% Regression for Covariate Adjustment
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Covariate Adjustment in sampling

- Freedman [Adv. in Appl. Math. 40 (2008) 180-193; Ann. Appl. Stat. 2 (2008) 176-196] showed that regression can be **biased** in small samples and is **inconsistent** for an experimental parameter in the case when interactions aren't included.
  - Unadjusted estimates also transparent and limit garden of forking paths concerns
  - ''The reason for the breakdown is not hard to find: randomization does not justify the assumptions behind the OLS model." Freedman[Adv. in Appl. Math. 40 (2008) 180-193]
- Lin, Winston. (2013) "Agnostic Notes on Regression Adjustments to Experimental Data: Reexamining Freedman's Critique" *Annals of Applied Statistics.* 7(1):295-318.
  - OLS adjustment cannot hurt asymptotic precision when a full set of treatment $\times$ covariate interactions is included
  - Huber-White sandwich standard error estimator is consistent or asymptotically conservative (regardless of whether interactions are included)
- An analogy. Imagine that we are biologists. We are interested in leaf size.
  - Finding the size of leaves is hard, but weighing leaves is easy.
  - Key insight is that we can use auxilliary information to be smarter:
  - Sample from leaves on a tree.
  - Measure their size and weight
  - Let $\bar{y}_s$ be the average size in the sample. We want $\bar{y}$.
  - Let $\bar{x}_s$ be the average weight in the sample.
  - We know that $\bar{y}_s$ unbiased and consistent for $\bar{y}$ but we have extra information- the mean population weight ($\bar{x}$)
  - $\hat{\bar{y}} = \bar{y}_s + q(\bar{x} - \bar{x}_s)$, with some q e.g. from a regssion of $\bar{y}_s$ on $\bar{x}_s$

Connection to Multiple Regression

- $Y_i = X_i Y_{1i} + (1 - X_i) Y_{0i}$
- We have auxiliary data on Z and by random assignment, $X_i \perp Z_i$
- Unlike leaves, we are sampling for both treatment and control potential outcomes
- For treated units: $\mathrm{E}[Y_i(1)]$ is unbiased for $Y(1)$ but it ignores information from Z_i, so we use $Y(1)_{reg} = Y_i(1) + \beta(Z_i - \bar{Z})$
- There's no reason to expect treatment and control groups to exhibit identical effects (form of omitted variable bias)
- Putting it altogether: $Y_i = \alpha\beta_1 X_i + \beta_2 Z_i + \beta_2 (X_i \times Z_i) + e_i$

Covariate Adjustment in Experiments

- Now imagine we are social scientists (hopefully this isn't hard)
- We are interested in the effects of a binary treatment on education, measured by a test.
- Let's set up a simulation.
- 250 students. Ten classes of 25 students each. Observed over two years.
- First year has half good teachers and half bad.
- We want to estimate the effect of the intervention in year 2.
- Treatment is assigned randomly by **individual**

- Note: This setup usually demands an accounting of clustering, which I'm ignoring. Maybe I'll bring it back later in the semester when we discuss SUTVA.

Simulation

```r
#Variables which govern the size of the simulation (and our causal effects)
nclass <- 5
nstudent <- 25
Eff <- 5
EffSD <- 3
# Simulate data
set.seed(1977)
Yr1ClassType <- rep(c(1,0),nclass*nstudent)
Yr2ClassType <- sample(Yr1ClassType,replace=FALSE)
Yr1Score <- rnorm(2*nclass*nstudent,76+Yr1ClassType*5,9)
# Fixed margins randomization
Trt  <- sample(Yr1ClassType,replace=FALSE)
# There is an independent effect of class type in each year
# Variance is different across class types in year 2
CtlOutcome <- rnorm(2*nclass*nstudent,Yr1Score+Yr2ClassType*3,9-Yr2ClassType*4)
# Treatment effect is random, but with expectation Eff
Yr2Obs <- CtlOutcome + Trt * rnorm(2*nclass*nstudent,Eff,EffSD)


#regression models
m1_unadj <- lm(Yr2Obs~Trt)
m1_adj <- lm(Yr2Obs~Trt+Yr1Score)

#results
summary(m1_unadj)$coefficients[2,]
```

```
##     Estimate   Std. Error      t value     Pr(>|t|)
## 7.479367e+00 1.666160e+00 4.488985e+00 1.096573e-05
```

```r
summary(m1_adj)$coefficients[2,]
```

```
##     Estimate   Std. Error      t value     Pr(>|t|)
## 4.605782e+00 1.058039e+00 4.353130e+00 1.966673e-05
```

```r
# We don't want the model-based SEs,
# we want the robust standard errors:
list.of.packages <- c("estimatr")
new.packages <- list.of.packages[!(list.of.packages %in% installed.packages()[,"Package"])]
if(length(new.packages)) install.packages(new.packages)
try(library('estimatr'),silent=TRUE)

#robust standard errors
commarobust(m1_adj) #default is HC2
```

```
##               Estimate Std. Error   t value     Pr(>|t|)    CI Lower CI Upper
## (Intercept) -6.003548 4.82176885 -1.245092 2.142777e-01 -15.5005749 3.493479
## Trt          4.605782 1.06012693  4.344557 2.039358e-05   2.5177406 6.693824
## Yr1Score     1.097974 0.06110947 17.967327 9.800090e-47   0.9776117 1.218336
##               DF
```

```
## (Intercept) 247
## Trt         247
## Yr1Score    247
```

```r
commarobust(m1_adj, se_type = "HC3")
```

```
##              Estimate Std. Error   t value     Pr(>|t|)    CI Lower CI Upper
## (Intercept) -6.003548  4.8790878 -1.230465 2.196935e-01 -15.6134712 3.606375
## Trt          4.605782  1.0669253  4.316874 2.292075e-05   2.5043504 6.707214
## Yr1Score     1.097974  0.0618359 17.756252 5.104105e-46   0.9761809 1.219767
##              DF
## (Intercept) 247
## Trt         247
## Yr1Score    247
```

```r
#robust standard errors
m1_unadj <- lm_robust(Yr2Obs~Trt,se_type = "HC3")
m1_adj <- lm_robust(Yr2Obs~Trt+Yr1Score,se_type = "HC3")
```

Exercise

In this exercise, we use replication data from Munger 2019. This study explores the effects of social sanctioning on racist online harassment. The author randomly assigns Twitter users with a histoty of racist behavior to receive messages from Twitter bots with different attributes - in-group/out-group (same race) and high (500-550)/low(0-10) number of followers. Here, we focus on just two groups - the control group (N=51) and the group that received tweets from bots that were in-group AND with a high number of followers (N=48).

Part A

Load the twitter_experiment file (using read.csv("twitter.csv")). The dataset is structured as follows:

Variable	Description
treat.f	Treatment variable
racism.scores.post.1wk	Racist harassment 1 week after treatment
log.followers	log(Number of Followers)
racism.scores.pre.2mon	Racist harassment 2 months before treatment

Part B

What is the unadjusted SATE? What is the standard error?

Part C

Replace missing potential outcomes for each unit, assuming that the individual average treatment effect is $Y_i(1) - Y_i(0) = \tau_i \sim \mathcal{N}(0.2, 0.2)$. Write a loop that generates a treatment vector 1000 times. Store the unadjusted SATE from each run. What is the unadjusted SATE? Where does the uncertainty arise from?

Part D

Include the regressors log.followers and racism.scores.pre.2mon in a regression for the SATE. What is the adjusted SATE? What is the standard error?

SOLUTIONS

```
munger2019 <- read.csv("munger2019.csv")

#unadjusted SATE
diff.in.means <- function(Y,D){
    diff.1 <- mean(Y[D == 1])- mean(Y[D == 0])
    se.1 <-sqrt(var(Y[D== 1])/sum(D==1)
                + var(Y[D==0])/sum(D == 0))
    t <- diff.1/se.1
    results <- c(diff.1,se.1,t)
  return(results)
}
diff.in.means(munger2019$racism.scores.post.1wk,munger2019$treat.f)
```

```
## [1] -0.2394689  0.1620180 -1.4780385
```

```
#potential outcomes
Y1 <- rep(NA,100)
Y0 <- rep(NA,100)

#consistency: E[Y_i(0)] = E[Y_i(0)|D_i=0] and E[Y_i(1)] = E[Y_i(1)|D_i=1]
Y1[munger2019$treat.f==1]=
  munger2019$racism.scores.post.1wk[munger2019$treat.f==1]
Y0[munger2019$treat.f==0]=
  munger2019$racism.scores.post.1wk[munger2019$treat.f==0]

#number of treated and control units
t <- nrow(munger2019[munger2019$treat.f==1,])
c <- nrow(munger2019[munger2019$treat.f==0,])

# individual treatment effect
Y0[munger2019$treat.f==1]=
  munger2019$racism.scores.post.1wk[munger2019$treat.f==1]-
  rnorm(t, mean = 0.2, sd = 0.2)
Y1[munger2019$treat.f==0]=
  munger2019$racism.scores.post.1wk[munger2019$treat.f==0]+
  rnorm(c, mean = 0.2, sd = 0.2)


tau <- Y1 - Y0 # individual treatment effect
## true value of the sample average treatment effect
SATE <- mean(tau)
SATE
```

```
## [1] 0.1941123
```

```r
#Generating the standard error of the SATE
sims <- 5000 # repeat 5000 times, we could do more
diff.means <- rep(NA, sims) # container
for (i in 1:sims) {
  ## randomize the treatment by sampling of a vector of 0s and 1s
  treat <- sample(c(rep(1, 50), rep(0, 50)), size = 100, replace = FALSE)
  ## difference-in-means
  diff.means[i] <- mean(Y1[treat == 1]) - mean(Y0[treat == 0])
}
## estimation error for SATE
est.error <- diff.means - SATE
summary(est.error)
```

```
##       Min.    1st Qu.     Median       Mean    3rd Qu.       Max.
## -0.5914045 -0.1214270  0.0032010 -0.0002225  0.1233642  0.6561840
```

```r
#adjusted and unadjusted via regression
summary(lm_robust(racism.scores.post.1wk~treat.f,data=munger2019))
```

```
##
## Call:
## lm_robust(formula = racism.scores.post.1wk ~ treat.f, data = munger2019)
##
## Standard error type:  HC2
##
## Coefficients:
##             Estimate Std. Error t value  Pr(>|t|) CI Lower CI Upper DF
## (Intercept)   0.6264     0.1422   4.404 2.711e-05   0.3441  0.90860 98
## treat.f      -0.2395     0.1620  -1.478 1.426e-01  -0.5610  0.08205 98
##
## Multiple R-squared:  0.02085 ,   Adjusted R-squared:  0.01086
## F-statistic: 2.185 on 1 and 98 DF,  p-value: 0.1426
```

```r
summary(lm_robust(racism.scores.post.1wk~treat.f*log.followers+
                    treat.f*racism.scores.pre.2mon,data=munger2019))
```

```
##
## Call:
## lm_robust(formula = racism.scores.post.1wk ~ treat.f * log.followers +
##     treat.f * racism.scores.pre.2mon, data = munger2019)
##
## Standard error type:  HC2
##
## Coefficients:
##                                Estimate Std. Error  t value Pr(>|t|) CI Lower
## (Intercept)                     0.01080    0.35625  0.03032  0.97587 -0.69654
## treat.f                         0.14021    0.46691  0.30029  0.76462 -0.78685
## log.followers                   0.06110    0.06694  0.91280  0.36368 -0.07181
## racism.scores.pre.2mon          1.18052    0.62240  1.89671  0.06094 -0.05528
## treat.f:log.followers          -0.05831    0.08489 -0.68683  0.49388 -0.22687
## treat.f:racism.scores.pre.2mon  0.97934    1.05685  0.92666  0.35648 -1.11906
##                                CI Upper DF
## (Intercept)                      0.7181 94
## treat.f                          1.0673 94
## log.followers                    0.1940 94
```

```
## racism.scores.pre.2mon           2.4163 94
## treat.f:log.followers            0.1103 94
## treat.f:racism.scores.pre.2mon   3.0777 94
##
## Multiple R-squared:  0.3495 ,    Adjusted R-squared:  0.3149
## F-statistic: 2.787 on 5 and 94 DF,  p-value: 0.02158
```