NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
MA1002D MATHEMATICS II, TUTORIAL 7
Winter Semester 2019-2020

Find the Laplace transform of the following functions. (Here a, b , c, ω, δ are constants).

1. $2t + 6$ 2. e^{a-bt}
3. $\sin \pi t$ 4. $\sin(\omega t + \delta)$

Find the inverse Laplace Transforms

5. $\dfrac{1-7s}{(s-3)(s-1)(s+2)}$ 6. $\dfrac{-s-10}{s^2-s-2}$ 7. $\dfrac{2.4}{s^4} - \dfrac{228}{s^6}$

Find the Laplace transform of

8. t^2e^{-3t} 9. $e^{2t}\sin^3 3t$

Find the inverse transform of

10. $\dfrac{1}{(s+1)^2}$ 11. $\dfrac{3}{s^2+6s+18}$

Solve the following initial value problems by the Laplace transform method.

12. $y' + 3y = 10\sin t$; $y(0) = 0$
13. $y'' + ay' - 2a^2y = 0$; $y(0)= 6, y'(0) = 0$
14. $y''+ 2y' - 3y = 6e^{-2t}$; $y(0) = 2, y'(0)= -14$
15. $y'+0.2y = 0.01t$; $y(0) = -0.25$
16. $y''-3y'+2y = 4t + e^{3t}$; $y(0) = 1$ and $y'(0) = -1$

Find the inverse transforms by Integration

17. $\dfrac{1}{s^2+4s}$ 18. $\dfrac{1}{s(s^2+\omega^2)}$
19. $\dfrac{1}{s^3-s}$ 20. $\dfrac{9}{s^2}\left[\dfrac{s+1}{s^2+9}\right]$

Find the Laplace transform of the following functions

21. te^t 22. $t^2\cosh \pi t$ 23. $t\cos \omega t$ 24. $te^{-t}\sin t$
25. $\dfrac{e^{2t}+\cos t}{t}$ 26. $\dfrac{e^{-t}\sin t}{t}$ 27. $\dfrac{e^{2t}+\cos t}{t}$

Find the inverse transform of the following functions.

28. $\dfrac{1}{(s-3)^3}$ 29. $\dfrac{s^2-\pi^2}{(s^2+\pi^2)^2}$ 30. $\ln\dfrac{s^2+1}{(s-1)^2}$
31. $\dfrac{s}{(s^2+4)^2}$ 32 . $\log(1+1/s^2)$ 33. $\cot^{-1}\dfrac{s}{a}$

(P.T.O)

Find the inverse transforms using convolution.

34. $\dfrac{1}{s^2(s-1)}$ 35. $\dfrac{s}{(s^2+\pi^2)^2}$ 36. $\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}$

37. $\dfrac{1}{s^2(s^2+9)}$ 38. $\dfrac{\omega}{s^2(s^2+\omega^2)}$

Using Laplace transforms, solve the integral equations

39 $y(t)=1+\int_0^t y(\tau)\,d\tau$ 40. $y(t) = 1-\int_0^t (t-\tau)y(\tau)d\tau$

Find the inverse transforms of the following functions

41. $\dfrac{6}{(s+2)(s-4)}$ 42. $\dfrac{2s^3}{(s^4-81)}$ 43. $\dfrac{s^3+6s^2+14s}{(s-2)^2}$ 44. $\dfrac{a(s^2-2a^2)}{s^4+4a^4}$

45. $\dfrac{s^2+9s\ -9}{s^3-9s}$ 46. $\dfrac{s^4+3(s+1)^3}{s^4(s+1)^3}$ 47. $\dfrac{1}{s(s+a)^3}$ 48. $\dfrac{2s^2+6s\ +5}{(s-1)(s-2)(s-3)}$

Find the Laplace transforms of the following functions

49. (t-1) u(t-1) 50. e^{-2t} u(t-3)

51. 4u(t-π) cost

52. sin ωt $(0<t<\pi/\omega)$ 53. e^t (0<t<1)

54. 10 cos πt (1<t<2) 55. $f(t) = \begin{cases} \dfrac{t}{m}, & 0<t<m \\ 1\ , & t>m \end{cases}$

Find the inverse Laplace transforms.

56. $3(1-e^{-\pi s})\ /\ (s^2+9)$ 57. $se^{-2s}/(s^2+\pi^2)$

Find the Laplace Transforms of the following periodic functions

58. $f(t) = \begin{cases} 1 & , \ if\ 0<t<\dfrac{a}{2} \\ -1 & , \ if\ \dfrac{a}{2}<t<a \end{cases}$, f (t + a) = f (t)

59. $f(t)= \begin{cases} \cos^2 t & , \ if\ 0<t<\pi \\ 0 & , \ if \pi<t<2\pi \end{cases}$ f (t +2π) = f (t).

60. $f(t)= \begin{cases} t & , \ if\ 0<t<a \\ 2a-t & , \ if\ a<t<2a \end{cases}$ when f(t+2a) = f(t)

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