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B190304CE
MATHEMATICS
Roll: 46

A)

1) $y'' + 9y = 0$; $y(0) = 4$, $y'(0) = -6$

Put $y = e^{\lambda x}$, $\therefore y' = \lambda e^{\lambda x}$, $y'' = \lambda^2 e^{\lambda x}$

$\therefore y'' + 9y = \lambda^2 e^{\lambda x} + 9 e^{\lambda x} = 0$

$e^{\lambda x}[\lambda^2 + 9] = 0$

$\lambda^2 = -9$

$\lambda = \pm 3i$

$y = e^{0x}[A\cos 3x + B\sin 3x]$

$y = A\cos 3x + B\sin 3x$

$y(0) \Rightarrow 4 = \underline{\underline{A}}$

$\cancel{y'(0)}$

$y' = -3A\sin 3x + 3B\cos 3x$

$y'(0) = -6 = 3B$

$\Rightarrow \underline{\underline{B = -2}}$

$\therefore y = \underline{\underline{4\cos 3x - 2\sin 3x}}$

2) $4x^2 y'' - 3y = 0$; $y(1) = 3$, $y'(1) = 2.5$

$y = x^m$, $y' = m x^{m-1}$, $y'' = m(m-1)x^{m-2}$

$\therefore 4x^2 y'' - 3y = x^m[4m(m-1) - 3]$

$4m^2 - 4m - 3 = 0$

$m^2 - m - 3/4 = 0$

$(m - 1/2)^2 = 1$

$m = 1/2 \pm 1$

$m = 3/2, -1/2$

$y = c_1 x^{3/2} + c_2 x^{-1/2}$

$y' = 3/2\, c_1 x^{1/2} - 1/2\, c_2 x^{-3/2}$

$y(1) = 3 = c_1 + c_2$

$y'(1) = 2.5 = 3/2\, c_1 - 1/2\, c_2$

$2.5 = 3/2\, c_1 - 1/2\, [3 - c_1]$

$5/2 = 3/2\, c_1 - 3/2 + c_1/2$

$4 = 2c_1 \Rightarrow \underline{\underline{c_1 = 2}}$

$c_2 = 1$

$\underline{\underline{y = 2x^{3/2} + x^{-1/2}}}$

B) 1) $8y'' - 2y' - y = 0$

$y = e^{\lambda x},\ y' = \lambda e^{\lambda x},\ y'' = \lambda^2 e^{\lambda x}$

$\therefore\ 8y'' - 2y' - y = [8\lambda^2 - 2\lambda - 1] e^{\lambda x} = 0$

$[\lambda^2 - 1/4\,\lambda - 1/8] = 0$

$(\lambda - 1/8)^2 = 1/64 + 8/64 = 9/64$

$\lambda = \frac{1}{8} \pm \frac{3}{8}$

$\lambda = 1/2,\ -1/4$

$\underline{\underline{y = c_1 e^{1/2 x} + c_2 e^{-1/4 x}}}$

2) $y'' + 9y' + 20y = 0$

$\lambda^2 + 9\lambda + 20 = 0$

$(\lambda + 9/2)^2 - \frac{81}{4} + \frac{80}{4} = 0$

$(\lambda + 9/2)^2 - 1/4 = 0$

$\lambda + 9/2 = \pm 1/2$

$\lambda = -9/2 \pm 1/2$

$\lambda = \underline{\underline{-4}}, -5$

$$y = c_1 e^{-4x} + c_2 e^{-5x}$$

3) $9y'' - 30y' + 25y = 0$

$9\lambda^2 - 30\lambda + 25 = 0$

$\lambda^2 - {}^{10}/_{3}\lambda + {}^{25}/_{9} = 0$

$(\lambda - \frac{5}{3})^2 = 0$

$\lambda = 5/3$

$$y = (c_1 x + c_2) e^{5/3 x}$$

4) $4y'' + 4y' - 3y = 0$

$4\lambda^2 + 4\lambda - 3 = 0$

$\lambda^2 + \lambda - 3/4 = 0$

$(\lambda + 1/2)^2 - 1/4 - 3/4 = 0$

$(\lambda + 1/2)^2 = 1$

$\lambda = -1/2 \pm 1$

$y = -3/2 ,\ 1/2$

$$y = c_1 e^{-3/2 x} + c_2 e^{1/2 x}$$

c) 1) $y'' + 4y' + 2y = 0$; $y(0) = -1$, $y'(0) = 2 + 3\sqrt{2}$

$\lambda^2 + 4\lambda + 2 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16 - 8}}{2}$

$\lambda = \dfrac{-4 \pm 2\sqrt{2}}{2}$

$\lambda = -2 \pm \sqrt{2}$

$\lambda = -2 + \sqrt{2} ,\ -2 - \sqrt{2}$

$y = c_1 e^{(-2+\sqrt{2})x} + c_2 e^{(-2-\sqrt{2})x}$

$y' = (-2 + \sqrt{2}) c_1 e^{(-2+\sqrt{2})x} + (-2 - \sqrt{2}) c_2 e^{(-2-\sqrt{2})x}$

$y(0) = -1 = c_1 + c_2$

$y'(0) = 2+3\sqrt{2} = (-2+\sqrt{2})c_1 + (-2-\sqrt{2})c_2$

$2+3\sqrt{2} = (-2+\sqrt{2})(-1-c_2) + (-2-\sqrt{2})c_2$

$= 2+2c_2-\sqrt{2}-\sqrt{2}c_2 - 2c_2 - \sqrt{2}c_2$

$2+3\sqrt{2} = -2\sqrt{2}c_2 + 2 - \sqrt{2}$

$4\sqrt{2} = -2\sqrt{2}c_2$

$c_2 = -2$

$c_1 = 1$

$$y = e^{(-2+\sqrt{2})x} - 2e^{-(2+\sqrt{2})x}$$

2) $4y'' - 4y' - 3y = 0$; $y(-2) = e$, $y'(-2) = -e/2$

$4\lambda^2 - 4\lambda - 3 = 0$

$\lambda^2 - \lambda - 3/4 = 0$

$(\lambda - 1/2)^2 = 1$

$\lambda = 1/2 \pm 1$

$\lambda = 3/2, -1/2$

$y = c_1 e^{3/2} + c_2 e^{-1/2x}$

$y' = \frac{3}{2}c_1 e^{3/2x} - 1/2 c_2 e^{-1/2x}$

$y(-2) = e = c_1 e^{3/2 \times -2} + c_2 e^{-1/2 \times -2}$

$e = c_1 e^{-3} + c_2 e$ —— 1

$y'(-2) = -\frac{e}{2} = \frac{3}{2}c_1 e^{-3} - \frac{1}{2}c_2 e$

$-e/2 = 3/2\,[e - c_2 e]e^{-3} - 1/2\, c_2 e$ —— 2

~~$-e^4 = 3(e - c_2 e) - c_2 e^4$~~

~~$-e^4$~~

solving 1 and 2, we get,

$c_2 = 1$, $c_1 = 0$

$\therefore y = e^{-1/2x}$

D] 1) $y'' + 4y = 0$; $y(0) = 3$, $y'(\pi/2) = -3$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = A\cos(2x) + B\sin(2x)$

$y' = -2A\sin(2x) + 2B\cos(2x)$

$y(0) = 3 \Rightarrow 3 = A$

$y'(\pi/2) = -3 \Rightarrow -3 = -2B$

$B = 3/2$

$y = 3\cos(2x) + \frac{3}{2}\sin 2x$

2) $y'' + 2y' + 2y = 0$; $y(0) = 1$, $y(\pi/2) = 0$

$\lambda^2 + 2\lambda + 2 = 0$

$(\lambda+1)^2 + 1 = 0$

$\lambda = -1 \pm i$

$y = e^{-x}[A\cos x + B\sin x]$

$y(0) = 1 \Rightarrow A = 1$

$y(\pi/2) = 0 \Rightarrow B = 0$

$\therefore y = e^{-x}\cos x$

E) 1) $y'' - y = 2e^x$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$y_h = c_1e^x + c_2e^{-x}$

$y_p = xe^x$

$y = c_1e^x + c_2e^{-x} + xe^x$

$y' = c_1e^x - c_2e^{-x} + xe^x + e^x$

$y(0) = -1 \Rightarrow -1 = c_1 + c_2$

$y'(0) = 0 \Rightarrow 0 = c_1 - c_2 + 1$

$\therefore c_1 = \cancel{0} -1$

$c_2 = 0$

$\therefore y = -e^{-x} + xe^{x}$

a) $x^2 y'' - 3xy' + 3y = 3\ln x - 4$; $y(1) = 0$, $y'(1) = 1$, $y_p = \ln x$

$y = x^m$, $y' = mx^{m-1}$, $y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - 3xmx^{m-1} + 3x^m = 0$

$x^m[m^2 - m - 3m + 3] = 0$

$m^2 - 4m + 3 = 0$

$m = 3, 1$

$y_h = c_1 x^3 + c_2 x$

$y_p = \ln x$

$y = y_h + y_p$

$= c_1 x^3 + c_2 x + \ln x$

$y' = 3c_1 x^2 + c_2 + \frac{1}{x}$

$y'(1) = 1 \Rightarrow 1 = 3c_1 + c_2 + 1$

$3c_1 - c_1 = 0$

$2c_1 = 0$

$\therefore c_1 = c_2 = 0$

$y = \ln x$

F) $y_1 = x^3$; $x^2 y'' - 5xy' + 9y = 0$ —— (1)

let $y_2 = vx^3$

$y_2' = 3vx^2 + x^3 v'$

$y_2'' = 3(2vx + x^2 v') + x^3 v'' + 3v'x^2$

$y_2'' = 6vx + 3x^2 v' + x^3 v'' + 3v'x^2$

$y_2'' = x^3 v'' + 6x^2 v' + 6vx$

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$\therefore$

① $\Rightarrow$

$x^2[x^3v''+6x^2v'+6vx]-5x[3vx^2+x^3v']+9vx^3=0$

$x^5v''+6x^4v'+6vx^3-15vx^3-5x^4v'+9vx^3=0$

$x^5v''+x^4v'=0$

$xv''+v'=0$

Put $v'=w$ and $w'=v''$

$\therefore xv''+v'= xw'+w$

$\Rightarrow x\frac{dw}{dx}+w=0$

$\frac{dw}{w}=-\frac{dx}{x}$

$\log|w|=-\log|x|+\log c$

$wx=c$

$w=c/x$

$v=\int w\,dx = c\int \frac{1}{x}dx = c\log x+k$

put $c=k=1$

$\therefore v=\log x$

Now $y_2=vx^3$

$y_2=\log x\times x^3$

$$y=c_1x^3\log x+c_2x^3$$

5) 1) $y''+1.5y'-y=12x^2+6x^3-x^4$ —①

$\lambda^2+1.5\lambda-1=0$

$(\lambda+3/4)^2-9/16-16/16=0$

$\lambda+3/4=\pm 5/4$

$\lambda=-3/4\pm 5/4$

$\lambda=1/2,-2$

$y_h=c_1e^{1/2x}+c_2e^{-2x}$

$y_p = k_4x^4 + k_3x^3 + k_2x^2 + k_1x + k_0$

$y_p' = 4k_4x^3 + 3k_3x^2 + 2k_2x + k_1$

$y_p'' = 12k_4x^2 + 6k_3x + 2k_2$

① $\Rightarrow$ $12k_4x^2 + 6k_3x + 2k_2 + 6k_4x^3 + 4.5k_3x^2 + 3k_2x + 1.5k_1$

$- k_4x^4 - k_3x^3 - k_2x^2 - k_1x - k_0 = 12x^2 + 6x^3 - x^4$

$-k_4x^4 + x^3(6k_4 - k_3) + x^2(12k_4 + 4.5k_3 - k_2)$

$+ x(6k_3 + 3k_2 - k_1) + (2k_2 + 1.5k_1 - k_0) = 12x^2 + 6x^3 - x^4$

$-k_4 = -1$

$\underline{\underline{k_4 = 1}}$

$12k_4 + 4.5k_3 - k_2 = 12$

$4.5k_3 = k_2$

$\underline{\underline{k_2 = 0}}$

$6k_4 - k_3 = 6$

$\underline{\underline{k_3 = 0}}$

$6k_3 + 3k_2 - k_1 = 0$

$\underline{k_1 = 0}$

$2k_2 + 1.5k_1 - k_0 = 0$

$\underline{\underline{k_0 = 0}}$

$y = y_h + y_p$

$\underline{\underline{= c_1e^{1/2x} + c_2e^{-2x} + x^4}}$

2) $y'' - 4y = e^{-2x} - 2x$ —①

$\lambda^2 - 4 = 0$

$\lambda = 2, -2$

$y_h = c_1e^{2x} + c_2e^{-2x}$

$y_p = cxe^{-2x} + k_1x + k_0$

$y_p = cxe^{-2x} + k_1x + k_0$

$y_p' = -2cxe^{-2x} + ce^{-2x} + k_1$

$y_p'' = 4cxe^{-2x} - 4ce^{-2x}$

① $\Rightarrow$ $4cxe^{-2x} - 4ce^{-2x} - 4(cxe^{-2x} + k_1x + k_0) = e^{-2x} - 2x$

$4cxe^{-2x} - 4ce^{-2x} - 4cxe^{-2x} - 4k_1x - 4k_0 = e^{-2x} - 2x$

$-4k_1 = -2$ $\quad -4ce^{-2x} = e^{-2x}$ $\quad k_0 = 0$

$k_1 = 1/2$ $\quad c_1 = -1/4$

$y = y_n + y_p$

$= c_1e^{2x} + c_2e^{-2x} - \frac{xe^{-2x}}{4} + \frac{x}{2}$

3) $y'' + y' = 2 + 2x + x^2$ —① $\quad ; \; y(0) = 8, \; y'(0) = -1$

$\lambda^2 + \lambda = 0$

$\lambda(\lambda + 1) = 0$

$\lambda = 0, -1$

$y_n = c_1 + c_2e^{-x}$

$y_p = k_3x^3 + k_2x^2 + k_1x + k_0$

$y_p' = 3k_3x^2 + 2k_2x + k_1$

$y_p'' = 6k_3x + 2k_2$

① $\Rightarrow$ $6k_3x + 2k_2 + 3k_3x^2 + 2k_2x + k_1 = 2 + 2x + x^2$

$x^2(3k_3) + x(6k_3 + 2k_2) + (k_1 + 2k_2) = 2 + 2x + x^2$

$3k_3 = 1$ $\quad 6k_3 + 2k_2 = 2$ $\quad k_1 + 2k_2 = 2$

$k_3 = 1/3$ $\quad k_2 = 0$ $\quad k_1 = 2$

$y = y_n + y_p$

$= c_1 + c_2e^{-x} + 1/3\, x^3 + 2x + k_0 = c_1 + c_2e^{-x} + \frac{1}{3}x^3 + 2x$

$y' = -c_2e^{-x} + x^2 + 2$

$y(0) = 8 \Rightarrow 8 = c_1 + c_2$

$y'(0) = -1 \Rightarrow -1 = -c_2 + 2$

$\therefore c_1 = 5, \; c_2 = 3$

$y = 5 + 3e^{-x} + 1/3\, x^3 + 2x$

4) $y'' + 4y' + 13y = 2e^{-x}$ —— (1)

$\lambda^2 + 4\lambda + 13 = 0$

$$\lambda = \frac{-4 \pm \sqrt{16-52}}{2}$$

$$= \frac{-4 \pm \sqrt{-36}}{2}$$

$\lambda = -2 \pm 3i$

$y_h = e^{-2x}\left(A\cos 3x + B\sin 3x\right)$

$y_p = ke^{-x}$

$y_p' = -ke^{-x}$

$y_p'' = ke^{-x}$

sub in (1) $\Rightarrow$ $ke^{-x} + 4(-ke^{-x}) + 13(ke^{-x}) = 2e^{-x}$

$10ke^{-x} = 2e^{-x}$

$\underline{k = 1/5}$

$y = y_h + y_p$

$= e^{-2x}\left[A\cos 3x + B\sin 3x\right] + \frac{1}{5}e^{-x}$

$y' = e^{-2x}\left(-3A\sin 3x + 3B\cos 3x\right)$

$- 2e^{-2x}\left[A\cos 3x + B\sin 3x\right] - \frac{1}{5}e^{-x}$

$y(0) = 0 \Rightarrow 0 = A + \frac{1}{5} \Rightarrow \underline{A = -\frac{1}{5}}$

$y'(0) = -1 \Rightarrow -1 = 3B - 2A$

$B = -2/5$

$\therefore \underline{y = e^{-2x}\left[-\frac{1}{5}\cos 3x - \frac{2}{5}\sin 3x\right] + \frac{e^{x}}{5}}$

5) $y'' + 9y = x^2 \cos 3x$

$m^2 + 9 = 0$

$m = \pm 3i$

$y_h = c_1 \cos 3x + c_2 \sin 3x$

$y_p = (A_2 x^3 + A_1 x^2 + A_0 x) \cos 3x$

$+ (B_2 x^3 + B_1 x^2 + B_0 x) \sin 3x$

$y_p' = (A_2 3x^2 + A_1 2x + A_0) \cos 3x \cancel{+ (B_2 x^3 + B_1 x^2 + B_0 x) \sin 3x}$

$+ (A_2 x^3 + A_1 x^2 + A_0 x)(-3\sin 3x)$

$+ (B_2 3x^2 + B_1 2x + B_0) \sin 3x + (B_2 x^3 + B_1 x^2 + B_0 x)$
$3\cos 3x$

$y_p'' = (A_2 6x + A_1(2)) + \cos 3x + A_2(3x^2) + A_1(2x) + A_0)(-6\sin 3x)$

$+ (A_2 x^3 + A_1 x^2 + A_0 x)(-9\cos 3x) + (B_2 6x + B_1(2))(\sin 3x)$

$+ (B_2(3x^2) + B_1(2x) + B_0)(6\cos 3x) = x^2 \cos 3x$

$18 B_2 = 1 \Rightarrow B_2 = 1/18$

$12 B_1 + 6A_2 = 0$ $\quad B_1 = 0$

$6B_0 + 2A_1 = 0$ $\quad 6B_0 = -1/18 \Rightarrow B_0 = -1/108$

$-18A_2 = 0 \Rightarrow A_2 = 0$

$-12A_1 + 6B_2 = 0$ $\quad A_1 = +1/36$

$-6A_0 + 2B_1 = 0$ $\quad A_0 = 0$

$\therefore \; y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{36} x^2 \cos 3x - \frac{1}{108} x \sin 3x$

$+ \frac{x^3}{18}$

6) $y'' - y = e^{x}\sin(2x)$ —— ①

$\lambda^2 - 1 = 0$

$\lambda = 1, -1$

$y_h = c_1 e^{x} + c_2 e^{-x}$

$y_p = e^{x}(A\cos 2x + B\sin 2x)$

$y_p' = e^{x}[-2A\sin 2x + 2B\cos 2x]$

$\quad + e^{x}[A\cos 2x + B\sin 2x]$

$y_p' = e^{x}[(B-2A)\sin 2x + (2B+A)\cos 2x]$

$y_p'' = e^{x}[2(B-2A)\cos 2x - 2(2B+A)\sin 2x]$

$\quad + e^{x}[(B-2A)\sin 2x + (2B+A)\cos 2x]$

① $\Rightarrow$

$e^{x}[\cos 2x(2B-4A+2B+A-A) +$

$\sin 2x(-4B-2A+B-2A+B)]$

$= e^{x}\sin 2x$

$e^{x}[(4B-4A)\cos 2x + (-4B-4A)\sin 2x] = e^{x}\sin 2x$

$-4A - 4B = 1$

$4(A+B) = -1$

$A+B = -1/4$

$A = B = -\frac{1}{8}$

$\therefore y = y_h + y_p$

$= c_1 e^{x} + c_2 e^{-x} - \frac{1}{8}e^{x}(\cos 2x + \sin 2x)$

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7) $y'' + 4y' + 4y = (6+x^2)e^{-4x}$ —— ①

$\lambda^2 + 4\lambda + 4 = 0$

$(\lambda+2)^2 = 0$

$\lambda + 2 = 0$

$\lambda = -2$

$y_h = (c_1x + c_2)e^{-2x}$

$y_p = e^{-4x}[k_2x^2 + k_1x + k_0]$

$y_p' = e^{-4x}[2k_2x + k_1] - 4[k_2x^2 + k_1x + k_0]e^{-4x}$

$= e^{-4x}[2k_2x + k_1 - 4k_2x^2 - 4k_1x - 4k_0]$

$y_p' = e^{-4x}[-8k_2x^2 + (2k_2 - 4k_1)x + (k_1 - 4k_0)]$

$y_p'' = e^{-4x}[-8k_2x + 2k_2 - 4k_1] - 4e^{-4x}[-4k_2x^2$
$+ (2k_2 - 4k_1)x + (k_1 - 4k_0)]$

$y_p'' = e^{-4x}[x^2(16k_2) + x(-8k_2 - 8k_2 + 16k_1)$
$+ (2k_2 - 4k_1 - 4k_1 + 16k_0)]$

① ⇒ $e^{-4x}[x^2(16k_2 - 16k_2 + 4k_2) + x[-16k_2 + 16k_1 + 8k_2 - 16k_1 + 4k_1]$
$+ (2k_2 - 4k_1 - 4k_1 + 16k_0 + 4k_1 - 16k_0 + 4k_0)]$
$= (6+x^2)e^{-4x}$

$x^2(4k_2) + x(-8k_2 + 4k_1) + (2k_2 - 4k_1 + 4k_0) = x^2 + 6$

$4k_2 = 1$

$k_2 = 1/4$

$4k_1 = 8k_2$

$k_1 = 2k_2$

$k_1 = 1/2$

$2k_2 - 4k_1 + 4k_0 = 6$

$\frac{1}{2} - 2 + 4k_0 = 6$

$4k_0 = 15/2$

$k_0 = 15/8$

$y = (c_1x + c_2)e^{-2x} + e^{-4x}\left(\frac{x^2}{4} + \frac{x}{2} + \frac{15}{8}\right)$

H) $y'' + ay' + by = 0$

$y = c_1 y_1 + c_2 y_2$

$\lambda = \dfrac{-a \pm \sqrt{a^2 - ab}}{2}$

$y_1 = e^{\left[\frac{-a+\sqrt{a^2-ab}}{2}\right]x}$, $y_2 = e^{\left[\frac{-a-\sqrt{a^2-ab}}{2}\right]x}$

$y_1' = \left(\dfrac{-a+\sqrt{a^2-ab}}{2}\right) e^{\left(\frac{-a+\sqrt{a^2-ab}}{2}\right)x}$

$y_2' = \left(\dfrac{-a-\sqrt{a^2-ab}}{2}\right) e^{-\left(\frac{-a-\sqrt{a^2-ab}}{2}\right)x}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$= e^{-ax}\left[\dfrac{-a-\sqrt{a^2-ab}}{2}\right] - e^{-ax}\left[\dfrac{-a+\sqrt{a^2-ab}}{2}\right]$

$= e^{-ax}\left(-\sqrt{a^2-ab}\right) = ce^{-ax}$

$\therefore \; W(y_1, y_2) = \underline{\underline{ce^{-ax}}}$

I) 1) $y'' - 4y' + 4y = e^{2x}/x$

$\lambda^2 - 4\lambda + 4 = 0$

$(\lambda - 2)^2 = 0$

$\lambda = 2$

$y_h = (c_1 x + c_2) e^{2x}$

$y_1 = xe^{2x}$, $y_1' = 2xe^{2x} + e^{2x}$

$y_2 = e^{2x}$, $y_2' = 2e^{2x}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} xe^{2x} & e^{2x} \\ 2xe^{2x} + & 2e^{2x} \end{vmatrix}$

$= 2xe^{4x} - e^{4x} - 2xe^{4x}$

$= \underline{\underline{-e^{4x}}}$

$$u(x) = \int \frac{-y_2 r(x)}{W} = \int \frac{-e^{2x} e^{2x}}{x \cdot e^{4x}}$$

$$= \int 1/x = \log|x| + C_1$$

$$v(x) = \int \frac{y_1 r(x)}{W} = \int \frac{x e^{2x} e^{2x}}{x \times e^{4x}} = \int -1\,dx = -x + C_2$$

$$y = u(x) y_1 + v(x) y_2$$

$$= (\log x + c_1) x e^{2x} + (-x + c_2) e^{2x}$$

$$= x e^{2x} [\log x - \log e] + (c_1 x + c_2) e^{2x}$$

$$= \underline{\underline{x e^{2x} \log(x/e) + (c_1 x + c_2) e^{2x}}}$$

2) $2y'' + 2y = \operatorname{cosec} x$

$2\lambda^2 + 2 = 0$

$\lambda^2 = -1$

$\lambda = \pm i$

$y = A\cos x + B\sin x$

$y_1 = \cos x \;,\; y_1' = -\sin x$

$y_2 = \sin x \;,\; y_2' = \cos x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1$$

$$u(x) = \int -\sin x \operatorname{cosec} x \, dx = \int -1\,dx = -x + C_1$$

$$v(x) = \int \cos x \operatorname{cosec} x \, dx \int \cot x \, dx = \log|\sin x| + c_2$$

$$y = u(x) y_1 + v(x) y_2$$

$$= (-x + c_1)\cos x + (\log|\sin x| + c_2)\sin x$$

$$= \underline{\underline{(-x\cos x + \log|\sin x| \sin x) + (c_1 \cos x + c_2 \sin x)}}$$

3) $y'' - 2y' + y = 3x^{3/2} e^{x}$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda = 1$

$y = (c_1 x + c_2) e^{x}$

$y_1 = x e^{x}$, $y_1' = x e^{x} + e^{x}$

$y_2 = e^{x}$, $y_2' = e^{x}$

$$W = \begin{vmatrix} x e^{x} & e^{x} \\ x e^{x} + e^{x} & e^{x} \end{vmatrix} = x e^{2x} - x e^{2x} - e^{2x}$$

$$= \underline{\underline{-e^{2x}}}$$

$$u(x) = \int \frac{-\cancel{x}e^{x}\, 3x^{3/2} e^{x}}{-e^{2x}} = 3\int x^{3/2}$$

$$= 3\,\frac{x^{5/2}}{5} \times 2$$

$$= 6/5\, x^{5/2} + c_1$$

$$v(x) = \int \frac{x e^{x}\, 3x^{3/2} e^{x}}{-e^{2x}} = -\int 3x^{5/2} = -3\frac{x^{7/2}}{7} \times 2$$

$$= -\frac{6}{7} x^{7/2} + c_2$$

$$y = (6/5\, x^{5/2} + c_1) x e^{x} + (-6/7\, x^{7/2} + c_2) e^{x}$$

$$= (6/5\, x^{7/2} e^{x} - 6/7\, x^{7/2} e^{x}) + (c_1 x + c_2) e^{x}$$

$$= \underline{\underline{\frac{12}{35} x^{7/2} e^{x} + (c_1 x + c_2) e^{x}}}$$

4) $y'' - y = \frac{2}{1+e^x}$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$y = c_1 e^x + c_2 e^{-x}$

$y_1 = e^x, \quad y_1' = e^x$

$y_2 = e^{-x}, \quad y_2' = -e^{-x}$

$W = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -1 - 1 = -2$

$u(x) = \int \frac{-e^{-x} \times 2}{-2\,(1+e^x)} = \int \frac{e^{-x}}{1+e^x} = \log(e^{-x}+1) - e^{-x} + c_1$

$v(x) = \int \frac{e^x \times 2}{-2(1+e^x)} = -\int \frac{e^x}{1+e^x} = -\log|1+e^x| + c_2$

$y = (\log(e^{-x}+1) - e^{-x} + c_1)\, e^x + (-\log(1+e^x) + c_2)\, e^{-x}$

$\cancel{= [c_1 e^x + c_2 e^{-x}] + [e^x(\log|e^{-x}+1|}$

$y = (c_1 e^x + c_2 e^{-x}) + (e^x \log|e^{-x}+1) - 1 - e^{-x}\log|1+e^x|)$

5) $y'' + 4y = 4\sec^2 2x$

$\lambda^2 + 4 = 0$

$\lambda = \pm 2i$

$y = A\cos 2x + B \sin 2x$

$y_1 = \cos 2x, \quad y_1' = -2\sin 2x$

$y_2 = \sin 2x, \quad y_2' = 2\cos 2x$

$W = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x = 2$

$u(x) = \int \frac{-\sin 2x \; 4\sec^2 2x}{2} = -2\int \frac{\sin 2x}{\cos^2 2x} = \frac{-1}{\cos 2x} = -\sec 2x + c_1$

$v(x) = \int \frac{\cos 2x \; 4\sec^2 2x}{2} = 2\int \sec 2x = \log|\sec 2x + \tan 2x| + c_2$

$$y = (-\sec 2x + c_1)\cos 2x + (\log|\sec 2x + \tan 2x| + c_2)\sin 2x$$

$$= (\sin 2x \cdot \log|\sec 2x + \tan 2x| - 1) + (c_1 \cos 2x + c_2 \sin 2x)$$

J) 1) $x^2y'' - 4xy' + 6y = 21x^{-4}$ —— ①

$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$

① $\Rightarrow$ $x^m[m^2 - 5m + 6] = 6$

$m = 2, 3$

$y = c_1x^2 + c_2x^3$

$y_p = kx^{-4}$

$y_p' = -4kx^{-5}$

$y_p'' = 20kx^{-6}$

sub in ∴ ① $\Rightarrow$ $x^2(20kx^{-6}) - 4x(-4kx^{-5}) + 6(kx^{-4}) = 21x^{-4}$

$20kx^{-4} + 16kx^{-4} + 6kx^{-4} = 21x^{-4}$

$42kx^{-4} = 21x^{-4}$

$k = 1/2$

$\therefore y = y_n + y_p$

$= c_1x^2 + c_2x^3 + x^{-4}/2$

2) $4x^2y'' + 8xy' - 3y = 7x^2 - 15x^3$ —— ①

$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$

① $\Rightarrow 4x^2m(m-1)x^{m-2} + 8xmx^{m-1} - 3x^m = 0$

$x^m(4m^2 - 4m + 8m - 3) = 0$

$x^m(4m^2 + 4m - 3) = 0$

$m = \frac{-4 \pm \sqrt{16 + 48}}{8} = \frac{-4 \pm 8}{8} = -3/2,\ 1/2$

$y_n = c_1x^{-3/2} + c_2x^{1/2}$

$y_p = k_3 x^3 + k_2 x^2 + k_1 x + k_0$

$y_p' = 3k_3 x^2 + 2k_2 x + k_1$

$y_p'' = 6k_3 x + 2k_2$

sub in ① $\Rightarrow$

$$4x^2(6k_3x + 2k_2) + 8x(3k_3x^2 + 2k_2x + k_1) - 3(k_3x^3 + k_2x^2 + k_1x + k_0)$$
$$= 7x^2 - 15x^3$$

$$x^3[24k_3 + 24k_3 - 3k_3] + x^2[8k_2 + 16k_2x^2 - 3k_2]$$
$$+ x[8k_1 - 3k_1] + [-3k_0] = 7x^2 - 15x^3$$

$-15 = 45k_3$ $21k_2 = 7$ $5k_1 = 0$ $-3k_0 = 0$

$k_3 = -1/3$ $k_2 = 1/3$ $k_1 = 0$ $k_0 = 0$

$y = y_h + y_p$

$$= c_1 x^{-3/2} + c_2 x^{1/2} - \frac{x^3}{2} + \frac{x^2}{3}$$

3) $x^2y'' - 2xy' + 2y = x^3\cos x$

$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$

$x^2 m(m-1)x^{m-2} - 2xmx^{m-1} + 2x^m = 0$

$x^m[m^2 - m - 2m + 2] = 0$

$x^m(m^2 - 3m + 2) = 0$

$m = 2, 1$

$y_h = c_1x^2 + c_2x$

$y_1 = x^2,\ y_1' = 2x$

$y_2 = x,\ y_2' = 1$

$$W = \begin{vmatrix} x^2 & x \\ 2x & 1 \end{vmatrix} = x^2 - 2x^2 = -x^2$$

Now $r(x) = \dfrac{x^3\cos x}{x^2} = x\cos x$

$$u(x) = \int \frac{-x\, x\cos x}{-x^2} = \int \cos x\, dx = \sin x + c_1$$

$$v(x) = \int \frac{x^2\, x\cos x}{-x^2} = -\int x\cos x = -x\sin x - \cos x + c_2$$

$$y = u(x)y_1 + v(x)y_2$$

$$= (\sin x + c_1)x^2 + (-x\sin x - \cos x + c_2)x$$

$$= c_1x^2 + c_2x - x\cos x$$

4) $x^2y'' - 2xy' - 4y = x^2 + 2\log x$ —①

$y = x^m,\ y' = mx^{m-1},\ y'' = m(m-1)x^{m-2}$

① → $x^m[m^2 - m - 2m - 4] = 0$

$x^m(m^2 - 3m - 4) = 0$

$m = 4, -1$

$y = c_1x^4 + c_2x^{-1}$

$y_1 = x^4,\ y_1' = 4x^3$

$y_2 = x^{-1},\ y_2' = -x^{-2}$

$$W = \begin{vmatrix} x^4 & x^{-1} \\ 4x^3 & -x^{-2} \end{vmatrix} = -x^2 - 4x^2 = -5x^2$$

$$r(x) = \cancel{\tan x}\ \frac{x^2 + 2\log x}{x^2}$$

$$u(x) = \int \frac{-x^{-1}}{-5x^2}\left(\frac{x^2 + 2\log x}{x^2}\right)$$

$$= \frac{1}{5}\int \frac{x^2 + 2\log x}{x^5}$$

$$= \frac{1}{5}\left[\int x^{-3} + 2x^{-5}\log x\right]$$

$$= \frac{1}{5}\left[-\frac{x^{-2}}{2} + 2\left(\frac{\log x \cdot x^{-4}}{-4} + \int \frac{1}{x}\frac{x^{-4}}{4}\right)\right.$$

$$= \cancel{\tfrac{1}{5}}\ \frac{-x^{-2}}{10} - \frac{\log x \cdot x^{-4}}{10} + \int \frac{x^{-5}}{10}$$

$$= \frac{-x^{-2}}{10} - \frac{\log x \cdot x^{-4}}{10} - \frac{x^{-4}}{40}$$

$$v(x) = \int \frac{x^4}{-5x^2}\left[\frac{x^2+2\log x}{x^2}\right]$$

$$= -\frac{1}{5}\int x^2 + 2\log x$$

$$= -\frac{1}{5}\left(\frac{x^3}{3} + 2x\log x - 2x\right)$$

$$= -\frac{x^3}{15} - \frac{2x\log x}{5} + \frac{2x}{5}$$

$$y = \left[-\frac{x^{-2}}{10} - \log x \cdot x^{-4} - \frac{x^{-4}}{40} + c_1\right]x^4$$

$$+ \left[-\frac{x^3}{15} - \frac{2x\log x}{5} + \frac{2x}{5} + c_2\right]x^{-1}$$

$$y = c_1x^4 + c_2x^{-1} - \frac{x^2}{10} - \frac{\log x}{10} - \frac{1}{40} - \frac{x^2}{15} - \frac{2\log x}{5} + \frac{2}{5}$$

$$y = c_1x^4 + c_2x^{-1} - \frac{x^2}{6} - \frac{\log x}{2} + \frac{3}{8}$$

---

k) $y(x) = e^x(A\cos 2x + B\sin 2x) + e^x + x$

$y_h = e^x(A\cos 2x + B\sin 2x)$

roots $= 1 \pm 2i$

$\therefore \quad x^2 - 2x + 5 = 0$

Homogenous part of ODE is

$$y'' - 2y' + 5y = 0$$

$r(x) = y_p'' - 2y_p' + 5y_p$

$(e^x+x)'' - 2(e^x+x)' + 5(e^x+x) = r(x)$

$r(x) = 5(e^x+x) - 2(e^x+1) + e^x$

$= 4e^x + 5x - 2$

$\therefore y'' - 2y' + 5y = 4e^{x} + 5x - 2$

M] $R = 16\,\Omega$, $C = 0.02$ fd, $L = 2H$, $E = 100 \sin(33t)$

$$L \frac{dI}{dt} + RI + \frac{1}{c}\int I\,dt = E_0 \sin\omega t$$

$$L \frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{I}{c} = E\omega \cos\omega t$$

$$2\frac{d^2I}{dt^2} + 16\frac{dI}{dt} + \frac{I}{0.02} = 3300 \cos(33t)$$

$$\frac{d^2I}{dt^2} + 8\frac{dI}{dt} + 25I = 1650 \cos(33t)$$

$$y'' + 8y' + 25y = 1650 \cos(33t \quad \text{—}(1)$$

$$\lambda^2 + 8\lambda + 25 = 0$$

$$(\lambda+4)^2 = -9$$

$$\lambda = -4 \pm 3i$$

$$y = e^{-4t}(A\cos 3t + B\sin 3t)$$

$$y_p = c_1\cos(33t) + c_2\sin(33t)$$

$$y_p' = -33c_1\sin(33t) + 33c_2\cos(33t)$$

$$y_p'' = -33^2c_1\cos(33t) - 33^2c_2\sin(33t)$$

Sub in (1),

$$-33^2c_1\cos(33t) - 33^2c_2\sin(33t) - (33\times8)c_1\sin(33t)$$
$$+ (33\times8)c_2\cos(33t) + 25c_1\cos(33t) + 25c_2\sin(33t)$$
$$= 1650\cos 33t$$

$$\cos(33t)[-1089c_1 + 264c_2 + 25c_1] + \sin(33t)[-1089c_2 - 264c_1 + 25c_2]$$
$$= 1650\cos(33t)$$

$$-1064c_1 + 264c_2 = 1650$$
$$-1064c_2 - 264c_1 = 0$$

$$C_1 = -\frac{1064}{264} C_2 = -4.03C_2 \approx -4C_2$$

$$-1064(-4C_2) + 264C_2 = 1650$$

$$4256C_2 + 264C_2 = 1650$$

$$4526C_2 = 1650$$

$$C_2 = 0.36$$

$$C_1 = -1.45$$

$$\therefore \; y_p = -1.45\cos 33t + 0.36\sin 33t$$

$$I = e^{-4t}[A\cos 33t + B\sin 33t) - 1.45\cos 33t + 0.36\sin 33t$$

@ $t = 0,\ I = 0 \Rightarrow 0 = A - 1.45 \Rightarrow A = \underline{\underline{1.45}}$

$$q = I' = e^{-4t}[-33A\sin 33t + 33B\cos 33t)$$

$$-4e^{-4t}[A\cos 33t + B\sin 33t) + 1.45\sin 33t \times 33$$

$$+ 0.36\cos 33t \times 33$$

@ $t = 0,\ q = 0$

$$0 = 33B - 4A + (0.36 \times 33)$$

$$B = \frac{4A - (0.36 \times 33)}{33}$$

$$= \frac{5.8 - 11.88}{33} = -0.186$$

$$I = e^{-4t}[1.45\cos 33t - 0.186\sin 33t)$$

$$- 1.45\cos 33t + 0.36\sin 33t$$

N) $m = 2\,kg$.

$l = m$ $\quad\quad$ $k = 128$

$F = 25.6N$ $\quad\quad$ $u = 0\,m/s$

$l_t = 0.7m$

$$my'' + ky = 0$$

$$y'' + k/m = 0$$

$$25.6 = 128(0.7 - \Delta x)$$

$0.2 = 0.7 - \Delta x$

$\Delta x = 0.5m$

$x(t=0) = 0.2m, \; v(t=0) = x'(t=0) = 0 m/s$

$-kx = mx''$

$128x + 2x'' = 0$

$x'' + 64x = 0$

$m^2 + 64 = 0$

$m = \pm 8i$

$\Rightarrow x = c_1 \cos(8t) + c_2 \sin(8t)$

$v = x' = c_1(-8)\sin 8t + c_2(8)\cos 8t$

$x(t=0) = 0.2 \Rightarrow c_1 = 0.2$

$v(t=0) = 0 \Rightarrow c_2 = 0$

$\Rightarrow x = 1/5 \cos(8t)$

$$\underline{\underline{x = 0.2 \cos(8t) m}}$$

o) $m = 16lb$ $\quad k = 5 \, lb/ft$

$b = 4$ $\quad F = 24\sin(10t)$

$t = 0, y = 0$

$F_{ext} + mg - 4v - k(x + x_0) = mx''$

$24 \sin 10t = 5x + 4x' + 1/2 x''$

$x'' + 8x' + 10x = 48\sin(10t)$

$m^2 + 8m + 10 = 0$

$$m = \frac{-8 \pm \sqrt{64 - 40}}{2} = \frac{-8 \pm 2\sqrt{6}}{2} = -4 \pm \sqrt{6}$$

$x_h = c_1 e^{(-4+\sqrt{6})t} + c_2 e^{(-4-\sqrt{6})t}$

$x_p = k_1(\sin 10t) + k_2 \cos(10t)$

$x_p' = 10k_1 \cos 10t - 10k_2 \sin 10t$

$x_p'' = -100k\sin 10t - 100k_2 \cos 10t$

$-k_1 100 \sin 10t - k_2 100 \cos 10t + 80k_1 \cos 10t - 80k_2 \sin 10t$

$+ 10k_1 \sin 10t + 10k_2 \cos 10t = 48 \sin 10t$

$-90k_1 - 80k_2 = 48$ —— ①

$80k_1 - 90k_2 = 0$ —— ②

$\Rightarrow k_1 = -\frac{216}{725}, \quad k_2 = \frac{-192}{725}$

$x = c_1 e^{(-4+5i)t} + c_2 e^{(-4-5i)t} - \frac{216}{725}\sin 10t - \frac{192}{725}\cos 10t$

$\Rightarrow v = x' = c_1(-4+5i)e^{(-4+5i)t} + c_2(-4-5i)e^{(-4-5i)t}$

$- \frac{216}{725}(10)\cos 10t + \frac{192}{725}(10)\sin(10t)$

$x(0) = 0 \Rightarrow c_1 + c_2 = 192/725$

$v(0) = 0 \Rightarrow c_1(-4+5i) + c_2(-4-5i) = \frac{216}{725}(10)$

$\therefore c_1 = 0.956, \quad c_2 = -0.692$

$\therefore x = 0.956e^{-1.55t} - 0.692e^{-6.45t} - 0.298\sin 10t$

$- 0.26\cos 10t$

@ steady state, $(t \to \infty)$

$x = -0.298\sin 10t - 0.265\cos 10t$

$x = \sqrt{0.298^2 + 0.265^2}\left[\frac{\sin(10t)(-0.298)}{\sqrt{0.298^2 + 0.265^2}} + \frac{\cos(10t)(-0.265)}{\sqrt{0.298^2 + 0.268^2}}\right]$

$= 0.3988(\sin 10t \cos(3.868) + \cos 10t \sin(3.868))$

$= 0.3988(\sin 10t + 3.868) - 0.265$

amplitude $= 0.3988$

$\text{Period} = \frac{2\pi}{10} = \pi/5 = 0.6283\ s$

$fr = 1/T = 1.591\ \text{cycles/s}$

Substituting and solving Fo x',

$V = x' = -1.483e^{-1.55t} + 4.462e^{-6.45t}$

$- 2.93\cos(10t) + 2.65\sin 10t)$ ft/s

L) 1) $Dx = y+1, \quad Dy = x+1$

$x' = y+1$

$y' = x+1$

$x'' = y'' = x+1$

$x'' - x = 1$

$m^2 - 1 = 0$

$m = \pm 1$

$x_n = c_1 e^{t} + c_2 e^{-t}$

$x_p = K_0$

$x_p' = x_p'' = 0$

$-K_0 = 1, \quad K_0 = -1$

$x_p = -1$

$\therefore x = c_1 e^{t} + c_2 e^{-t} - 1$

$x' - y = y$

$c_1 e^{t} - c_2 e^{-t} - 1 = y$

$x = c_1 e^{t} + c_2 e^{-t} - 1$

$y = c_1 e^{t} - c_2 e^{-t} - 1$

2) $x' + 5x + y' + 3y = e^{-x}$ —— ①

$2x' + x + y' + y = 3$ —— ②

① - ② $\quad -x' + 4x + 2y = e^{-x} - 3$

$y = \frac{x' - 4x + e^{-x} - 3}{2}$ —— ③

①' => $x'' + 5x' + y'' + 3y' = -e^{-x}$ —— ④

$2x'' + x' + y'' + y' = 0$ —— ⑤

$① ' - ② ' \Rightarrow -x' + 4x' + 2y' = -e^{-x}$ —— ⑥

$① - 3(2) \Rightarrow x' + 5x + y' + 3y - 6x' - 3x - 3y' - 3y = e^{-x} - 9$

$-5x' + 2x - e^{-x} + 9 = 2y'$ —— ⑦

$⑥ \text{ in } ⑦ \Rightarrow -x'' + 4x' - 5x' + 2x - e^{-x} + 9 = -e^{-x}$

$9 = x'' + x' - 2x$

$m^2 + m - 2 = 0$

$m = -2, 1$

$x_h = c_1 e^{-2t} + c_2 e^{t}$

$x_p = k_0$

$x_p' = x_p'' = 0$

$-2k_0 = 9$

$k_0 = -9/2$

$x = c_1 e^{-2t} + c_2 e^{t} - 9/2$

$x' = -2c_1 e^{-2t} + c_2 e^{t}$

$x = c_1 e^{-2t} + c_2 e^{t} - 9/2$

$y = -3c_1 e^{-2t} - 1.5 c_2 e^{t} + 0.5 e^{-t} + 7.5$

3) $3x + 8y = x''$ —— ①

$y' = -x - 3y$ —— ②

$①' \Rightarrow x'' = 3x + 8y'$ —— ③

Put ② in ③, $x'' 3x' + 8(-x - 3y)$

$\Rightarrow x'' = 3x' - 8x - 24y$ —— ④

$\Rightarrow x'' = 3x' - 8x - 24\left(\frac{x^{a} - 3x}{8}\right)$

$x'' = 3x' - 8x - 3x' + 9x$

$x'' - x = 0$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$x = c_1 e^{t} + c_2 e^{-t}$

$x' = c_1 e^{t} - c_2 e^{-t}$

$8y = c_1 e^{t} - c_2 e^{-t} - 3c_1 e^{t} - 3c_2 e^{-t}$

$= -2c_1 e^{t} - 4c_2 e^{-t}$

$y = -1/4\, c_1 e^{t} - 1/2\, c_2 e^{-t}$

$x = c_1e^{t} + c_2e^{t}$

$y = -\frac{1}{2}c_1e^{t} - \frac{1}{2}c_2e^{-t}$

$x(0) = 6 \Rightarrow \quad c_1 + c_2 = 6$ —— (1)

$y(0) = -2 \Rightarrow \quad c_1 + 2c_2 = 8$ —— (2)

(2) − (1) $\Rightarrow \quad c_2 = 2, \; c_1 = 4$

$x = 4e^{t} + 2e^{-t}$

$y = -e^{t} - e^{-t}$