Department of Mechanical Engineering (NITC)

S1ME **ZZ1001D ENGINEERING MECHANICS** **Tutorial Test 3-Set 4**

Time: One Hour **Answer Key** Maximum Marks: 20

1. The plate has a thickness of 0.5 in. and is made of steel having a specific weight of 490 lb/ft^3. Determine the horizontal and vertical components of reaction at the pin *A* and the force in the cord at *B*.

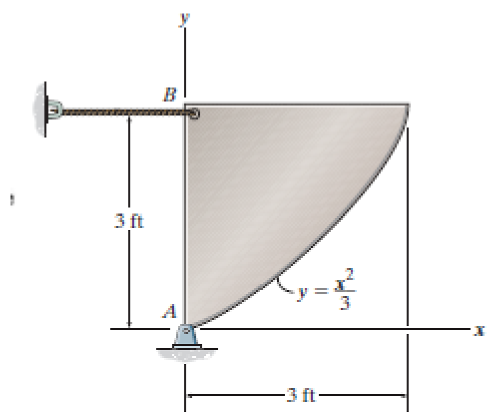


Figure 1

Solution:

Differential Element: The element parallel to the x axis shown shaded in Fig. *a* will be considered. The area of this differential element is given by

$$dA = x\,dy = \sqrt{3}y^{1/2}\,dy$$

Centroid: The centroid of the element is located at $\tilde{x} = x/2 = \dfrac{\sqrt{3}}{2}y^{1/2}$ and $y_c = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^{3\text{ ft}} \sqrt{3}y^{1/2}\,dy = \frac{2\sqrt{3}}{3}y^{3/2}\Big|_0^{3\text{ ft}} = 6\text{ ft}^2$$

Thus, the weight of the plate can be obtained from

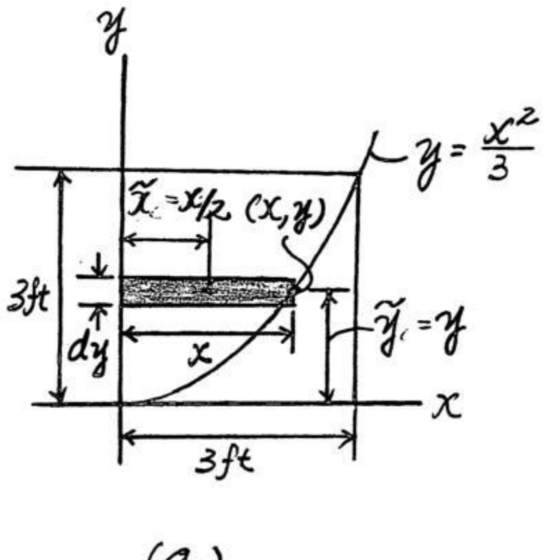
$$W = \gamma At = 490(6)\left(\frac{0.5}{12}\right) = 122.5\text{ lb}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{3\text{ ft}}\left(\frac{\sqrt{3}}{2}y^{1/2}\right)\left(\sqrt{3}y^{1/2}\,dy\right)}{6} = \frac{\int_0^{3\text{ ft}}\frac{3}{2}y\,dy}{6} = \frac{\frac{3}{4}y^2\Big|_0^{3\text{ ft}}}{6} = 1.125\text{ ft}$$

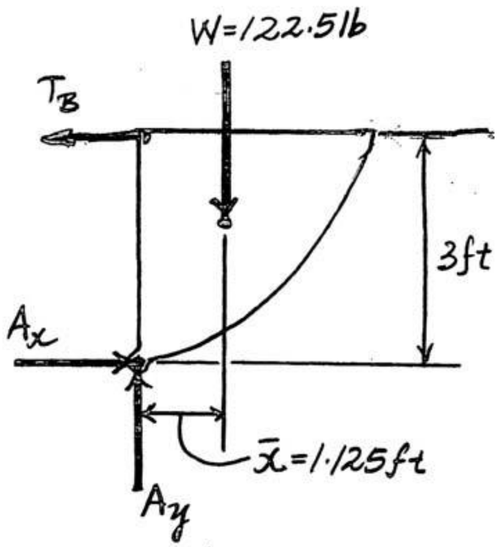
Since the plate has a uniform thickness, its center of gravity coincides with its centroid.

Equations of Equilibrium: By referring to the free body diagram shown in Fig. *b*,

$+\Sigma M_A = 0;$	$T_B(3) - 122.5(1.125) = 0$	$T_B = 45.94\text{ lb} = 45.9\text{ lb}$	**Ans.**
$\xrightarrow{+}\Sigma F_x = 0;$	$A_x - 45.94\text{ lb} = 0$	$A_x = 45.94\text{ lb} = 45.9\text{ lb}$	**Ans.**
$+\uparrow\Sigma F_y = 0;$	$A_y - 122.5 = 0$	$A_y = 122.5\text{ lb}$	**Ans.**



(a)



(b)

Solution:

2. Determine the location of the centre of mass of the cylinder shown in Fig. 2 if its density varies directly with the distance from its base, i.e., density = 200z kg/m^3.

Solution:

EXAMPLE 9.8

Determine the location of the center of mass of the cylinder shown in Fig. 9–15 if its density varies directly with the distance from its base, i.e., $\rho = 200z \text{ kg/m}^3$.

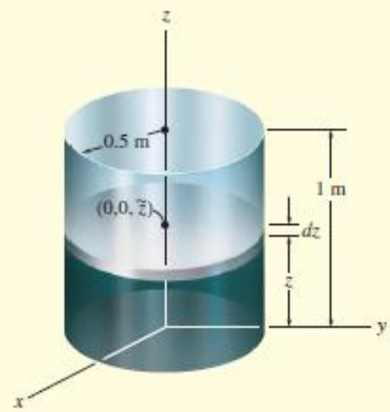


Fig. 9–15

SOLUTION

For reasons of material symmetry,

$$\bar{x} = \bar{y} = 0 \qquad \textit{Ans.}$$

Differential Element. A disk element of radius 0.5 m and thickness dz is chosen for integration, Fig. 9–15, since the *density of the entire element is constant* for a given value of z. The element is located along the z axis at the *arbitrary point* $(0, 0, z)$.

Volume and Moment Arm. The volume of the element is $dV = \pi(0.5)^2\,dz$, and its centroid is located at $\tilde{z} = z$.

Integrations. Using an equation similar to the third of Eqs. 9–2 and integrating with respect to z, noting that $\rho = 200z$, we have

$$\bar{z} = \frac{\int_V \tilde{z}\rho\,dV}{\int_V \rho\,dV} = \frac{\int_0^{1\text{ m}} z(200z)\left[\pi(0.5)^2\,dz\right]}{\int_0^{1\text{ m}} (200z)\pi(0.5)^2\,dz}$$

$$= \frac{\int_0^{1\text{ m}} z^2\,dz}{\int_0^{1\text{ m}} z\,dz} = 0.667 \text{ m} \qquad \textit{Ans.}$$

3. A plate of thickness 0.25ft and specific weight 180 lb/ft. determine the center of gravity and tension in chords used for supports fig3

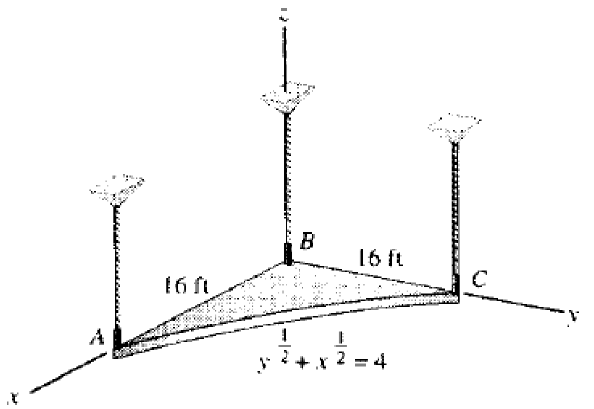
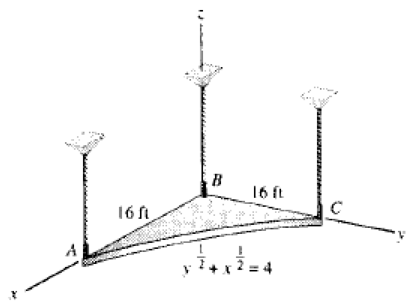


Figure 3

Solution:

9-13. The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180\ \text{lb/ft}^3$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.



Area and Moment Arm: Here, $y = x - 8x^{\frac{1}{2}} + 16$. The area of the differential element is $dA = y\,dx = (x - 8x^{\frac{1}{2}} + 16)dx$ and its centroid is $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)$. Evaluating the integrals, we have

$$A = \int_A dA = \int_0^{16\text{ ft}} (x - 8x^{\frac{1}{2}} + 16)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{16}{3}x^{\frac{3}{2}} + 16x\right)\Bigg|_0^{16\text{ ft}} = 42.67\ \text{ft}^2$$

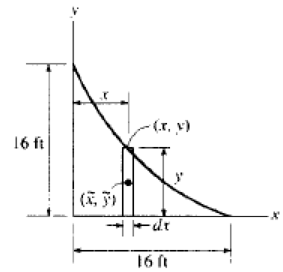
$$\int_A \tilde{x}\,dA = \int_0^{16\text{ ft}} x[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \left(\frac{1}{3}x^3 - \frac{16}{5}x^{\frac{5}{2}} + 8x^2\right)\Bigg|_0^{16\text{ ft}} = 136.53\ \text{ft}^3$$

$$\int_A \tilde{y}\,dA = \int_0^{16\text{ ft}} \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - \frac{32}{5}x^{\frac{5}{2}} + 48x^2 - \frac{512}{3}x^{\frac{3}{2}} + 256x\right)\Bigg|_0^{16\text{ ft}}$$

$$= 136.53\ \text{ft}^3$$

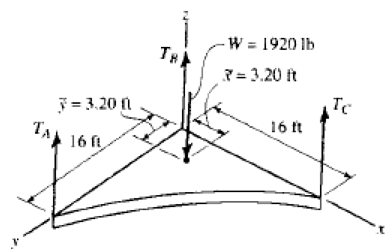


Centroid: Applying Eq. 9-6, we have

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \quad \textbf{Ans}$$

Equations of Equilibrium: The weight of the plate is $W = 42.67(0.25)(180) = 1920$ lb.



$\Sigma M_x = 0;\quad 1920(3.20) - T_A(16) = 0 \quad T_A = 384$ lb **Ans**

$\Sigma M_y = 0;\quad T_C(16) - 1920(3.20) = 0 \quad T_C = 384$ lb **Ans**

$\Sigma F_z = 0;\quad T_B + 384 + 384 - 1920 = 0$

$T_B = 1152\ \text{lb} = 1.15$ kip **Ans**

4. Express the position vector **r** in the Cartesian vector form; then determine its magnitude and coordinate direction angles.

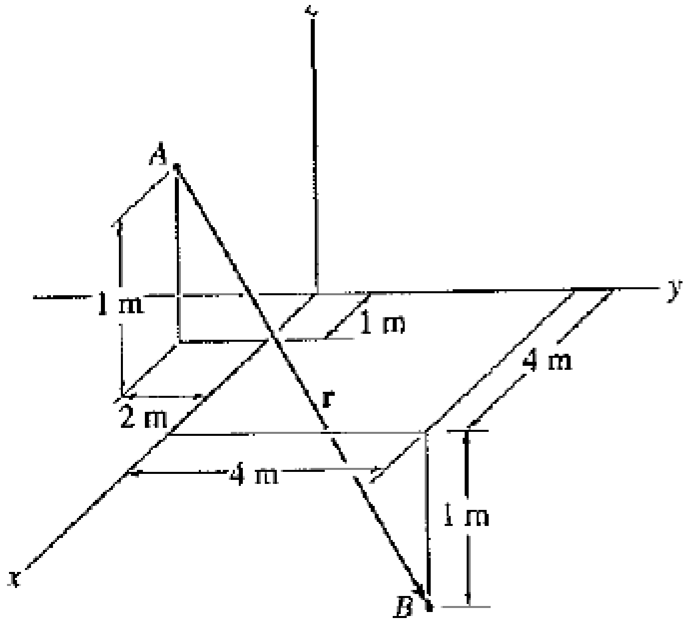


Figure 4

Solution:

(9) In order to find the position vector $\vec{r}$ of line AB

1) Find out the cordinates of points A & B

point A $= \{1, -2, 1\}$

point B $= \{4, 4, -1\}$

2) Express the coordinates in vector form

$$\vec{AB} = (4-1)i + (4-2)j + (-1-1)k$$

$$= 3i + 6j - 2k$$

$$|\vec{AB}| = \sqrt{9+36+4} = \underline{\underline{7}}$$

$\cos\alpha = \frac{3}{7}$ $\quad\cos\beta = \frac{6}{7}$ $\quad\cos\gamma = \frac{-2}{7}$

$\alpha = 64.6^\circ$ $\quad\beta = 31^\circ$ $\quad\gamma = \underline{\underline{106.6^\circ}}$