Department of Mechanical Engineering (NITC)

S1ME **ZZ1001D ENGINEERING MECHANICS** **Tutorial Test 3-Set 5**

Time: One Hour **Answer Key** Maximum Marks: 20

1. Locate the centroid of the semi-elliptical area shown in Fig.1.

Solution:

EXAMPLE 9.6

Locate the centroid of the semi-elliptical area shown in Fig. 9–13*a*.

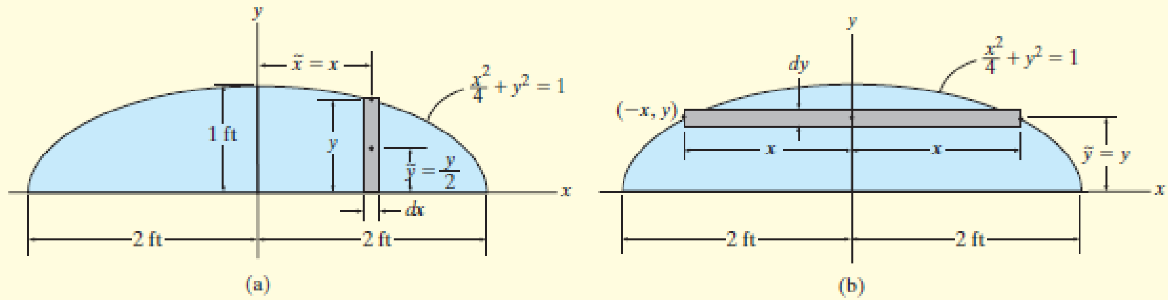


Fig. 9–13

SOLUTION I

Differential Element. The rectangular differential element parallel to the y axis shown shaded in Fig. 9–13*a* will be considered. This element has a thickness of dx and a height of y.

Area and Moment Arms. Thus, the area is $dA = y\,dx$, and its centroid is located at $\tilde{x} = x$ and $\tilde{y} = y/2$.

Integration. Since the area is symmetrical about the y axis,

$$\bar{x} = 0 \qquad \textit{Ans.}$$

Applying the second of Eqs. 9–4 with $y = \sqrt{1 - \frac{x^2}{4}}$, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-2\text{ ft}}^{2\text{ ft}} \frac{y}{2}(y\,dx)}{\int_{-2\text{ ft}}^{2\text{ ft}} y\,dx} = \frac{\frac{1}{2}\int_{-2\text{ ft}}^{2\text{ ft}}\left(1 - \frac{x^2}{4}\right)dx}{\int_{-2\text{ ft}}^{2\text{ ft}} \sqrt{1 - \frac{x^2}{4}}\,dx} = \frac{4/3}{\pi} = 0.424\text{ ft} \qquad \textit{Ans.}$$

SOLUTION II

Differential Element. The shaded rectangular differential element of thickness dy and width $2x$, parallel to the x axis, will be considered, Fig. 9–13*b*.

Area and Moment Arms. The area is $dA = 2x\,dy$, and its centroid is at $\tilde{x} = 0$ and $\tilde{y} = y$.

Integration. Applying the second of Eqs. 9–4, with $x = 2\sqrt{1 - y^2}$, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{1\text{ ft}} y(2x\,dy)}{\int_0^{1\text{ ft}} 2x\,dy} = \frac{\int_0^{1\text{ ft}} 4y\sqrt{1 - y^2}\,dy}{\int_0^{1\text{ ft}} 4\sqrt{1 - y^2}\,dy} = \frac{4/3}{\pi}\text{ ft} = 0.424\text{ ft} \qquad \textit{Ans.}$$

2. Determine the magnitude of the hydrostatic force acting on the glass window if it is circular, A. The specific weight of seawater is gw = 63.6 lb/ft^3.

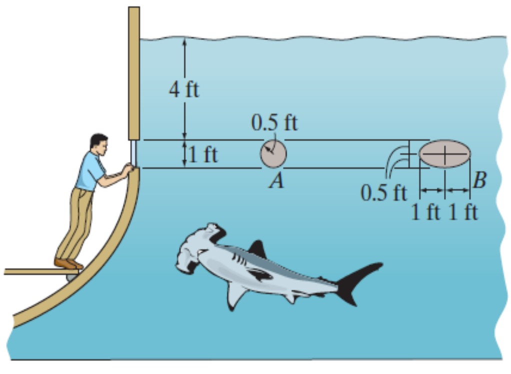
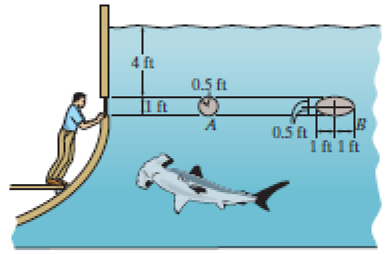


Figure 2

Solution:

9–110. Determine the magnitude of the hydrostatic force acting on the glass window if it is circular, A. The specific weight of seawater is $\gamma_w = 63.6\ \text{lb/ft}^3$.



Loading: By referring to the geometry of Fig. a, the depth h expressed in terms of y is

$$h = 4 + 0.5 - y = (4.5 - y)\ \text{ft}$$

Thus, the water pressure at the depth h is

$$p = \gamma_w h = 63.6(4.5 - y)\ \text{lb/ft}^2$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the differential area dA shown shaded in Fig. a is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left(2\sqrt{0.25 - y^2}\right)dy$$

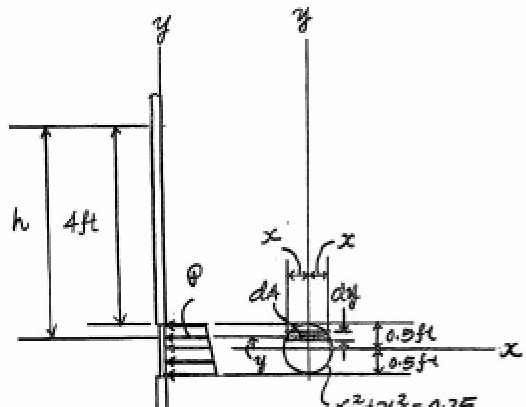
$$= \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\ \text{ft}}^{0.5\ \text{ft}} \left(572.4\sqrt{0.25 - y^2} - 127.2y\sqrt{0.25 - y^2}\right)dy$$

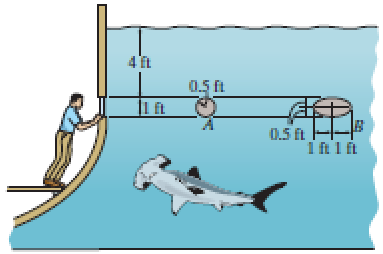
$$= \left[286.2\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 42.4\sqrt{(0.25 - y^2)^3}\right]_{-0.5\ \text{ft}}^{0.5\ \text{ft}}$$

$$= 224.78\ \text{lb} = 225\ \text{lb}$$ **Ans.**



(a)

9–111. Determine the magnitude and location of the resultant hydrostatic force acting on the glass window if it is elliptical, B. The specific weight of seawater is $\gamma_w = 63.6\ \text{lb/ft}^3$.



Loading: By referring to the geometry of Fig. a, the depth h expressed in terms of y is

$$h = 4 + 0.5 - y = (4.5 - y)\ \text{ft}$$

Thus, the water pressure at the depth h is

$$p = \gamma_w h = 63.6(4.5 - y)\ \text{lb/ft}^2$$

Resultant Force: The differential force $d\mathbf{F}_R$ acting on the area dA shown shaded in Fig. a is

$$dF_R = p\,dA = p(2x)\,dy = 63.6(4.5 - y)\left[2\left(2\sqrt{0.25 - y^2}\right)\right]dy$$

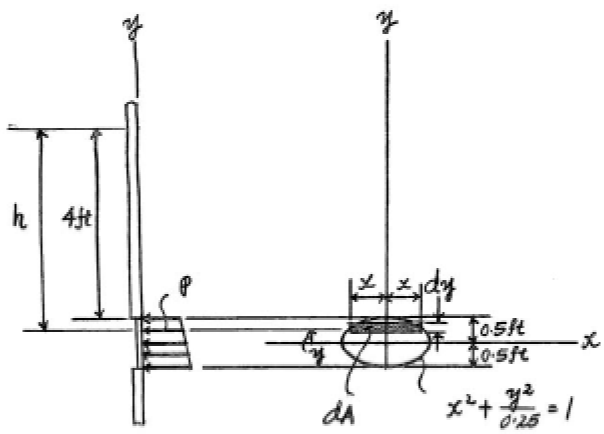
$$= \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$

Integrating $d\mathbf{F}_R$ from $y = -0.5$ ft to $y = 0.5$ ft,

$$F_R = \int dF_R = \int_{-0.5\ \text{ft}}^{0.5\ \text{ft}} \left(1144.8\sqrt{0.25 - y^2} - 254.4y\sqrt{0.25 - y^2}\right)dy$$

$$= \left[572.4\left(y\sqrt{0.25 - y^2} + 0.25\sin^{-1}\frac{y}{0.5}\right) + 84.8\sqrt{(0.25 - y^2)^3}\right]_{-0.5\ \text{ft}}^{0.5\ \text{ft}}$$

$$= 449.56\ \text{lb} = 450\ \text{lb}$$ **Ans.**



(a)

3. Locate the centre of gravity of the volume the material is homogeneous Fig. 3.

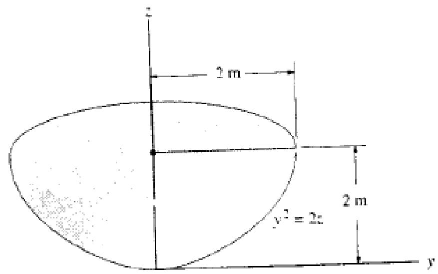
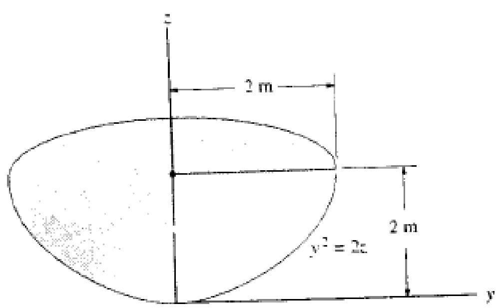


Figure 3

Solution:

9-33. Locate the center of gravity of the volume. The material is homogeneous.



Volume and Moment Arm: The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi(2z)dz = 2\pi z dz$ and its centroid $\tilde{z} = z$.

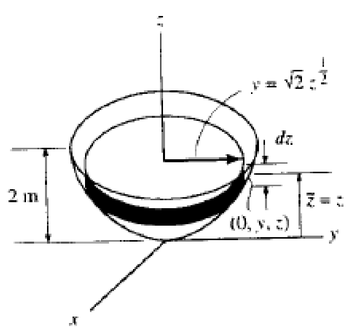
Centroid: Due to symmetry about z axis

$\bar{x} = \bar{y} = 0$ **Ans**

Applying Eq. 9-5 and performing the integration, we have

$$\bar{z} = \frac{\int_v \tilde{z}\,dV}{\int_v dV} = \frac{\int_0^{2m} z(2\pi z dz)}{\int_0^{2m} 2\pi z dz}$$

$$= \frac{2\pi\left(\frac{z^3}{3}\right)\Big|_0^{2m}}{2\pi\left(\frac{z^2}{2}\right)\Big|_0^{2m}} = \frac{4}{3}\text{m} \qquad \textbf{Ans}$$



4. Determine the projected component of the force F_{AB} = 560N acting along cable AC. Express the result as a Cartesian vector. (Fig. 4).

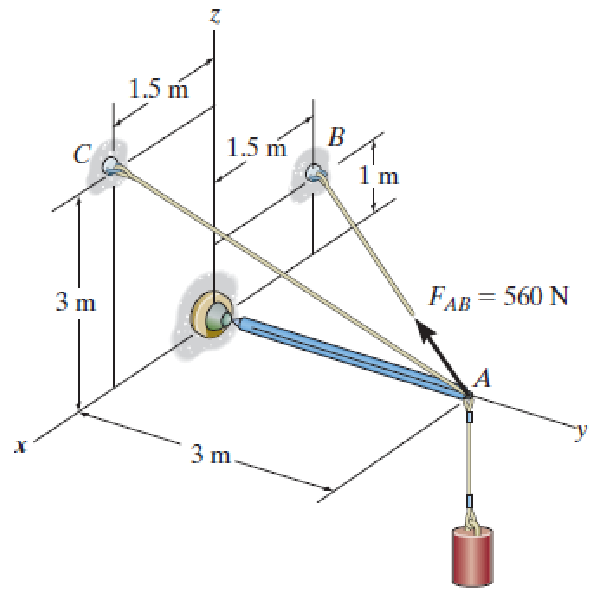


Figure 4

Solution:

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$$U_{AB} = \frac{r_{AB}}{|r_{AB}|} = \frac{(-1.5-0)i + (0-3)j + (1-0)k}{\sqrt{1.5^2 + 3^2 + 1^2}}$$

$$= -\frac{3}{7}i - \frac{6}{7}j + \frac{2}{7}k$$

$$U_{AC} = \frac{r_{AC}}{|r_{AC}|} = \frac{(1.5-0)i + (0-3)j + (3-0)k}{\sqrt{1.5^2 + 3^2 + 3^2}}$$

$$= \frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k$$

Thus the force vector F_{AB} is given by

$$F_{AB} = F_{AB} \cdot U_{AB} = 560\left[-\frac{3}{7}i - \frac{6}{7}j + \frac{2}{7}k\right]$$

$$= [-240i - 480j + 160k]\ N$$

Vector Dot Product

$$(F_{AB})_{AC} = F_{AB} \cdot U_{AC} = (-240i - 480j + 160k) \cdot \left(\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k\right)$$

$$= 346.67\ N$$

Thus $(F_{AB})_{AC}$ expressed in Cartesian vector form

$$= (F_{AB})_{AC}\ U_{AC} = 346.67\left(\frac{1}{3}i - \frac{2}{3}j + \frac{2}{3}k\right)$$

$$= (116i - 231j + 231k)\ N$$