+1} a_k x^k, \quad a_{2n+1} \neq 0$$

$$a \times x^0 = a_0$$

$$\lim_{x \to -\infty} P_n = \lim a_{2n+1} x^{2n+1}$$

1ère cas

$a_{2n+1} < 0$ $\qquad (-) \times (-) = (+)$

$$\lim_{x \to -\infty} a_{2n+1} x^{2n+1} = +\infty$$

$$\lim_{x \to +\infty} a_{2n+1} x^{2n+1} = -\infty$$

$a_{2n+1} > 0$

$$\lim_{x \to -\infty} a_{2n+1} x^{2n+1} = \boxed{-\infty}$$

$$\lim_{x \to +\infty} a_{2n+1} x^{2n+1} = \boxed{+\infty}$$

D'après la question précédente,

$\exists c \in \mathbb{R}, \; P_n(c) = 0$

Ex3.

$f : [a,b] \longrightarrow \mathbb{R}$ continue

$f(a) \neq f(b)$ Mq $\exists c \in]a,b[$

$$(p+q) f(c) = p f(a) + q f(b)$$

Soit $h = \dfrac{p f(a) + q f(b)}{p+q}$

1er cas $f(a) < f(b)$

$$h = \frac{p f(a) + q f(b)}{p+q} < \frac{p f(b) + q f(b)}{p+q} = \frac{f(b)\cancel{(p+q)}}{\cancel{p+q}} = f(b) \quad h < f(b)$$

$$h = \frac{p f(a) + q f(b)}{p+q} > \frac{p f(a) + q f(a)}{p+q} = \frac{\cancel{(p+q)} f(a)}{\cancel{p+q}} = f(a)$$

donc $f(a) < h < f(b)$

$f(b) < f(a) \Rightarrow f(b) < h < f(a)$

h : la valeur intermediaire

$\exists c \in [a,b]$, $h = f(c)$

$$f(c) = \frac{p f(a) + q f(b)}{p+q}$$

$$\Rightarrow (p+q) f(c) = p f(a) + q f(b)$$

Ex4 :

Ex.

$f : [a,b] \to \mathbb{R}$ continue

$f(a) \neq f(b)$ Mq $\exists c \in]a,b[$

$$(p+q) f(c) = p f(a) + q f(b)$$

Soit
1er cas $h = \dfrac{p f(a) + q f(b)}{p+q}$

$f(a) < f(b)$

$$h = \frac{p f(a) + q f(b)}{p+q} < \frac{p f(b) + q f(b)}{p+q} = \frac{f(b)(p+q)}{p+q} = f(b). \quad h < f(b)$$

$$h = \frac{p f(a) + q f(b)}{p+q} > \frac{p f(a) + q f(a)}{p+q} = \frac{(p+q) f(a)}{p+q} = f(a)$$

donc $f(a) < h < f(b)$

$f(a) < f(a) \Rightarrow f(a) < h < f(b)$

h : la valeur intermédiaire

$\exists c \in [a,b]$, $h = f(c)$

$$f(c) = \frac{p f(a) + q f(b)}{p+q}$$

$$\Rightarrow (p+q) f(c) = p f(a) + q f(b)$$

Ex. $f : \mathbb{R}_+ \to \mathbb{R}$

$\forall a, b \in \mathbb{R}_+$

continue

Mq $\exists c \in]a, b[$

$f(c) = \dfrac{p f(a) + q f(b)}{p+q}$

$\dfrac{\ldots f(a) + q f(b)}{p+q}$

$\dfrac{\ldots + q f(b)}{p+q}$

$\dfrac{\ldots) + q f(b)}{p+q}$

$\dfrac{\ldots}{\ldots q} = f(b) \quad h < f(b)$

$h = \dfrac{p f(a) + q f(b)}{p+q}$

$> \dfrac{p f(a) + q f(a)}{p+q}$

$= \dfrac{(p+q) f(a)}{p+q} = f(a)$

donc $f(a) < h < f(b)$

$f(b) < f(a) \Rightarrow f(b) < h < f(a)$

h : la valeur intermediaire

$\exists c \in [a, b]$, $h = f(c)$

$f(c) = \dfrac{p f(a) + q f(b)}{p+q}$

$\Rightarrow (p+q) f(c) = p f(a) + q f(b)$

Ex4 : $f : \mathbb{R}_+ \longrightarrow \mathbb{R}$

$\exists\, a, b \in \mathbb{R}_+$ tq $|f(x)| < ax + b$

f uniformement continue

$\forall \varepsilon > 0, \exists \eta > 0 ; |x - y| \leq \eta \Rightarrow |f(x) - f(y)| \leq \varepsilon$

soit $\varepsilon = 1$, $\exists \eta > 0$, $|x - y| \leq \eta \Rightarrow |f(x) - f(y)| \leq 1$

$|f(\eta) - f(0)| \leq 1$

$|f(2\eta) - f(\eta)| \leq 1$

$|f(3\eta) - f(2\eta)| \leq 1$

$|f(n\eta) - f((n-1)\eta)| < 1 \qquad n = E\left(\dfrac{x}{\eta}\right)$

$|f(x) - f(n\eta)| < 1$, tq

$\Rightarrow |f(x) - f(0)| \leq n + 1 \Rightarrow f(x) \leq n + 1 + f(0)$

$E(x) \leq x$

$\Rightarrow |f(x)| \leq E\left(\dfrac{x}{\eta}\right) + 1 + f(0)$

$\leq \dfrac{x}{\eta} + \underbrace{1 + f(0)}$

$= \underbrace{\dfrac{1}{\eta}}_{a} x + \underbrace{1 + f(0)}_{b}$

$= ax + b \quad \begin{cases} a = \dfrac{1}{\eta} \\ b = 1 + f(0) \end{cases}$

$n\eta = E(x)$

$\dfrac{E(x)}{j}$

Exo.

$f : [a,b] \to \mathbb{R}$ continue

$f(a) \neq f(b)$ Mq $\exists c \in]a,b[$

$(p+q) f(c) = p f(a) + q f(b)$

Soit $h = \dfrac{p f(a) + q f(b)}{p+q}$

1er cas $f(a) < f(b)$

$h = \dfrac{p f(a) + q f(b)}{p+q}$

$< \dfrac{p f(b) + q f(b)}{p+q}$

$= \dfrac{f(b)\cancel{(p+q)}}{\cancel{p+q}} = f(b)$. $h < f(b)$

$h = \dfrac{p f(a) + q f(b)}{p+q}$

$> \dfrac{p f(a) + q f(a)}{p+q}$

$= \dfrac{\cancel{(p+q)} f(a)}{\cancel{p+q}} = f(a)$,

donc $f(a) < h < f(b)$

$f(b) < f(a) \Rightarrow f(b) < h < f(a)$

h : la valeur intermédiaire

$\exists c \in [a,b]$, $h = f(c)$

$f(c) = \dfrac{p f(a) + q f(b)}{p+q}$

$\Rightarrow (p+q) f(c) = p f(a) + q f(b)$