Lab 11: Sensitivity Analysis

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Sensitivity Analysis

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  - An assumption that is unverifiable from observable data
  - Balance tests and placebo tests are consistent with ignorability but are not dispositive

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- Observational research assumes some form of ignorability
  - An assumption that is unverifiable from observable data
  - Balance tests and placebo tests are consistent with ignorability but are not dispositive
- How can we improve on this?
  - Describe the type of confounder that would substantively change our conclusions about the ATE
  - Assess how plausible such confounding is given expert knowledge and research design

Empirical application

- What is the effect of exposure to violence on attitudes towards peace (in Darfur, Sudan) (Cinelli and Hazlett (2020))
- In 2003 and 2004, the Darfurian government killed an estimated two hundred thousand people.
  - Villages were aerially bombarded and then razed/looted by the *Janjaweed*, a pro-government militia.
  - Violence within villages was indiscriminate. With one exception, women were targeted and often subjected to sexual violence.
- The outcomes of interest: individual willingness to make peace with those who perpetrated this violence

The threat of unobserved confounders

What if bombs were still more likely to hit the center of the village, and those in the center would also likely hold different attitudes towards peace?

What if the *Janjaweed* targetted the wealthy and wealth is correlated with attitudes towards peace?

How strong would these unobserved confounders (or *all* reamaining unobserved confounders?

Can we use the fact that any unobservable variable (e.g. wealth) is unlikely to as strongly affect the outcome/treatment as gender would be?

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Remember OVB = impact times imbalance

$$\underbrace{E[Y_i|D_i = 1] - E[Y_i|D_i = 0]}_{\text{Difference-in-means}} =$$

$$\underbrace{E[Y_i(1) - Y_i(0)|D_i = 1]}_{\text{ATT}} + \underbrace{E[Y_i(0)|D_i = 1] - E[Y_i(0)|D_i = 0]}_{\text{Selection bias}}$$

We saw that selection bias
$= \sum_{d=0}^{1} \{E[Y_i(0)|X_i = 1] - E[Y_i(0)|X_i = 0]\} \times \mathbb{P}(X_i = 0|D_i = d)$

OLS expression of bias

$$\underbrace{Y = \hat{\tau} D_i + \mathbf{X}_i \hat{\beta} + \hat{\gamma} Z_i + \epsilon_i}_{\text{Full regression}}$$

$$\underbrace{Y = \hat{\tau}_{res} D_i + \mathbf{X}_i \hat{\beta} + \epsilon_i}_{\text{Available Regression}}$$

If omitted variable has $Z = \hat{\delta} D_i + \mathbf{X}_i \hat{\mu} + \nu_i$

- selection bias $= \hat{\gamma}\hat{\delta}$

Issues with traditional OVB formulation:

- If omitted variable is nonbinary, then covariate scaling effects $\hat{\delta}$
- If there are more than one confounders, especially if they interact non-linearly, $\hat{\delta}$ is hard to assess

Reformulating traditional OVB

$$\hat{bias} = \underbrace{\frac{cov(E[D|X], E[Z|X])}{Var(E[D|X])}}_{\hat{\gamma}} \underbrace{\frac{cov(E[Y|X,D], E[Z|X,D])}{Var(E[Z|X,D])}}_{\hat{\delta}}$$

Noting that $cov(A, B) = cor(A, B)/(\sigma_A \sigma_B)\ldots$

$$= \frac{cor(E[D|X], E[Z|X]) \times sd(E[Z|X])}{sd(E[D|X])}$$
$$\times \frac{cor(E[Y|X,D], E[Z|X,D]) \times sd(E[Y|X,D])}{sd(E[Z|X,D])}$$
$$= \frac{cor(E[D|X], E[Z|X]) cor(E[Y|X,D], E[Z|X,D]) \times sd(E[Z|X,D])}{\frac{sd(E[Z|X,D])}{sd(E[Z|X])}}$$
$$\times \frac{sd(E[Y|X,D])}{sd(E[D|X])}$$

Partial R^2 formulation

partial $R^2_{Y\sim Z|D,X} = 1 - \frac{Var(E[Y|D,X])}{Var(Y)} = cor(E[Y|D,X],Y)^2$

$$\hat{bias} = \sqrt{\frac{R^2_{Y\sim Z|D,X} \times R^2_{D\sim Z|X}}{1 - R^2_{D\sim Z|X}}} \frac{sd(E[Y|X,D])}{sd(E[D|X])}$$

Noting that $sd(\hat{\tau}_{res}) = \frac{sd(E[Y|X,D])}{sd(E[D|X])}\sqrt{\frac{1}{df}}$

$$\hat{bias} = sd(\hat{\tau}_{res})\sqrt{\frac{R^2_{Y\sim Z|D,X} \times R^2_{D\sim Z|X}(df)}{1 - R^2_{D\sim Z|X}}}$$

Comments;

Partial $R^2_{Y\sim Z|D,X}$

- Proportion of Y that is explained by Z, D and X. We can reason on about this parameter without Z.
- formulation is scale-free i.e. does not matter whether Z enters linearly or quadratically
- Gives us two different ways to think about sensitivity

Extreme scenario case

If there were an *extreme* confounder that explained all the residual variance of the outcome, how strongly associated would it need to be in order to eliminate the estimated ATE (τ_{res})?

- Explains all the residual variance means $R^2_{Y\sim Z|D,X} = 1$
- To eliminate estimate effect means bias $= \tau_{res}$.
- $\frac{\tau_{res}}{sd(\hat{\tau}_{res})} = t = \sqrt{\frac{R^2_{Y\sim D|X}}{1-R^2_{Y\sim D|X}} \times df}$
- Implies the unknown value $R^2_{D\sim Z|X}$ must be $R^2_{Y\sim D|X}$
- How sensitive are our results to including an extreme confounder s.t., $R^2_{Y\sim Z|D,X} = 1$

The robustness value

If there were an confounder with an equal association to outcome and treatment (RV_q), how strong would that association have to be in order to reduce the estimated ATE by $(100 \times q)\%$

- Equal association means $R^2_{D\sim Z|X} = R^2_{Y\sim Z|X,D} = RV_q$
- Re-writing above expressions reveals this is $RV_q = 1/2(\sqrt{f_q^4 + 4f_q^2} - f_q^2)$, where
    - $f_{D\sim Z|X} = \sqrt{\frac{R^2_{D\sim Z|X}}{1-R^2_{D\sim Z|X}}}$
- RV close to one means the ATE is robust to confounders explaining almost all the residual variation of the treatment and outcome. RV close to zero means a weak confounder could eliminate the ATE.

Load the Data

```r
# loads package
#install.packages("sensemakr")
library(sensemakr)
```

```
## Warning: package 'sensemakr' was built under R version 3
```

```r
# loads data
data("darfur")
```

The variables are:

`directlyharmed` - indicates whether the individual was physically injured or maimed during the attack on her or his village in Darfur.

`peacefactor` - main outcome: an index measuring pro-peace attitudes.

OLS model

```r
library(estimatr)
# runs regression model
darfur.model <- lm(peacefactor ~ directlyharmed+ village +
                   female+age+ farmer_dar+ herder_dar +
                   pastvoted+hhsize_darfur,data = darfur)
summary(darfur.model)$coefficients[c(1:2),c(1:3)]
```

```
##                  Estimate Std. Error  t value
## (Intercept)    1.08189175 0.31491596 3.435494
## directlyharmed 0.09731582 0.02325654 4.184450
```

Sensitivity statistics for routine reporting

```r
#compute partial R-squared
partial_r2(t_statistic = 0.097/0.023, dof = 783)
```

```
## [1] 0.02221115
```

```r
# robustness value
robustness_value(t_statistic = 0.097/0.023, dof = 783)
```

```
## [1] 0.1397868
## Parameters: q = 1
```

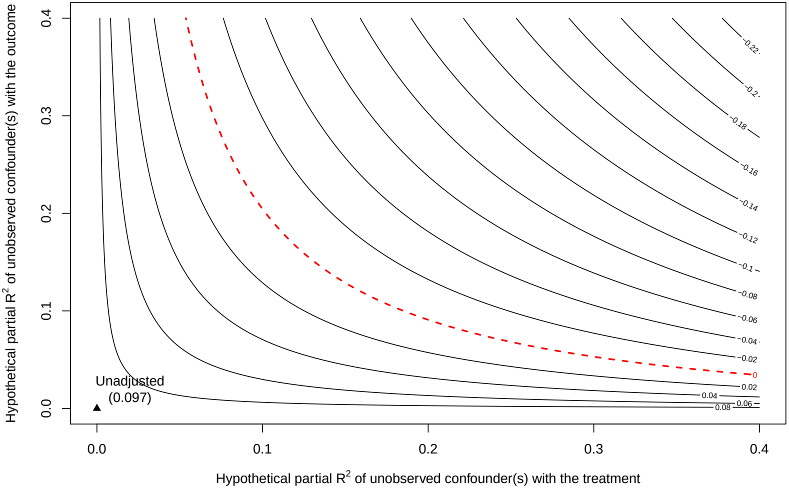
```r
#minimum strength of confounding to bring the lower bound
#of the 95 CI to half the current estimate

robustness_value(t_statistic = 0.097/0.023, dof = 783,
                 q = 1/2, alpha = 0.05)
```

```
## [1] 0.005148514
## Parameters: q = 0.5, alpha = 0.05
```

Plot the sensitivity results

```r
ovb_contour_plot(estimate = 0.097, se = 0.023, dof = 783)
```



Sensitivity Analysis

```r
# runs sensemakr for sensitivity analysis in the example
darfur.sensitivity <-
  sensemakr(model = darfur.model,
            treatment = "directlyharmed",
            benchmark_covariates = "female",
            kd = 1:3, ky = 1:3, q = 1,
            alpha = 0.05, reduce = TRUE)
```

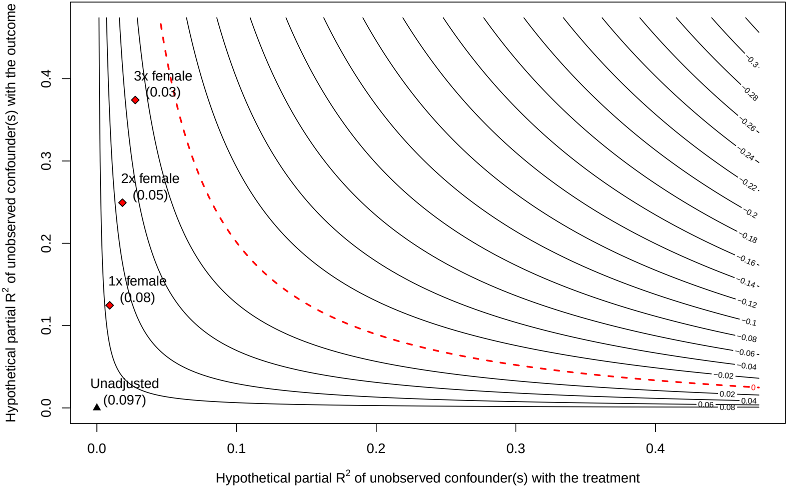
Minimal sensitivity reporting

```
darfur.sensitivity
```

```
## Sensitivity Analysis to Unobserved Confounding
##
## Model Formula: peacefactor ~ directlyharmed + village +
##     herder_dar + pastvoted + hhsize_darfur
##
## Unadjusted Estimates of ' directlyharmed ':
##   Coef. estimate: 0.09732
##   Standard Error: 0.02326
##   t-value: 4.18445
##
## Sensitivity Statistics:
##   Partial R2 of treatment with outcome: 0.02187
##   Robustness Value, q = 1 : 0.13878
##   Robustness Value, q = 1 alpha = 0.05 : 0.07626
##
## For more information, check summary.
```

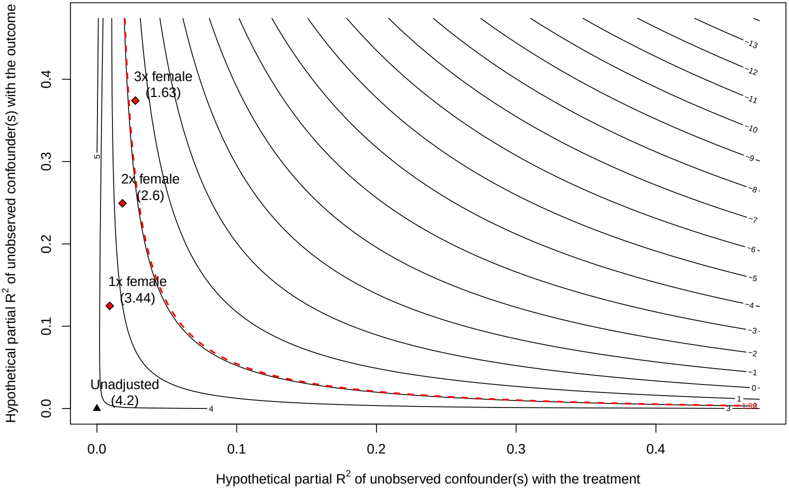
Sensitivity contour plots with comparison to female covariate

```
plot(darfur.sensitivity)
```



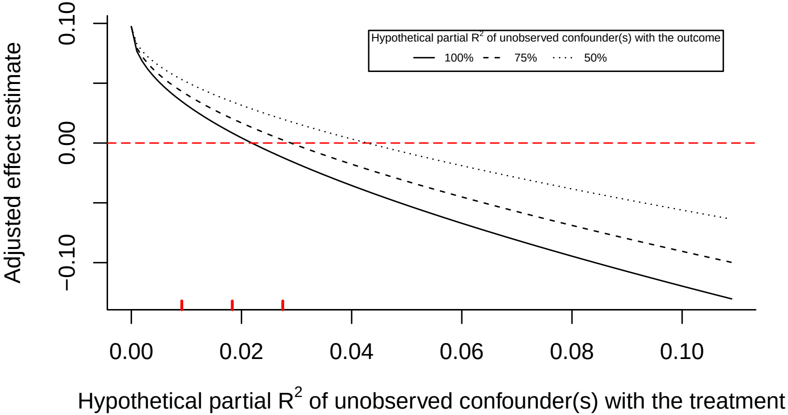
Sensitivity of the t-value

```
plot(darfur.sensitivity, sensitivity.of = "t-value")
```



Sensitivity plots of extreme scenarios

```
plot(darfur.sensitivity, type = "extreme")
```



Discussion

- How strong would an unobserved confounder (or a group of confounders) have to be to change our research conclusions?
- In a *worst-case* scenario, how robust are our results to *all* unobserved confounders acting together, possibly non-linearly?
- How strong would confounding need to be *relative* to the strength of observed covariates, to change our answer a certain amount?