Day 6

Equivalent Force Systems

Equivalent vectors - same capacity in some given situation

Rigid body model used

2 force systems are equivalent if they are capable initiating the same motion of the rigid body.

Conditions: (i) addition of forces in each system results equal force vectors

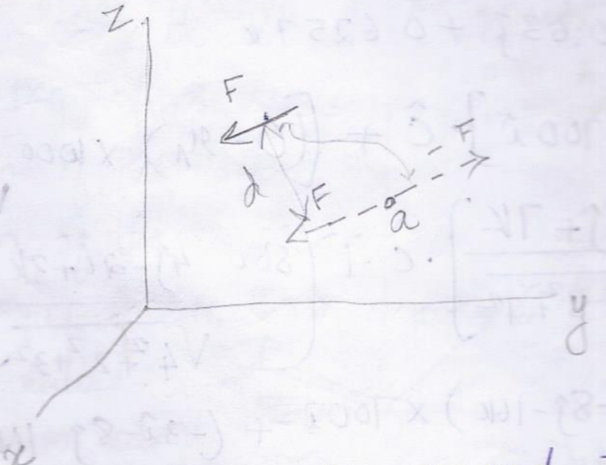
(ii) equal turning action about any point in space

① Sum of a set of concurrent forces = single force equiv

② Forces are transmissible vector

③ Couple moment - free vector

Translation of a Force to a Parallel Position

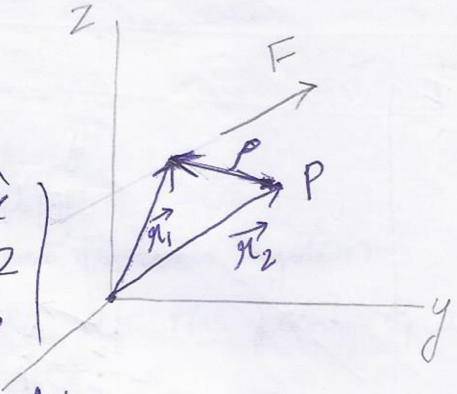


A force may be moved to any parallel p provided that a couple moment of the corre orientation & magnitude is simultaneously provi

$$M = \rho \times F = C$$

1. A force $\vec{F} = (6\hat{i} + 3\hat{j} + 6\hat{k})N$ goes thru a point whose position vector is $\vec{r}_1 = (2\hat{i} + \hat{j} + 10\hat{k})m$. Replace this force by an equivalent force system, for purposes of rigid-body mechanics, going thru position P, whose position vector is $\vec{r}_2 = (6\hat{i} + 10\hat{j} + 12\hat{k})\ m$.

Sol: Move $\vec{F}$ to point P.

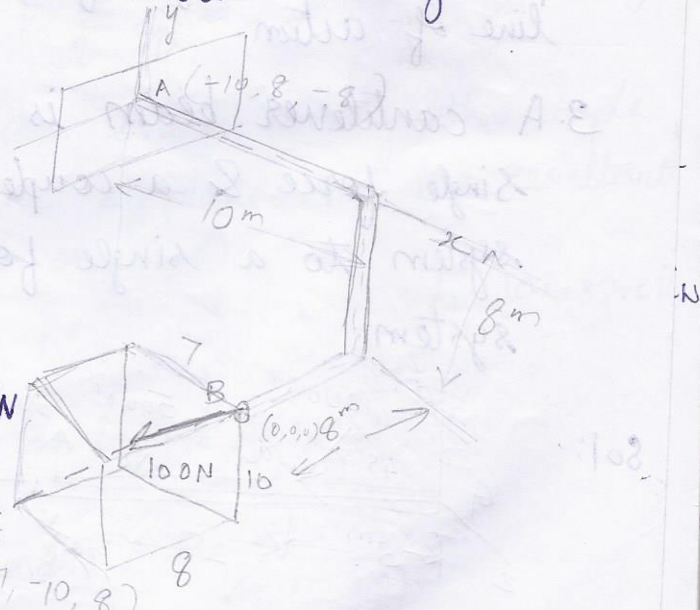


$$C = \rho \times F = (\vec{r}_1 - \vec{r}_2) \times \vec{F}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & -9 & -2 \\ 6 & 3 & 6 \end{vmatrix}$$

$$C = (-48\hat{i} + 12\hat{j} + 42\hat{k})\ N\text{-}m$$

2. What is the equivalent force system at position A for the 100-N force shown in Fig?



Sol: $\vec{F} = F \dfrac{\overrightarrow{BE}}{|\overrightarrow{BE}|}$

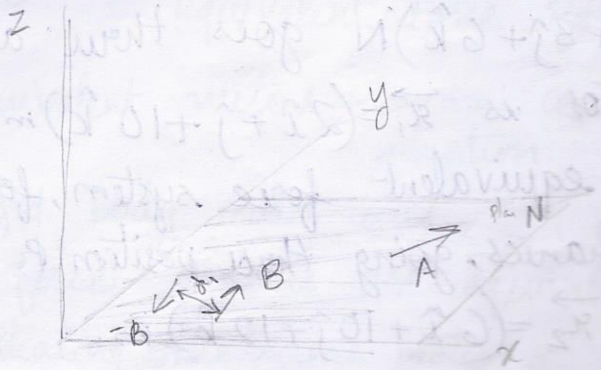
$$= 100 \frac{(-7\hat{i} - 10\hat{j} + 8\hat{k})}{\sqrt{7^2 + 10^2 + 8^2}}$$

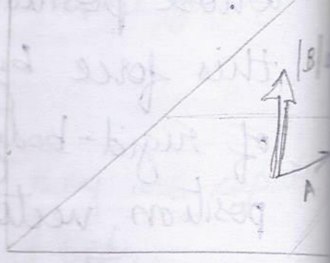
$$\vec{F} = (-48\hat{i} - 68.5\hat{j} + 54.8\hat{k})N$$

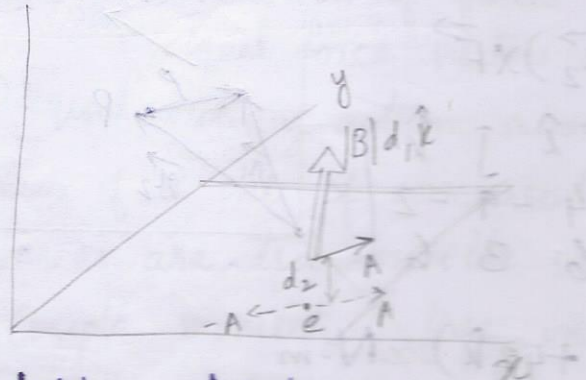
$$C = \vec{r}_{AB} \times \vec{F}$$

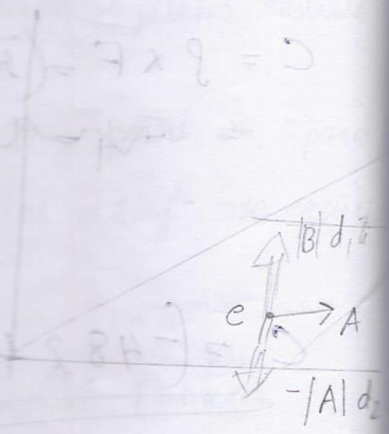
$$= (10\hat{i} - 8\hat{j} + 8\hat{k}) \times \vec{F}$$

$$= (109.6\hat{i} - 932\hat{j} - 1069\hat{k})\ N\text{-}m$$







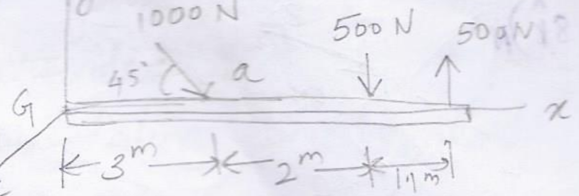


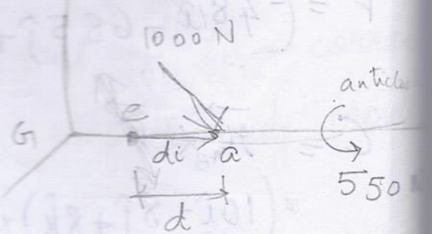
$|A| d_2 = |B| d_1$

We can reduce a force & a couple in the sam plane to a single force which must have a sp line of action.

3. A cantilever beam is shown in Fig, supporting single force & a couple in the xy plane. Reduce system to a single force equivalent to the give system.

Sol:

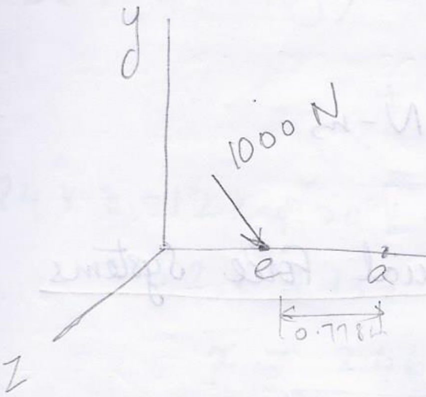




$$d\hat{i} \times 1000(0.707\hat{i} - 0.707\hat{j}) + 550\hat{k} = 0$$

$$-(707d)\hat{k} + 550\hat{k} = 0$$

$d = \underline{0.778\,m}$



4-14

Resultant of a Force System

It is a simpler equivalent force system

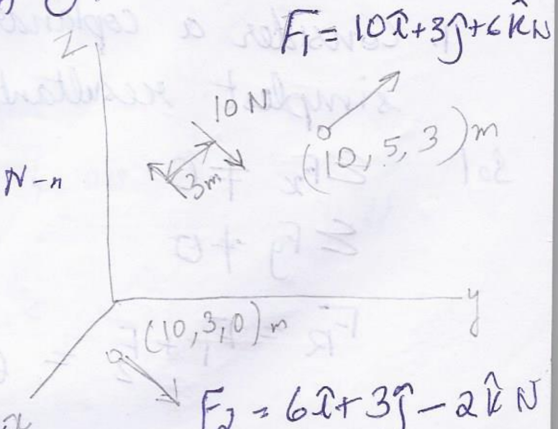
System of concurrent forces at the point & a system of concurrent couple moments

$$F_R = \left[\sum_p (F_p)_x\right]\hat{i} + \left[\sum_p (F_p)_y\right]\hat{j} + \left[\sum_p (F_p)_z\right]\hat{k}$$

$$C_R = \left[r_1 \times F_1 + r_2 \times F_2 + \ldots\right] + \left[C_1 + C_2 + \cdots\right]$$

$$C_R = \sum_p \vec{r}_p \times F_p + \sum_q C_q$$

4. 2 forces & couple are shown in Fig, the couple being positioned in the zy plane. Find the resultant of the system at the origin O.



Sol: $F_R = 16\hat{i} + 6\hat{j} + 4\hat{k}$ N

$C_R = r_1 \times F_1 + r_2 \times F_2 - 30\hat{i}$ N-m

$$r_1 \times F_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 5 & 3 \\ 10 & 3 & 6 \end{vmatrix} \cancel{=21\hat{i}}$$

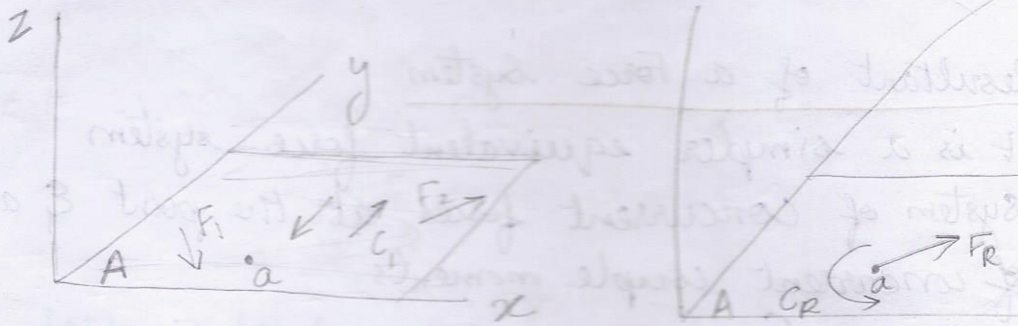
$$= 21\hat{i} - 30\hat{j} - 20\hat{k} \text{ N-m}$$

$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 10 & 3 & 0 \\ 6 & 3 & -2 \end{vmatrix} = -6\hat{i} + 20\hat{j} + 12\hat{k} \text{ N-m}$$

$$C_R = \underline{\underline{-15\hat{i} - 10\hat{j} - 8\hat{k} \text{ N-m}}}$$

Day 7 **Simplest Resultants of Special Force Systems**

$\boxed{A}$ **Coplanar Force Systems**



$$F_R = \left[\sum_p (F_p)_x\right]\hat{i} + \left[\sum_p (F_p)_y\right]\hat{j}$$

$$C_R = (F_1 d_1 + F_2 d_2 + \dots)\hat{k} + (C_1 + C_2 + \dots)k$$

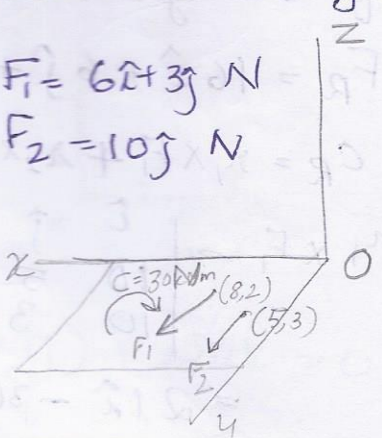
$F_R \neq 0 \Rightarrow$ single force with a specific line of ac

$\sum F_x = 0$ & $\sum F_y = 0 \Rightarrow$ Resultant - couple mome zero

1. Consider a coplanar force system shown in Fig. Find simplest resultant

$F_1 = 6\hat{i} + 3\hat{j}$ N

$F_2 = -10\hat{j}$ N



Sol $\sum F_x \neq 0$

$\sum F_y \neq 0$

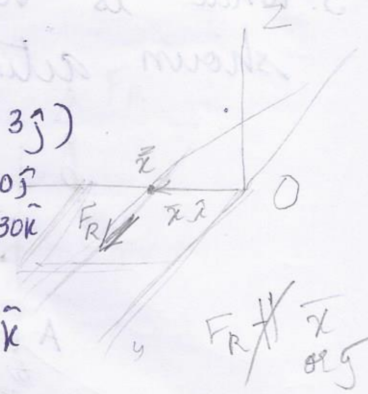
$F_R = F_1 + F_2 = 6\hat{i} + 13\hat{j}$ N

To find the line of action

$\vec{r}\times\vec{F} = \vec{r_1}\times\vec{F_1} + \vec{r_2}\times\vec{F_2}$

$\bar{x}\hat{i}\times(6\hat{i}+13\hat{j}) = (8\hat{i}+2\hat{j})\times(6\hat{i}+3\hat{j})$

$+ \cancel{10\hat{j}\times}(5\hat{i}+3\hat{j})\times 10\hat{j} \;\; -30\hat{k}$



$24\hat{k} \cancel{8} - 12k + 50\hat{k} - 30\hat{k} = 13\bar{x}\hat{k}$

$32\hat{k} = 13\bar{x}\hat{k}$

$\bar{x} = 2.46\,m$ x intercept

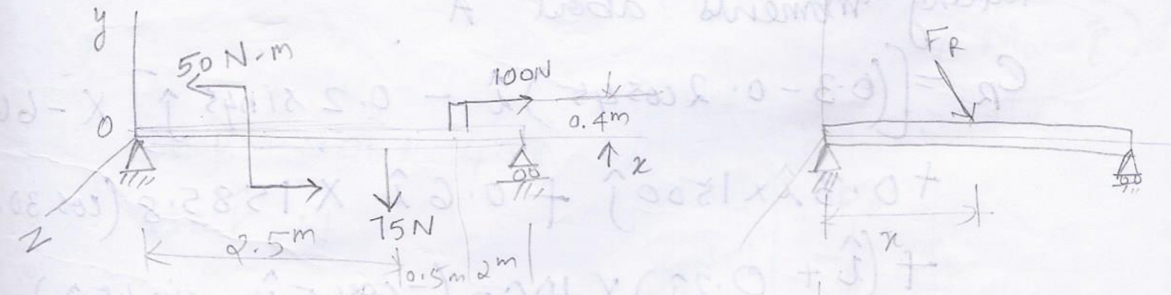
or

$\bar{y}\hat{j}\times(6\hat{i}+13\hat{j}) = (8\hat{i}+2\hat{j})\times(6\hat{i}+3\hat{j}) + (5\hat{i}+3\hat{j})\times 10\hat{j} - 30\hat{k}$

$-6\bar{y}\hat{k} = 32\hat{k}$

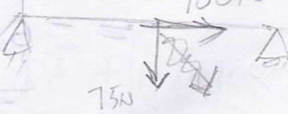
$\bar{y} = -5.33\,m$ y intercept

2. Compute the simplest resultant for the loads shown acting on the beam in Fig. Give the intercept with x-axis



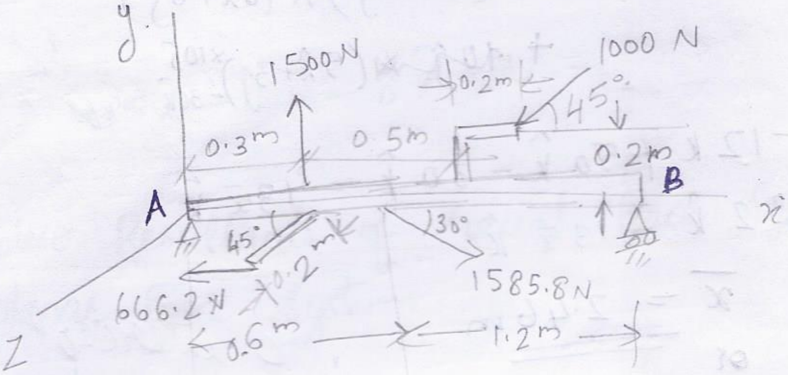
$F_R = 100\hat{i} - 75\hat{j}$ N

$-75\bar{x} = 50 - 2.5\times 75 - 0.4\times 100$



$\bar{x} = 2.37\,m$

3. What is the simplest resultant for the forc
shown acting on beam AB in Fig?



Sol: $F_R = 1500\hat{j} - 666.2\hat{i} - 1585.8\sin 30\hat{j} + 1585.8$

$-1000\cos 45\hat{i} - 1000\sin 45\hat{j}$

$F_R = (-666.2 + 1373.3 - 707.1)\hat{i} + (1500 - 792.9-$

$= 0$

⇒ either a couple moment or null vector

Taking moments about A

$C_R = [(0.3 - 0.2\cos 45)\hat{i} - 0.2\sin 45\hat{j}] \times -6$

$+ 0.3\hat{i} \times 1500\hat{j} + 0.6\hat{i} \times 1585.8(\cos 30$

$+ (\hat{i} + 0.2\hat{j}) \times 1000(-\cos 45\hat{i} - \sin 45\hat{j})$

$= -94.2\hat{k} + 450\hat{k} - 475.7\hat{k} - 707.1k$

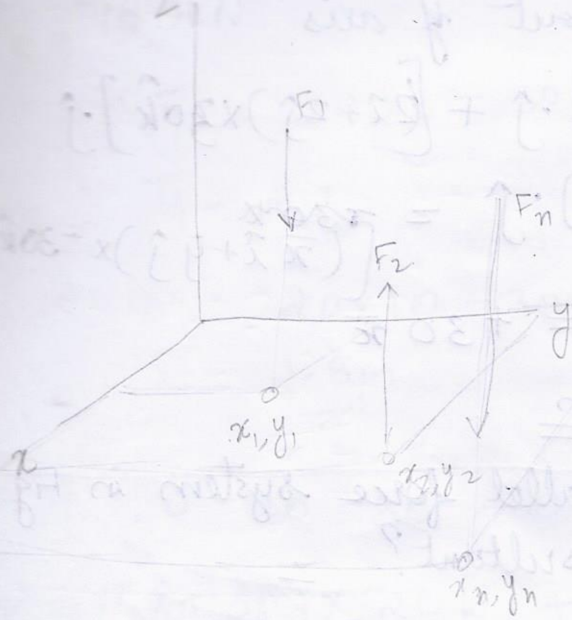
$= -685.6\,\hat{k}$ Nm

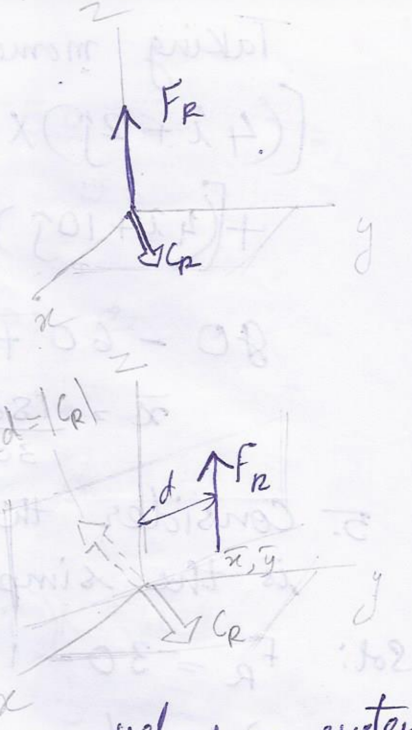
HW
Ex 4.20

$\boxed{B}$ Parallel Force Systems in Space

$F_R = \Sigma(F_p)$

$C_R = \Sigma\, r \times F + \Sigma C_p$

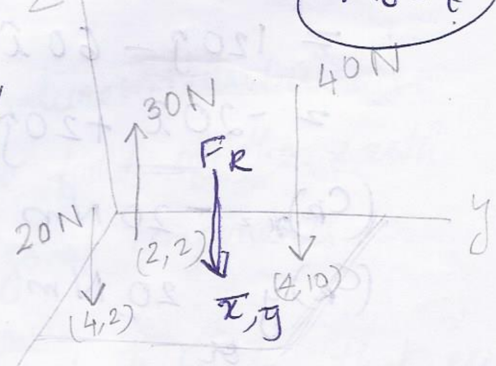




The simplest resultant system of a ||el force system is either a force with a specific line of action, a single couple moment, or a null vector.

4. Find the simplest resultant of the parallel force system in Fig.

$M_o = ?$



Sol: $F_R = -20\hat{k} + 30\hat{k} - 40\hat{k}$ N

$= -30\hat{k}$ N

Taking moment about x-axis,

$$\left[(4\hat{i}+2\hat{j})\times -20\hat{k}\right]\cdot\hat{i} + \left[(2\hat{i}+2\hat{j})\times 30\hat{k}\right]\cdot\hat{i} + \left[(4\hat{i}+10\hat{j})\times -40\hat{k}\right]\cdot\hat{i} = \left[(\bar{x}\hat{i}+\bar{y}\hat{j})\times -30\hat{k}\right]\cdot\hat{i}$$

$$-40 + 60 - 400 = -30\bar{y}$$

$$\bar{y} = -\frac{-380}{30} = 12.7\text{ m}$$

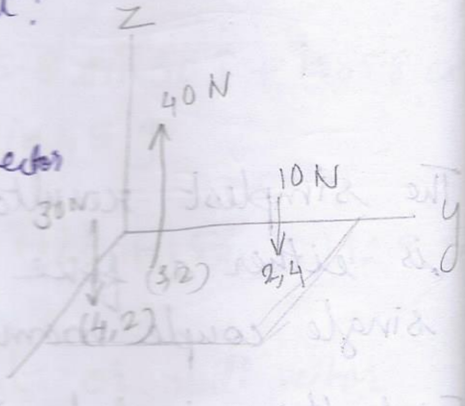
Taking moment about y axis

$$\left[(4\hat{i}+2\hat{j})\times -20\hat{k}\right]\cdot\hat{j} + \left[(2\hat{i}+2\hat{j})\times 30\hat{k}\right]\cdot\hat{j} + \left[(4\hat{i}+10\hat{j})\times 40\hat{k}\right]\cdot\hat{j} = \left[(\bar{x}\hat{i}+\bar{y}\hat{j})\times -30\hat{k}\right]\cdot\hat{j}$$

$$80 - 60 + 160 = +30\bar{x}$$

$$\bar{x} = \frac{180}{30} = \underline{6\ m}$$

5. Consider the parallel force system in Fig. What is the simplest resultant?



Sol: $F_R = -30 - 10 + 40 = 0$

$\Rightarrow$ couple moment or null vector

$$C_R = \left[(4\hat{i}+2\hat{j})\times -30\hat{k}\right] + \left[(3\hat{i}+2\hat{j})\times 40\hat{k}\right] + \left[(2\hat{i}+4\hat{j})\times -10\hat{k}\right]$$

$$= 120\hat{j} - 60\hat{i} - 120\hat{j} + 80\hat{i} + 20\hat{j} - 40\hat{i}$$

$$= \underline{-20\hat{i} + 20\hat{j}}\ Nm$$

$(C_R)_x = -20\ Nm$

$(C_R)_y = 20\ Nm$

or

Find moment about x & y axes – $(C_R)_x$ & $(C_R)_y$

HW

4.32 20N force @ $x = 10m$, $y = 3m$;

30N @ $x = 5m$, $y = -3m$

50N $x = -2m$, $y = 5m$

(a) All forces $-z \rightarrow$ simplest resultant force & its line

Sol: $F_R = -100\hat{k}$ N

$$-100\,\bar{y} = -20\times 3 - 30\times -3 - 50\times 5$$

$$\bar{y} = 2.2\,m$$

$$-100\,\bar{x} = -20\times 10 - 30\times 5 - 50\times -2$$

$$\bar{x} = 2.5\,m$$

(b) $F_R = 50 - 20 - 30 = 0$

$$C_R = \left[(10\hat{i}+3\hat{j})\times -20\hat{k}\right] + (5\hat{i}-3\hat{j})\times -30\hat{k} + (-2\hat{i}+5\hat{j})\times 50\hat{k}$$

$$= 200\hat{j} - 60\hat{i} + 150\hat{j} + 90\hat{i} + 100\hat{j} + 250\hat{i}$$

$$= 280\hat{i} + 450\hat{j} \text{ Nm}$$

⇒ Wrench Resultant

Day 8

Distributed Force Systems

Vector & scalar fields

Ex for scalar field – temperature distribution

vector field – gravitational force field of the earth

↳ 3 scalar fields – orthogonal components

Not restricted to forces

(1) Body force distribution – force distribution that exert influence directly on elements of mass distributed thruout the body – per unit of mass Eg:- gravitational force

(2) Surface force distribution – per unit of area of surface

Eg:- force on a surface submerged in fluid

Pressure - scalar qty $\quad df = -p\,dA$

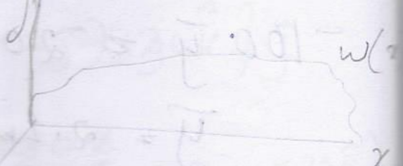
(3) line forces
Continuous load on beam

$\parallel^{el}$ loading distribution

symmetrical about center plane xy

intensity of loading

$F = w\,dx$



$\boxed{A}$ Parallel Body Force System - Centre of Gravity

Gravity body force $= -g\hat{k}$

$$F_R = -\int_V g(\rho dV)\hat{k}$$

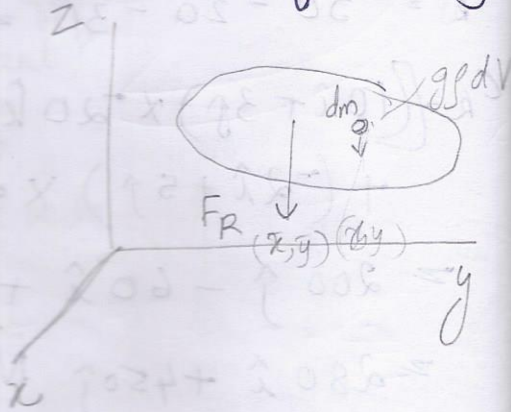
$$= -g\hat{k}\int_V \rho\, dV$$

$$= -gM\hat{k}$$

$$-F_R\bar{y} = -g\int_V y\rho\, dv$$

$$-F_R\bar{x} = -g\int_V x\rho\, dv$$

$$\bar{x} = \frac{\int x\rho\, dV}{M} \qquad \bar{y} = \frac{\int y\rho\, dV}{M}$$

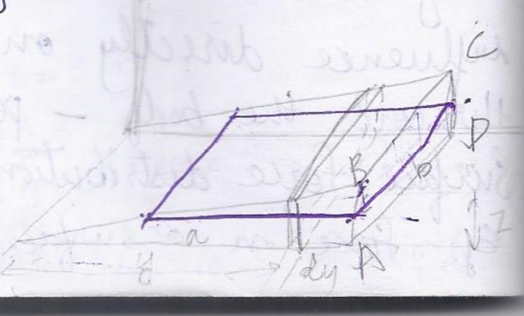


Lines of action for simplest resultants for all orientations of the body intersect at the point center of gravity - all wt concentrated here

1. Find CG of triangular block having a uniform density ρ shown in Fig

Sol: (1) Total wt. of body

$$F_R = g\rho\,\frac{abc}{2}$$



② To find $\bar{y}$ ⇒ Moment of F_R about x axis

Wt. of slice $= (z\,b\,dy)\rho g$

$$-F_R\,\bar{y} = -\int_0^a y\,(z\,b\,dy)\,\rho g$$

$$\frac{z}{c} = \frac{y}{a} \Rightarrow z = \left(\frac{y}{a}\right)c$$

$$\bar{y} = \frac{1}{g\rho\,(abc/2)}\, g \int_0^a \rho\, y^2 \frac{bc}{a}\, dy$$

$$= \frac{2}{abc} \times \frac{bc}{a} \left.\frac{y^3}{3}\right|_0^a = \frac{2}{3}a$$

③ Reorientation of body

To find $\bar{z}$ ⇒ Moment of F_R about x axis

$$-F_R\,\bar{z} = -\int_0^c \cancel{g}\,(z\,b\,dx)\,\rho g$$

Wt. of slice $= (y\,b\,dz)\,\rho g$

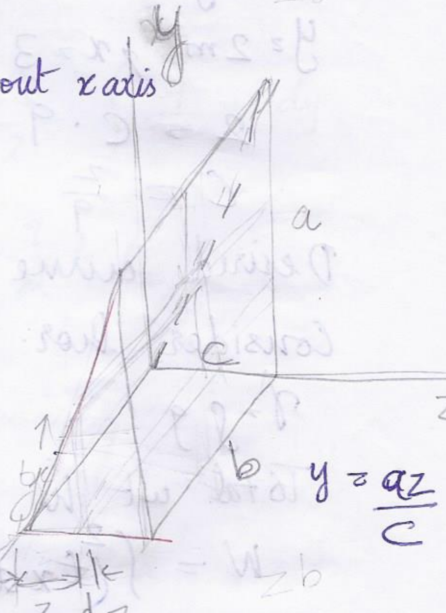
$$-F_R\,\bar{z} = -\int_0^c z\,y\,b\,dz\,\rho g$$

$$= -\frac{ab}{c}\rho g \int_0^c z^2\,dz$$

$$= -\frac{ab}{c}\rho g \left.\frac{z^3}{3}\right|_0^c$$

$$\bar{z} = \frac{1}{g\rho\,\frac{abc}{2}} \times \frac{abc^3}{3c}\,\rho g$$

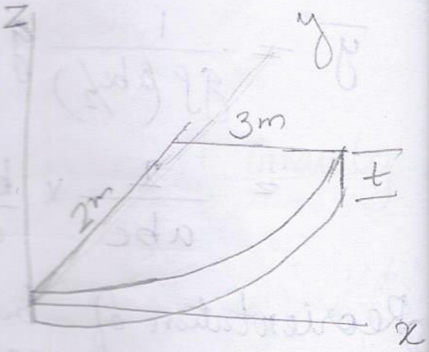
$$\bar{z} = \frac{2}{3}c$$



$$-F_R\,\bar{x} = \int x$$

$$\bar{x} = \frac{b}{2}$$

2. A plate is shown in Fig lying flat on the ground. The plate is 60mm thick & has a uniform density. The curved edge is that of a parabola with zero slope at the origin. Find the coordinates of the CG.



Sol: Eqn of parabola oriented like that of curved edge of plate is

$$y = Cx^2$$

$y = 2m$, $x = 3m$

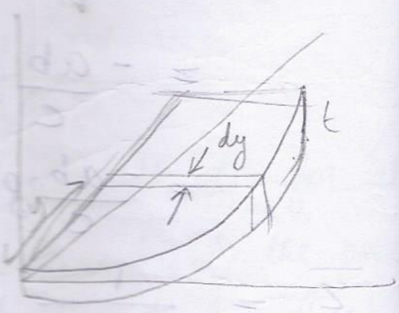
$$2 = C \cdot 9$$

$$C = \frac{2}{9}$$

Desired curve is $y = \frac{2}{9}x^2 \Rightarrow x = \frac{3}{\sqrt{2}} y^{1/2}$

Consider hor. strips of plate of width dy

$\gamma = \rho g$



Total wt W of plate

$$W = \int_0^2 (tx\,dy)\gamma$$

$$= t\gamma \int_0^2 \left(\frac{3}{\sqrt{2}} y^{1/2}\right) dy$$

$$= t\gamma \frac{3}{\sqrt{2}} \frac{y^{3/2}}{3} 2 \Big|_0^2 = t\gamma \sqrt{2} \times 2^{3/2} = 4t\gamma$$

Taking moments about x axis to get $\bar{y}$.

$$-W\bar{y} = -\int_0^2 y\,(tx\,dy)\,\gamma$$

$$= -\gamma t \int_0^2 y \left(\frac{3}{\sqrt{2}} y^{1/2}\right) dy$$

$$= -\gamma t \frac{3}{\sqrt{2}} \frac{y^{5/2}}{5} 2 \Big|_0^2 = -\gamma t \frac{3\sqrt{2}}{5} 4\sqrt{2}$$

$$= -\frac{24}{5} \gamma t$$

$$\bar{y} = \frac{24}{5} \frac{\gamma t}{\times 4 t \gamma} = \frac{6}{5} m = 1.2 m$$

Taking moments about y axis - to get $\bar{x}$

CG of strip is at its center since γ is const & $LA = \frac{x}{2}$

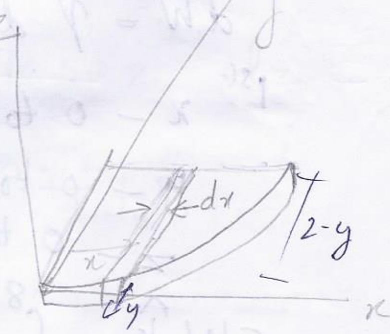
$$W\bar{x} = \int_0^2 \frac{x}{2} (t x \, dy) \gamma$$

$$= \frac{t\gamma}{2} \int_0^2 x^2 dy = \frac{t\gamma}{2} \int_0^2 \frac{9}{2} y \, dy$$

$$= \frac{9}{4} t\gamma \frac{y^2}{2}\Big|_0^2 = \frac{9}{2} t\gamma$$

$$\bar{x} = \frac{9}{2} \frac{t\gamma}{\times 4t\gamma} = \frac{9}{8} m$$

or



$$W\bar{x} = \int_0^3 x\gamma(2-y) t \, dx$$

$$= \int_0^3 x\gamma\left(2 - \frac{2}{9}x^2\right) t \, dx$$

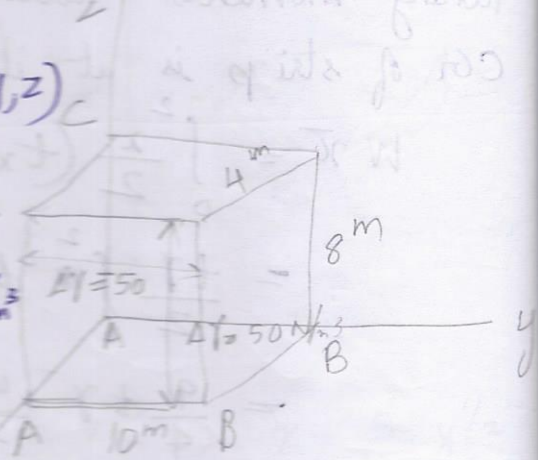
$$= 2\gamma t \int_0^3 x - \frac{x^3}{9} dx$$

$$= 2\gamma t \left[\frac{x^2}{2} - \frac{x^4}{9 \times 4}\right]_0^3$$

$$= 2\gamma t \left[\frac{9}{2} - \frac{9}{4}\right] = \frac{9}{2}\gamma t$$

$$\bar{x} = \frac{9}{8} m$$

3. Consider a block wherein the specific wt γ at A is 200 N/m³. The specific wt in the block [does] not change in the x direction. However, it de[creases] linearly by 50 N/m³ in 10 m in y direction [and] linearly by 50 N/m³ in 8 m in z direction, [as] shown in Fig. What are the coordinates $\bar{x}, \bar{y}$ for this block?



Sol: γ at any position $P(x,y,z)$

$$\gamma = 200 - \frac{y}{10} \times 50 + \frac{z}{8} 50$$

$$= 200 - 5y + 6.25z \text{ N/m}^3$$

Wt. of the block

Instead of slice, we use an infinitesimal rectangular parallelopiped having $dx\,dy\,dz$, located at (x,y,z)

$$dW = \gamma\, dx\, dy\, dz = (200 - 5y + 6.25z)\, dx\, dy\, dz$$

1st x – 0 to 4 m y & z fixed

y – 0 to 10 z const

z – 0 to 8

$$-W\hat{k} = \int_0^8 \int_0^{10} \int_0^4 (200 - 5y + 6.25z)\, dx\, dy\, dz\ (-k)$$

$$= \int_0^8 \int_0^{10} (200x - 5xy + 6.25xz)\Big|_0^4 dy\, dz\ (-k)$$

$$W = \int_0^8 \int_0^{10} 800 - 20y + 25z\ dy\, dz$$

$$= \int_0^8 800y - 20\frac{y^2}{2} + 25zy\Big|_0^{10} dz$$

$$= \int_0^8 8000 - 1000 + 250z \, dz$$

$$= 7000z + 250\frac{z^2}{2}\Big|_0^8$$

$$= 56000 + 8000 = \underline{64000} \text{ N}$$

To get $\bar{y}$, take moment about x axis

$$-64000\,\bar{y} = -\int_0^8\int_0^{10}\int_0^4 y(200 - 5y + 6.25z)\,dx\,dy\,dz$$

$$64000\,\bar{y} = \int_0^8\int_0^{10} 200xy - 5xy^2 + 6.25xzy\Big|_0^4 dy\,dz$$

$$= \int_0^8\int_0^{10} 800y - 20y^2 + 25yz \; dy\,dz$$

$$= \int_0^8 800\frac{y^2}{2} - 20\frac{y^3}{3} + 25z\frac{y^2}{2}\Big|_0^{10} dz$$

$$= \int_0^8 40000 - 6667 + 1250z \; dz$$

$$= 33333z + 1250\frac{z^2}{2}\Big|_0^8$$

$$= 307000$$

$$\bar{y} = \underline{4.79 \text{ m}}$$

Because γ does not depend on x, $\bar{x} = 2$ m

$$64000\,\bar{x} = \int_0^8\int_0^{10}\int_0^4 x(200 - 5y + 6.25z)\,dx\,dy\,dz$$

$$= \int_0^8\int_0^{10} 1600 - 40y + 50z \; dy\,dz$$

$$= \int_0^8 1600y - \frac{40y^2}{2} + 50zy \Big|_0^{10} dz$$

$$= \int_0^8 16000 - 2000 + 500z \; dz$$

$$= 14000z + 500\frac{z^2}{2}\Big|_0^8$$

$$= 128000$$

$$\bar{x} = 2m$$

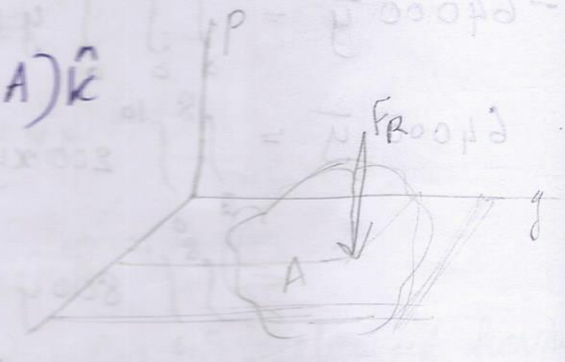
Day 9

[B] Parallel Force Distribution over a Plane Surface - Ce[ntre] of Pressure

$$F_R = -\int p\,dA = -\left(\int p\,dA\right)\hat{k}$$

$$\bar{x} = \frac{\int p\,x\,dA}{\int p\,dA}$$

$$\bar{y} = \frac{\int p\,y\,dA}{\int p\,dA}$$



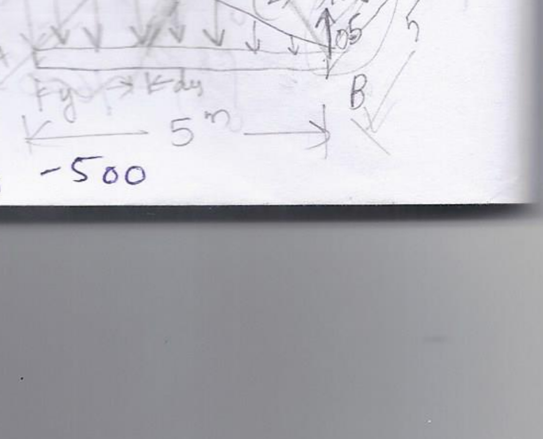
1. A plate ABCD on which both distributed & point force systems act is shown in Fig. The pressure distribution is given as $p = -4y^2 + 100 \; N/m^2$. Find the simplest resultant for the system.

Sol: Consider a strip dy along the plate.

p-uniform along this strip

Force from pressure on strip

$= p\,dA = p\,dy\,5$



$$F_R = -\int_0^5 p\,5\,dy - 500$$

$$= -\int_0^5 (-4y^2 + 100)\,5\,dy - 500$$

$$= -\left(-\frac{20y^3}{3} + 500y\Big|_0^5\right) - 500$$

$$= +833.33 - 2500 - 500 = -2167\,N$$

To get $\bar{x}, \bar{y}$ of F_R without a couple moment

Taking moment about x axes

$$-2167\,\bar{y} = -\int_0^5 y\,p(5\,dy) - 500 \times 2$$

$$= -\int_0^5 5y(-4y^2+100)\,dy - 1000$$

$$= 20\frac{y^4}{4} - 500\frac{y^2}{2}\Big|_0^5 - 1000$$

$$= 3125 - 6250 - 1000$$

$$= -4125$$

$$\bar{y} = 1.904\,m$$

Taking moment about y axes

$$2167\,\bar{x} = \int_0^5 \frac{5}{2}\,p\,(5\,dy) + 500 \times 2 - 0.5 \times 100$$

$$= \frac{25}{2}\int_0^5 (-4y^2+100)\,dy + 1000 - 50$$

$$= \frac{25}{2}\left[-\frac{4y^3}{3} + 100y\right]_0^5 + 950$$

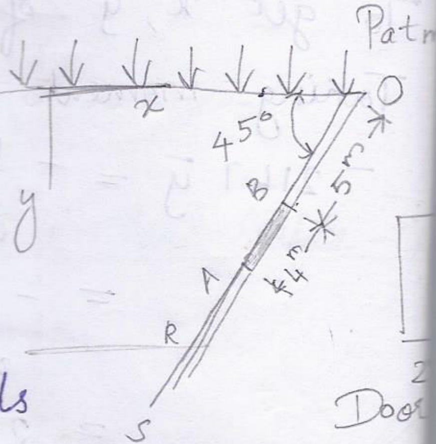
$$= \frac{25}{2} \times \left[-\frac{4}{3} \times 5^3 + 500\right] + 950$$

$$= 5116.67$$

$$\bar{x} = 2.36\,m$$

2. Find the force on the door AB from water whose specific wt is $9806\ N/m^3$ & on whose surface there is atmospheric pressure $= 101325$
Also find the center of pressure

Sol: The pressure on the door AB



$$P = P_{atm} + \gamma y$$

$$= P_{atm} + \gamma s \sin 45°$$

$s \rightarrow$ distance from O along the inclined wall OR

$$F = \int_5^9 \left[P_{atm} + \gamma s \sin 45\right] 2\, ds$$

$$= \int_5^9 \left[101325 + 9806 \times \frac{1}{\sqrt{2}} s\right] 2\, ds$$

$$= 2\left[101325 s + 9806 \frac{1}{\sqrt{2}} \frac{s^2}{2}\right]_5^9$$

$$= 1.199 \times 10^6\ N$$

$$1.199 \times 10^6\, \bar{s} = \int_5^9 s \left[P_{atm} + \gamma s \sin 45\right] 2\, ds$$

$$= 2\left[101325 \frac{s^2}{2} + \frac{9806}{\sqrt{2}} \frac{s^3}{3}\right]_5^9$$

$$= 8.466 \times 10^6$$

$$\bar{s} = 7.061\ m$$

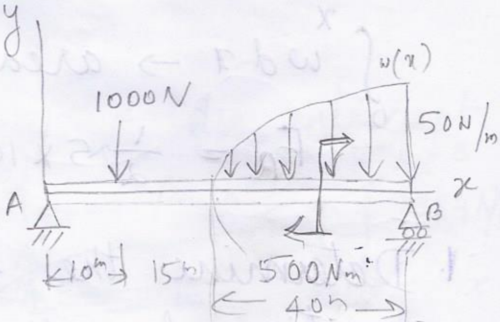
[C] Coplanar Parallel Force Distribution

$\Rightarrow$ beams loaded symmetrically over longitudinal of the beam

$$F_R = -\int w(x)\,dx \uparrow$$

$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx}$$

3. A simply supported beam is shown in Fig supporting 1000 N point force, 500 Nm couple & a coplanar, parabolic, distributed load w N/m. Find the simplest resultant of this force system



Sol $w^2 = ax + b$

$x = 25 \quad w = 0$

$x = 65 \quad w = 50$

$0 = a\,25 + b$

$2500 = 65a + b$

$40a = 2500$

$\underline{a = 62.5} \qquad \underline{b = -1562.5}$

$w^2 = 62.5x - 1562.5$

$$F_R = -1000 - \int_{25}^{65} \underbrace{\sqrt{62.5x - 1562.5}\,dx}_{}$$

$w(x)\,dx$

$\mu = 62.5x - 1562.5 \qquad x = 25 \quad \mu = 0$

$d\mu = 62.5\,dx \qquad x = 65 \quad \mu = 2500$

$$F_R = -1000 - \int_0^{2500} \sqrt{\mu}\,\frac{d\mu}{62.5}$$

$$= -1000 - \frac{1}{62.5}\,\frac{\mu^{3/2}\,2}{3}\Big|_0^{2500} = \underline{-2333\text{ N}}$$

$$-2333\bar{x} = -10 \times 1000 - \int_{25}^{65} x\sqrt{62.5x - 1562.5}\, dx - 5$$

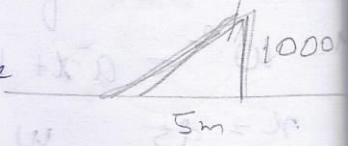
$$\int x\sqrt{a+bx}\, dx = -\frac{2(2a-3bx)}{15b^2}\sqrt{(bx+a)^3}$$

$a = $
$b = $

$$\bar{x} = \frac{-1}{2333}\left[-10 \times 1000 - 65300 - 500\right]$$

$$\bar{x} = \underline{32.5 m}$$

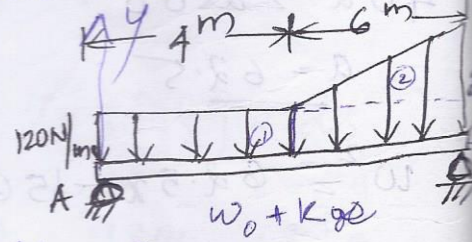
$\int_0^x w\, dx \rightarrow$ area under loading curve



$$F_R = \frac{1}{2} \times 5 \times 1000 = 2500 N$$

1. Determine the equivalent concentrated load & reactions for the simply supported beam which subjected to the distributed load shown.

Sol: $R_1 = 120 \times 10 = 1800 N$



ⓐ $\frac{10}{2} = 5m$ $1200\bar{x} =$

$R_2 = \frac{1}{2} \times 6 \times 160 = 480 N$ @

$w = k$

$w = w_0 + k$

1. Find the simplest resultant & its intercept with the X-axis

$120 = 120$

$w = 120$

Sol: $R = \int w\, dx = \int_0^4 120\, dx + \int_4^{10} 120 + \frac{160}{6}(x-4)\, dx$

$$= 120 \times 4 + 120 \times 6 + \frac{160}{6}\frac{x^2}{2} - \frac{160}{6} \times 4x \Big|_4^{10}$$

$$= \underline{1680 N}$$

$R\bar{x} = \int w x\,dx$

$1680\,\bar{x} = \int_0^4 120x\,dx + \int_4^{10} 120x + \frac{160}{6}x(x-4)\,dx$

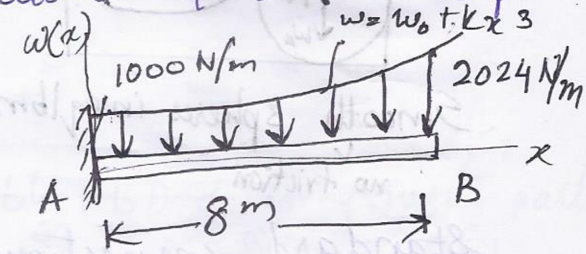
$= 120\frac{x^2}{2}\Big|_0^4 + 120\frac{x^2}{2}\Big|_4^{10} + \frac{160}{6}\frac{x^3}{3}\Big|_4^{10} - \frac{160}{6}\times 4\frac{x^2}{2}\Big|_4^{10}$

$= 60\times16 + 60\times(10^2-4^2) + \frac{160}{18}\times(10^3-4^3) - \frac{160}{3}(10^2-4^2)$

$= 9840$

$\bar{x} = 5.857\,m$

2. Find the simplest resultant & its intercept with the X-axis.



Sol: $w_0 = 1000\ N/m$

$w = w_0 + kx^3$

$x = 0 \Rightarrow w = 1000$

$x = 8 \Rightarrow 2024 = 1000 + k\times 8^3$

$k = 2\ N/m^4$

$R = \int w\,dx = \int_0^8 1000 + 2x^3\,dx$

$= 1000x + 2\frac{x^4}{4}\Big|_0^8$

$= 1000\times 8 + \frac{8^4}{2} = 10\,048\,N$

$\bar{x} = \frac{\int x w\,dx}{R} = \frac{1}{10048}\int_0^8 1000x + 2x^4\,dx$

$= \frac{1}{10048}\left[1000\frac{x^2}{2} + 2\frac{x^5}{5}\right]_0^8 = 4.489\,m$