NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Mathematics, Winter Semester 2019-2020
MA1002D MATHEMATICS-II, Tutorial-6

A. Verify that the given functions form a basis of solution of the given differential equations and solve the initial values problems.
   1) $y'' + 9y = 0$, $y(0) = 4$, $y'(0) = -6$; cos 3x, sin3x 2) $4x^2 y'' - 3y = 0$, $y(1) = 3$, $y'(1) = 2.5$; $x^{-1/2}, x^{3/2}$.

B. Find a general solution of the following differential equations
   1) $8y'' - 2y' - y = 0$ 2) $y'' + 9y' + 20y = 0$ 3) $9y'' - 30y' + 25y = 0$ 4) $4y'' + 4y' - 3y = 0$.

C. Solve the initial value problems. Check that your answer satisfies the equation as well as the initial conditions
   1) $y'' + 4y' + 2y = 0$, $y(0) = -1$, $y'(0) = 2 + 3\sqrt{2}$ 2) $4y'' - 4y' - 3y = 0$, $y(-2) = e$, $y'(-2) = -e/2$.

D. Solve the following boundary values problems
   1) $y'' + 4y = 0$, $y(0) = 3$, $y\left(\frac{\pi}{2}\right) = -3$ 2) $y'' + 2y' + 2y = 0$, $y(0) = 1$, $y(\pi/2) = 0$.

E. Verify that y_p is a solution of the given equation. Solve the initial value problems
   1) $y'' - y = 2e^x$, $y(0) = -1$ $y'(0) = 0$, $y_p = x e^x$
   2) $x^2 y'' - 3xy' + 3y = 3\ln x - 4$, $y(1) = 0$, $y'(1) = 1$, $y_p = \ln(x)$.

F. If $y(x) = x^3$ is a solution of $x^2 y'' - 5xy' + 9y = 0$, using the method of reduction of order, find the general solution of the given differential equation.

G. Solve by the method of undermined coefficients.
   1) $y'' + 1.5y' - y = 12x^2 + 6x^3 - x^4$ 2) $y'' - 4y = e^{-2x} - 2x$
   3) $y'' + y' = 2 + 2x + x^2$, $y(0) = 8$, $y(0) = -1$ 4) $y'' + 4y' + 13y = 2e^{-x}$, $y(0) = 0$, $y'(0) = -1$
   5) $y'' + 9y = x^2 \cos(3x)$ 6) $y'' - y = e^x \sin(2x)$
   7) $y'' - 4y' + 4y = (6 + x^2)e^{-4x}$.

H. If $y1$ and $y2$ are two linearly independent solutions of $y'' + a y' + \mathrm{b} y = 0$, then show that the Wronskian of $y1$ and $y2$ is ce^{-ax}, c is any constant.

I. Solve by the method of variation of parameters
   1) $y'' - 4y' + 4y = e^{2x}/x$ 2) $2y'' + 2y = \operatorname{cosec} x$ 3) $y'' - 2y' + y = 3x^{3/2} e^x$
   4) $y'' - y = \frac{2}{1+e^x}$ 5) $y'' + 4y = 4\sec^2 2x$.

J. Solve
   1) $x^2 y'' - 4xy' + 6y = 21x^{-4}$ 2) $4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3$
   3) $x^2 y'' - 2xy' + 2y = x^3 \cos x$ 4) $x^2 y'' - 2xy' - 4y = x^2 + 2\log x$.

K. Construct a non-homogeneous second order differential equation with constant coefficients whose general solution is $y(x) = e^x(A\cos(2x) + B\sin(2x)) + e^x + x$.

L. Solve the following system of differential equations, where D=d/dt
   1) $Dx = y + 1$, $Dy = x + 1$ 2) $(D+5)x + (D+3)y = e^{-t}$, $(2D+1)x + (D+1)y = 3$
   3) $Dx = 3x + 8y$, $Dy = -x - 3y$; $x(0)=6$, $y(0) = -2$.

M. An inductor of 2 henries, resistor of 16 ohms and capacitor of 0.02 farads are connected in series with a battery of e.m.f E = 100sin33t. At t =0, the charge on the capacitor and current in the circuit are zero. Find the charge and current at time t.

N. A spring with a mass of 2 kg has natural length m. A force of 25.6 N is required to maintain it stretched to a length of 0.7m (The spring constant is 128 by Hook's law). If the spring is stretched to a length of 0.7m and then released with initial velocity 0, find the position of the mass at any time.

O. A 16 lb weight is suspended from a spring having spring constant 5 lb/ft. Assume that an external force given by 24 sin (10t) and a damping force with damping constant 4, are acting on the spring. Initially the weight is at rest at its equilibrium position. Find the position of the weight at any time. Find the steady state solution. Find the amplitude, period and frequency of the steady state solution. Determine the velocity of the weight at any time.