Methods of Momentum for Particles

Linear Momentum

Impulse & Momentum Relations for a Particle

$F = f(t) \Rightarrow$ Methods of momentum

$$\vec{F} = m\frac{d\vec{V}}{dt}$$

Multiply both sides by dt,

$$\boxed{\int_{t_1}^{t_2} \vec{F} \cdot dt = \int_{t_1}^{t_2} m\frac{d\vec{V}}{dt} \cdot dt = m\vec{V}_f - m\vec{V}_i}$$

This is a vector eqn.

$I = \int_{t_i}^{t_f} \vec{F} \cdot dt \rightarrow$ impulse of force $\vec{F}$ during the time interval $t_f - t_i$.

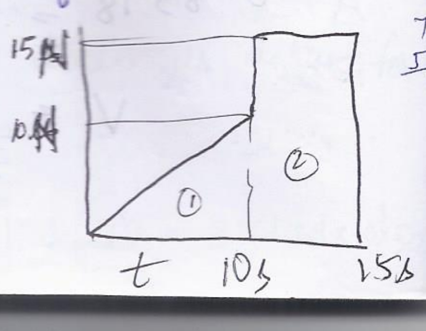
$m\vec{V}$ – linear-momentum vector of the particle

The impulse I over a time interval equals the change in linear momentum of a particle during that time interval.

1. A particle initially at rest is acted on by a force whose variation with time is shown graphically Fig. If the particle has a mass of 1 kg & is constrained to move rectilinearly in the direction of the force, what is the speed after 15 sec?

Sol: Impulse $= \frac{1}{2} \times 10 \times 10 + 5 \times 15$

$= 125$ N-s



75
50
5

Impulse $= mV_f - mV_i$

$125 = 1 \times V_f - 0$

$V_f = 125$ m/s

Day 4

2. A particle A with a mass of 1 kg has an initial velocity $\vec{V}_0 = 10\hat{i} + 6\hat{j}$ m/s. After particle A strikes particle B, the velocity becomes $\vec{V} = 16\hat{i} - 3\hat{j} + 4\hat{k}$ m/s. If the time of encounter is 10 msec, what avg force was exerted on the particle A? What is the change of linear momentum of particle B?

Sol: The impulse I acting on A

$$I_A = 1(16\hat{i} - 3\hat{j} + 4\hat{k}) - 1(10\hat{i} + 6\hat{j})$$
$$= 6\hat{i} - 9\hat{j} + 4\hat{k} \text{ N-sec}$$

Since $\int_{t_i}^{t_f} \vec{F}_A \, dt = (\vec{F}_{av})_A \Delta t$

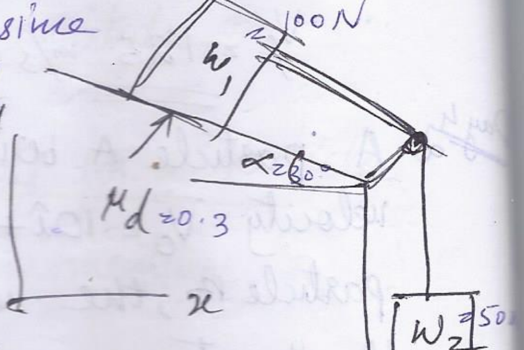
$$(F_{av})_A \times 0.01 = 6\hat{i} - 9\hat{j} + 4\hat{k}$$
$$(F_{av})_A = 600\hat{i} - 900\hat{j} + 400\hat{k} \text{ N}$$

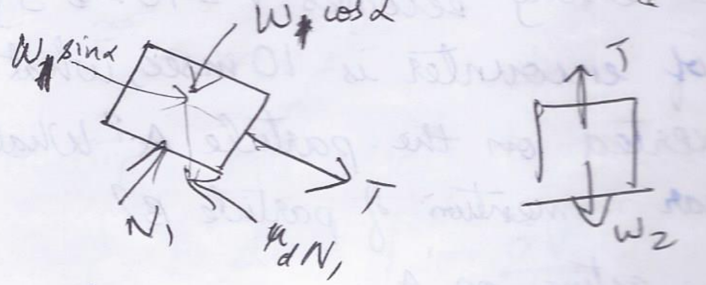
By Newton's 3rd law, an equal but opp avg force act on B during 10-msec time interval.

$$\Delta(mV)_B = -I_A = -6\hat{i} + 9\hat{j} - 4\hat{k} \text{ N-sec}$$

3. Two bodies, 1 & 2, are connected by an inextensible & wtless cord. Initially, the bodies are at rest. If the dynamic coefficient of friction is $\mu_d = 0.3$ for body 1 on the surface inclined at angle $\alpha = 30°$, compute the velocity of the bodies ~~at any time t~~ after $t = 10$s before body 1 has reached the end of the incline.

Sol: Only const. forces exist & since a time interval is specified we can use momentum considerations.





$N_1 = W_1 \cos\alpha = 86.6$

$f_1 = \mu_d N_1 = \mu_d W_1 \cos\alpha = 25.98$

For body 1, use linear impulse-momentum eqn along the incline:

$$\int_0^t (-\mu_d W_1 \cos\alpha + W_1 \sin\alpha + T)\,dt = \frac{W_1}{g}(V - 0)$$

$$(-\mu_d W_1 \cos\alpha + W_1 \sin\alpha + T)\,t = \frac{W_1}{g} V$$

$$(\underbrace{-25.98 + 50}_{24.02} + T) \times 10 = \frac{100}{9.81} V$$

For body 2, use momentum eqn in vertical direction

$$\int_0^t (W_2 - T)\,dt = \frac{W_2}{g}(V - 0)$$

V & T same for bodies 1 & 2

$$(W_2 - T)\,t = \frac{W_2}{g} V$$

$$(50 - T) \times 10 = \frac{50}{9.81} V$$

$$V = \frac{gt}{W_1 + W_2}\left[W_2 + W_1 \sin\alpha - \mu_d W_1 \cos\alpha\right]$$

$T = 25.33\,N$

$V = 48.4$ m/s

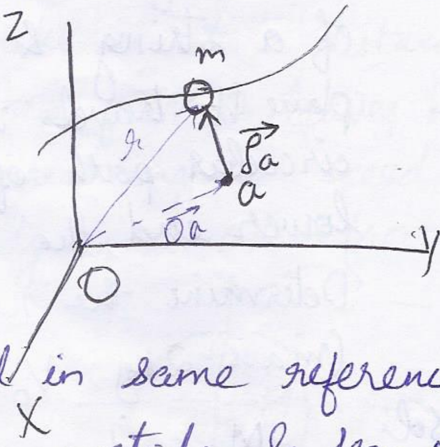
Moment of Momentum

$$\vec{F} = \frac{m\,d\vec{V}}{dt} = \frac{d}{dt}(m\vec{V}) = \dot{\vec{P}}$$

$\vec{P}$ – linear momentum of the particle

Take moment of each side of eqn about a in space

$$\vec{\rho_a} \times \vec{F} = \vec{\rho_a} \times \dot{\vec{P}}$$



$$\frac{d}{dt}(\vec{\rho_a} \times \vec{P}) = \dot{\rho_a} \times \vec{P} + \rho_a \times \dot{P}$$

$$\dot{\rho_a} \times P = \dot{\rho_a} \times m\dot{r}$$

Vectors $\vec{\rho_a}$ & r are measured in same reference from a fixed point 'a' to the particle & from origin to particle.

$$\vec{r} = \underbrace{\vec{Oa}}^{\text{const vector}} + \vec{\rho_a}$$

$$\dot{r} = \dot{\rho_a}$$

$$\therefore \dot{\rho_a} \times m\dot{r} = 0$$

$$\Rightarrow \frac{d}{dt}(\vec{\rho_a} \times \vec{P}) = \vec{\rho_a} \times \dot{\vec{P}}$$

$$\vec{\rho_a} \times \vec{F} = \vec{M_a} = \frac{d}{dt}(\rho_a \times P) = \dot{H_a}$$

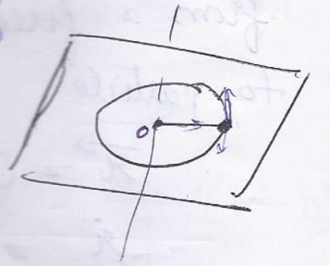
$$\boxed{M_a = \dot{H}_a}$$

H_a – moment about point a of the linear momentum vector or moment of momentum
– angular momentum vector

The moment $\vec{M_a}$ of the resultant force on a particle about a point 'a', fixed in an inertial reference, equals the time rate of change of the moment about point a of the linear momentum of the particle relative to the inertial reference

1. A small ball of wt 'W' is attached to the end of a string & is supported on a smooth horizontal plane. It travels with a uniform speed V_0 in a circular path of radius 'R'. By pulling string on lower end, the radius of path is reduced to $\frac{R}{2}$. Determine the new velocity of body. (mass = 2kg; $V_0 = 2$ m/s, R = 3m)

Sol: $M_o = \dot{H}_o$ $\qquad F_{ext}=0 \quad M_o = 0$



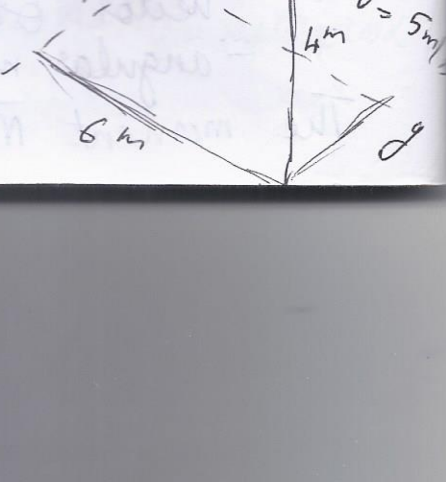
$H_i = H_f$

$r \times mv = \text{const}$

$3 \times 2 \times 2 = 1.5 \times 2 \times V$

$\underline{V = 4 \text{ m/s}}$

2. A small sphere has the position & velocity indicated in the Fig & is acted upon by the force F. Determine the angular momentum H_o about point O & the time derivative $\dot{H}_o$.



Sol: $H_o = r \times mv$ $\qquad$ kg m×m/s $\quad$ kg m/s² ×m/s

$= (3\hat{i} + 6\hat{j} + 4\hat{k}) \times 2(5\hat{j})$

$= (30\hat{k} - 40\hat{i})$ kg m²/s

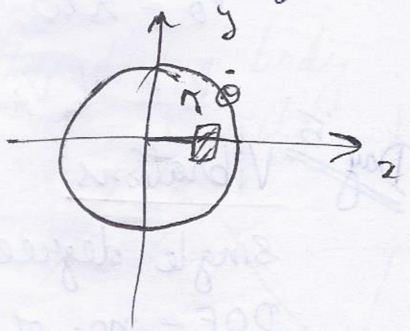
$\dot{H}_0 = M_0 = \vec{r} \times \vec{F} = (3\hat{i} + 6\hat{j} + 4\hat{k}) \times 10\hat{k}$

$= -30\hat{j} + 60\hat{i}$ Nm

3. A rod is rotating freely in a horizontal plane about a centerline O, with an angular speed of 0.3 rad/sec. A body of mass 16 kg is at a distance of 0.6 m from O. If the radial force from O leaves body at rate of 0.3 m/s, what is the angular speed & angular acceleration when body is at 0.3 m from O.

Sol:

$m r_1^2 \omega_1 = m r_2^2 \omega_2$



$(0.6)^2 \times 0.3 = (0.3)^2 \times \omega_2$

$\omega_2 = 1.2$ rad/s

H const $\Rightarrow \frac{d}{dt}(m r^2 \dot{\theta}) = 0$

$m r^2 \ddot{\theta} + m \dot{\theta} \times 2r \times \dot{r} = 0$

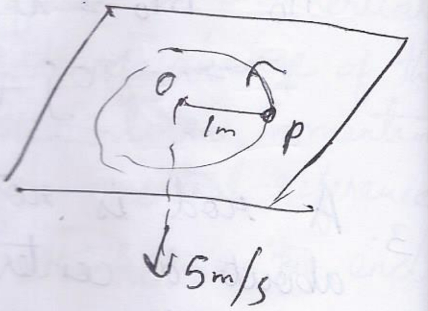
$(0.3)^2 \ddot{\theta} + 1.2 \times 2 \times 0.3 \times 0.3 = 0$

$\ddot{\theta} = -2.4$ rad/s^2

HW

4. A particle of mass 2 kg tied at end of an inextensible string is rotated at 2 rad/s along a circle of radius 1m over a smooth hor. table top. The string is pulled down thru a slot at center at a speed of 5 m/s. Calculate the speed of the particle when it reaches 0.5m from the centre.

Sol: $V = V_r e_r + V_\theta e_\theta$



$r_0 \times F = 0 \Rightarrow$ only force tension

$r \times mV = \text{const}$

$m r_1^2 \omega_1 = m r_2^2 \omega_2$

$1^2 \times 20 = 0.5^2 \times \omega_2$

$\omega_2 = 80$ rad/s

$V_r = 5$ m/s

$V_\theta = r\omega = 0.5 \times 80 = 40$ m/s

$V = \sqrt{V_r^2 + V_\theta^2} = 40.3$ m/s

Day 5 Vibrations

Single degree of freedom systems

DOF - no. of independent coordinates necessary to specify the configuration or position of a system at any time

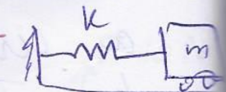
Elements: 1) mass - m - inertial characteristics

2) K - spring element - elastic restoring force PE capacity

3) damping element - $c \rightarrow$ frictional characteristics

4) excitation force $F(t)$ - external forces on system

Undamped Free Vibration



$F = ma$

$F = cV$

$F = kx$

$\boxed{m\ddot{x} + kx = 0}$

$\ddot{x} + \frac{k}{m}x = 0$

$\ddot{x} + \omega^2 x = 0$

linear
2nd order
homogenous
const - coeff

Natural frequency $\boxed{\omega = \sqrt{\frac{k}{m}}}$

$x = A\cos\omega t + B\sin\omega t$

$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$

$t = 0;\ x = x_0\ \&\ \dot{x} = v_0 \Rightarrow A = x_0\ \&\ B = \frac{v_0}{\omega}$

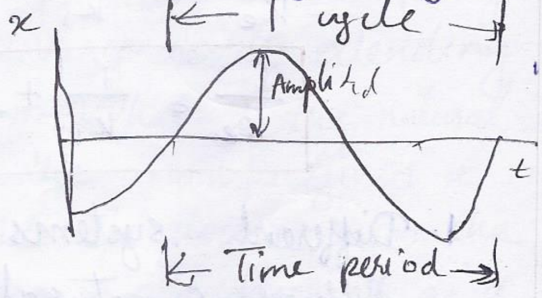
$\boxed{x = x_0\cos\omega t + \frac{v_0}{\omega}\sin\omega t}$ — Harmonic Motion

→ Time period, T is the time of one cycle. $T = \frac{2\pi}{\omega}$

→ Frequency – no. of cycles per unit time

$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

→ Amplitude – largest displacement attained by body during a cycle



Damped Free Vibration

$m\ddot{x} + c\dot{x} + kx = 0$

Undamped Forced Vibration

$m\ddot{x} + kx = F(t)$

Damped forced vibration

$m\ddot{x} + c\dot{x} + kx = F(t)$

→ $y = Ce^{pt}$

$mCp^2e^{pt} + cCpe^{pt} + kCe^{pt} = 0$

$mp^2 + cp + k = 0$

$p_1, p_2 = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$

Critically damped → $\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0 \rightarrow c_c = 2\sqrt{km}$

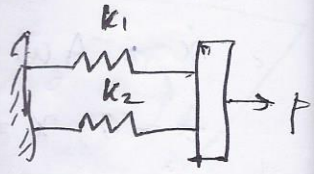
Overdamped — $c > c_{cr}$

Underdamped — $c < c_{cr}$

Springs in Parallel or in Series

Parallel

Displacement is same.



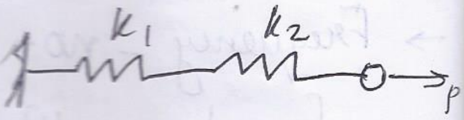
$F = F_1 + F_2$

$K_e x = K_1 x_1 + K_2 x_2$ $\qquad x = x_1 = x_2$

$$\boxed{K_e = K_1 + K_2}$$

Series

Force is same on 2 springs

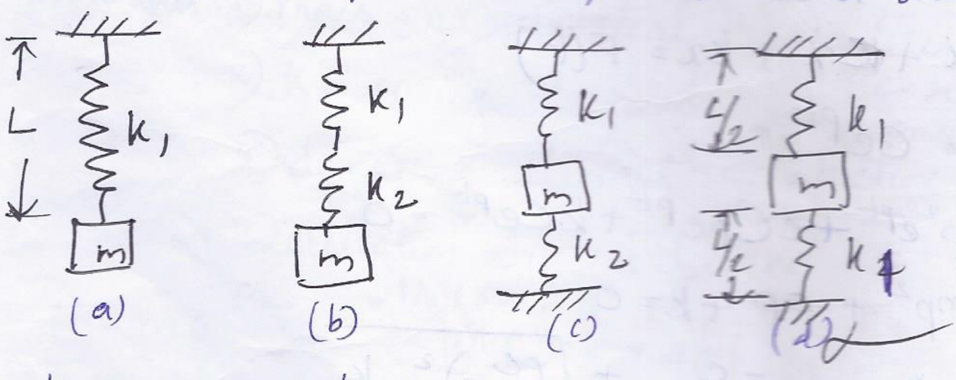


$x = x_1 + x_2$ $\qquad F = F_1 = F_2$

$\frac{F}{K_e} = \frac{F_1}{K_1} + \frac{F_2}{K_2}$

$$\boxed{\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2}}$$

1. Different systems are shown in Fig, find out spring stiffness & natural frequency of the system. Take $k_1 = 8$ N/mm & $k_2 = 12$ N/mm, $m = 10$ kg



$\frac{kg\,m}{s^2\,m \times kg}$

Sol (a) $K = k_1 = 8 \times 10^3$ N/m

$\sqrt{4 \times 10^2}$ $\quad 4 \times 10^2 \times 10$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8 \times 10^3}{10}} = 20\sqrt{2}$ rad/s $= 28.28$ rad/s

(b) $\frac{1}{K_e} = \frac{1}{k_1} + \frac{1}{k_2} = \frac{1}{8} + \frac{1}{12} = 0.2083$

$K_e = 4.8 \times 10^3$ N/m

$\omega = \sqrt{\frac{k_e}{m}} = \sqrt{\frac{4.8\times10^3}{10}} = 21.91$ rad/s

(c) $K = K_1 + K_2 = 8 + 12 = 20\times10^3$ N/m

$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{20\times10^3}{10}} = 44.72$ rad/s

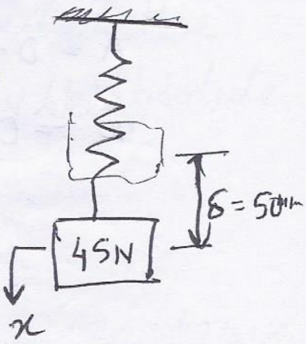
(d) $K_1 = 2\times8\times10^3$ N/m $= 16\times10^3$ N/m

$K_{eq} = 32\times10^3$ N/m

$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{32\times10^3}{10}} = 56.56$ rad/s

2. A mass weighing 45 N is placed on the spring shown in Fig & is released very slowly, extending the spring a distance of 50 mm. What is the natural frequency of the system? If the mass is given a velocity instantaneously of 1.6 m/s down from the eqlbm position, what is the eqn for displacement as a function of time?

Sol: Spring const $k = \frac{F}{\delta} = \frac{45}{50} = 0.9$ N/mm



Natural frequency, $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{0.9\times10^3}{45/9.81}}$

$= 14.01$ rad/s

$x = A \cos 14.01t + B \sin 14.01t$

$\dot{x} = -A\times14.01 \sin 14.01t + B\times14.01 \cos 14.01t$

At $t=0$, $x=0$ & $\dot{x} = 1.6$ m/s $\Rightarrow A = 0$

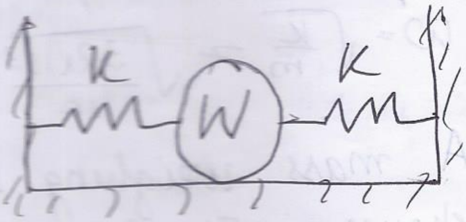
$B = \frac{1.6}{14.01} = 0.1142$

The eqn is $x = \underline{0.1142 \sin 14.01 t}$ m

3. The 2 springs shown in Fig have a spring const K = 178 N/m & the attached ball has wt W = 4.45 N. If the ball is initially ~~pos~~ displaced 25 to the right, find the period of oscillation of the ball & velocity with which it passes thru its middle position. Neglect friction.

Sol: $K_{eq} = K_1 + K_2 = 178 \times 2$

$= 356$ N/m



$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{356 \times 9.81}{4.45}}$

$= \underline{28 \text{ rad/s}}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{28} = 0.224$ s

$x = A\cos\omega t + B\sin\omega t$

$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$

At $t = 0$, $x = 0.025$ m & $\dot{x} = 0$

$A = 0.025$ & $B = 0$

$\ddot{x} = 0 \Rightarrow$ $0 = 0.025\cos\omega t$

$\omega t = \frac{\pi}{2}$

$t = \frac{\pi}{2\omega}$

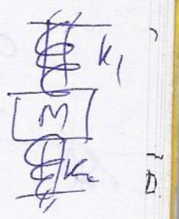
$\dot{x} = -0.025\,\omega \sin \omega \times \frac{\pi}{2\omega}$

$= -0.025 \times 28$

$= \underline{-0.7 \text{ m/s}}$

HW

19.4 A mass M of 100g rides on a frictionless guide rod. If the natural frequency with spring k_1 attached is 5 rad/s, what must k_2 be to increase the natural frequency to 8 rad/sec?



Sol:

$$\omega_1 = \sqrt{\frac{k_1}{m}} \qquad \omega_2 = \sqrt{\frac{k_1+k_2}{m}}$$

$$5 = \sqrt{\frac{k_1}{0.1}} \qquad 8 = \sqrt{\frac{2.5+k_2}{0.1}}$$

$$k_1 = 2.5\,N/m \qquad k_2 = 3.9\,N/m$$

5. A platform of wt $W = 4000$ ~~lb~~ is being supported by 4 equal columns which are clamped to the foundation as well as to the platform. Experimentally, it has been determined that a static force of $F = 1000N$ applied horizontally to the platform produces a displacement of $\Delta = 0.1$ m. It is estimated that damping in the structures is of the order of 5% of the critical damping. Determine for this structure the following: (a) undamped natural frequency (b) absolute damping coefficient

Sol:
(a) $k = \frac{F}{\Delta} = \frac{1000}{0.1} = 10000\ N/m$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10000}{4000/9.81}} = 4.95\ rad/s$$

$kg\frac{m}{s^2} \times /y$

$F = cv$

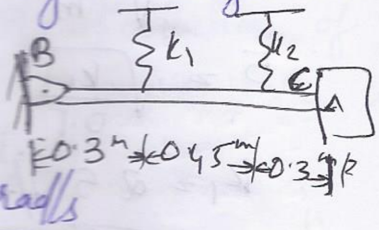
$\frac{Ns}{m}$

(b) $C_{cr} = 2\sqrt{km} = 2\sqrt{10000 \times \frac{4000}{9.81}} = 4038.55$ ~~kg~~/s $\left(\frac{Ns}{m}\right)$

absolute damping $= \xi C_{cr} = 0.05 \times 4038.55 = 201.93\ kg/s$ $\frac{Ns}{m}$

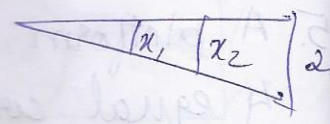
6. What is the natural frequency of motion for block
19.12 A for small oscillation? Consider BC to have negligible
mass & body A to be a particle. When body A
attached to the rod, the static deflection is 25 mm.
spring const k_1 is 1.75 N/mm. Body A weighs 110N. What

Sol:



$$k = \frac{F}{\delta} = \frac{110}{25} = 4.4\,N/mm$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4.4\times10^3}{110/9.81}} = \underline{19.81\ rad/s}$$



$$\frac{x_1}{0.3} = \frac{25}{1.05} \Rightarrow x_1 = 7.14\,mm$$

$$\frac{x_2}{0.75} = \frac{25}{1.05} \Rightarrow x_2 = 17.85\,mm$$

$$k_1 = \frac{F_1}{x_1} \Rightarrow F_1 = 1.75\times 7.14 = \underline{12.5\,N}$$

$$\Sigma M_B = 0 \Rightarrow 12.5\times0.3 + F_2\times0.75 - 110\times1.05 = 0$$

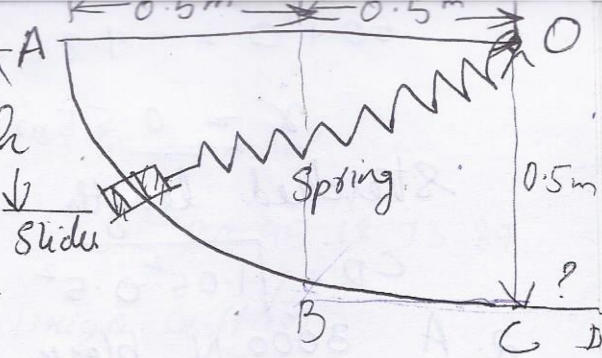
$$\underline{F_2 = 149\,N}$$

$$k_2 = \frac{149}{17.85} = \underline{8.34\ N/mm}$$

Revision

1. A slider of mass 2kg is attached to a spring of stiffness 400 N/m & unstretched length 0.5 is released from rest at A as shown in Fig. Determine the velocity of the slider as it passes thru B & C. Assume that slider moves over the bent rod with negligible friction. Also compute the distance beyond C where the slider should come to rest. Use law of conservation of energy

Sol: Slider can be considered to be in a conservative force field comprising of gravitational & spring forces.



Take A as datum.

$PE = -mgh + \frac{1}{2}kx^2$

h - ht of slider below reference pt.

x - compression of spring for position considered

$PE_A = \frac{1}{2} \times 400 \times (1-0.5)^2 = 50 \text{ Nm}$

$KE_A = 0$

$KE_B = \frac{1}{2} m V_B^2 = \frac{1}{2} \times 2 V_B^2 = V_B^2$

$PE_B = -2 \times 9.81 \times 0.5 + \frac{1}{2} \times 400 (0.707 - 0.5)^2$

$= -1.24 \text{ Nm}$

$PE_C = -2 \times 9.81 \times 0.5 = -9.81 \text{ Nm}$

$KE_C = \frac{1}{2} m V_C^2 = \frac{1}{2} \times 2 \times V_C^2 = V_C^2$

$(PE + KE) = \text{const}$

$50 + 0 = -1.24 + V_B^2 = -9.81 + V_C^2$

$V_B = 7.16 \text{ m/s}$

$V_C = 7.73 \text{ m/s}$

Consider a point D where slider would come to rest again

$PE_D = -2 \times 9.81 \times 0.5 + \frac{1}{2} \times 400 x^2$

$KE_D = 0$

$$50 + 0 = -9.81 + 200x^2$$

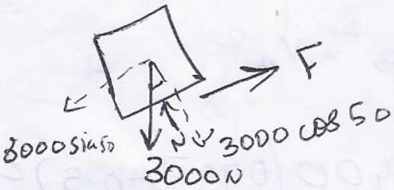
$$x = 0.55 m$$

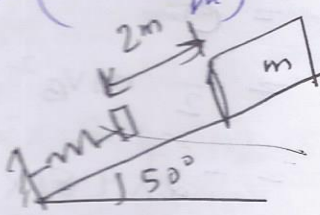
Stretched length $= 0.55 + 0.5 = 1.05 m$

$$CD = \sqrt{1.05^2 - 0.5^2} = 0.92 m$$

2. A 3000 N block starting from rest moves dow a 50° incline. After moving a distance of 2m, it ~~stretches~~ strikes a spring whose stiffness is 20 N/mm. If the coefficient of friction b/n block & incline is determine the max. deformation of the spring & max velocity of the block (WE eqn)

Sol:





$$N = 3000 \cos 50$$

$$F = \mu N = 0.2 \times 3000 \cos 50 = 385.67 N$$

Let max. deformation of spring be 's' mm

Body was at rest & is again at rest when it mo a distance of $(2000+s)$ mm

Applying WE eqn, we have

$$\underbrace{(3000 \sin 50 - F)(2000+s)}_{W_{1-2}} - \frac{1}{2} k s^2 = 0$$

$$s = 721.4 mm$$

Max velocity of block, $V_{max} \rightarrow \frac{dV}{dt} = 0$

Net force on system should be 0.

Let x = deformation corresponding to max velocity

At this instant, net force acting on system is 0.

$3000 \sin 50 - F - kx = 0$

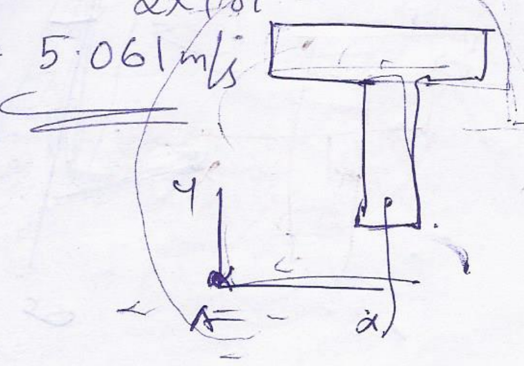
$2298.13 - 385.67 - 20x = 0$

$x = 95.62\,mm$

WE eqn,

$(3000 \sin 50 - F)(2000 + x) - \frac{1}{2}kx^2 = \frac{3000}{2 \times 9.81}(V^2 - 0^2)$

$V = 5.061\,m/s$



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8 Energy {WE / conse

8 Impulse

14 23rd - internals

20

11-12

$(3000 \sin 50 - F)s - \frac{1}{2}ks^2 = \frac{1}{2}mv^2$

$1912.46s - 10^4 s^2 = \frac{1}{2} 3822.63$

$S = 0.721$

$-Fs = (0 - \frac{1}{2}mv^2) + (0 - 3000 \sin 50\, s) + (0 - \frac{1}{2}ks^2)$