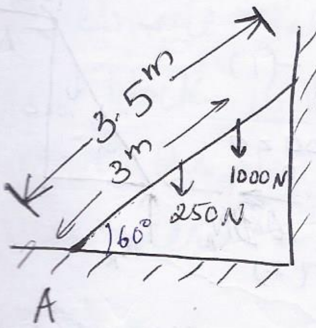
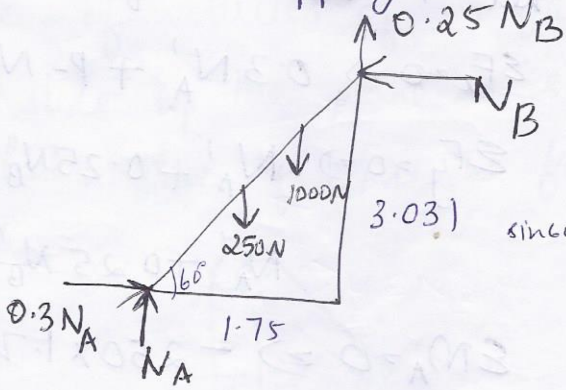
4. A ladder AB ~~is~~ 3.5 m long & 250 N in wt. is placed against the wall touching at B. It makes 60° with the floor at point A. If coefficients of friction b/n wall & ladder is 0.25 & that b/n floor & ladder is 0.3, investigate the eqlbm of the ladder when supporting a vertical 1000N force at a distance of 3m measured along the ladder. Find the horizontal force, if any, reqd to be applied at A to prevent slipping.

Sol:





$\sin 60 = \frac{x}{3.5}$

$\Sigma F_x = 0 \Rightarrow +0.3N_A - N_B = 0 \quad -(1)$

$\Sigma F_y = 0 \Rightarrow N_A + 0.25N_B - 250 - 1000 = 0$

$N_A + 0.25N_B = 1250 \quad -(2)$

$N_A = 1162.79\,N \qquad N_B = 348.84\,N$

2 ways:

block (1) $\Sigma M_A = N_B \times 3.031 + 0.25N_B \times 1.75 - 1000 \times 3 \cos 60$

$- 250 \times \frac{3.5}{2} \cos 60$

$= -508.8\ Nm \neq 0 \Rightarrow$ system unstable

not in eqlbm

② Let x be distance along ladder upto w
~~man can climb~~ load can be placed without ladder slipping.

$$\Sigma M_A = 0 \Rightarrow N_B \times 3.031 + 0.25 N_B \times 1.75 - 250 \times \frac{3.5}{2} \cos$$

$$-1000 \times x \cos 60 = 0$$

$$\cancel{10}500x = 991.2$$

$$x = \underline{1.98m}$$

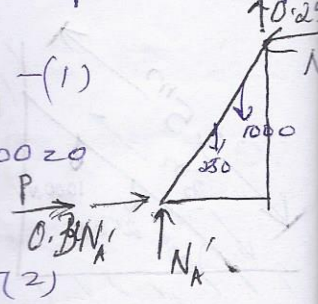
∴ Ladder will be slip.

Let P be hor. force at A to prevent slipp

$$\Sigma F_x = 0 \Rightarrow 0.3 N_A' + P - N_B' = 0 \quad -(1)$$

$$\Sigma F_y = 0 \Rightarrow N_A' + 0.25 N_B' - 250 - 1000 = 0$$

$$N_A' + 0.25 N_B' = 1250 \quad -(2)$$



$$\Sigma M_A = 0 \Rightarrow -250 \times 1.75 \cos 60 - 1000 \times 3 \cos 60 +$$

$$+ 0.25 N_B' \, 3.5 \cos 60 = 0$$

$$N_B' = \frac{1718.75}{3.5 \sin 60 + 0.25 \times 3.5 \cos 60} = \underline{\underline{495.52}}$$

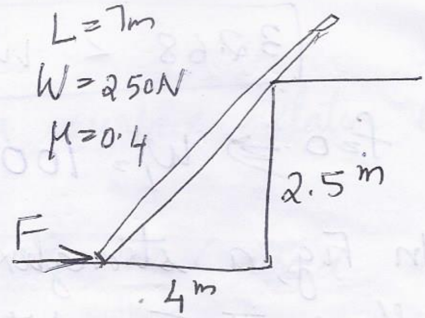
$$N_A' = \underline{\underline{1126.12 \, N}}$$

$$P = 495.52 - 0.3 \times 1126.12 = \underline{\underline{157.68 \, N}}$$

HW
7.30

$N_2 \times 4.717 = 250 \times 2.967 = 0$

$N_2 = \underline{157.3\,N}$



$N_1 + N_2 \cos 32 - 0.4 N_2 \sin 32 + 250 = 0$

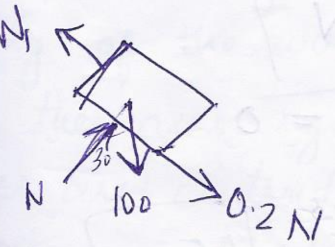
$N_1 = 150 N$

$F - 0.4 N_1 - N_2 \sin 32 - 0.4 N_2 \cos 30 = 0$

$F = \underline{196.7 N}$

7.34 Determine the range of values of W_1 for which the block will either slide up the plane or slide down the plane. At what value of W_1 is the friction force zero? $W_2 = 100$ lb

Sol: (a) slide up



$W_1 - 0.2N - 100 \sin 30 = 0$

$N = 100 \cos 30 = 50\sqrt{3} = 86.6$ lb

$W_1 = \underline{67.32 \text{ lb}}$

(b) slide down

$N = 86.6$ lb

$W_1 + 0.2N - 100 \sin 30 = 0$

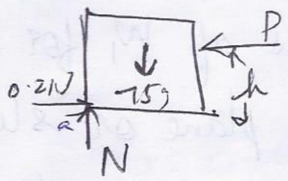
$W_1 = 32.68$ lb

$$\boxed{32.68 < W_1 < 67.32}$$

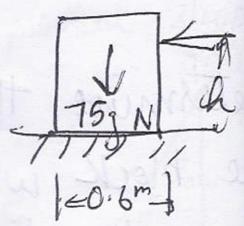
$f = 0 \Rightarrow W_1 = 100 \sin 30 = \underline{50\ lb}$

5. In Fig, a strongbox of mass 75 kg rests on floor. The static coefficient of friction for contact surface is 0.20. What is the largest P. & what is the highest position h for applying this force that will not allow the strongbox to slip on the floor or to tip?

Sol:



impending tipping



$\Sigma F_y = 0 \Rightarrow N = 75g = 735.75 N$

$\Sigma F_x = 0 \Rightarrow P = 0.2N = \boxed{147\ N}$

$\Sigma M_a = 0 \Rightarrow -75g \times 0.3 + P h = 0$

$$h = \frac{735.75 \times 0.3}{147} = \boxed{1.5\ m}$$

Day 5

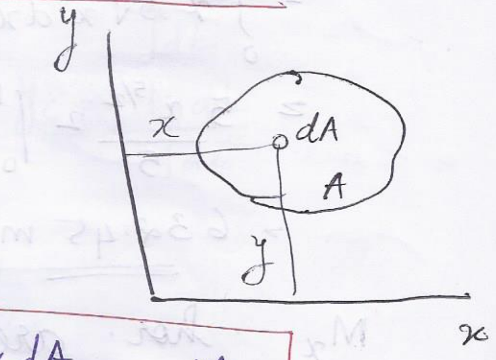
Properties of Surfaces

→ shape & disposition of a surface relative to some reference

First Moment of an Area & the Centroid

$M_x = \int_A y\,dA$

$M_y = \int_A x\,dA$



Centroid - $\bar{x}, \bar{y}$

$A y_c = \int_A y\,dA$

$$\boxed{y_c = \frac{M_x}{A}}$$

$$\boxed{x_c = \frac{\int_A x\,dA}{A} = \frac{M_y}{A}}$$

The location of the centroid of an area is independent of the reference axes. It is a property only of the area itself.

If the axes xy have their origin at centroid, these axes are centroidal axes.

First moments of an area about any of its centroidal axes must be zero.

1. A plane surface is shown in Fig bounded by the x axis, the curve $y^2 = 25x$, & a line ||el to the y axis. What are the first moments of the area about x & y axes & what are the centroidal coordinates?

Sol: Using vertical infinitesimal area elements of width dx & ht y. $y = 5\sqrt{x}$

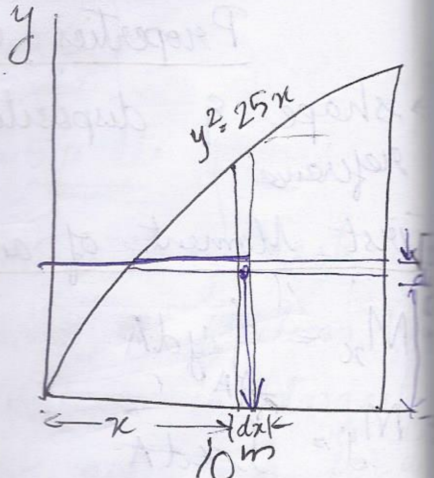
$$M_y = \int_A x\,dA$$

$$= \int_0^{10} x\,y\,dx$$

$$= \int_0^{10} x\,5\sqrt{x}\,dx$$

$$= \frac{5\,x^{5/2}}{5}\,2\,\Big|_0^{10}$$

$$= \underline{\underline{632.45\ m^3}}$$



M_x -- hor. area elements of width dy & length $x = \frac{y^2}{25}$

$$M_x = \int_0^{\sqrt{250}} y\,(10-x)\,dy = \int_0^{\sqrt{250}} \left(10y - \frac{y^3}{25}\right) dy$$

$$= \frac{10y^2}{2} - \frac{y^4}{100}\Big|_0^{\sqrt{250}} = \underline{\underline{625\ m^3}}$$

Using vertical strips,

$$M_x = \int_0^{10} \frac{y}{2}\,y\,dx = \int_0^{10} \frac{25x}{2}\,dx = \frac{25}{2}\,\frac{x^2}{2}\Big|_0^{10}$$

To compute (x_c, y_c)

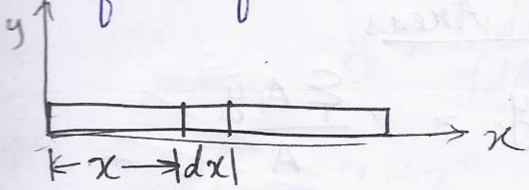
$$A = \int_0^{10} y\,dx = \int_0^{10} 5\sqrt{x}\,dx = 5\,\frac{x^{3/2}\times 2}{3}\Big|_0^{10} = 105.4$$

$$x_c = \frac{M_y}{A} = \frac{632.45}{105.4} = \underline{\underline{6m}}$$

$$y_c = \frac{M_x}{A} = \frac{625}{105.4} = \underline{\underline{5.93m}}$$

Centre of Gravity by Geometrical considerations

① CG of uniform rod is at its middle point



$$x_c = \frac{\int x\,dL}{\int dL}$$

$$y_c = \frac{\int y\,dL}{\int dL}$$

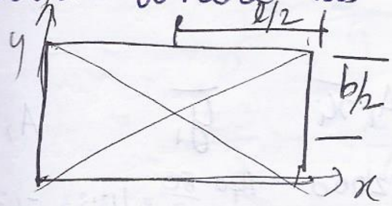
$dL = dx$

$x = x$

$$x_c = \frac{\int_0^L x\,dx}{\int_0^L dx} = \frac{L^2}{2L} = \frac{L}{2}$$

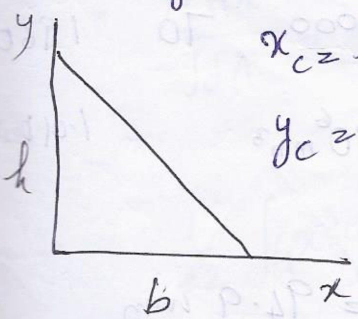
$y_c = 0$

② CG of a rectangle (or parallelogram) is at the point where its diagonals meet each other.



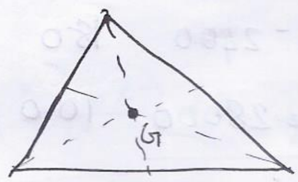
$$x_c = \frac{l}{2} \qquad y_c = \frac{b}{2}$$

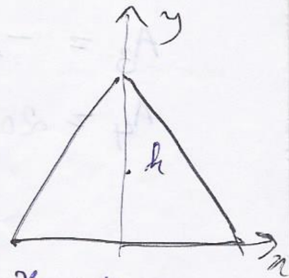
③ CG of triangle is at the point where 3 medians of triangle meets.



$$x_c = \frac{b}{3}$$

$$y_c = \frac{h}{3}$$

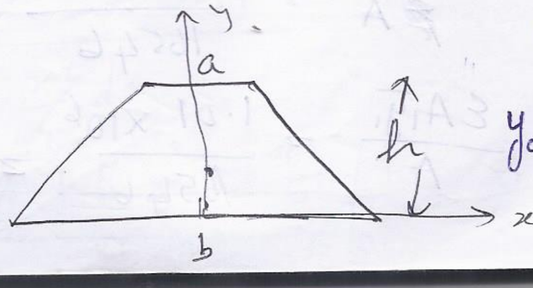




$x_c = 0$

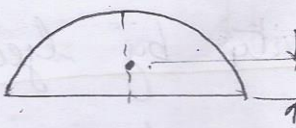
$$y_c = \frac{h}{3}$$

④ Trapezium



$$y_c = \frac{h}{3}\frac{(b+2a)}{b+a}$$

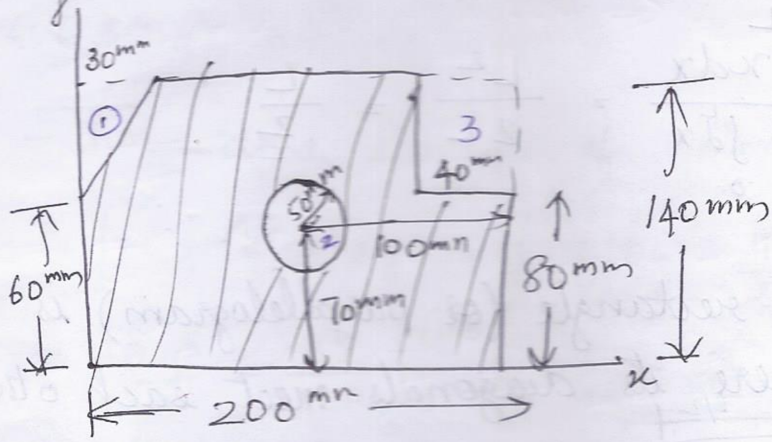
(5) Semi circle



Centroid of Composite Areas

$$x_c = \frac{\sum_i A_i \bar{x}_i}{A} \qquad y_c = \frac{\sum_i A_i \bar{y}_i}{A}$$

2. Find the centroid of the shaded section shown



Sol

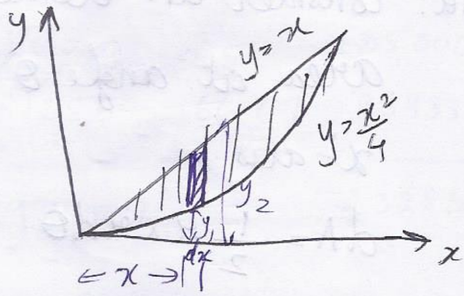
A_i	$\bar{x}_i$	$A_i \bar{x}_i$	$\bar{y}_i$	A_i
$A_1 = -\frac{1}{2} \times 30 \times 80 = -1200$	$\frac{30}{3} = 10$	-12000	$140 - \frac{80}{3} = 113.33$	-1
$A_2 = -\pi \times 50^2 = -7854$	100	-785400	70	-54
$A_3 = -40 \times 60 = -2400$	180	-432000	110	-26
$A_4 = 200 \times 140 = 28000$	100	2800000	70	1960
$A = 16546\ mm^2$		$1.571 \times 10^6\ mm^3$		1.01×10

$$x_c = \frac{\sum A_i \bar{x}_i}{A} = \frac{1.571 \times 10^6}{16546} = 94.9\ mm$$

$$y_c = \frac{\sum A_i \bar{y}_i}{A} = \frac{1.01 \times 10^6}{16546} = 61.1\ mm$$

3. Locate the centroid of the shaded area b/n the line $y = x$ & curve $y = x^2/4$.

Sol: $x_c = \frac{\int x\,dA}{\int dA}$ $y_c = \frac{\int y\,dA}{\int dA}$



$y' = y_2 - y_1 = x - \frac{x^2}{4}$

$dA = y'\,dx = \left(x - \frac{x^2}{4}\right)dx$

$y = x = \frac{x^2}{4}$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$, $x = 4$

$x = x$

$y = \frac{y'}{2} + y_1 = \frac{x - \frac{x^2}{4}}{2} + \frac{x^2}{4} = \frac{4x - x^2}{8} + \frac{2x^2}{8} = \frac{x^2 + 4x}{8}$

$$x_c = \frac{\int_0^4 x\left(x - \frac{x^2}{4}\right)dx}{\int_0^4 x - \frac{x^2}{4}\,dx} = \frac{\int_0^4 x^2 - \frac{x^3}{4}\,dx}{\int_0^4 x - \frac{x^2}{4}\,dx}$$

$$= \frac{\frac{x^3}{3} - \frac{x^4}{16}\Big|_0^4}{\frac{x^2}{2} - \frac{x^3}{12}\Big|_0^4} = \underline{\underline{2}} \text{ units}$$

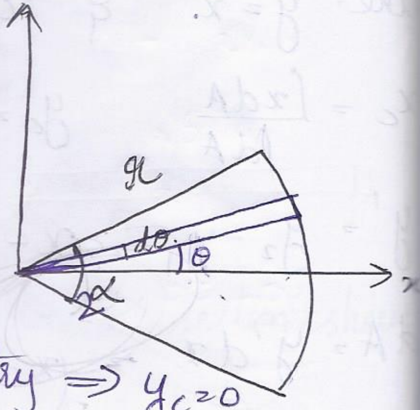
$$y_c = \frac{\int y\,dA}{\int dA} = \frac{\int_0^4 \left(\frac{x^2 + 4x}{8}\right)\left(x - \frac{x^2}{4}\right)dx}{\int_0^4 x - \frac{x^2}{4}\,dx}$$

$$= \frac{\frac{1}{32}\int 4x^3 + 16x^2 - x^4 - 4x^3\,dx}{\int x - \frac{x^2}{4}\,dx}$$

$$= \frac{\frac{1}{32}\left[x^4 + \frac{16x^3}{3} - \frac{x^5}{5}\right]_0^4}{\frac{x^2}{2} - \frac{x^3}{12}} = \frac{8}{5} = 1.6 \text{ units}$$

4. Find CG of lamina in form of circular se[ctor]

Sol: Consider an elementary area at angle θ from x-axis.



$dA = \frac{1}{2} r \times r d\theta$

x-axis is line of symmetry $\Rightarrow y_c = 0$

$$x_c = \frac{\int x\, dA}{\int dA}$$

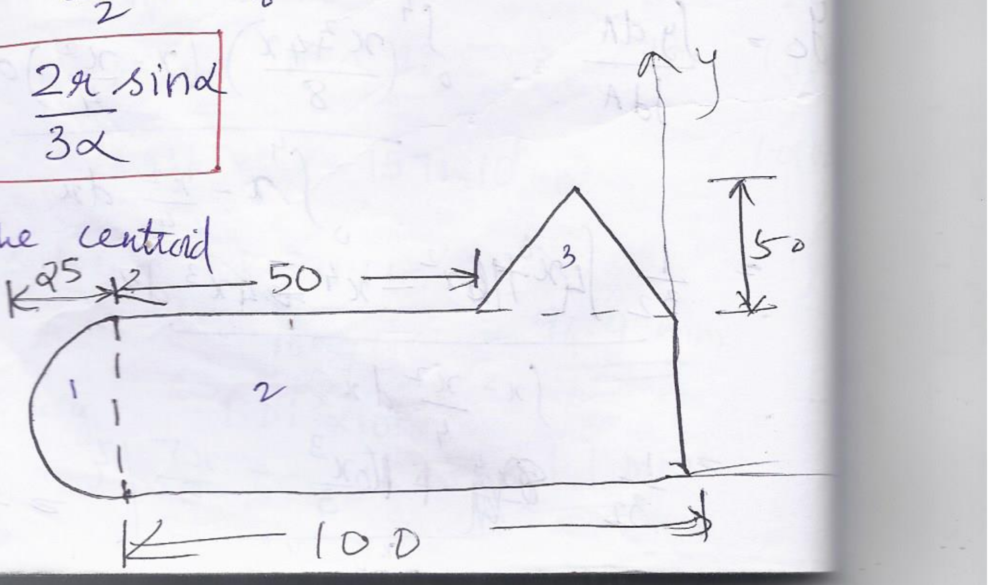
$$x = \frac{2}{3} r\cos\theta$$

$$x_c = \frac{\int_0^{\alpha} \frac{2r}{3}\cos\theta \, \frac{1}{2} r^2 d\theta}{\int_0^{\alpha} \frac{1}{2} r^2 d\theta} = \frac{\frac{r^3}{3}\int_0^{\alpha}\cos\theta\, d\theta}{\frac{r^2}{2}\int_0^{\alpha} d\theta}$$

$$= \frac{\frac{r^3}{3}\sin\theta\Big|_0^{\alpha}}{\frac{r^2}{2}\theta\Big|_0^{\alpha}} = \frac{2r}{3}\frac{\sin\alpha}{\alpha}$$

$$\boxed{x_c = \frac{2r\sin\alpha}{3\alpha}}$$

HW 5. Locate the centroid



A_i	x_i	$A_i x_i$	y_i	$A_i y_i$
$A_1 = \frac{\pi \times 25^2}{2} = 981.74$	$100 + \frac{4r}{3\pi} = 110.61$	108590.26	25	24543.5
$A_2 = 100 \times 50 = 5000$	50	250000	25	125000
$A_3 = \frac{1}{2} \times 50 \times 50 = 1250$	25	31250	66.67	83333.33
$A = 7231.74$		389840.26		232876.83

$$x_c = \frac{\Sigma A_i x_i}{A} = 53.91\ m$$

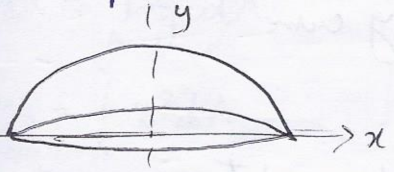
$$y_c = \frac{\Sigma A_i y_i}{A} = 32.2\ m$$

Day 6

Centroid of Composite Volumes

$$x_c = \frac{\Sigma V_i x_i}{V} \qquad y_c = \frac{\Sigma V_i y_i}{V}$$

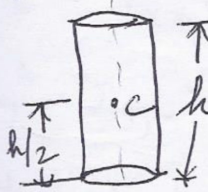
(1) Hemisphere



$V = \frac{2}{3}\pi r^3$

$y_c = \frac{3r}{8}$

(2) Cylinder

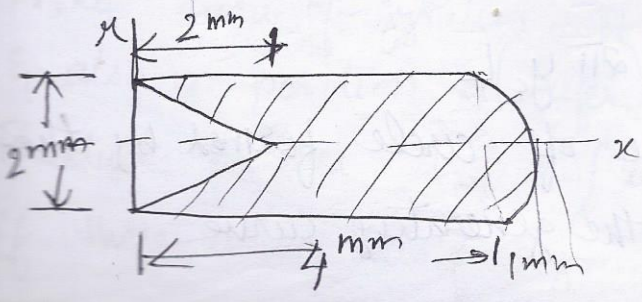


$V = \pi r^2 h$

(3) Cone

$V = \frac{1}{3}\pi r^2 h$

6.



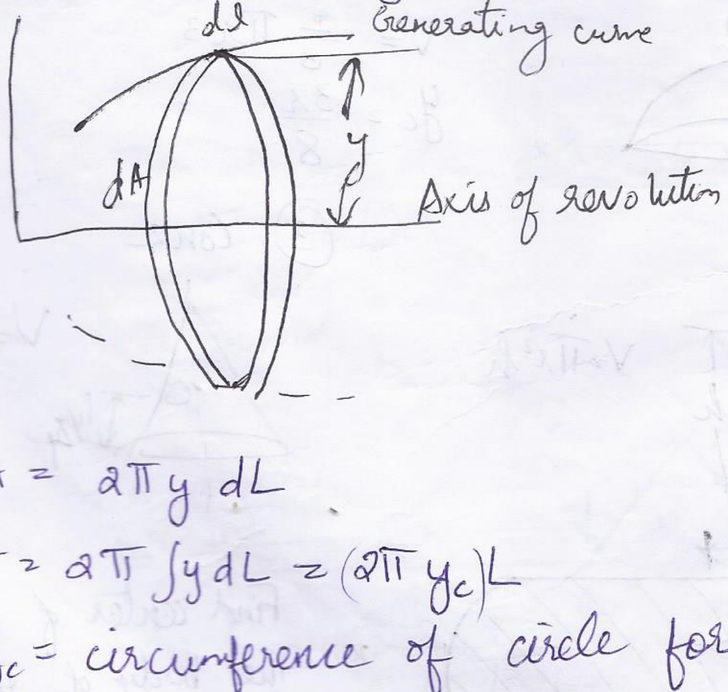
Find center of volume of the body of revolution shown in Fig

V_i	$\bar{x}_i$ (mm)	$V_i \bar{x}_i$
$V_1 = -\frac{1}{3}\pi 1^2 \times 2 = -2.09$	$\frac{2}{4}$	-1.045
$V_2 = \pi . 1^2 \times 4 = 12.57$	2	25.14
$V_3 = \frac{2}{3}\pi 1^2 = 2.09$	4.375	9.14
$V = 12.57 \, mm^3$		33.235

$$x_c = \frac{\Sigma V_i x_i}{V} = 2.64 \text{ mm}$$

Theorems of Pappus - Guldinus

Theorem I: Area of surface generated by rotat[ing a] plane curve about a non-intersecting axes in i[ts plane] is given by the product of length of the cu[rve] & distance moved by its centroid.



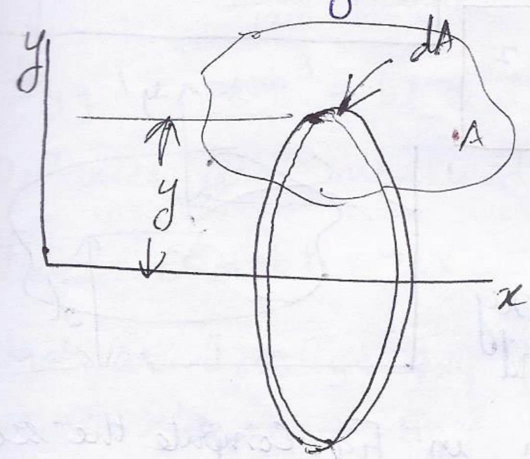
$$dA = 2\pi y \, dL$$

$$A = 2\pi \int y \, dL = (2\pi y_c) L$$

$2\pi y_c$ = circumference of circle formed by the centroid of the generating curve

Theorem II: Volume of the solid generated by rotating a plane figure about a non-intersecting axes in its plane is given by product of area of the plane figure & distance moved by its centroid.

$$dV = 2\pi y \, dA$$



$$dV = 2\pi y \, dA$$

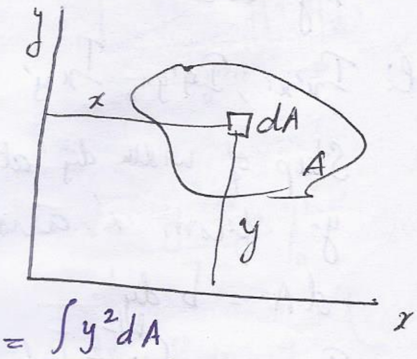
$$V = 2\pi \int_A y \, dA = 2\pi y_c A$$

$(2\pi y_c) \rightarrow$

Second Moments & the Product of Area of a Plane Area

$$I_{xx} = \int_A y^2 dA$$

$$I_{yy} = \int_A x^2 dA$$



Second moment of area $\neq$ -ve

$$A K_x^2 = I_{xx} = \int_A y^2 dA \Rightarrow K_x^2 = \frac{\int y^2 dA}{A}$$

$$A k_y^2 = I_{yy} = \int_A x^2 dA \Rightarrow k_y^2 = \frac{\int x^2 dA}{A}$$

k_x, k_y - radii of gyration - It depends on shape of area & also on position of reference

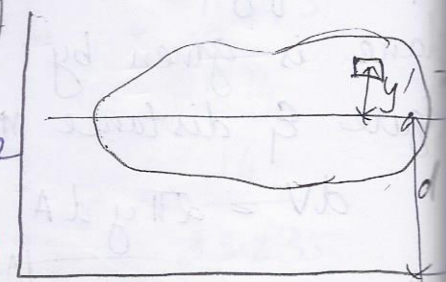
Product of Area $I_{xy} = \int_A xy \, dA \rightarrow$ may be -ve

If there is an axis of symmetry, $I_{xy} = 0$.

Transfer Theorems

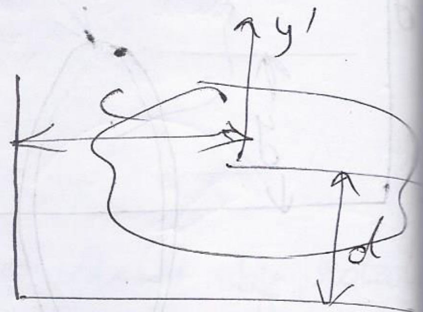
Parallel Axes Theorem

$$I_{xx} = \int_A y^2 dA = \int_A (y'+d)^2 dA$$



$$= \int_A y'^2 dA + 2d \int y' dA + Ad^2$$

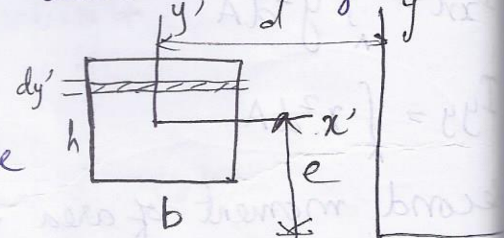
$$\boxed{I_{\text{about any axis}} = I_{\text{about }\parallel^{\text{el}}\text{ axis at centroid}} + Ad^2}$$



$$I_{xy} = I_{x'y'} + Adc$$

c, d - measured from xy axes to the centroid

1. A rectangle is shown in Fig. Compute the se[cond] moments & products of area about the centro[idal] $x'y'$ axes as well as about the xy axes.



Sol: $I_{x'x'}$, $I_{y'y'}$, $I_{x'y'}$

Strip of width dy' at distance y' from x' axis.

$dA = b\, dy'$

$$I_{x'x'} = \int_{-h/2}^{h/2} y'^2 b\, dy' = b \left.\frac{y'^3}{3}\right|_{-h/2}^{h/2} = 2b\frac{h^3}{24}$$

$$\boxed{I_{x'x'} = \frac{bh^3}{12}}$$

$$I_{y'y'} = \int_{-b/2}^{b/2} x'^2 h\, dx' = h \left.\frac{x'^3}{3}\right|_{-b/2}^{b/2} = \frac{2h}{3}\frac{b^3}{8}$$

$$\boxed{I_{y'y'} = \frac{hb^3}{12}}$$

$I_{x'y'} = 0 \Rightarrow$ Since there is a line of symmetry

I_{xx}, I_{yy}, I_{xy}

Using transfer theorems

$$I_{xx} = I_{x'y'} + A \times d^2$$

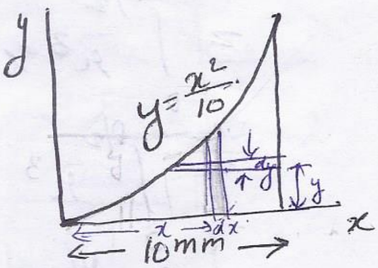
$$= \frac{bh^3}{12} + bhe^2$$

$$I_{yy} = hb^3 + bhd^2$$

Distances are measured from noncentroidal axes to C.

$$I_{xy} = 0' + bh \times -d \times e = \underline{-bhde}$$

2. What are $I_{xx}, I_{y'y'}, I_{xy}$ for the area under the parabolic curve shown in Fig?



Sol: I_{xx} – use hor. strip of width dy

$$I_{xx} = \int_0^{10} y^2 (10 - x)\, dy$$

$$x = \sqrt{10y}$$

$$I_{xx} = \int_0^{10} y^2 (10 - \sqrt{10y})\, dy = 10\frac{y^3}{3} - \sqrt{10}\, \frac{y^{7/2}}{7} 2 \Big|_0^{10}$$

$$= \frac{10 \times 10^3}{3} - \sqrt{10} \times \frac{2}{7} \times 10^{7/2} = \underline{476.2 \text{ mm}^4}$$

I_{yy} - vertical strip

$$dI_x = \frac{1}{3}(dx)y^3 \rho$$

$$I_x = \frac{1}{3}\int_0^{10} \frac{x^6}{10^3}\, dx = \frac{1}{3} \frac{x^7}{7 \times 10^3}\Big|_0^{10}$$

$$I_{yy} = \int_0^{10} x^2\, y\, dx = \int_0^{10} x^2 \times \frac{x^2}{10}\, dx = \frac{x^5}{50}\Big|_0^{10} = \frac{10^5}{50} = 476.19 \text{ mm}^4$$

$$= \underline{2000 \text{ mm}^4}$$

I_{xy} – infinitesimal area element $dx\, dy$

$$I_{xy} = \int_0^{10}\int_{y=0}^{y=x^2/10} xy \, dy \, dx$$

$$= \int_0^{10} x \frac{y^2}{2}\Big|_0^{\frac{x^2}{10}} dx = \int_0^{10} \frac{x}{2} \frac{x^4}{100} dx$$

$$= \frac{x^6}{200 \times 6}\Big|_0^{10} = 833.33 \, mm^4$$

3. Compute the second moment of area of a circ[ular] area about a diameter. Also specify radii [of] gyration

Sol: Using polar coordinates,

$$I_{xx} = \int_0^{D/2}\int_0^{2\pi} (r\sin\theta)^2 \, r \, d\theta \, dr$$

$$= \int_0^{D/2} r^3 \sin^2\theta \, d\theta \, dr$$

$$\cos 2\theta = \ldots \qquad \sin^2\theta = \ldots$$

$$= \int_0^{D/2} r^3 \frac{1-\cos 2\theta}{2} d\theta \, dr$$

$$= \int_0^{D/2} \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr$$

$$= \int_0^{D/2} \frac{r^3}{2}\left[2\pi - 0\right] dr$$

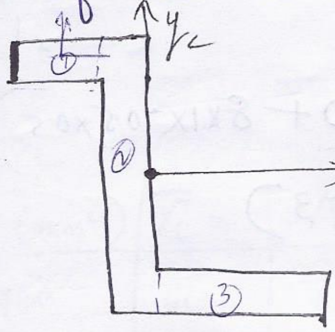
$$= \frac{\pi r^4}{4}\Big|_0^{D/2} = \frac{\pi}{4}\frac{D^4}{16}$$

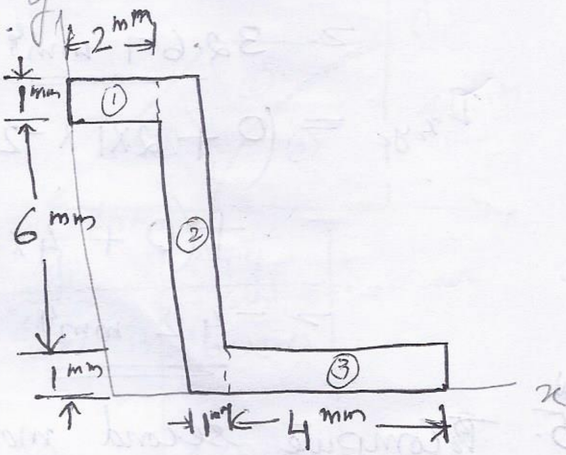
$$I_{xx} = \frac{\pi D^4}{64} \qquad I_{xy} = 0 \Rightarrow \text{symmetry}$$

$$k_x = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\pi D^4 \times 4}{64 \times \pi D^2}} = \sqrt{\frac{D^2}{16}} \Rightarrow \frac{D}{4}$$

4 Find the centroid of the area of the unequal leg Z section shown in Fig. Determine the second moment of area & product of area about the centroidal axes ‖el to the sides of the Z section.

Sol





	$A_i\ (mm^2)$	$\bar{x}_i$	$A_i\bar{x}_i$	$\bar{y}_i$	$A_i\bar{y}_i$
1.	$2\times1=2$	1	2	7.5	15
2.	$1\times8=8$	2.5	20	4	32
3.	$4\times1=4$	5	20	0.5	2
	14		$\Sigma A_i\bar{x}_i=42$		$\Sigma A\bar{y}_i=49$

$$x_c = \frac{\Sigma A_i\bar{x}_i}{A} = \frac{42}{14} = \underline{3\ mm}$$

$$y_c = \frac{\Sigma A_i\bar{y}_i}{A} = \frac{49}{14} = \underline{3.5\ mm}$$

$$I_{x_c x_c} = \left(\frac{2\times1^3}{12} + 2\times1\times4^2\right) + \left(\frac{1\times8^3}{12} + 8\times1\times0.5^2\right) + \left(\frac{1^3\times4}{12} + 4\times1\times3^2\right) = \underline{113.2\ mm^4}$$

$$I_{y_c y_c} = \left(\frac{2^3 \times 1}{12} + 2\times1\times2^2\right) + \left(\frac{1^3\times8}{12} + 8\times1\times0.5^2\right) + \left(\frac{4^3\times1}{12} + 4\times1\times2^2\right)$$

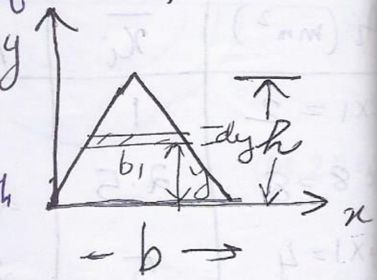
$$= \underline{32.67\ mm^4}$$

$$I_{x_c y_c} = (0 + 2\times1\times-2\times4) + (0 + 8\times1\times-0.5\times0.5) + (0 + 4\times1\times2\times-3)$$

$$= \underline{\underline{-42\ mm^4}}$$

HW

5. Compute second moments of area about x, y & centroidal axes.



Sol: Consider a hor. strip at a distance y from x-axis with a thickness of dy.

$$b_1 = \frac{b}{h}(h-y)$$

$$dA = b_1 dy$$

$$I_{xx} = \int y^2\, dA = \int_0^h y^2 \frac{b}{h}(h-y)\, dy$$

$$= \frac{b}{h}\, \frac{hy^3}{3} - \frac{y^4}{4}\Big|_0^h = \frac{b}{h}\left[\frac{h^4}{3} - \frac{h^4}{4}\right]$$

$$\boxed{I_{xx} = \frac{bh^3}{12}}$$

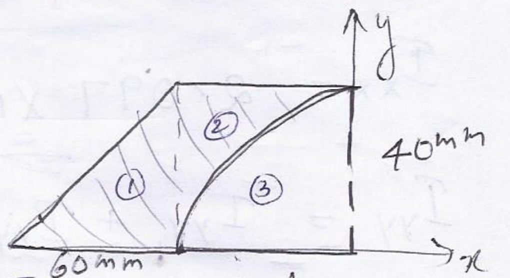
$$I_{x'x'} = I_{xx} - A\left(\frac{h}{3}\right)^2$$

$$= \frac{bh^3}{12} - \frac{1}{2}bh \times \frac{h^2}{9} = \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$\boxed{I_{x_1 y_1} = \frac{bh^3}{36}}$$

6. Calculate MI of shaded area about centroidal x & y axes

Sol



A_i (mm²)	$\bar{x}_i$	$A_i\bar{x}_i$	$\bar{y}_i$	$A_i\bar{y}_i$
1. $\frac{1}{2}\times 60\times 40 = 1200$	60	72000	$\frac{40}{3} = 13.33$	16000
2. $40^2 = 1600$	20	32000	20	32000
3. $-\frac{\pi}{4}\times 40^2 = -1256.64$	$\frac{4\times 40}{3\pi} = 16.97$	-21333.33	16.97	-21333.33
~~4056.64~~ 1543.36		~~125333.33~~ 82666.67		~~69333.33~~ 26666.67

$$x_c = \frac{\Sigma A_i x_i}{\Sigma A_i} = \cancel{\frac{125333.33}{4056.64}} = 53.56 \text{ mm}$$

$$y_c = \frac{26666.67}{1543.36} = 17.28 \text{ mm}$$

$$I_{xx} = I_{xx_1} + I_{xx_2} - I_{xx_3}$$

$$I_{xx_1} = \frac{60\times 40^3}{36} + \frac{1}{2}\times 60\times 40\times\left(17.28 - \frac{40}{3}\right)^2$$

$$= 125358.08 \text{ mm}^4$$

$$I_{xx_2} = \frac{40\times 40^3}{12} + \cancel{40}\, 40^2\times(17.28-20)^2$$

$$= 225170.77 \text{ mm}^4$$

$$I_{xx_3} = \frac{0.11 \times 40^4}{2} + \frac{\pi}{4} \times 40^2 (17.23 - 16.97)$$

$$= 140834.43 \text{ mm}^4$$

$$I_{xx} = 2.097 \times 10^5 \text{ mm}^4$$

$$I_{yy} = I_{yy_1} + I_{yy_2} - I_{yy_3}$$

$$I_{yy_1} = \frac{60^3 \times 40}{36} + \frac{1}{2} \times 60 \times 40 \times (53.56 - 60)^2$$

$$= 289768.32 \text{ mm}^4$$

$$I_{yy_2} = \frac{40 \times 40^3}{12} + 40^2 \times (53.56 - 20)^2$$

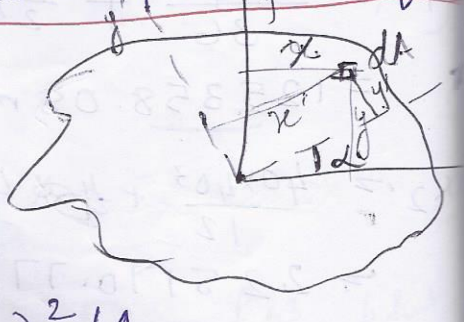
$$= 2015371.09 \text{ mm}^4$$

$$I_{yy_3} = \frac{0.11 \times 40^4}{2} + \frac{\pi}{4} \times 40^2 (53.56 - 16.97)$$

$$= 1823221.04 \text{ mm}^4$$

$$I_{yy} = 481918.4 \text{ mm}^4 = 4.819 \times 10$$

Day 7 Relation b/n second Moments & Products of A



$$x' = x\cos\alpha + y\sin\alpha$$

$$y' = -x\sin\alpha + y\cos\alpha$$

$$I_{x'x'} = \int_A (y')^2 \, dA$$

$$= \int (-x\sin\alpha + y\cos\alpha)^2 \, dA$$

$$= \sin^2\alpha \int_A x^2\, dA - 2\sin\alpha\cos\alpha \int_A xy\, dA + \cos^2\alpha \int_A y^2\, dA$$

$$I_{x'x'} = I_{yy}\sin^2\alpha + I_{xx}\cos^2\alpha - 2 I_{xy}\sin\alpha\cos\alpha$$

$$\cos^2\alpha = \frac{1}{2}(1+\cos 2\alpha)$$

$$\sin^2\alpha = \frac{1}{2}(1-\cos 2\alpha)$$

$$2\sin\alpha\cos\alpha = \sin 2\alpha$$

$$I_{x'x'} = I_{yy}\frac{(1-\cos 2\alpha)}{2} + I_{xx}\frac{(1+\cos 2\alpha)}{2} - I_{xy}\sin 2\alpha$$

$$\boxed{I_{x'x'} = \frac{I_{xx}+I_{yy}}{2} + \frac{I_{xx}-I_{yy}}{2}\cos 2\alpha - I_{xy}\sin 2\alpha}$$

To determine $I_{y'y'}$, replace α by $\alpha + \frac{\pi}{2}$

$$\boxed{I_{y'y'} = \frac{I_{xx}+I_{yy}}{2} - \frac{I_{xx}-I_{yy}}{2}\cos 2\alpha + I_{xy}\sin 2\alpha}$$

$$I_{x'y'} = \int_A x'y'\, dA = \int_A (x\cos\alpha + y\sin\alpha)(-x\sin\alpha + y\cos\alpha)\, dA$$

$$= \int -x^2\sin\alpha\cos\alpha - xy\sin^2\alpha + xy\cos^2\alpha + y^2\sin\alpha\cos\alpha\, dA$$

$$= \sin\alpha\cos\alpha\,(I_{xx} - I_{yy}) + (\cos^2\alpha - \sin^2\alpha)\, I_{xy}$$

$$\boxed{I_{x'y'} = \frac{I_{xx}-I_{yy}}{2}\sin 2\alpha + I_{xy}\cos 2\alpha}$$

These eqns are called transformation equations.

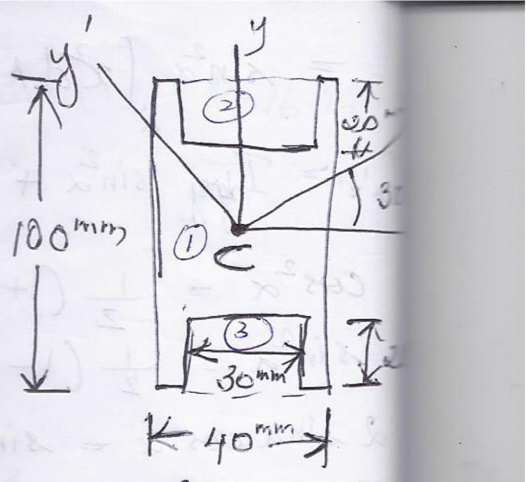
1. Find $I_{x'x'}$, $I_{y'y'}$ & $I_{x'y'}$ for the cross section of the beam shown in fig.

Sol: 3 rectangles

R_1 : 100 mm × 40 mm

R_2 : 20 mm × 30 mm

R_3 : 20 mm × 30 mm



$$I_{xx} = \frac{1}{12} \times 40 \times 100^3 - 2\left[\frac{1}{12} \times 30 \times 20^3 + 20 \times 30 \times 40^2\right]$$

$$= 1.373 \times 10^6 \text{ mm}^4$$

$$I_{yy} = \frac{1}{12} \times 100 \times 40^3 - 2\left[\frac{1}{12} \times 20 \times 30^3\right] = 4.4$$

$$I_{xy} = 0 \text{ (symmetry)}$$

$$I_{x'x'} = \frac{I_{xx} + I_{yy}}{2} + \frac{I_{xx} - I_{yy}}{2} \cos 2\alpha - I_{xy}$$

$$= \frac{1.373 \times 10^6 + 4.433 \times 10^5}{2} + \frac{1.373 \times 10^6 - 4.433 \times 10^5}{2}$$

$$= 1.141 \times 10^6 \text{ mm}^4$$

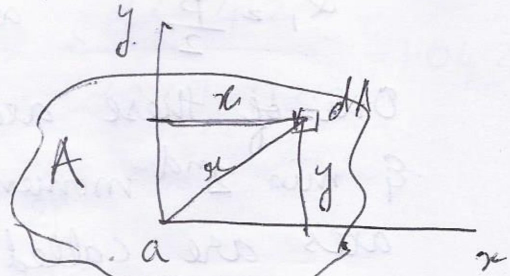
$$I_{y'y'} = \frac{I_{xx} + I_{yy}}{2} - \frac{I_{xx} - I_{yy}}{2} \cos 2\alpha + I_{xy} \sin$$

$$= 6.757 \times 10^5 \text{ mm}^4$$

$$I_{x'y'} = \frac{I_{xx} - I_{yy}}{2} \sin 2\alpha + I_{xy} \cos 2\alpha$$

$$= 4.026 \times 10^5 \text{ mm}^4$$

Polar Moment of Area



$$I_{xx} + I_{yy} = \int_A y^2 dA + \int_A x^2 dA$$

$$= \int_A (x^2 + y^2)\, dA$$

$$= \int_A r^2 dA$$

r^2 is independent of the orientation of the coordinate system, thus sum $I_{xx} + I_{yy}$ is independent of the orientation of reference.

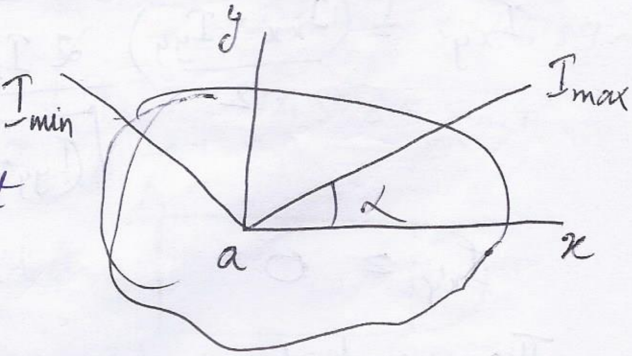
$$\boxed{J = I_{xx} + I_{yy}}$$

It is only function of position of origin 'a' for axes.

$$I_{xx} + I_{yy} = I_{x'x'} + I_{y'y'} \quad \text{— invariant}$$

Principal Axes

Find α at which there is max 2nd moment of area



$$\frac{\partial I_{x'x'}}{\partial \alpha} = 0$$

$$\left(\frac{I_{xx} - I_{yy}}{2}\right) - \sin 2\alpha \times 2 - I_{xy} \cos 2\alpha \times 2 = 0$$

$$\boxed{\tan 2\alpha = \frac{2 I_{xy}}{I_{yy} - I_{xx}}}$$

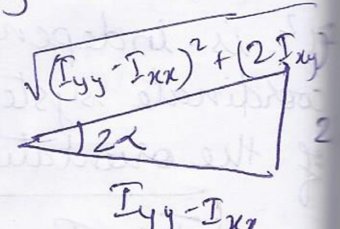
$$2\alpha = \beta = \tan^{-1} \frac{2 I_{xy}}{I_{yy} - I_{xx}}$$

$$\alpha_1 = \frac{\beta}{2}, \quad \alpha_2 = \frac{\beta}{2} + \frac{\pi}{2}$$

One of these axes is max 2^{nd} moment of & min 2^{nd} moment of area on other axis – axes are called principal axes.

$$I_{x'y'} = \frac{I_{xx} - I_{yy}}{2} \sin 2\bar{\alpha} + I_{xy} \cos 2\bar{\alpha}$$

$$\sin 2\alpha = \frac{2I_{xy}}{\sqrt{(I_{yy}-I_{xx})^2 + 4I_{xy}^2}}$$



$$\cos 2\alpha = \frac{I_{yy} - I_{xx}}{\sqrt{(I_{yy}-I_{xx})^2 + 4I_{xy}^2}}$$

$$I_{x'y'} = \left(\frac{I_{xx} - I_{yy}}{2}\right) \frac{2I_{xy}}{\sqrt{(I_{yy}-I_{xx})^2 + 4I_{xy}^2}} + I_{xy} \frac{(I_{yy}}{\sqrt{(I_{yy}-I_{xx})}}$$

$$I_{x'y'} = 0$$

The product of area corresponding to princip is zero.

2. Find the principal second moments of area centroid of the Z section.

Sol: $I_{x_c x_c} = 113.2 \text{ mm}^4$

$I_{y_c y_c} = 32.67 \text{ mm}^4$

$I_{x_c y_c} = -42 \text{ mm}^4$

$$\tan 2\alpha = \frac{2I_{x_cy_c}}{I_{y_cy_c} - I_{x_cx_c}} = \frac{2\times -42}{32.67-113.2} = 1.043$$

$$2\alpha = 46.21^\circ,\ 226.21^\circ$$

For $2\alpha = 46.2^\circ$;

$$I_1 = \frac{113.2+32.67}{2} + \frac{113.2-32.67}{2}\cos 46.21 - (-42)\sin 46.21$$

$$= 131.1\ mm^4$$

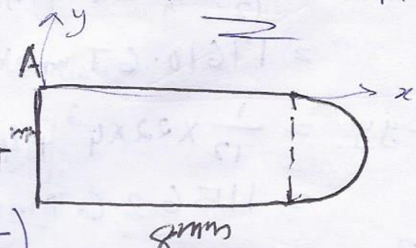
$$I_2 = \frac{113.2+32.67}{2} + \frac{113.2-32.67}{2}\cos 226.21 - (-42)\sin 226.21$$

$$= 14.75\ mm^4$$

Check: $I_{x_cx_c} + I_{y_cy_c} = 113.2 + 32.67 = 145.87\ mm^4$

$I_1 + I_2 = 131.1 + 14.75 = 145.85\ mm^4$

HW 8.90 Determine the principal 2nd moments of area at point A.



Sol: $I_{xx} = \left(\frac{1}{12}\times 8\times 4^3 + 4\times 8\times 2^2\right) + \frac{\pi D^4}{128} + \frac{\pi D^2}{8}\times\left(\frac{D}{2}\right)^2$

$$= 170.67 + 31.42 = 202.08\ mm^4$$

$I_{yy} = \left(\frac{1}{12}\times 8^3\times 4 + 4\times 8\times 4^2\right) + 0.11 r^4 + \frac{\pi D^2}{8}\times\left(8+\frac{4r}{3\pi}\right)^2$

$$= 682.67 + 57.36 = 1176.41\ mm^4$$

$I_{xy} = 0 + 4\times 8\times -2\times 4 + 0 + \frac{\pi D^2}{8}\times\left(8+\frac{4r}{3\pi}\right)\times -2$

$I_{xy} = -367.2\ mm^4$

S A / T C

~~Pxx~~ $\tan 2\alpha = \dfrac{2I_{xy}}{I_{yy} - I_{xx}} = \dfrac{2 \times -367.2}{~~240.03~~ - 202.08}$

1176.41 -0.7 143

$2\alpha = ~~53.77~~\ 37^\circ \quad 143^\circ, 323^\circ$

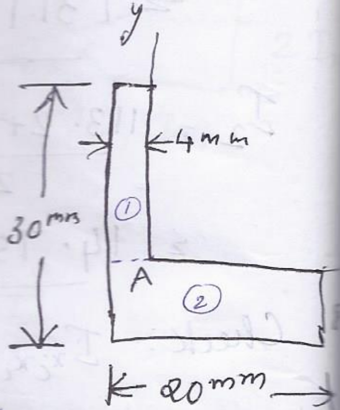
$323^\circ \to I_{min} = 79.2\ mm^4$

$143^\circ \to I_{max} = 1299\ mm^4$

AW

8.96 (a)

	A_i	$\bar{x}_i$	$A_i\bar{x}_i$	$\bar{y}_i$	$A_i\bar{y}_i$
1.	88	-2	-176	11	968
2.	160	6	960	-4	-640
	$248\ mm^2$		784		328



$x_c = \dfrac{784}{248} = 3.16\ mm \qquad y_c = \dfrac{328}{248} = 1.32\ mm$

(b) $I_{xx} = \frac{1}{12} \times 4 \times 22^3 + 4 \times 22 \times 11^2 + \frac{1}{12} \times 20 \times 8^3 + 20 \times 8 \times 4^2$

$= 17610.67\ mm^4$

$I_{yy} = \frac{1}{12} \times 22 \times 4^3 + 4 \times 22 \times 2^2 + \frac{1}{12} \times 8 \times 20^3 + 20 \times 8 \times 6^2$

$= 11562.67\ mm^4$

$I_{xy} = 4 \times 22 \times 11 \times -2 + 20 \times 8 \times 6 \times -4 = -5776\ mm^4$

$\tan 2\alpha = \dfrac{2I_{xy}}{I_{yy} - I_{xx}} = \dfrac{2 \times -5776}{11562.67 - 17610.67} = 1.91$

$2\alpha = 62.36^\circ, 242.36^\circ$

$I_{xx} = \dfrac{I_{xx} + I_{yy}}{2} + \dfrac{I_{xx} - I_{yy}}{2} \cos 2\alpha - I_{xy} \sin 2\alpha$

$I_1 = 21106.39\ mm^4 \qquad I_2 = 8066.95\ mm^4$