Department of Mechanical Engineering (NITC)

S1ME **ZZ1001D ENGINEERING MECHANICS** **Tutorial Test 2-Set 3**

Time: One Hour **Answer Key** Maximum Marks: 20

1. At what height *h* (Fig. 1) will the water cause the door to rotate clockwise? The door is 3 m wide. Neglect friction and the weight of the door.

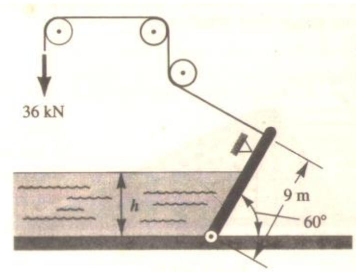


Figure 1

Solution:

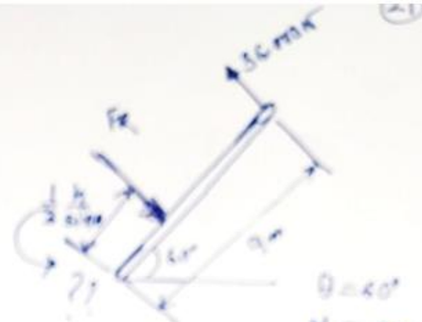
$$F_R = \frac{1}{2}\rho g h \frac{h}{\sin\theta} w \quad \text{(derive this relation)}$$

$$\left(\frac{1}{3}\frac{h}{\sin\theta}\right) F_R = 9\,(36\,000)$$

$$\Rightarrow \frac{1}{3}\frac{h}{\sin\theta}\,\frac{1}{2}\rho g h \frac{h}{\sin\theta} w = 9\,(36\,000)$$

$$\Rightarrow \frac{1}{6}\,\frac{1000\,(9.81)\,(3)}{\sin^2(60^\circ)}\,h^3 = 9\,(36\,000)$$

$$\Rightarrow h = 3.673 \text{ m}$$



2. The beam is subjected to the two forces shown. Express each force in Cartesian vector form and determines the magnitude and coordinate direction angles of the resultant force.

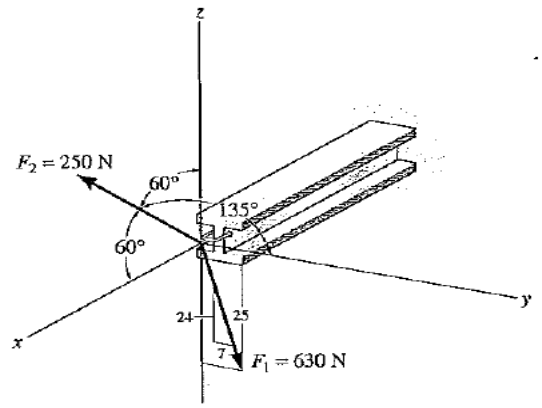
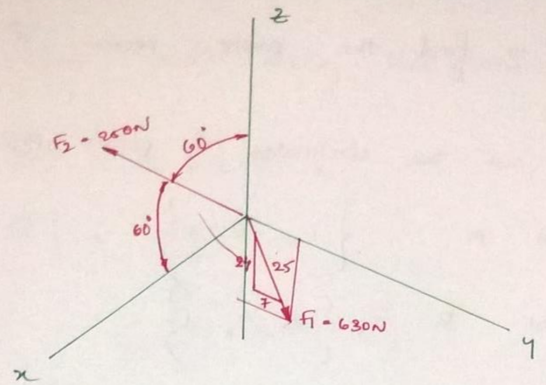


Figure 2

Solution:

(8)



$$\vec{F_1} = 630\left[\frac{7}{25}j - \frac{24}{25}k\right]$$

$$= \left[176j - 605k\right]N$$

$$F_2 = 250\left[\cos 60\, i + \cos 135\, j + \cos 60\, k\right]$$

$$= \left[125i - 177j + 125k\right]N$$

$$F_R = F_1 + F_2$$

$$= 125i - 0.877j - 480k$$

$$\left|F_R\right| = \sqrt{125^2 + 0.377^2 + 480^2} = \underline{\underline{496}}\ N$$

$$\cos\alpha = \frac{125}{496} \Rightarrow \alpha = 75.4^0$$

$$\cos\beta = \frac{-0.377}{496} \Rightarrow \beta = 90^0$$

$$\cos\gamma = \frac{-480}{496} \Rightarrow \gamma = \underline{\underline{165^0}}$$

3. Locate the centroid for the paraboloid of revolution, shown in Fig. 3.

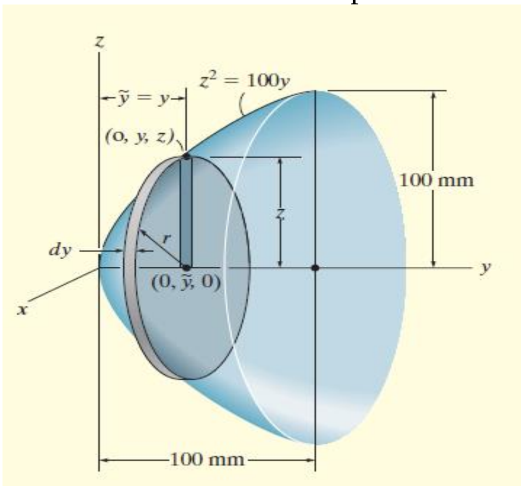


Figure 3

Solution:

EXAMPLE 9.7

Locate the $\overline{y}$ centroid for the paraboloid of revolution, shown in Fig. 9–14.

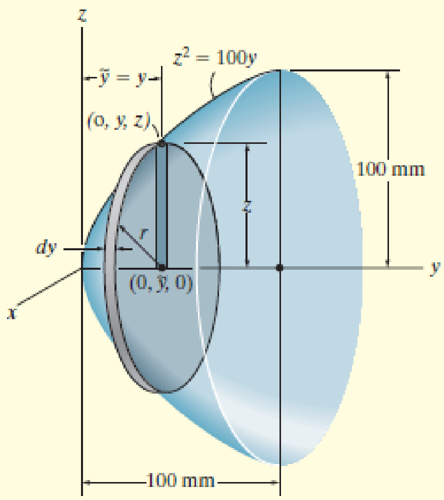


Fig. 9–14

SOLUTION

Differential Element. An element having the shape of a *thin disk* is chosen. This element has a thickness dy, it intersects the generating curve at the *arbitrary point* $(0, y, z)$, and so its radius is $r = z$.

Volume and Moment Arm. The volume of the element is $dV = (\pi z^2)\, dy$, and its centroid is located at $\tilde{y} = y$.

Integration. Applying the second of Eqs. 9–3 and integrating with respect to y yields

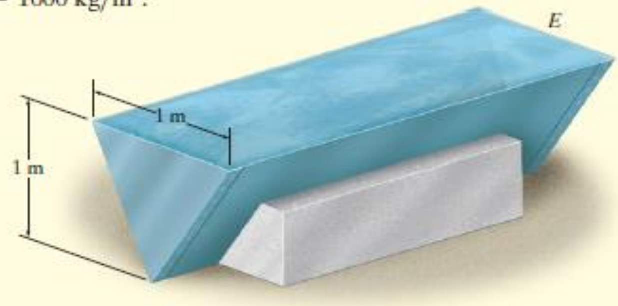
$$\overline{y} = \frac{\int_V \tilde{y}\, dV}{\int_V dV} = \frac{\int_0^{100\text{ mm}} y(\pi z^2)\, dy}{\int_0^{100\text{ mm}} (\pi z^2)\, dy} = \frac{100\pi \int_0^{100\text{ mm}} y^2\, dy}{100\pi \int_0^{100\text{ mm}} y\, dy} = 66.7\text{ mm} \quad \textit{Ans.}$$

4. Determine the magnitude and location of the resultant force acting on the triangular end plates of the water trough shown in Fig. 4; density = 1000 kg/m^3 .

Solution:

EXAMPLE 9.16

Determine the magnitude and location of the resultant force acting on the triangular end plates of the water trough shown in Fig. 9–30*a*; $\rho_w = 1000\ \text{kg/m}^3$.



(a)

SOLUTION

The pressure distribution acting on the end plate *E* is shown in Fig. 9–30*b*. The magnitude of the resultant force is equal to the volume of this loading distribution. We will solve the problem by integration. Choosing the differential volume element shown in the figure, we have

$$dF = dV = p\,dA = \rho_w g z(2x\,dz) = 19\,620zx\,dz$$

The equation of line *AB* is

$$x = 0.5(1 - z)$$

Hence, substituting and integrating with respect to z from $z = 0$ to $z = 1$ m yields

$$F = V = \int_V dV = \int_0^{1\text{ m}} (19\,620)z[0.5(1 - z)]\,dz$$

$$= 9810 \int_0^{1\text{ m}} (z - z^2)\,dz = 1635\text{ N} = 1.64\text{ kN} \qquad \textit{Ans.}$$

This resultant passes through the *centroid of the volume*. Because of symmetry,

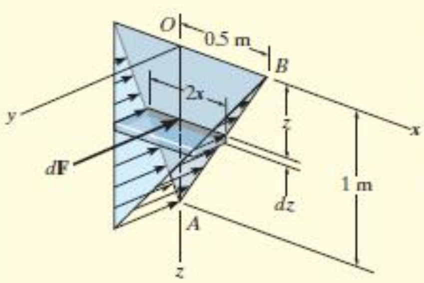
$$\bar{x} = 0 \qquad \textit{Ans.}$$

Since $\tilde{z} = z$ for the volume element, then

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{1\text{ m}} z(19\,620)z[0.5(1 - z)]\,dz}{1635} = \frac{9810\int_0^{1\text{ m}} (z^2 - z^3)\,dz}{1635}$$

$$= 0.5\text{ m} \qquad \textit{Ans.}$$

NOTE: We can also determine the resultant force by applying Eq. 9–14, $F_R = \gamma\bar{z}A = (9810\text{ N/m}^3)(\tfrac{1}{3})(1\text{ m})[\tfrac{1}{2}(1\text{ m})(1\text{ m})] = 1.64\text{ kN}$.



(b)

Fig. 9–30