Department of Mechanical Engineering (NITC)

S1ME **ZZ1001D ENGINEERING MECHANICS** **Tutorial Test 3-Set2**

Time: One Hour **Answer Key** Maximum Marks: 20

1. The pressure p_o at the corner O of the rectangular plate shown in Fig. 1 is 75 Pa and increases linearly in the y-direction by 10 Pa/m. In the x-direction, it increases parabolically starting with zero slope so that in 30 m the pressure has changed from 75 Pa to 750 Pa. Determine the simplest resultant pressure force for this distribution on the plate. Give the coordinates of the centre of pressure, relative to O.

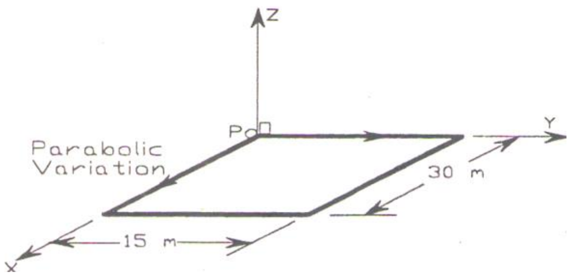

Figure 1

Solution:

25) $p(x,y) = p_0 + 10y + ax^2$

$p(0,0) = p_0 = 75 \text{ Pa}$

$p(30,0) = 75 + a(30)^2 = 750 \Rightarrow a = 0.75$

$\therefore p(x,y) = p_0 + 10y + ax^2 = 75 + 10y + 0.75x^2 \text{ Pa}$

$$F_R = \int_0^{15}\left(\int_0^{30} (75 + 10y + 0.75x^2)\,dx\right)dy$$

$$= \int_0^{15} \left[75(30) + 10y(30) + \frac{0.75}{3}(30)^3\right] dy$$

$$= 75(30)(15) + \frac{10(30)}{2}(15)^2 + 0.25(30)^3(15)$$

$$= 33,750 + 33,750 + 101,250$$

$$= 168,750 \text{ N}$$

$$= \underline{168.75 \text{ kN}}$$

$$x_p F_R = \int_0^{15}\left(\int_0^{30} (75x + 10xy + 0.75x^3)\,dx\right)dy$$

$$= \int_0^{15} \left[(75)\frac{30^2}{2} + \frac{10}{2}(30)^2 y + \frac{0.75}{4}30^4\right] dy$$

$$= \frac{1}{2}(75)(30^2) + \frac{1}{2}10(30)^2\frac{15^2}{2} + \frac{1}{4}(0.75)(30^4)(15)$$

$$\Rightarrow x_p(168750) = 33750 + 506,250 + 2278125$$

$$\Rightarrow \underline{x_p = 16.7 \text{ m}}$$

$$y_p F_R = \int_0^{15}\left(\int_0^{30} (75y + 10y^2 + 0.75x^2y)\,dx\right)dy$$

$$= \int_0^{15} \left[75(30)y + 10(30)y^2 + \frac{0.75}{3}(30)^3 y\right) dy$$

$$= 75(30)\frac{15^2}{2} + 10(30)\frac{15^3}{3} + 0.25(30)^3\frac{15^2}{2}$$

$$\Rightarrow y_p(168750) = 253125 + 337500 + 759375$$

2. Calculate the distance h_c measured from the base to the centroid of the volume of the frustum of the right-circular cone shown in Fig. 2.

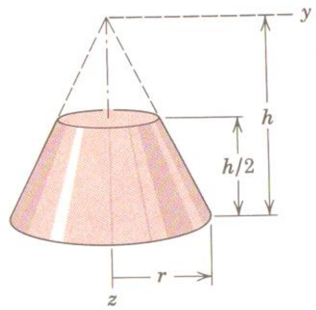

Figure 2

Solution:

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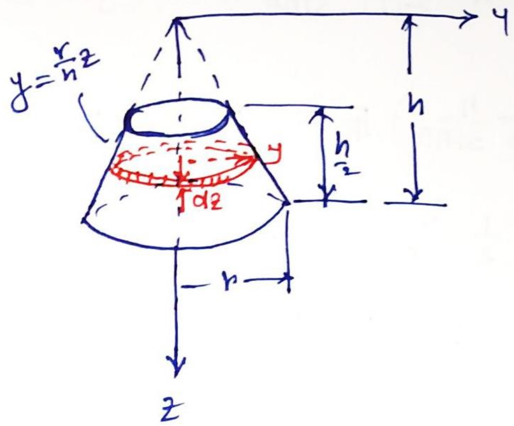
$$dV = \pi y^2 dz$$
$$= \frac{\pi r^2}{h^2} z^2 dz$$



$$z_c = \frac{\int \tilde{z}\, dV}{\int dV}$$

$$= \frac{\int_{h/2}^{h} z \frac{\pi r^2}{h^2} z^2 dz}{\int_{h/2}^{h} \frac{\pi r^2}{h^2} z^2 dz}$$

$$= \frac{\int_{h/2}^{h} z^3 dz}{\int_{h/2}^{h} z^2 dz} = \frac{\frac{1}{4} z^4 \Big|_{h/2}^{h}}{\frac{1}{3} z^3 \Big|_{h/2}^{h}}$$

$$= \frac{3}{4} \frac{\left(h^4 - \frac{h^4}{16}\right)}{\left(h^3 - \frac{h^3}{8}\right)} = \frac{3}{4} \cdot \frac{15}{\cancel{16}_2} \cdot \frac{\cancel{8}}{7} h$$

$$\Rightarrow z_c = \frac{45}{56} h$$

$$\therefore h_c = h - z_c = \frac{11}{56} h$$

3. The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determines the magnitude and coordinate direction angles of the resultant force Fig 3.

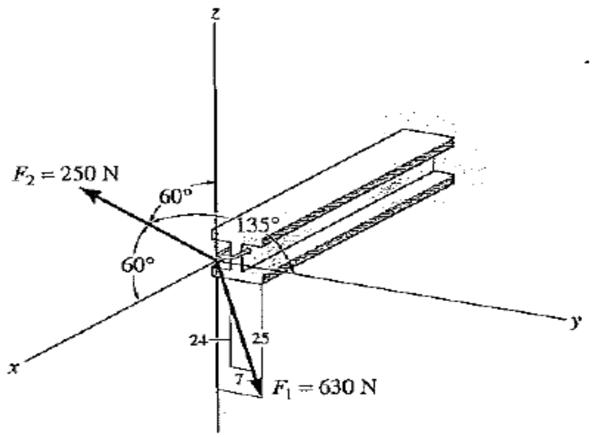


Figure 3

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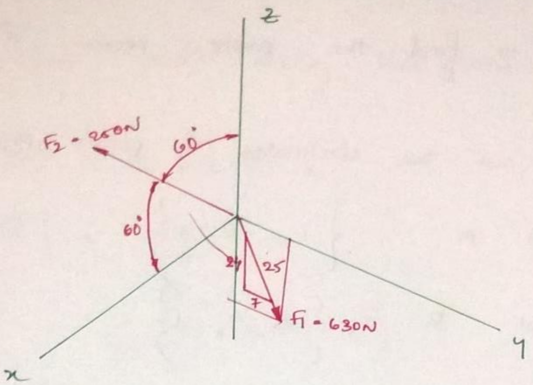


$$\vec{F_1} = 630\left[\frac{7}{25}j - \frac{24}{25}k\right]$$

$$= \left[176j - 605k\right] N$$

$$F_2 = 250\left[\cos 60\, i + \cos 135\, j + \cos 60\, k\right]$$

$$= \left[125i - 177j + 125k\right] N$$

$$F_R = F_1 + F_2$$

$$= 125i - 0.377j - 480k$$

$$\left|F_R\right| = \sqrt{125^2 + 0.377^2 + 480^2} = \underline{496}\ N$$

$$\cos\alpha = \frac{125}{496} \Rightarrow \alpha = 75.4^\circ$$

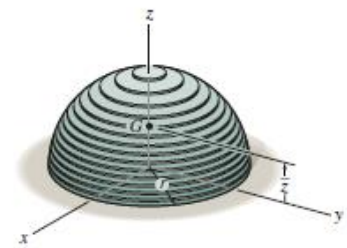
$$\cos\beta = \frac{-0.377}{496} \Rightarrow \beta = 90^\circ$$

$$\cos\gamma = \frac{-480}{496} \Rightarrow \gamma = \underline{165^\circ}$$

4. The hemisphere of radius *r* is made from a stack of very thin plates such that the density varies with height, density = kz , where *k* is a constant. Determine its mass and the distance $\bar{z}$ to the centre of mass *G*..Fig. 4?

Solution:

9–43. The hemisphere of radius r is made from a stack of very thin plates such that the density varies with height, $\rho = kz$, where k is a constant. Determine its mass and the distance $\bar{z}$ to the center of mass G.



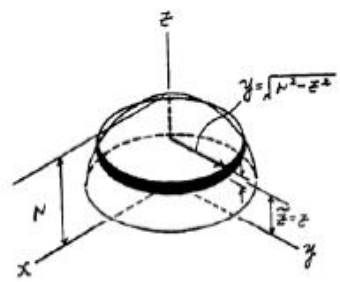
Mass and Moment Arm : The density of the material is $\rho = kz$. The mass of the thin disk differential element is $dm = \rho dV = \rho \pi y^2 dz = kz\left[\pi\left(r^2 - z^2\right) dz\right]$ and its centroid $\tilde{z} = z$. Evaluating the integrals, we have

$$m = \int_m dm = \int_0^r kz\left[\pi\left(r^2 - z^2\right) dz\right]$$

$$= \pi k\left(\frac{r^2 z^2}{2} - \frac{z^4}{4}\right)\Bigg|_0^r = \frac{\pi k r^4}{4} \qquad \textbf{Ans}$$

$$\int_m \tilde{z}\, dm = \int_0^r z\left\{kz\left[\pi\left(r^2 - z^2\right) dz\right]\right\}$$

$$= \pi k\left(\frac{r^2 z^3}{3} - \frac{z^5}{5}\right)\Bigg|_0^r = \frac{2\pi k r^5}{15}$$



Centroid : Applying Eq. 9-3, we have

$$\bar{z} = \frac{\int_m \tilde{z}\, dm}{\int_m dm} = \frac{2\pi k r^5/15}{\pi k r^4/4} = \frac{8}{15} r \qquad \textbf{Ans}$$