National Institute of Technology Calicut

Department of Mathematics

Second Semester B.Tech., End Semester Examination, Winter Semester 2016-2017

MA 1002 - MATHEMATICS II

Duration: 3 Hours **Max. Marks: 50**

1. The trajectory of a particle is given by $\vec{r}(t) = \cos(\pi t)\hat{\imath} + \sin(\pi t)\hat{\jmath} + t\hat{k}$.

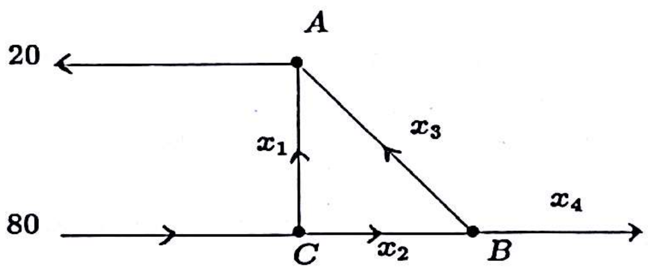
   (a) Determine the velocity, $\vec{v}(t)$, and acceleration, $\vec{a}(t)$, of the particle at $t = 1$.

   (b) Determine the tangential and normal components of $\vec{a}(t)$ at $t = 1$. 3

2. If $\vec{F} = x^2y\hat{\imath} - 2xz\hat{\jmath} + 2yz\hat{k}$, find (a) $\nabla \cdot \vec{F}$ (b) $\nabla \times \vec{F}$ (c) $\nabla(\nabla \cdot \vec{F})$ (d) $\nabla \cdot (\nabla \times \vec{F})$. 3

3. Verify Green's Theorem in plane for $\int_C y^2dx + x^2dy$ where C is the boundary of the triangle with vertices $(0,0), (1,0)$ and $(1,1)$ oriented in the anticlockwise direction. 3

4. Find the general flow pattern of the network shown in the figure given below. Assuming that all flows are nonnegative, what is the largest possible value for x_3?. 4



5. Let the concentration of an air pollutant at a point (x, y, z) is given by $f(x, y, z) = xy^2z$ particles/m^3. Find the total amount of pollutant in a region R where $x, y, z \geq 0$ and $x^2 + y^2 + z^2 \leq 25$. 3

6. Determine whether the following statements are true or false and justify your answer.

   (a) If S_1 and S_2 are linearly independent subsets of a vector space so is their union $S_1 \cup S_2$.

(b) The set of all $n \times n$ matrices such that $A^T = A^{-1}$ is a subspace of the set of all $n \times n$ matrices $M_{n\times n}(R)$.

(c) If U is a subspace of a vector space V and v_1, v_2 are vectors in V such that $v_1+v_2 \in U$ then $v_1, v_2 \in U$.

(d) Let A be an $m \times n$ matrix with rank m. Then the column vectors of A span R^m.

(e) Sum of any two eigenvectors of A is also an eigenvector of A.

(f) Similar matrices have same eigenvectors. 6

7. Let $T : R^4 \to R^3$ be a linear transformation defined by $T(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find basis and dimension of null space ($Ker(T)$) and range space ($Im(T)$) and verify Rank-Nullity Theorem. 4

8. Let V and W be real vector spaces with respective bases $v_1 = (1, 2, 3), v_2 = (2, 1, 0), v_3 = (1, -1, 2)$ and $w_1 = (1, 0, 0), w_2 = (0, 1, 0), w_3 = (1, 1, 1)$. Find a linear transformation $T : V \to W$ such that $T(v_i) = w_i, i = 1, 2, 3$. Is T invertible? If so find T^{-1}. 4

9. Find a linear transformation $T : R^3 \to R^3$, in each case, such that, (a) $Ker(T) = \{(x, y, z) |\ x + y = 0\}$. (b) $Im(T) = \{(x, y, z) |\ x - 2y + z = 0\}$. 4

10. Define an inner product space. Let V be an inner product space of set of all real polynomials of degree at most 2 with inner product $< p, q >= p(0)q(0) + p(1)q(1) + p(2)q(2), \forall\ p, q \in V$. Using Gram-Schmidt orthogonalization process, construct an orthonormal basis of V from the basis $\{1, x, x^2\}$. 4

11. Given the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Using Cayley Hamilton Theorem, evaluate A^{-1} and $A^4 + A^3 - A^2 + 4A + 6I$. 4

12. (a) Show that $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ if $a + b = c + d; a, b, c, d \in \mathbb{R}$. Determine the eigenvalues of A.

(b) Identify the conic section represented by the quadratic form $7x_1^2 + 6x_1x_2 + 7x_2^2 = 200$ and obtain the principal axes. 4

13. Determine the nature of the quadratic form $Q = X^TAX = \frac{3}{2}x_1^2 + 3x_2^2 + \frac{3}{2}x_3^2 + x_1x_3$ by reducing it to the form Y^TDY where $X = PY$ and D is a diagonal matrix. Write down the matrix P. Also, find the rank, index and signature of the quadratic form. 4