MODULE 1

Day 1

Introduction

What is Mechanics?

- physical science - greater role - oldest

Newtonian mechanics

Divisions of Engineering Mechanics < Statics - at rest / Dynamics < Kinetics / Kinematics ↳ ≠ forces

Units < primary or basic dimensions - L, T, M / secondary dimensions

Law of Dimensional Homogeneity

Idealizations of Mechanics

(1) Continuum

Pressure, density, temp - gross effects

(2) Rigid body

A continuum that undergoes theoretically no deformation

→ Accuracy of results

(3) Point force

infinitesimal area or point

(4) Particle

An object that has no size but that has a mass

Eg - trajectory of planet

skater spinning on ice

- Centre of mass

Eg - frictionless fluid

perfectly elastic body

⇒ Mechanics is the physical science concerned with the dynamical behavior of bodies that are acted on by mechanical disturbances

Day 2

Vector & scalar Quantities

→ Scalar - quantities which have magnitude only such as L, T, distance, ρ, temp, speed etc

→ Vector - quantities that have magnitude & direction & add according to parallelogram law.

Eg:- force, displacement, velocity, acceleration, momentum

Velocity ⟶

length of line segment - speed - how fast? magnitude

directed line segment - velocity - which way? < angular or toward or from

current?

Parallelogram law - commutative

If not, ≠ vector - Eg:- book along x & Z-axes

finite angle of rotation of body about an axis

Eg:- current - scalar quantity

Equality & Equivalence of vectors

→ 2 vectors are equal if they have the same dimensions, magnitude & direction.

→ 2 vectors are equivalent in a certain capacity if each produces the very same effect in capacity.

It depends on the situation at hand

Equality determined by vectors themselves & equivalence b/n 2 vectors is determined by involving the vectors

① Free vectors - anywhere in space, magnitude direction intact

② Transmissible vectors - moved along their line action

③ Bound vector – applied at definite points

Laws of Mechanics

① Newton's 1^{st} & 2^{nd} laws of motion
② Newton's 3^{rd} law
③ The gravitational law of attraction
④ The parallelogram law

Newton's 1^{st} & 2^{nd} Laws of motion

1: Every particle continues in a state of rest or uniform motion in a straight line unless it is compelled to change that state by forces imposed on it.

2: The change of motion is proportional to the natural force impressed & is made in a direction of the straight line in which the force is impressed.

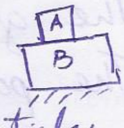
→ inertial references – fixed stars
– other system moving uniformly & without rotation relative to fixed stars
– earth's surface – error is small

Equilibrium – state of a body in which all its constituent particles are at rest or moving uniformly along a straight line relative to an inertial reference

→ realitivistic effects
laws of Newton becomes approximate as speed of body ↑ → speed of light
→ quantum mechanics

Newton's 3rd law:

To every action there is always opposed an equal reaction, or the mutual actions of 2 b[illegible] upon each other are always equal & 'directe[d] contrary points' → collinear



Eg:- gravitational attraction

electrostatic force b/n charged particles

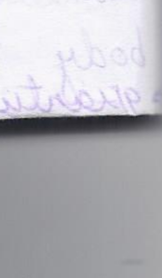
Law of Gravitational Attraction

2 particles will be attracted toward each other along their connecting line with a fo[rce] whose magnitude is directly proportional to product of the masses & inversely propor[tional] to the distance squared b/n the particles

$$F = G \frac{m_1 m_2}{r^2}$$ G- Gravitational con[stant]

Parallelogram law

If two forces, acting simultaneously on a parti[cle] be represented in magnitude & direction by the adjacent sides of a parallelogram; their resulta[nt] may be represented in magnitude & direction b[y] diagonal of the parallelogram; which passes thru point of intersection



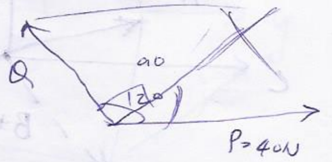
$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta}$$

1. 2 forces of 100 N & 150 N are acting simultaneously at a point. What is the resultant of these 2 forces, if angle b/n them is 45°?

Ans: $F_1 = 100\,N$ $F_2 = 150\,N$ $\theta = 45°$ $\Rightarrow$ $R = 232\,N$

2. Two forces act at an angle of 120°. The bigger force is of 40 N & resultant is $\perp^r$ to smaller one. Find the smaller force.

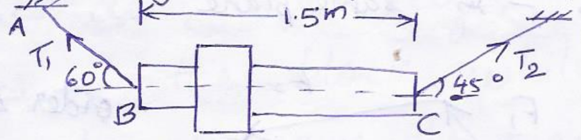


Ans: $\alpha = 120 - 90 = 30°$ $\theta = 120°$ $F_1 = 40\,N$

$$\tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2\cos\theta} \Rightarrow \underline{\underline{F_2 = 20\,N}}$$

Resolution of a force

3. A machine component 1.5m long & wt 1000N is supported by 2 ropes AB & CD as shown in Fig below



Calculate the tensions T_1 & T_2 in the ropes AB & CD.

Ans: $W = 1000\,N$ $T_1 \cos 60° = T_2 \cos 45° \Rightarrow T_1 = 1.414\, T_2$

$T_1 \sin 60° + T_2 \sin 45° = 1000$

$\underline{\underline{T_1 = 732.6\,N}}$ $\underline{\underline{T_2 = 518.1\,N}}$

Ex 2.6
4. Force A (10 N) & B (vertical) add up to a force C that has a magnitude of 20 N. What is the magnitude of force B & the direction of force C?

$B = 10\sqrt{3}\,N$

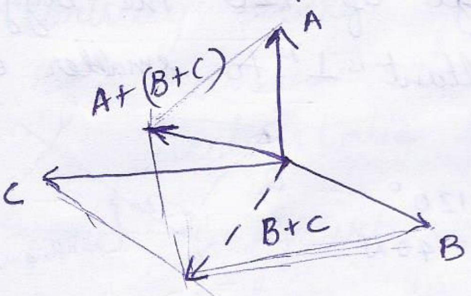
$\alpha = 60°$

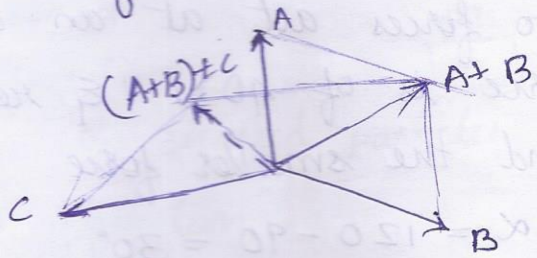
Day 3 Elements of Vector Algebra

① Magnitude & multiplication of a vector by a sca

Magnitude of $\vec{A} = |\vec{A}| \equiv A$ $\qquad (m\vec{A})$

② Addition & subtraction of Vectors



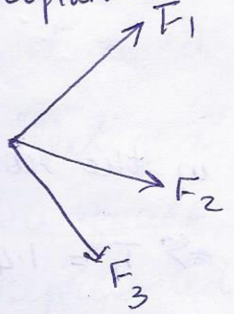


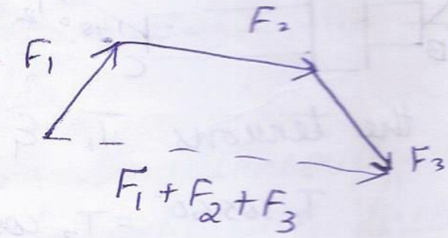
$A + (B + C) = (A + B) + C$ — commutative associative

* lines of action can be changed

Addition by "closing the polygon"

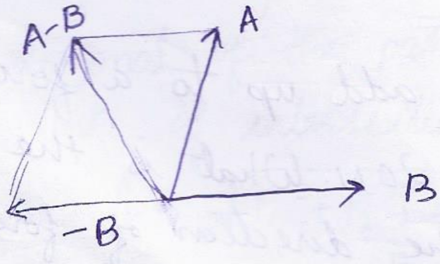
3 coplanar vectors - in same plane



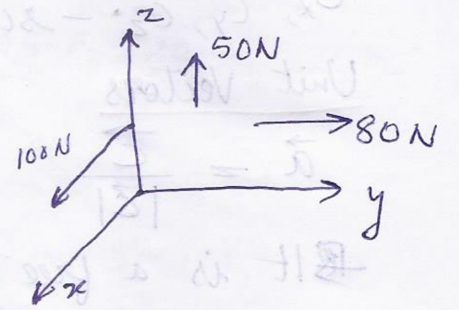


order is ≠ sig

Subtraction of vectors



2.16 Add the 3 vectors. The 100 N force is in xz plane, while the other 2 forces are ||el to yz plane & do not intersect. Give the magnitude of sum & the angle it forms with the x axis



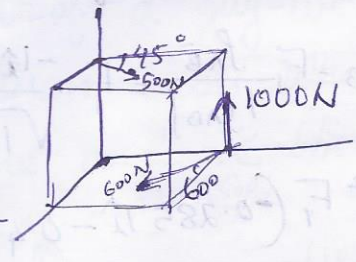
Sol: $R = \sqrt{X^2+Y^2+Z^2}$

$= \sqrt{100^2+50^2+80^2} = 137.5\,N$

$\cos\alpha = \frac{X}{R} = \frac{100}{137.5}$

$\alpha = 43.34^\circ$ with x-axis

2.4 3 forces act on the block. The 500 N & 600 N forces act, respectively, on the upper & lower faces of the block, while the 1000 N force acts along the edge. Give the magnitude of the sum of these forces.



Sol: $R = \sqrt{\left(\frac{500}{\sqrt{2}} - 600\sin 60\right)^2 + 1000^2 + \left(600\cos 60 + 500\sin 45\right)^2}$

$= 1206.1\,N$

Resolution of Vectors

Orthogonal or rectangular component vectors

Direction cosines - direction of a vector relative to an orthogonal reference is given by cosines of angles formed by the vector & respective coordinate axes

$l = \cos\alpha \; ; \; m = \cos\beta \; ; \; n = \cos\gamma$

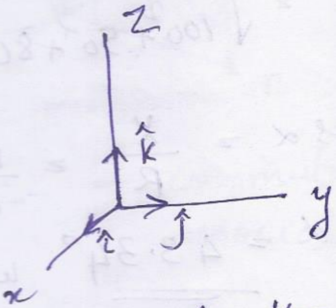
C_x, C_y, C_z - scalar quantities

Unit Vectors

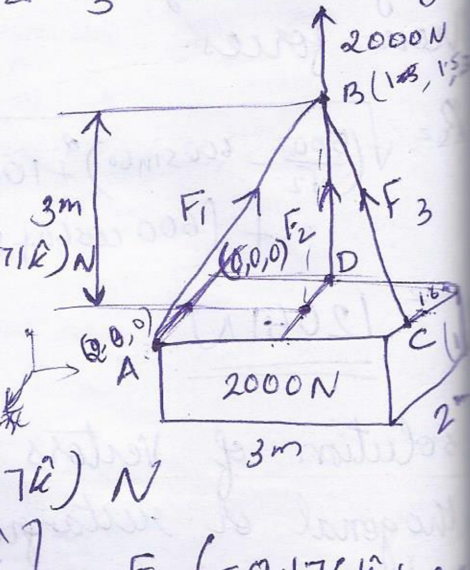
$$\vec{a} = \frac{\vec{C}}{|\vec{C}|} \quad \text{no dimensions}$$

It is a free vectors

$$C = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$$



1. A crane is supporting a 2000N crate thru 3 AB, CB & DB. Note that D is at the centre of the outer edge of the crate; C is 1.6m from corner edge; & B is directly above the center of the What are the forces F_1, F_2 & F_3 transmitted by



Sol: $F_{AB} = F_1 \frac{\rho_{AB}}{|\rho_{AB}|} = F_1 \frac{-1\hat{i} + 1.5\hat{j} + 3\hat{k}}{\sqrt{1^2 + 1.5^2 + 3^2}}$

$= F_1(-0.2857\hat{i} - 0.4286\hat{j} + 0.8571\hat{k})\,N$

$F_{DB} = F_2\left[\frac{1\hat{i} + 0\hat{j} + 3\hat{k}}{\sqrt{1^2 + 3^2}}\right]$

$= F_2(0.3162\,\hat{i} + 0.9487\hat{k})\;N$

$F_{CB} = F_3\left[\frac{-0.6\hat{i} - 1.5\hat{j} + 3\hat{k}}{\sqrt{0.6^2 + 1.5^2 + 3^2}}\right] = F_3(-0.1761\hat{i} + 0.

$F_{AB} + F_{OB} + F_{CB} = 2000\,\hat{k}$

$-0.2857\,F_1 + 0.3162\,F_2 - 0.1761\,F_3 = 0$

$+0.4286\,F_1 + 0 - 0.4402\,F_3 = 0$

$0.8571\,F_1 + 0.9487\,F_2 + 0.8805\,F_3 = 2000$

$F_1 = 648.1\ N$

$F_2 = 937\ N$

$F_3 = 631\ N$

Scalar or Dot Product of Two Vectors

Eg. $F \times d$ = work - scalar

$A \cdot B = AB\cos\alpha$ — commutative $A\cdot B = B\cdot A$; associative $A(B+C)$; distributive

$i \cdot j = 0 \qquad \alpha = 90°$

$i \cdot i = 1 \qquad \alpha = 0$

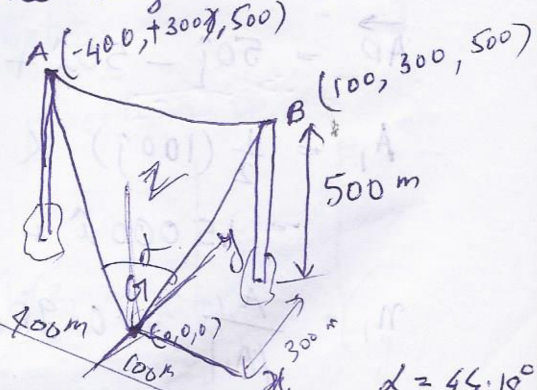
$A \cdot A = A^2$

$\hat{r} = (\hat{r}\cdot\hat{i})\,\hat{i} + (\hat{r}\cdot\hat{j})\,\hat{j} + (\hat{r}\cdot\hat{k})\,\hat{k}$

unit vec. $= l\,\hat{i} + m\,\hat{j} + n\,\hat{k}$

$\sqrt{l^2+m^2+n^2} = 1$

2. Cables GA & GB are part of a guy-wire system supporting 2 radio transmission towers. What are the lengths of GA & GB & the angle α b/n them?



Sol: $\vec{GA} = 300j - 400i + 500k$ m

$\vec{GB} = 300j + 100i + 500k$ m

$GA = \sqrt{300^2 + 400^2 + 500^2} = 707$ m

$GB = \sqrt{300^2 + 100^2 + 500^2} = 592$ m

$\cos\alpha = \dfrac{\vec{GA}\cdot\vec{GB}}{|GA||GB|} = \dfrac{90000 - 40000 + 250000}{707 \times 592} = 0.717$

$\alpha = 44.18°$

Cross Product of 2 Vectors

$\vec{A} \times \vec{B} = |A||B| \sin\alpha \; \hat{n}$

$\hat{n}$ - vector $\vec{C}$ has an orientation normal to the p
of the vectors $\vec{A}$ & $\vec{B}$.

$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ $\quad \neq$ commutative

$\vec{C} \times (A+B) = C \times A + C \times B$ - distributi

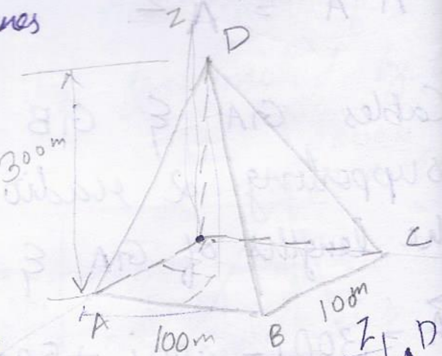
$i \times j = \hat{k} \quad \alpha = 90$ $\qquad i \times j = \hat{k}$

$i \times i = 0 \quad \alpha = 0$ $\qquad j \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ \hat{i} & \hat{j} & \hat{k} \end{vmatrix}$$

3. A pyramid is shown in Fig. If ht. of pyramid 300 m, find angle b/n outward normals to planes AD



Sol: Find unit normals to these planes
then dot product - desired angle

Area vector $A_1 = \frac{1}{2} \vec{AB} \times \vec{AD}$

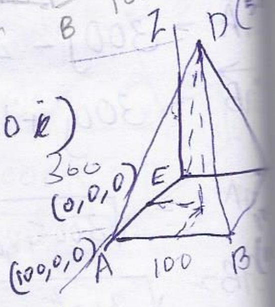
$\vec{AB} = 100\hat{j}$ m

$\vec{AD} = 50\hat{j} - 50\hat{i} + 300\hat{k}$ m

$A_1 = \frac{1}{2}(100\hat{j}) \times (-50\hat{i} + 50\hat{j} + 300\hat{k})$

$= 15000\hat{i} + 2500\hat{k} \quad m^2$

$n_1 = \frac{A_1}{|A_1|} = 0.9864\hat{i} + 0.1644\hat{k}$



$A_2 = \frac{1}{2} \vec{BC} \times \vec{BD}$

$\vec{BC} = -100\hat{i}$ m

$\vec{BD} = -50\hat{j} - 50\hat{i} + 300k$ m

$\vec{A}_2 = \frac{1}{2}(-100\hat{i}) \times (-50\hat{i} - 50\hat{j} + 300k) = 15000\hat{j} + 2500\hat{k}$ m^2

$n_2 = \frac{A_2}{|A_2|} = 0.9864\hat{j} + 0.1644\hat{k}$ $\quad \cos\beta = \frac{A_1 \cdot A_2}{|A_1| \cdot |A_2|}$

$\vec{n}_1 \cdot \vec{n}_2 = \cos\beta = 0.027 \Rightarrow \beta = 88.5°$

Scalar Triple Product

$(\vec{A} \times \vec{B}) \cdot \vec{C} \rightarrow$ scalar product $= \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$

Vector triple product $A \times (B \times C)$

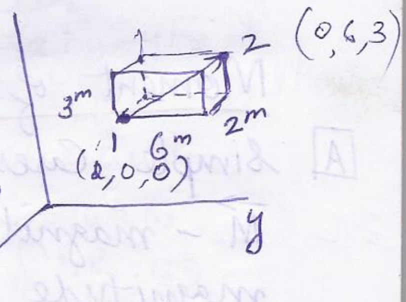
$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$

Day 3

Important Vector Quantities

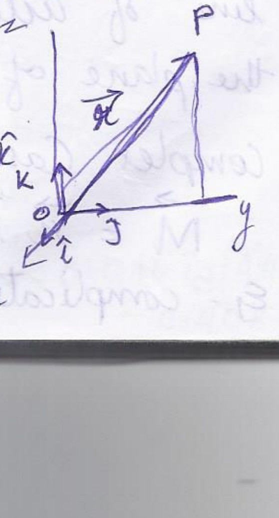
Position vector

→ Displacement vector – shortest movement of the particle to get from one position on the path of motion to another



$\rho_{12} = -2\hat{i} + 6\hat{j} + 3\hat{k}$ m

→ The directed line segment $\vec{r}$ from the origin of a coordinate system to a point P in space – position vector.



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\rho = r_2 - r_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

1. 2 sets of references, xyz & XYZ are show
Fig. The position vector of the origin O of xyz
relative to XYZ is given as

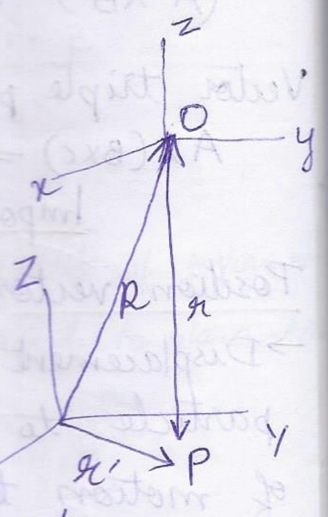
$$\vec{R} = 10\hat{i} + 6\hat{j} + 5\hat{k} \text{ m}$$

The position vector, $\vec{r}'$, of a point P relative
XYZ is $\vec{r}' = 3\hat{i} + 2\hat{j} - 6\hat{k}$ m
What is the position vector $\vec{r}$ of point P rel
to xyz? What are the coordinates x, y, z of

Sol: $\vec{r}' = \vec{R} + \vec{r}$

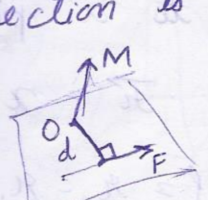
$\vec{r} = \vec{r}' - \vec{R} = -7\hat{i} - 4\hat{j} - 11\hat{k}$ m

$x = -7$ m ; $y = -4$ m ; $z = -11$ m



Moment of a Force about a Point

[A] Simple Cases
$\vec{M}$ - magnitude equals the product of the force
magnitude times the $\perp^r$ distance d from O to
line of action of force. & the direction is $\perp^r$
the plane of the point & the force

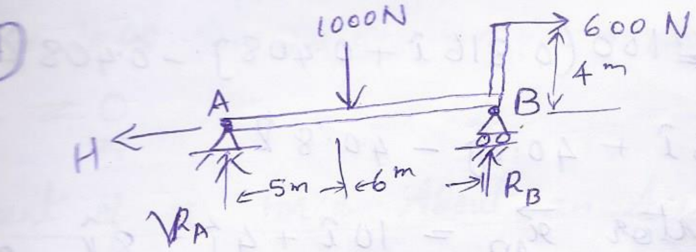


[B] Complex Cases

$$\vec{M} = \vec{r} \times \vec{F} = rF\sin\alpha$$

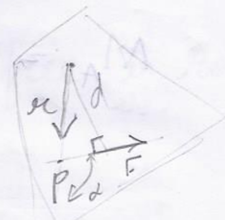
Eg:- complicated coplanar cases & for 3-d cases

Ex: (A)



$M_A = -5 \times 1000 + R_B \times 11 - 600 \times 4$

$= 11 R_B - 7400 \quad Nm$



Varignon's Theorem

Sum of the moments about a point of a system of concurrent forces is the same as the moment about the point of the sum of the forces.

$M = M_1 + M_2 + M_3 + \dots + M_n$

$= r \times F_1 + r \times F_2 + r \times F_3 + \dots + r \times F_n$

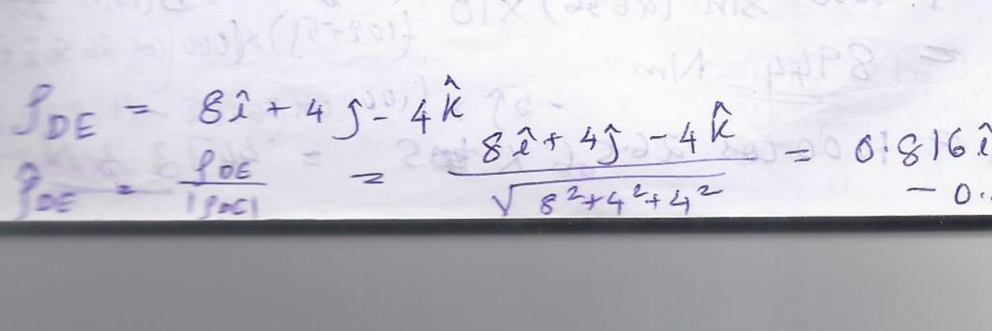
$= r \times (F_1 + F_2 + F_3 + \dots + F_n)$

$M = r \times F = r \times (F_x \hat{i} + F_y \hat{j} + F_z \hat{k}) \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Here F is a transmissible vector

$M = (yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$

2. Determine the moment of the 100N force F, shown in Fig, about points A & B respectively



Sol: $\rho_{DE} = 8\hat{i} + 4\hat{j} - 4\hat{k}$

$\hat{\rho}_{DE} = \frac{\rho_{DE}}{|\rho_{DE}|} = \frac{8\hat{i} + 4\hat{j} - 4\hat{k}}{\sqrt{8^2 + 4^2 + 4^2}} = 0.816\hat{i} + 0.408\hat{j} - 0.408\hat{k}$

$$\vec{F} = F\,\rho_{DE} = 100(0.816\hat{i} + 0.408\hat{j} - 0.408\hat{k})$$

$$= 81.6\hat{i} + 40.8\hat{j} - 40.8\hat{k}$$

Position vector $\vec{r}_{AD} = 10\hat{i} + 4\hat{j} - 8\hat{k}$ m

$$M_A = r_{AD} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 4 & -8 \\ 81.6 & 40.8 & -40.8 \end{vmatrix}$$

$$= (4 \times -40.8 + 8 \times 40.8)\hat{i} - \hat{j}(10 \times -40.8 + 8 \times \ldots$$

$$+ \hat{k}(10 \times 40.8 - 4 \times 81.6)$$

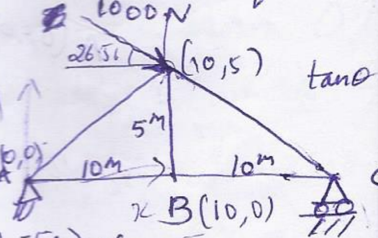
$$= \underline{163.2\hat{i} - 245\hat{j} + 81.6\hat{k} \text{ Nm}}$$

$$r_{BD} = 4\hat{j} - 8\hat{k} \text{ m}$$

$$M_B = r_{BD} \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & -8 \\ 81.6 & 40.8 & -40.8 \end{vmatrix}$$

$$= \underline{163.2\hat{i} - 653\hat{j} - 326\hat{k} \text{ Nm}}$$

3.16 Compute the moment of the 1000 N force a… points A, B & C.



tanθ …

$r \times F$

Sol: $M_A = 1000\cos(26.56) \times 5$

$+ 1000\sin(26.56) \times 10$

$= \underline{8944}$ Nm

$(10\hat{i} + 5\hat{j}) \times 1000[\cos 26.56\,\hat{i} - \sin \ldots$

$-5\hat{j} \times (1000\cos 26.56\,\hat{i} - 1000\sin 26.5\ldots$

$M_B = 1000\cos 26.56 \times \cancel{10}\,5 = \underline{4472.3 \text{ Nm}}$

$(10\hat{i} - 5\hat{j}) \times 1000(\cos 26.56\hat{i} - \sin 26.56\hat{j})$

$M_c = 1000 \cos 26.56 \times 5 - 1000 \sin 26.56 \times 10$

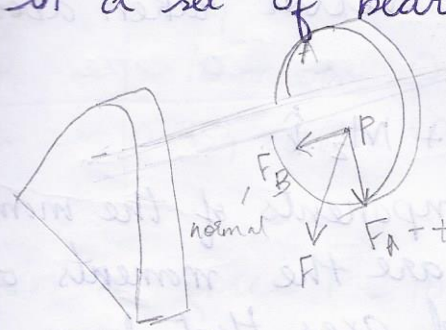
Ex 3.18 $= \underline{\underline{0}}$

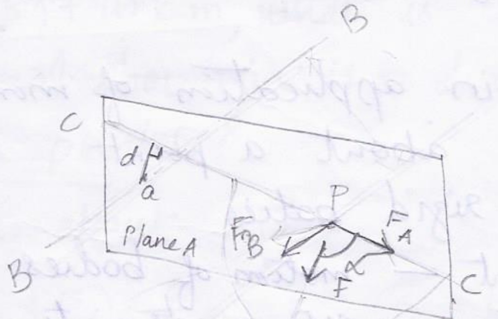
Day 5

Moment of a Force About an Axis

A For Simple Cases

Eg:- disc mounted on shaft that is free to rotate in a set of bearings.





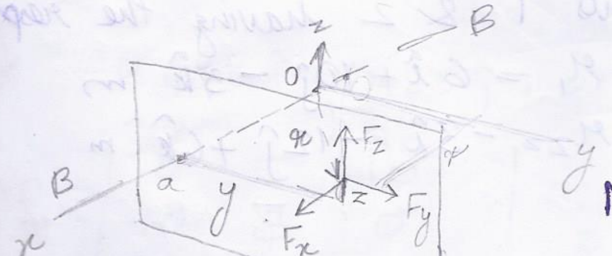
Moment about axis B-B

Moment about axis B-B

$= F_A d = |F| \cos\alpha \, d$

→ Moment about an axis is a <u>scalar</u>

B For complex cases



$M_x = yF_z - zF_y$

Moment about origin O

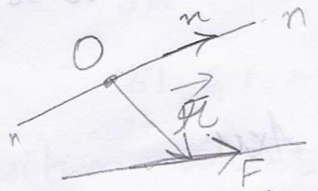
$M = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$

$= \vec{r} \times \vec{F}$

$= (yF_z - zF_y)\hat{i} + (zF_x - xF_z)\hat{j}$

$+ (xF_y - yF_x)\hat{k}$

Moment about x-axis $= M_x = M_0 \cdot \hat{i} = (r \times F) \cdot \hat{i}$



$$\boxed{M_n = (r \times F) \cdot n}$$

Scalar t[...] produ[...]

* The moment of a force about an axis equa[...] the scalar component in the direction of th[...] axis of the moment vector taken about an[...] point along the axis.

$$M = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

* The 3 orthogonal components of the moment of [...] force about a point are the moments of this [...] about the 3 orthogonal axes that have the po[...] as an origin.

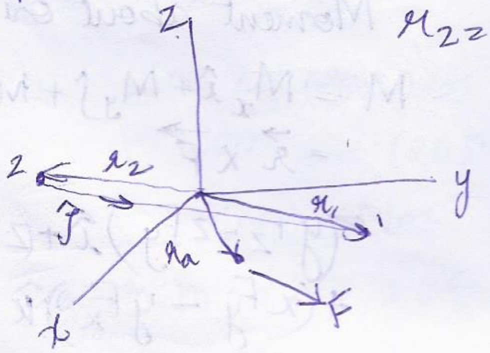
Physical differences in application of moments ab[...] an axis & moments about a point

Eg:- dynamics of rigid bodies

Moment about point — motion of bodies that ha[...] no constraints - missiles [...]

1. compute the moment of a force $F = 10\hat{i} + 6\hat{j}$ N, wh[...] goes through position $r_a = 2\hat{i} + 6\hat{j}$ m, about a li[...] going through points 1 & 2 having the respective position vectors $r_1 = 6\hat{i} + 10\hat{j} - 3\hat{k}$ m

$r_2 = -3\hat{i} - 12\hat{j} + 6\hat{k}$ m



Sol: $M_\rho = [(r_a - r_1) \times F] \cdot \hat{\rho}$

$r_a - r_1 = -4\hat{i} - 4\hat{j} + 3\hat{k}$ m

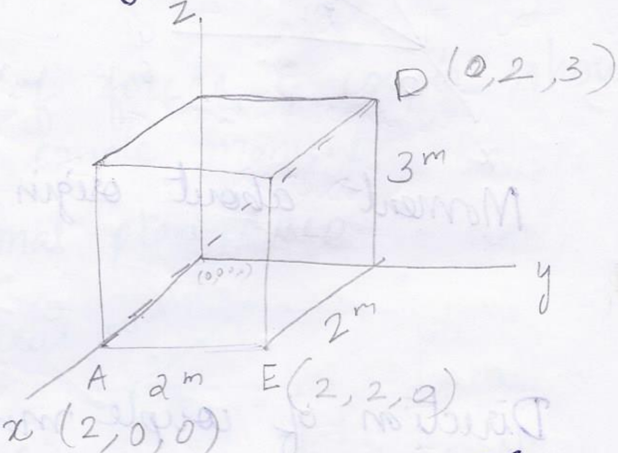
$F = 10\hat{i} + 6\hat{j}$

$$\hat{\rho} = \frac{r_1 - r_2}{|r_1 - r_2|} = \frac{9\hat{i} + 22\hat{j} - 9\hat{k}}{\sqrt{81+484+81}} = 0.354\hat{i} + 0.866\hat{j} - 0.35\hat{k}$$

$$M_\rho = \begin{vmatrix} -4 & -4 & 3 \\ 10 & 6 & 0 \\ 0.354 & 0.866 & -0.354 \end{vmatrix} = \underline{13.94 \text{ Nm}}$$ - clockwise about line 2-1

3.28 Given a force $\vec{F} = 10\hat{i} + 3\hat{j}$ N acting at position $\vec{r} = 5\hat{j} + 10\hat{k}$ m, what is the torque about the diagonal shown in the diagram? What is the moment about point E?

Sol: Torque about AD



$= (r \times F) \cdot \hat{\rho}$

$$\hat{\rho}_{AD} = \frac{-2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{2^2 + 2^2 + 3^2}}$$

$= -0.485\hat{i} + 0.485\hat{j} + 0.728\hat{k}$ $\quad \vec{r} = 2\hat{i}$

$$T_{AD} = \begin{vmatrix} -2 & 5 & 10 \\ 10 & 3 & 0 \\ -0.485 & 0.485 & 0.728 \end{vmatrix} = \begin{matrix} 22.282 \\ \cancel{26.65} \text{ Nm} \end{matrix} \quad 22.295 \text{ Nm}$$

$\vec{r}_{EF} = -2\hat{i} + 3\hat{j} + 10\hat{k}$

$$M_E = \vec{r} \times F = \begin{vmatrix} \hat{i} & \hat{j} & k \\ -2 & 3 & 10 \\ 10 & 3 & 0 \end{vmatrix}$$

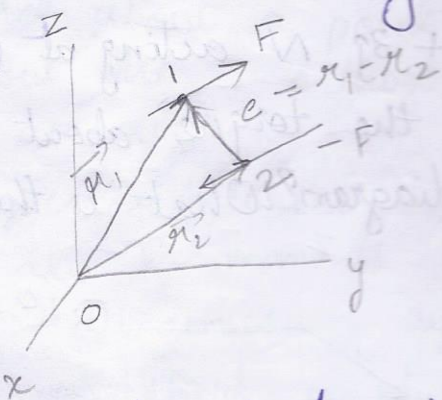
$$= \hat{i} \times -30 - \hat{j}(-100) + \hat{k}(-6-30)$$

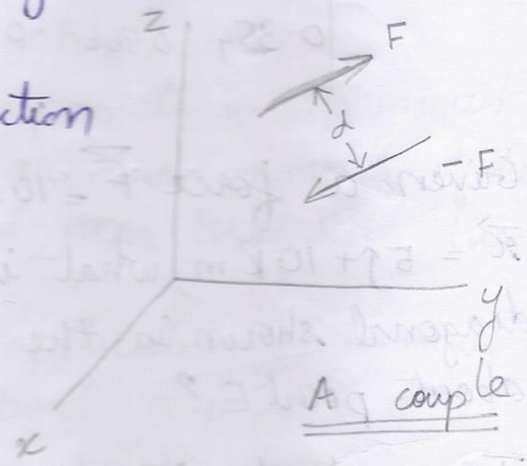
$$= (-30\hat{i} + 100\hat{j} - 36\hat{k}) \text{ Nm}$$

Couple & Couple Moment

A couple is formed by any two equal $\parallel^{el}$ forces have opposite senses.

- related to turning action





A couple

Moment about origin $O, M = r_1 \times F + r_2 \times -F$

$$= (r_1 - r_2) \times F$$

$$= e_1 \times F$$

Direction of couple moment $\rightarrow$ right hand screw

$$|M| = |e_1| |F| \sin 90 = |F| d$$

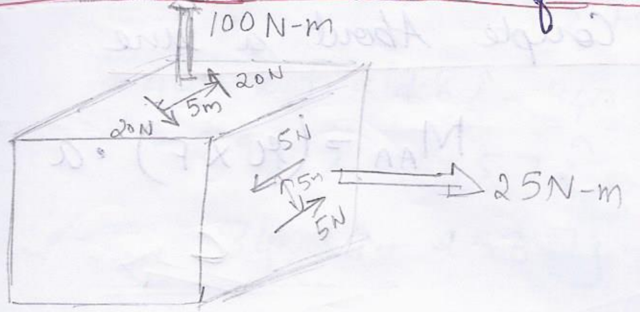
Position of point O ∉ involved → couple has same m about every pt i

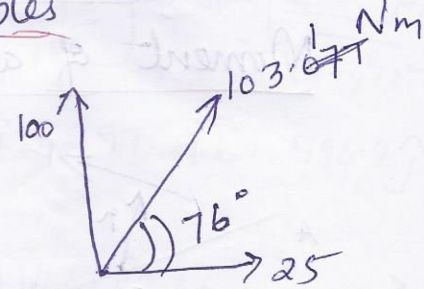
The Couple Moment as a Free Vector

The couple has the same moment about ever point in space.

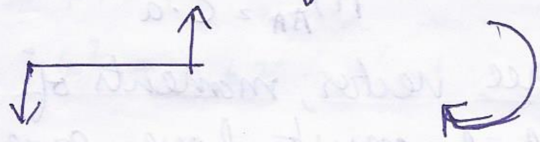
Move the couple in its own or ||el plane
$-F\uparrow, d\downarrow - |F|d$ remains same.
Couple represented by moment - magnitude & direction

Addition & Subtraction of couples





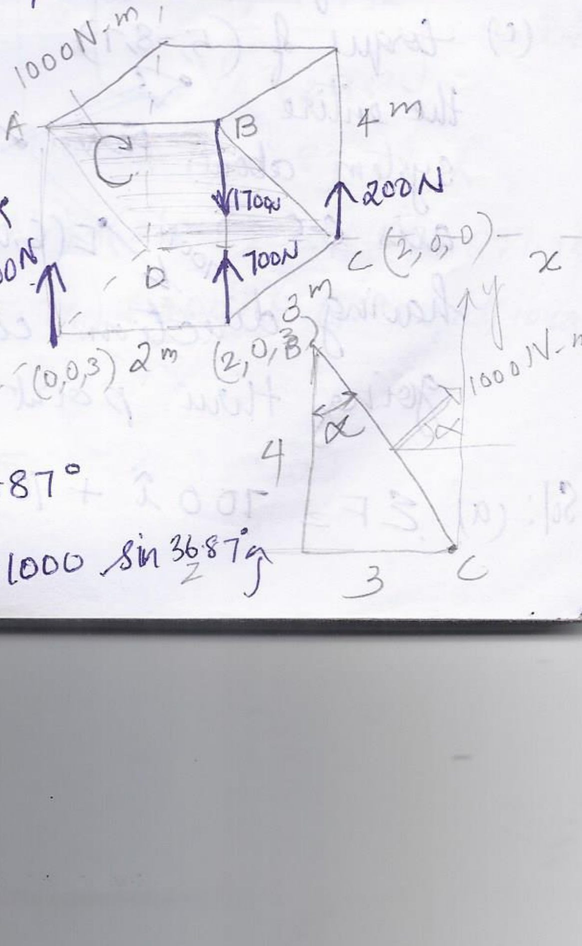
Representation of couple moments in a plane



2. Replace the system of forces & couple shown in Fig by a single couple moment.

Sol: 1000 N-m is in diagonal plane ABCD.

$F_y = +1700\text{ N}, -1700\text{ N}$ } not collinear



$C_1 = 3\hat{k} \times 800\hat{j} + (3\hat{k}+2\hat{i}) \times (700-1700)\hat{j} + 2\hat{i} \times 200\hat{j}$

$= (600\hat{i} - 1600\hat{k})$ N-m

$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.87°$

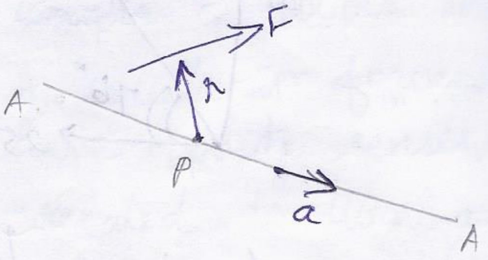
$C_2 = -1000\cos 36.87\,\hat{k} + 1000\sin 36.87\,\hat{j}$

$= -800\hat{k} + 600\hat{j}$ N-m

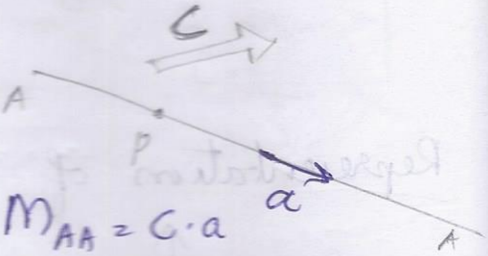
$C_{total} = C_1 + C_2 = (600\hat{i} - 1600\hat{k}) + (-800\hat{k} + 600\hat{j})$

$= (600\hat{i} + 600\hat{j} - 2400\hat{k})$ N-m

Moment of a Couple About a line



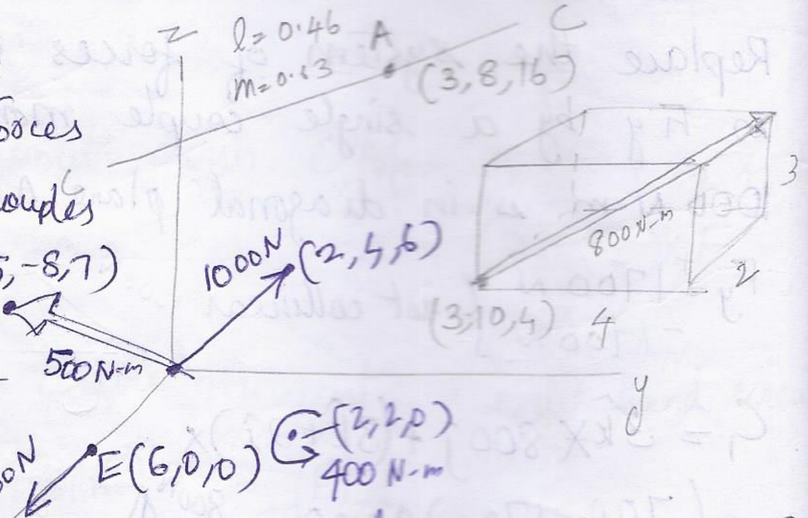
$M_{AA} = (r \times F) \cdot a$



$M_{AA} = C \cdot a$

Since C is a free vector, moments of C about all lines $\parallel^{el}$ to A-A must have same value

3 In Fig, find
(a) sum of the forces
(b) sum of the couples
(c) torque of the entire system about axis C-C having direction cosines $l = 0.46$ & $m = 0.63$ going thru point A



Sol: (a) $\Sigma F = 700\hat{i} + 1000\left[\dfrac{2\hat{i} + 4\hat{j} + 6\hat{k}}{\sqrt{2^2 + 4^2 + 6^2}}\right]$

$$= 700\hat{i} + 267.3\hat{i} + 534.5\hat{j} + 801.8\hat{k}$$

$$\Sigma F = 967.3\hat{i} + 534.5\hat{j} + 801.8\hat{k} \ N$$

(b) $$\Sigma C = 400\hat{k} + 500\left[\frac{5\hat{i} - 8\hat{j} + 7\hat{k}}{\sqrt{5^2+8^2+7^2}}\right] + 800\left[\frac{-2\hat{i}+4\hat{j}+3\hat{k}}{\sqrt{2^2+4^2+3^2}}\right]$$

$$= 400\hat{k} + 212.8\hat{i} - 340.5\hat{j} + 297.9\hat{k} + 594.2\hat{j} - 297.1\hat{i} + 445.7\hat{k}$$

$$\Sigma C = -84.3\hat{i} + 253.7\hat{j} + 1144\hat{k} \ N\text{-}m$$

(c) M_{cc}

$$l^2 + m^2 + n^2 = 1$$

$$0.46^2 + 0.63^2 + n^2 = 1 \Rightarrow n = 0.6257$$

$$\hat{c} = 0.46\hat{i} + 0.63\hat{j} + 0.6257\hat{k}$$

$$M_{cc} = \left[(r_E - r_A) \times 700\hat{i}\right]\cdot\hat{c} + \left[(0 - r_A) \times 1000\frac{2\hat{i}+4\hat{j}+6\hat{k}}{\sqrt{2^2+4^2+6^2}}\right]\cdot\hat{c} + \left(500\frac{5\hat{i}-8\hat{j}+7k}{\sqrt{5^2+8^2+7^2}}\right)\cdot\hat{c} + \left(800\frac{4\hat{j}-2\hat{i}+3k}{\sqrt{4^2+2^2+3^2}}\right)\cdot\hat{c} + 400\hat{k}\cdot\hat{c}$$

$$M_{cc} = \{(6\hat{i} - 3\hat{i} - 8\hat{j} - 16k) \times 700\hat{i} + (-3\hat{i} - 8\hat{j} - 16k) \times (267.3\hat{i} + 534.5\hat{j} + 801.8\hat{k}) + (212.8\hat{i} - 340.5\hat{j} + 297.9\hat{k}) + (594.2\hat{j} - 297.1\hat{i} + 445.7\hat{k}) + 400\hat{k}\}\cdot(.46\hat{i} + 0.63\hat{j} + 0.6257\hat{k})$$

$$M_{cc} = -2576 \ Nm$$

HW
3.72