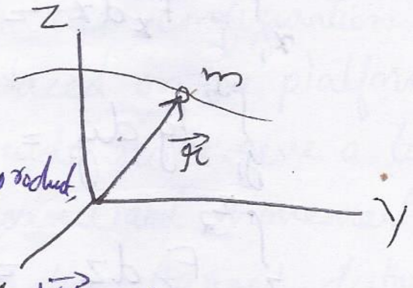
Day 1

MODULE IV

Energy Methods for Particles

$$F = m\frac{d^2\vec{r}}{dt^2} = m\frac{d\vec{V}}{dt}$$



Multiply both sides by $d\vec{r}$ as dot product.

$$\int_{r_1}^{r_2}\vec{F}\cdot d\vec{r} = m\int_{r_1}^{r_2}\frac{d\vec{V}}{dt}\cdot d\vec{r} = m\int_{t_1}^{t_2}\frac{d\vec{V}}{dt}\,\frac{d\vec{r}}{dt}\cdot dt$$

$$\frac{d\vec{r}}{dt} = \vec{V}$$

$$\int_{r_1}^{r_2}\vec{F}\cdot d\vec{r} = m\int_{t_1}^{t_2}\left(\frac{d\vec{V}}{dt}\cdot\vec{V}\right)dt = \frac{m}{2}\int_{t_1}^{t_2}\frac{d}{dt}(\vec{V}\cdot\vec{V})\,dt$$

$$= \frac{m}{2}\int_{t_1}^{t_2}\frac{d}{dt}V^2\cdot dt$$

$$= \frac{m}{2}\int_{V_1}^{V_2}d(V^2)$$

$\frac{1}{2}\times V^2$

$\frac{1}{2}\times 2\times V\times\frac{dV}{dt}$

$$\boxed{\int_{r_1}^{r_2}\vec{F}\cdot d\vec{r} = \frac{1}{2}m(V_2^2 - V_1^2)}$$

W_{1-2} = change in KE

Velocity is that of the mass center, force is the resultant external force on the system & the path of integration is that of mass center.

Single particle model $\begin{cases}\text{rigid body moving without rotation}\\ \text{size is small compared to trajectory}\end{cases}$

Newton's law in one direction.

$$F_x\hat{i} = m\frac{dV_x}{dt}\hat{i}$$

$$dx\hat{i} + dy\hat{j} + dz\hat{k} = d\vec{r}$$

$$\int_{x_1}^{x_2} F_x\,dx = \frac{m}{2}\left[(V_x)_2^2 - (V_x)_1^2\right]$$

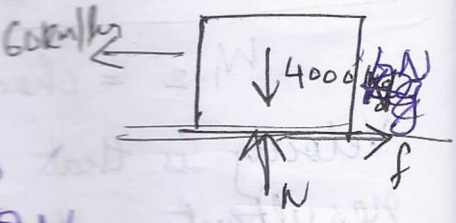
$$\int_{y_1}^{y_2} F_y\,dy = \frac{m}{2}\left[(V_y)_2^2 - (V_y)_1^2\right]$$

$$\int_{z_1}^{z_2} F_z\,dz = \frac{m}{2}\left[(V_z)_2^2 - (V_z)_1^2\right]$$

Work done on a particle in any direction equals the change in KE associated with the component of velocity in that direction.

Energy approach useful - velocities desired & $F = f(x)$

1. An automobile is moving at 60km/hr when the driver jams on his brakes & goes into a skid in the direction of motion. The car weighs 4000 N & the dynamic coefficient of friction b/n the rubber tires & the concrete road is 0.60. How far, l, will the car move before stopping?



Sol: Const. friction force $= \mu_d N$

$$= 0.6 \times 4000$$

$$= \underline{2400\ N}$$

This is the only force performing work-changes KE 60 km/hr to 0.

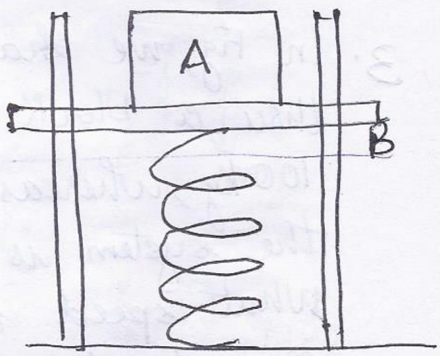
Work-energy eqn: $F \cdot x = \frac{1}{2} m (V_2^2 - V_1^2)$

$$-2400\,l = \frac{1}{2}\,\frac{4000}{9.81}\left(0 - \left(60 \times \frac{5}{18}\right)^2\right)$$

$$l = \underline{\underline{23.59\ m}}$$

2. Shown in Fig is a light platform B guided by vertical rods. The platform is positioned so that the spring has been compressed 10mm. In this configuration a body A weighing 100 N is placed on the platform & released suddenly. If the guide rods give a total const resistance force f to downward movement of the platform of 5N, what is the largest distance that the wt falls? The spring used here is a nonlinear spring requiring $0.5x^2$ N of force for a deflection of x mm.

Sol: Position of interest - δ below the initial configuration - A reaches 0 velocity for 1st time after being released.



Change in KE = 0

Zero net work done by forces acting on body A during displacement δ.

Forces - force of gravity, friction force from guides, force from spring

$$\int_{10}^{10+\delta} \vec{F}\cdot d\vec{r} = \int_{10}^{10+\delta} (W_A - f - 0.5x^2)\,dx$$

$$= \int_{10}^{10+\delta} (100 - 5 - 0.5x^2)\,dx = 0$$

$$95x - 0.5\frac{x^3}{3}\Big|_{10}^{10+\delta} = 0$$

$$95\left[10+\delta-10\right] - \frac{0.5}{3}\left[(10+\delta)^3 - 10^3\right] = 0$$

$$95\delta - \frac{0.5}{3}\left[\cancel{10^3} + 3\times10^2\delta + 3\times10\,\delta^2 + \delta^3 - \cancel{10^3}\right] = 0$$

$$\delta^3 + 30\delta^2 + 300\delta = \frac{\overset{(570)}{3\times95}}{0.5}\,\delta$$

$$\delta^3 + 30\delta^2 - 270\delta = 0$$

$$\delta\left(\delta^2 + 30\delta - 270\right) = 0$$

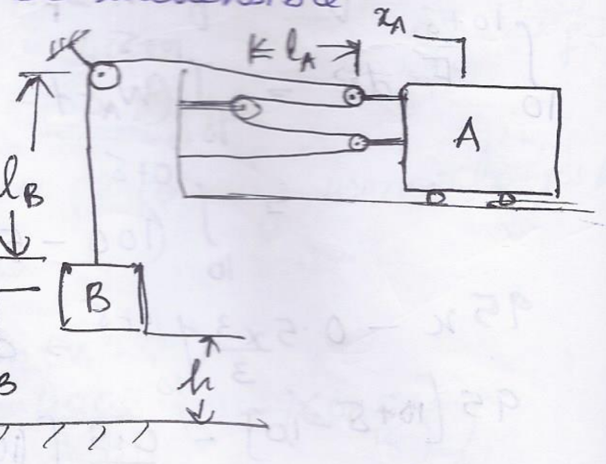
$$\delta = 0\ , \quad \delta = 7.25\text{ mm}, \quad \delta = -37.25\text{ mm}$$

$\delta = 0 \Rightarrow$ No work done if there is no deflection

Force in spring $= 0.5x^2 = 50N \Rightarrow$ No physical meaning

3. In Fig, we have shown bodies A & B interconnected thru a block & pulley system. Body B has a mass = 100 kg, whereas body A has a mass of 900 kg. Initially the system is stationary with B held at rest. What speed will B have when it reaches the ground at a distance $h = 3$m below after being released? What will be the corresponding speed of A? Neglect the masses of the pulleys & the rope. Consider the rope to be inextensible.

Sol: As the bodies move, only distance l_A & l_B change.



Since rope is inextensible,

$l_B + 4l_A = \text{const}$ — (a)

Differentiating wrt time

$$\dot{l}_B + 4\dot{l}_A = 0$$

$\dot{l}_B = -4\dot{l}_A \quad -(b)$

$\dot{l}_A = -V_A \quad \& \quad \dot{l}_B = V_B$

$V_B = 4V_A \quad -(c)$

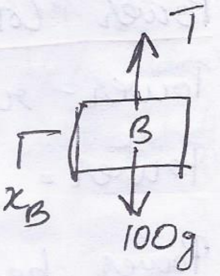
Taking differential of eqn (a): $dl_B + 4dl_A = 0$

$dl_B = -4\,dl_A \quad (d)$

$dx_B = dl_B \quad \& \quad dx_A = -dl_A$

$\Delta_B = 4\Delta_A \quad -(e)$

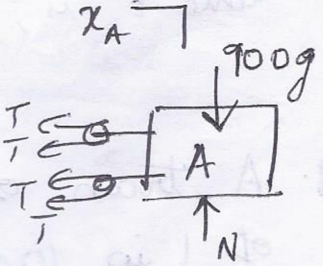
Work-energy eqn for B



$(100g - T)h = \frac{1}{2}100V_B^2$

$(981 - T)\times 3 = \frac{1}{2}100V_B^2 \quad (f)$

Work-energy eqn for A



$4T\cdot\Delta_A = \frac{1}{2}900V_A^2 \quad (g)$

According to eqn (e), $\Delta_A = \frac{\Delta_B}{4} = \frac{h}{4} = \frac{3}{4}\,m \quad (h)$

eqn (c), $V_A = \frac{1}{4}V_B \quad (i)$

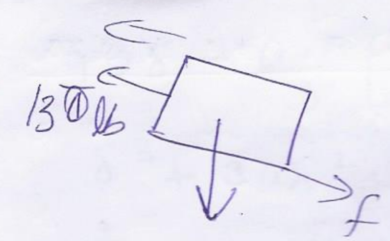
Substituting in (g),

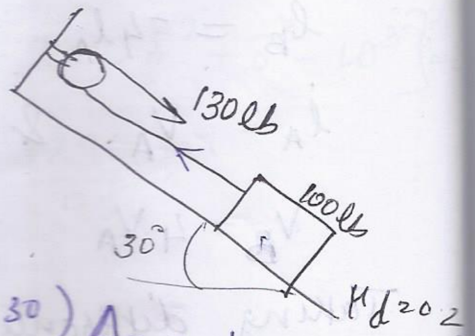
$4T\times\frac{3}{4} = \frac{1}{2}900\times\frac{V_B^2}{16} \quad (j)$

$(f)+(j) \Rightarrow 981\times 3 = \frac{1}{2}V_B^2\left(100 + \frac{900}{16}\right)$

$V_B = \underline{6.137}\ m/s \downarrow \qquad V_A = \underline{1.534}\ m/s$ left $\leftarrow$

HW 13.2





$$(130 - 100 \sin 30 - 0.2 \times 100 \cos 30)\,\Delta_B = \frac{1}{2} \times \frac{100}{32.2} \cdot V_B^2$$

$\Delta_B = 30$ ft

$V_B = 34.8$ ft/s

Day 2

Power Considerations

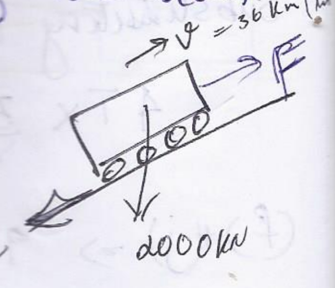
Power - rate at which work is performed

$$\text{Power} = \frac{dW_k}{dt}$$

Power being developed by a system of n forces at time t is,

$$P = \frac{\sum_{i=1}^{n} F_i \, dx_i}{dt} = \sum_{i=1}^{n} F_i \cdot V_i$$

1. A train of wt 2000 kN is ascending a slope of 1 in 100 with a uniform speed of 36 km/h. Find the power exerted by the engine if the resistance is 8 N per kN wt of the train.



Sol: $v = 36 \times \frac{5}{18} = 10$ m/s

$W = 2000$ kN

Slope = 1 in 100

$\tan\theta = \frac{1}{100} \approx \sin\theta$

Force of friction, $F = 8 \times 2000 = 16000$ N

$F - f - 2000 \sin\theta = \cancel{m} \times a^{0}$ $\quad$ Frna

$F = \cancel{2}16000 + 2000 \times 10^3 \times \frac{1}{100} = 36000\ N$

Power exerted $= F \times V$

$= 36000 \times 10 = \underline{\underline{360\ kW}}$

HW

13.36 A ~~m~~ 7500 kg streetcar starts from rest when the conductor draws 5 kW of power from the line. If this input is maintained const & if the mechanical efficiency of motors is 90%, how long does the streetcar take to reach a speed of 10 km/hr? Neglect wind resistance.

Sol: $\frac{\text{Output}}{0.9} = \text{Input}$

Output ~~power~~ $= F \times V = 5\ kW \times 0.9$

$F = ma$

$FV = mV\frac{dV}{dt}$

$5 \times 1000 \times 0.9 = 7500\, V \frac{dV}{dt}$

$\frac{3}{5} dt = V\,dV$

$\frac{V^2}{2} = \frac{3}{5} t + C_1$

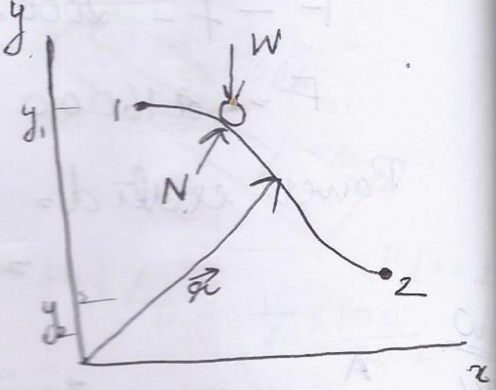
At $t = 0$, $V = 0 \Rightarrow C_1 = 0$

At $V = 10$ km/hr, $\frac{\left(10 \times \frac{5}{18}\right)^2}{2} = \frac{3}{5} t$

$t = 6.43\ s$ //

Conservative Force Fields

Gravity W - only active force



$$W_{1-2} = \int_1^2 \vec{F}\cdot d\vec{r} = \int_1^2 (-W\hat{j})\cdot d\vec{r}$$

$$= -W\int_1^2 dy$$

$$= -W(y_2 - y_1)$$

$$= W(y_1 - y_2)$$

Work done does not depend on path, but depends only on positions of end points of the path.

Force fields whose work like gravity is independent of the path are called conservative force fields.

$$W_{1-2} = \int_1^2 \vec{F}\cdot d\vec{r} = V_1(x,y,z) - V_2(x,y,z)$$

V – fn of position of end points – potential energy fn

$$-\int_1^2 \vec{F}\cdot d\vec{r} = V_2(x,y,z) - V_1(x,y,z) = \Delta V$$

PE $V(x,y,z)$ depends on datum

but change in PE, ΔV independent of datum used

Change in PE, $\Delta V = V_2 - V_1$ of a conservative force field is negative of work done by this conservative force field on a particle going from position 1 to 2 along any path.

For any closed path, $\oint \vec{F}\cdot d\vec{r} = 0$

$$\vec{F}\cdot d\vec{r} = -dV$$

$$F_x\,dx + F_y\,dy + F_z\,dz = -\left[\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right]$$

$$F_x = -\frac{\partial V}{\partial x}\ ;\ F_y = \frac{-\partial V}{\partial y}\ ;\ F_z = -\frac{\partial V}{\partial z}$$

$$\vec{F} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

$$= -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V = -\text{grad } V = -\nabla V$$

Conservative force field must be a fn of position & expressible as the gradient of a scalar fn.
Inverse also true.

Constant Force field

If force field is const at all positions, it can be expressed as gradient of a scalar fn of form

$V = -(ax + by + cz)$

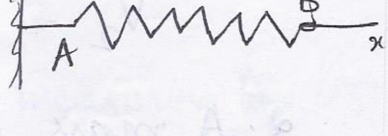
$\vec{F} = a\hat{i} + b\hat{j} + c\hat{k}$

Gravitational force $= -mg\hat{k}$, $PE = mgz$

Force $\propto$ Linear Displacements

Consider a body limited by constraints to move along a st. line. Restoring force eg. linear spring

$\vec{F} = -kx\,\hat{i}$



$PE = \frac{Kx^2}{2}$

PE is the energy stored in the force field as measured from a given datum.

Conservation of Mechanical Energy

Consider the motion of a particle upon which only a conservative force field does work.

$$\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$

Using definition of potential energy,

$$(PE)_1 - (PE)_2 = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$$

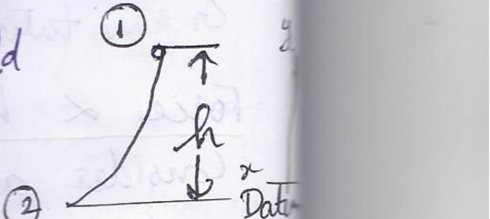
$$\boxed{(PE)_1 + \frac{1}{2} m V_1^2 = (PE)_2 + \frac{1}{2} m V_2^2}$$

Sum of PE & KE for a particle remains con at all times during the motion of the vehicle. Law of conservation of mechanical energy for conservative systems.

1. A particle is dropped with zero initial velocity dow a frictionless chute. What is the magnitude of its velocity if the vertical drop during the motion is h

Sol: For small trajectories, uniform force field

$= -mg\hat{j}$



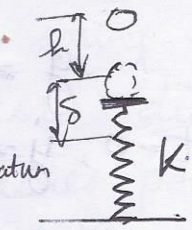
Conservation of ME eqn

$$mgh + 0 = 0 + \frac{1}{2} m V_2^2$$

$$V_2 = \sqrt{2gh}$$

2. A mass is dropped onto a spring that has a spring const K & a negligible mass. What is the max deflection δ? Neglect the effects of permanent deformation of the mass & any vibration that ma occur.

Sol: Lowest position of body as datum, body falls a distance $(h+\delta)$



ME at uppermost & lowest positions of body

$$\underbrace{mg(h+\delta)}_{\text{PE gravity}} + \underbrace{0}_{\text{PE spring}} + \underbrace{0}_{\text{KE}} = \underbrace{0}_{\text{PE gravity}} + \underbrace{\tfrac{1}{2}k\delta^2}_{\text{PE spring}} + \underbrace{0}_{\text{KE}}$$

$$\frac{1}{2}k\delta^2 - mgh - mg\delta = 0$$

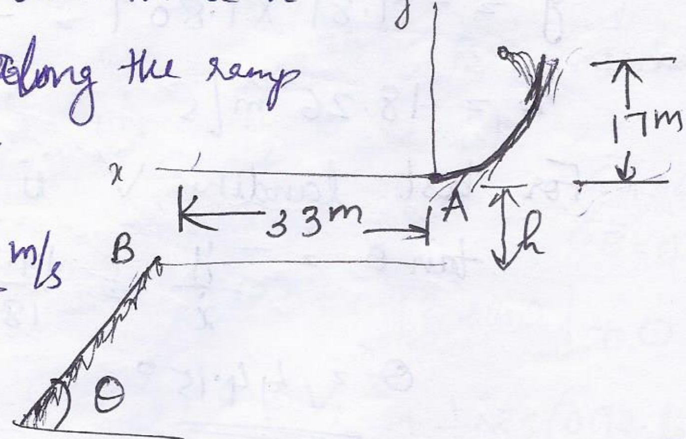
$$\delta^2 - \frac{2mg}{k}\delta - \frac{2mgh}{k} = 0$$

3. A ski jumper moves down the ramp aided only by gravity. If the skier moves 33m in the hor. direction & is to land very smoothly at B, what must be the angle θ for the landing incline? Neglect friction. Also determine h.

Sol: Conservation of ME along the ramp

$$mg \times 17 = \frac{1}{2}mV^2$$

$$V = \sqrt{2g \times 17} = \underline{\underline{18.26\ m/s}}$$



Using a reference xy at A & measuring time from the instant that the skier is at the origin, use Newton's law for free flight.

$$\ddot{y} = -9.81$$

$$\dot{y} = -9.81t + C_1$$

$$y = -9.81\frac{t^2}{2} + c_1 t + c_2$$

$t=0,\ \dot{y}=0$ & $y=0 \Rightarrow c_1=0\ ;\ c_2=0$

$\ddot{x} = 0$

$\dot{x} = c_3$

$x = c_3 t + c_4$

$t=0,\ \dot{x} = 18.26$ & $x=0$ $\quad \therefore c_3 = 18.26$

$c_4 = 0$

$\dot{y} = -9.81t$ (a) $\qquad \dot{x} = 18.26$ (c)

$y = -9.81\frac{t^2}{2}$ (b) $\qquad x = 18.26t$ (d)

To get h, set $x = 33$ in eq(d) & solve for time t

$\therefore 33 = 18.26\,t \Rightarrow \underline{t = 1.807\,s}$

$h = -9.81 \frac{\times 1.807^2}{2} = \underline{16.01\ m}$

$\dot{y} = -9.81 \times 1.807 = -17.73\ m/s$

$\dot{x} = 18.26\ m/s$

For best landing, V is $\parallel^{el}$ to incline

$\tan\theta = -\frac{\dot{y}}{\dot{x}} = \frac{17.73}{18.26} = 0.97$

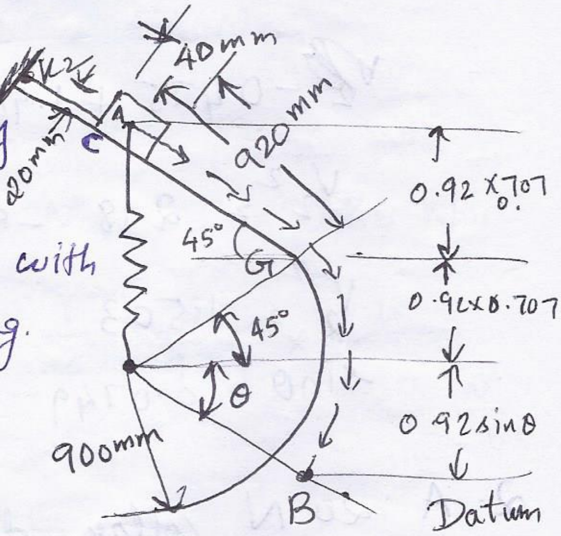
$\underline{\theta = 44.15^\circ}$

<u>Day 3</u>

1. A block A of mass 0.2 Kg slides on a frictionless surface as shown in Fig. The spring const k_1 is 25 N/m & initially, at the position shown, it is stretched 0.4 m. An elastic cord connects the top

support to point C on A. It has a spring const. K_2 of 10.26 N/m. Furthermore, the cord disconnects from C at the instant that C reaches point G at the end of the straight portion of the incline. If A is released from rest at the indicated position, what value of θ corresponds to the end position B where A just loses contact with the surface? The elastic cord (at the top) is initially unstretched.

Sol: Conservation of Mechanical Energy



Using datum at B & l_0 as unstretched length of spring with δ as elongation of the spring.

$$mgz_1 + \frac{mV_1^2}{2} + \frac{1}{2}k_1\delta_1^2$$

$$= mgz_2 + \frac{mV_2^2}{2} + \frac{1}{2}k_1\delta_2^2 + \frac{1}{2}k_2(\overline{CG})^2$$

$\overline{CG} = 0.94\text{m}$

$\overline{OB} = 0.92\text{m}$

$$0.200\times 9.81\left[0.92\times 0.707 + 0.92\times 0.707 + 0.92\sin\theta\right] + 0$$
$$+ \frac{1}{2}\times 25\times 0.4^2 = 0 + \frac{1}{2}\times 0.2 V_2^2 + \frac{1}{2}\times 25(0.92 - l_0)^2$$
$$+ \frac{1}{2}\times 10.26\times 0.94^2$$

$$l_0 = [0.92\times 0.707 + 0.92\times 0.707] - 0.4 = 0.901\text{m}$$

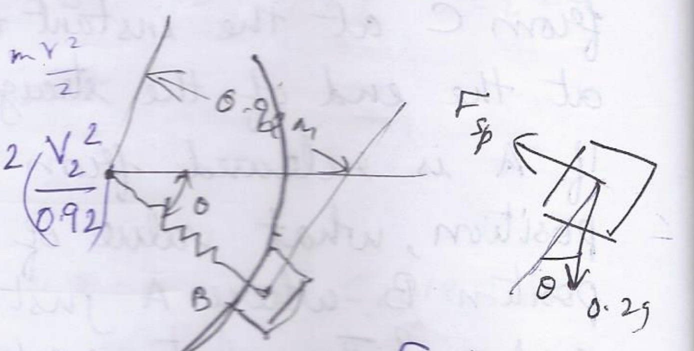
$$2.552 + 1.80504\sin\theta = 0.1V_2^2 + 2.537$$
$$0.015 + 1.805\sin\theta = 0.1V_2^2$$

$V_2^2 = 0.15 + 18.05 \sin\theta$ — ①

Use Newton's law at B where A just loses conta

In radial direction,

$-F_{sp} + 0.2g\sin\theta = -0.2\,\frac{V_2^2}{0.92}$



$-25(0.92-0.901) + 0.2\times 9.81\sin\theta = -\frac{V_2^2}{4.60}$

$F = ma_r$ $a =$ $V = R\omega$

$a_r = -r\dot{\theta}^2$

$= -r\omega^2$

$= -r\times\frac{v^2}{r^2}$

$= -\frac{v^2}{r}$

$-0.475 + 1.962\sin\theta = -\frac{V_2^2}{4.60}$

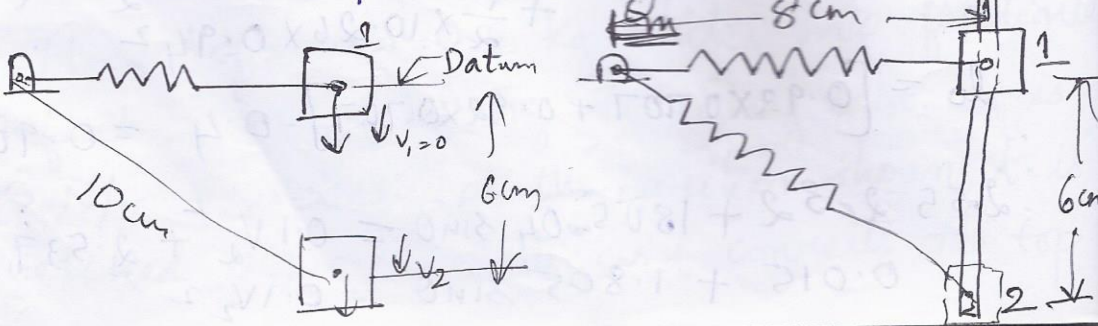
$V_2^2 = 2.18 - 9.03\sin\theta$ — ①

$V_2^2 = 1.503$ $V_2 = 1.226$ m/s

$\sin\theta = 0.0749 \Rightarrow \underline{\theta = 4.3^\circ}$

2. A 20 N collar slides without friction along a vertical rod as shown. The spring attached to the collar has an undeformed length of 4 cm & a const of 3 N/cm. If the collar is released from rest in position 1, determine its velocity after it has moved 6 cm to position 2.

Sol:



Position 1: PE: Elongation of spring is $x_1 = 8 - 4 = 4$ cm

(PEs) $V_s = \frac{1}{2} k x_1^2 = \frac{1}{2} \times 3 \times 4^2 = 24$ N cm

Choosing datum as shown, (PEg) $V_g = 0$

PE $V_1 = V_e + V_g = 24$ N cm $= 0.24$ Nm

KE: $V_1 = 0$, $KE_1 = 0$

Position 2: PE: $x_2 = 10 - 4 = 6$ cm

$V_s = \frac{1}{2} k x_2^2 = \frac{1}{2} \times 3 \times 6^2 = 54$ N cm

$V_g = Wy = 20 \times -6 = -120$ Ncm

$V_2 = 54 - 120 = -66$ Ncm $= -0.66$ Nm

KE: $KE_2 = \frac{1}{2} m V_2^2 = \frac{1}{2} \times \frac{20}{9.81} \times V_2^2$ $\frac{N}{m/s^2}$

Conservation of Energy

$PE_1 + KE_1 = PE_2 + KE_2$

$0.24 + 0 = -0.66 + \frac{10}{9.81} V_2^2$

$0.9 = \frac{10}{9.81} V_2^2$

$V_2 = 0.939$ m/s

HW 13.50

Alternative Form of Work-Energy Equation

Resembles 1st law of thermodynamics

Consider certain forces are conservative while others are not.

For conservative forces, −ve of change in PE = work done

$$\int_1^2 \vec{F}\cdot d\vec{r} - \Delta(PE)_{1,2} = \Delta(KE)_{1,2}$$

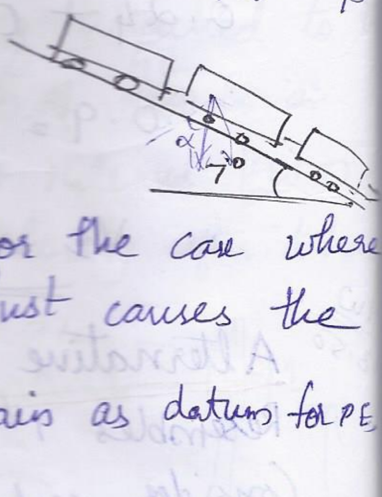
Integral represents work of non conservative forces & Δ represents final state − initial state

$$\boxed{\Delta(KE + PE) = W_{1-2}}$$

1. 3 coupled streetcars are moving at a speed of 32 km/hr down a 7° incline. Each car has a wt. of 198 kN. Specifications from the buyer requires that the cars must stop within 50m beyond the position where the brakes are fully applied so as to cause the wheels to lock. What is the max no. of brake failures that can be tolerated & still satisfy this specification? Assume the wt. is loaded equally among all the wheels of the system. There are 24 brake systems, one for each wheel. $\mu_d = 0.45$

Sol: f on any one wheel,

$$f = \frac{W\cos\alpha}{8} \times 0.45$$



$$= \underline{\underline{11.05\ kN}}$$

Consider work-energy relation for the case where a min no. of good brakes n, just causes the trains to stop in 50m.

Using end configuration of the train as datum for PE,

$$\Delta KE + \Delta PE = W_{1-2}$$

$$\left[0 - 3\times\frac{1}{2}\times\frac{198\times10^3}{9}\times\left(\frac{32\times5}{18}\right)^2\right] + \left(0 - 3\times198\times10^3\times50\,\sin\right.$$

$= -n \times 11050 \times 50$

$n = 10.88$

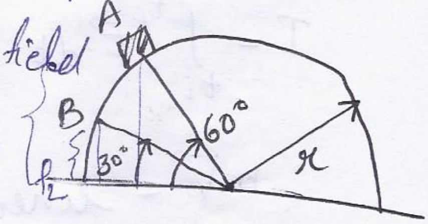
No. of brake failures to be tolerated $= 24 - 11$

$= 13$

HW

13.62 A body A is released from rest on a vertical circular path as shown. If a const resistance force of 1N acts along the path, what is the speed of the body when it reaches B? The mass of the body is 0.5 kg & the radius r of the path is 1.6 m.

Sol: Work done by non-conservative force field



$W_{NC} = \Delta PE + \Delta KE$

$W_{NC} = -1 \times \underbrace{(r\theta)}_{F \times \text{distance}}$

$= -1 \times 1.6 \times 30 \times \frac{\pi}{180} = -0.8378 \text{ Nm}$

$\Delta PE = -mgh$

$= -0.5 \times 9.81 \times 0.59$

$= -2.894 \text{ Nm}$

$h = h_1 - h_2$

$= 1.6 \sin 60 - 1.6 \sin 30$

$= 1.39 - 0.8$

$= 0.59 \text{ m}$

$\sin 60 = \frac{h_1}{1.6}$

$\Delta KE = \frac{1}{2} m V^2$

$-0.8378 = -2.894 + \frac{1}{2} \times 0.5 V^2$

$V = 2.86 \text{ m/s}$