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22.2.18 # System of Linear Equations

For chemical equations:

$$\overset{x_1}{C_3H_8} + \overset{x_2}{O_2} \longrightarrow \overset{x_3}{CO_2} + \overset{x_4}{H_2O}$$

C $3x_1 - x_3$

H $8x_1 = 2x_4$

O $2x_2 = 2x_3 + x_4$ Smallest +ve integers

System of Linear Eqs.

$a_{11}x_1 + \cdots\cdots a_{1n}x_n = b_1$

$a_{21}x_1 + \cdots\cdots a_{2n}x_n = b_2$

$\vdots$

$a_{m1}x_1 + \cdots\cdots a_{mn}x_n = b_m$

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}_{m\times n}$$

$$\underline{x} = \begin{pmatrix} x \\ \vdots \\ x_n \end{pmatrix}_{n\times 1} \qquad \underline{b} \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}_{m\times 1}$$

$$A\,\underline{x} = \underline{b}$$

Q1 Solve:

$$3x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + x_2 + x_3 = 0$$
$$4x_1 + 3x_2 + x_3 = 4$$

$$\left(\begin{array}{ccc|c} 3 & 2 & 1 & 3 \\ 2 & 1 & 1 & 0 \\ 4 & 3 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 2 & 1 & 1 & 0 \\ 4 & 3 & 1 & 4 \end{array}\right)$$

Divide I row by 3

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 4R_1$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & -1/3 & 1/3 & -2 \\ 0 & 1/3 & -1/3 & 0 \end{array}\right)$$

$1 - \frac{2}{3}$

$3 - \frac{8}{3}$

$\frac{9-8}{3}$

$R_2 \to \cancel{R_3 + R_2}\ -3R_2$ $\sim \cancel{\begin{matrix} 1 & 2/3 & 1/3 & 1 \end{matrix}}$

$1 - \frac{4}{3}$

$3-4$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & 1 & -1 & 6 \\ 0 & 1/3 & -1/3 & 0 \end{array}\right)$$

$R_3 \to R_3 - \frac{1}{3}R_2$

$$\sim \left(\begin{array}{ccc|c} 1 & 2/3 & 1/3 & 1 \\ 0 & 1 & -1 & 6 \\ 0 & 0 & 0 & 8 \end{array}\right)$$

$-\frac{1}{3} + \frac{1}{3}$

∴ This system cannot have a solution

Hence ~~no solution~~ called inconsistent

~~..~~ Augmented matrix

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Q. Solve.

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 - x_2 + 4x_3 = 5$$
$$3x_1 - x_2 - x_3 = 1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -1 & 4 & 5 \\ 3 & -1 & -1 & 1 \end{array}\right)$$

$R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 3R_1$

$-1-4$

$4-6$

$-1-6$

$-1-9$

$5-12$

$1-18$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -7 \\ 0 & -7 & -10 & -17 \end{array}\right)$$

$R_2 \rightarrow \frac{R_2}{-5}$

$\frac{7\times 7}{5}$ $\frac{49}{5}$ $\frac{85}{36}$ $-17 + \frac{49}{5}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & -7 & -10 & -17 \end{array}\right)$$

$R_3 \rightarrow R_3 + 7R_2$

$-7+7$

$-10 + \frac{14}{5}$ $\frac{14}{36}$

$\frac{-50+14}{5}$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & 0 & -36/5 & -36/5 \end{array}\right)$$

$-36/5\, x_3 = -36/5$

$x_3 = 1$

$x_2 + \frac{2}{5} \times 1 = \frac{7}{5}$

$x_2 = \frac{7}{5} - \frac{2}{5} = \frac{5}{5}$

$x_1 + 2x_2 + 3x_3 = 6$

$x_1 + 5 = 6$

$\underline{x_1 = 1}$.

Q. Solve. $x_1 + 2x_2 + 3x_3 = 6$

$2x_1 - x_2 + 4x_3 = 5$

$3x_1 + x_2 + 7x_3 = 11$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & -1 & 4 & 5 \\ 3 & 1 & 7 & 11 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$-2 \; -4$
$4 - 6$
$1 - 6$
$\cancel{11} \; 9$
$5 - 12$
$11 - 18$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -5 & -2 & -7 \\ 0 & -5 & -2 & -7 \end{array}\right]$$

$R_2 \to R_2/-5 \quad R_3 \to R_3/-5$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & 1 & 2/5 & 7/5 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2/5 & 7/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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∴ infinitely many solutions.

Echelon Form

A matrix A is said to be in Echelon form provided the following conditions are satisfied

1) The first non-zero element in every row is unity.
2) The number of zeros before the first non-zero element should be in the increasing order
3) If at all there is a zero row, it should occur at the end.

Rank of a matrix when it is in Echelon form is the number of non-zero rows.

Consistency: consistent if)

→ $R(A) = R(A, \underline{b})$

→ If $R(A) = R(A, b) = n$ (no. of unknowns)
Unique solution.

→ If $R(A) = R(A, b) = r <$ no. of unknowns.
Infinitely many solutions.

Gauss Elimination.

26.2.18 Q. Determine the value of λ such that.

$x + y + z = 3$

$2x + y + z = 5$

$x + 2y + \lambda z = \lambda^2$.

has i) no solution

ii) unique solution.

iii) many solution

$$(A, b) = \left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 2 & 1 & 1 & 5 \\ 1 & 2 & \lambda & \lambda^2 \end{array}\right)$$

$R_2 \rightarrow R_2 - 2R_1$ $\quad$ $R_3 \rightarrow R_3 - R_1$

$$\sim \left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & \lambda-1 & \lambda^2-3 \end{array}\right)$$

$R_2 \rightarrow \cancel{R_2/-1}\ -1R_1$

$$\sim \left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & \lambda-1 & \lambda^2-3 \end{array}\right)$$

$\cancel{R_3 \rightarrow R_2}$

$R_3 \rightarrow R_3 - R_2$.

$$\sim \left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \lambda-2 & \lambda^2-4 \end{array}\right)$$

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i) No solution

$\lambda - 2 = 0$

$\underline{\lambda = 2} \Rightarrow \lambda^2 - 4 = 0$

$R(A) = 2.$

$R(A, \underline{b}) = 2$

$R(A) = R(A, \underline{b}) = 2 < 3,$ no. of unknowns.

W

Infinite no. of solutions.

ii) unique solution

if $\lambda \neq 2.$

$R(A) = R(A, b) = 3 =$ no. of unknowns.

unique solution

$\therefore$ No solution is not possible

Whatever be λ, system is consistent.

Q. Determine the values of 'a' for which the system

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2.$$

has i) no solution ii) unique solu iii) many solu

$$\left[\begin{array}{ccc:c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & (a^2-14) & a+2 \end{array}\right]$$

$R_2 \rightarrow R_2 - 3R_1.$

$R_3 \rightarrow R_3 - 4R_1$

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$$\left[\begin{array}{cccc} 1 & 2 & -3 & : 4 \\ 0 & -7 & 14 & : -10 \\ 0 & -7 & a^2-2 & : a-14 \end{array}\right]$$

-1-6
5+9
2-12
1-5

$R_2 \rightarrow R_2/-7 \qquad R_3 \rightarrow R_3/-7$

$a^2-14+12$
a^2-2-16
$a+2$
$a-14$

$$\left[\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & : 10/7 \\ 0 & 1 & \frac{a^2-2}{-7} & : \frac{a-14}{-7} \end{array}\right]$$

$2-12$

$R_3 \rightarrow R_3 - R_2$.

$$\left[\begin{array}{cccc} 1 & 2 & -3 & : 4 \\ 0 & 1 & -2 & : 10/7 \\ 0 & 0 & \frac{a^2-2}{-7}+2 & : \frac{a-14}{-7} - \frac{10}{7} \end{array}\right]$$

$\frac{a^2-2}{-7}+2 = 0$

$\frac{a^2-2}{-7} = -2 \Rightarrow a^2-2 = 14$

$a^2 = 16$

$a = \pm 4$

$\frac{4-14}{-7} - \frac{10}{7}$

$\frac{-10}{-7} - \frac{10}{7}$

If $a=4$ then ~~wo~~ we will have infinite solution.

$R(A) = R(A, \underline{b}) = 2 < 3$.

$\therefore$ Many solutions.

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b) i) If $a = -4$

$R(A) = 2$.

$R(A, \underline{b}) = 3$ Hence system is inconsistent $\because R(A) \neq$

$\therefore$ No solution

ii) $\cancel{a^2 - 16 \neq 0}$

$\underbrace{a^2 - 2}_{-7} + 2 \neq 0$.

$a^2 - 2 \neq -14$

$a^2 \neq 16$

$R(A) = R(A, \underline{b}) = 3$.

Unique solution.

Q. Determine the condition on b_1, b_2, b_3. such that the system

$2x + y + 7z = b_1$

$6x - 2y + 11z = b_2$.

$2x - y + 3z = b_3$

i) have no solution 2) many solution

$$\left[\begin{array}{ccc|c} 2 & 1 & 7 & b_1 \\ 6 & -2 & 11 & b_2 \\ 2 & -1 & 3 & b_3 \end{array}\right]$$

$R_1 \to R_1/2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 7/2 & b_1/2 \\ 6 & -2 & 11 & b_2 \\ 2 & -1 & 3 & b_3 \end{array}\right]$$

$R_2 \to R_2 - 6R_1$; $R_3 \to R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 7/2 & b_1/2 \\ 0 & -10/2 & \cancel{-41/2}\ -10 & b_2 - 3b_1 \\ 0 & -2 & -4 & b_3 - b_1 \end{array}\right]$$

$R_2 \to -\frac{2}{10} R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 7/2 & b_1/2 \\ 0 & 1 & -2 & (b_2-3b_1)\left(-\frac{2}{10}\right) \\ 0 & -2 & -4 & b_3 - b_1 \end{array}\right]$$

$R_3 \to R_3 + 2R_2$

$$\left[\begin{array}{ccc|c} 1 & 1/2 & 7/2 & b_1/2 \\ 0 & 1 & -2 & (b_2-3b_1)\left(-\frac{2}{10}\right) \\ 0 & 0 & -8 & b_3 - b_1 - \frac{2}{5}(b_2 - 3b_1) \end{array}\right]$$

i) $b_3 - b_1 - \frac{2}{5}(b_2 - 3b_1) \neq 0$

~~$b_3 - b_1 \neq \frac{2}{5}(b_2 - 3b_1)$~~

$4b_1 - 2b_2 + 5b_3 \neq 0$

$R(A) = 2 \neq 3 = R(A, b)$

no solution.

$-2 - \frac{6}{2}$

$\frac{-4-6}{2}$

$\frac{11-42}{2}$

-31

$b_2 - \frac{6b}{2}$

$\frac{22-42}{2}$ $\frac{2b_2-6b}{2}$

$-1-1$

$3-7$

$b_3 - b_1$

$-4+4$

$b_3 - b_1 - \frac{2}{5}(b_2 - 3$

ii) if $b_1 - 2b_2 + 5b_3 = 0$

$R(A) = R(A, \underline{b})$, many solution.

System of Linear Homogeneous Eq.

$a_{11}x_1 + \dots\dots a_{1n}x_n = 0$

$\dots\dots\dots$

$a_{m1}x_1 + \dots\dots a_{mn}x_n = 0$

$m < n \rightarrow R(A) <$ No. of unknowns Underdetermined

$m = n \rightarrow R(A) \leq n$ → Unique → Infinite

$m > n \rightarrow$ Overdetermined.

$x + y = 2$

$2x + y = 3$

$x + 3y = 4$ → Redundant (3^{rd} eq. is not required)

→ Homogenous system of eqs. $R(A) = R(A, \underline{0})$
System of homogenous sys. will always be consistent.

→ i) $R(A) = R(A, \underline{0})$; Consistent.

→ ii) $R(A) = R(A, \underline{0}) = r < n$, many solution

→ iii) $R(A) = R(A, \underline{0}) = r = n$, unique solution

$\Rightarrow$ $x_1 = x_2 = \ldots\ldots x_n = 0$, trivial solution, zerosolution

$A_{n \times n}$ $\quad$ $AX = 0$

~~X X~~

Order of the largest non-vanishing ~~det~~ determinant

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ \hline 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right] \qquad \left(\begin{array}{cc|c|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

$R(A) = n$, unique solution.

$|A| \neq 0$ (Tri vial ~~sg~~ solution)

$m = n$

For Non Trivial solution $|A| = 0$.

If $m - n$ the homogenous eq will have a trivial solution.

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27.2.18 $|A| = 0$ non trivial sol (many)

$|A| \neq 0$ trivial sol (unique)

Q. Find the constant k such that the system $kx - 2y - z = 0$

$(k+1)y + 4z = 0$

$(k-1)z = 0$

has a non-trivial soln and find all such solutions.

$$\begin{pmatrix} k & -2 & -1 \\ 0 & k+1 & 4 \\ 0 & 0 & k-1 \end{pmatrix}$$

Non trivial $|A| = 0$.

$$\begin{vmatrix} k & -2 & -1 \\ 0 & k+1 & 4 \\ 0 & 0 & k-1 \end{vmatrix} = 0$$

$k(k+1)(k-1) = 0$

$k(k^2-1) = 0$

$k = 0, -1, +1$

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→ If $k=0$

$x=0,\ y=0$

$kx=0$ where $k=0 \Rightarrow x$ can take any value

$x=k_1$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} k_1 \\ 0 \\ 0 \end{pmatrix} = k_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

→ If $k=1$

$\cancel{z=0} \Rightarrow z=k_2.$

$2y+4z=0 \Rightarrow y=-2z=-2k_2.$

$kx-2y-z=0$

$x-2y-z=0$

$x=2y+z=-4k_2+k_2=-3k_2.$

solution $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -3k_2 \\ -2k_2 \\ k_2 \end{pmatrix} = k_2 \begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix}$

→ If $k=-1$

$(-1-1)z=0 \rightarrow \cancel{z=0}\ z=0$

$(k+1)y+0=0$

$(1+1)y=0 \Rightarrow y=k_3.$

$kx-2y-z=0$

$-x-2k_3=0$

$x=-2k_3$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2k_3 \\ k_3 \\ 0 \end{pmatrix} = k_3 \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}$$

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Q. Solve $2x+2y+z+3w=0$

$3x+2y+2z+w=0$

$x+4y+5z+2w=0$

$4x-2z+2w=0$

$$\begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 2 & 1 & 3 \\ 3 & 2 & 2 & 1 \\ 4 & 0 & -2 & 2 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 3R_1$; $R_4 \to R_4 - 4R_1$.

$$\begin{pmatrix} 1 & 4 & 5 & 2 \\ 0 & -6 & -9 & -1 \\ 0 & -10 & -13 & -5 \\ 0 & -16 & -22 & -6 \end{pmatrix}$$

$R_2 \to R_2/-6$ ~~$R_3 \to R_3/$~~

$$\begin{pmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 3/2 & 1/6 \\ 0 & -10 & -13 & -5 \\ 0 & -16 & -22 & -6 \end{pmatrix} \sim \begin{pmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 3/2 & 1/6 \\ 0 & 0 & 2 & -10/3 \\ 0 & 0 & 0 & -10/3 \end{pmatrix}$$

$-13-\frac{30}{2}$

$\frac{.26+30}{2}$

$-20+\frac{8}{16}\times\frac{3}{2}$

$-20+24$

~~$R_4 \to R_4 - R_3$~~ $R_3 \to R_3/2$; $R_4 \to R_4 - R_3$

$$\begin{pmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 3/2 & 1/6 \\ 0 & 0 & 1 & -5/3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R(A) = 3 \neq 4 \rightarrow$ no. of variables)

$\cancel{z - 5/3\,\boxed{w = 0}}$ $\quad z - \frac{5}{3}w = 0$

let $w = a$

$$\boxed{z = \frac{5}{3}a}$$

$$y + \frac{3}{2}z + \frac{1}{6}w = 0$$

$$y = -\frac{3}{2} \times \frac{5}{3}a - \frac{1}{6}a \cancel{-0}$$

$$y = -\frac{5}{2}a - \frac{1}{6}a = -\frac{8}{3}a \Rightarrow \boxed{y = -\frac{8}{3}a}$$

$-15 - 1$ / 6 / 3

$$x + 4y + 5z + 2w = 0$$

$\cancel{-16}$ $\cancel{6}$ 3

$$x + 4 \times -\frac{8}{3}a + 5 \times \frac{5}{3}a + 2a = 0$$

$$x = \frac{a}{3}$$

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} a/3 \\ -8/3a \\ 5/3^{a} \\ a \end{pmatrix} \Rightarrow b \begin{pmatrix} 1 \\ -8 \\ 5 \\ 3 \end{pmatrix}$$

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Q. $\begin{pmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 3/2 & 1/6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

$y + \frac{3}{2}z + \frac{1}{6}w = 0$

Let $z = k_1$; $w = k_2$.

$y = -\frac{3}{2}k_1 - \frac{1}{6}k_2$

$x = -4y - 5z - 2w = \frac{12}{2}k_1 + \frac{4}{6}k_2 - 5k_1 - 2k_2$

$= 6k_1 + \frac{2}{3}k_2 - 5k_1 - 2k_2$

$= k_1 + \left(\frac{2}{3} - 6\right)k_2$

$= k_1 + -\frac{4}{3}k_2$

$$\begin{pmatrix} k_1 - 4/3k_2 \\ -3/2k_1 - 1/6k_2 \\ k_1 \\ k_2 \end{pmatrix} = k_1\begin{pmatrix} 1 \\ -3/2 \\ 1 \\ 0 \end{pmatrix} + k_2\begin{pmatrix} -4/3 \\ -1/6 \\ 0 \\ 1 \end{pmatrix}$$

→ If rank of coefficient eq. is r ~~then we~~ which ~~arbitary~~ a is less than n then assign values for $(n-r)$ variables. and solve the remaining r variables interms of these $(n-r)$ variables.

→ $(n-r)$ is called the free variables

Homogenous & Non-homogenous.

$A\underline{X} = 0 \longrightarrow \underline{X}_h$ all solutions.

$A\underline{X} = \underline{b} \longrightarrow \underline{X}_p$ any one solution.

* let $\underline{X} = \underline{X}_h + \underline{X}_p$

consider $A\underline{X} = A(\underline{X}_h + \underline{X}_p)$

$= A\underline{X}_h + A\underline{X}_p = \underline{0} + \underline{b} = \underline{b}$

Q. Solve.

$x+y+z=3$

$2x+y+z=4$

$3x+2y+2z=7$

$$\left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 2 & 1 & 1 & 4 \\ 3 & 2 & 2 & 7 \end{array}\right) \sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & -1 & -1 & -2 \\ 0 & -1 & -1 & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{pmatrix} \sim \left(\begin{array}{ccc:c} 1 & 1 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

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$y + z = 2.$

Let $\boxed{z = k}$

$y + k = 2$

$\boxed{y = 2 - k}$

$x + y + z = 3$

$x + 2 - k + k = 3$

$\boxed{x = 1}$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2-k \\ k \end{pmatrix} = \underbrace{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}_{\underline{X}_p} + \underbrace{\boxed{k\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}}}_{\underline{X}_h}$$

One solution of given diff eq.

1.3.18 Linear Independence.

$$\underline{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\underline{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

if $R(A) = 3$ vectors are linearly independent

$R(A) < 3$, vectors are linearly dependent.

$$\underline{v}_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\alpha_1 \underline{v_1} + \alpha_2 \underline{v_2} + \alpha_3 \underline{v_3} = 0$$

$$\alpha_1\begin{pmatrix}1\\2\\3\end{pmatrix}+\alpha_2\begin{pmatrix}2\\3\\5\end{pmatrix}+\alpha_3\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\alpha_1+2\alpha_2+\alpha_3=0$$
$$2\alpha_1+3\alpha_2+\alpha_3=0$$
$$3\alpha_1+5\alpha_2+\alpha_3=0$$

$$\begin{vmatrix}1&2&1\\2&3&1\\3&5&1\end{vmatrix}\neq 0$$

$$a_{11}x_1+\cdots a_{1n}x_n=b_1$$
$$\vdots$$
$$a_{m1}x_1+\cdots\cdots a_{mn}x_n=b_m$$

$$\begin{pmatrix}a_{11}\\ \vdots\\ a_{m1}\end{pmatrix}x_1+\begin{pmatrix}a_{12}\\ \vdots\\ a_{m2}\end{pmatrix}x_2+\cdots\begin{pmatrix}a_{1n}\\ \vdots\\ a_{mn}\end{pmatrix}x_n=\begin{pmatrix}b_1\\ \vdots\\ b_m\end{pmatrix}$$

$$\underline{a}_1x_1+\underline{a}_2x_2+\cdots+\underline{a}_nx_n=\underline{b}$$

$$\underline{a}_1=\begin{pmatrix}1\\2\end{pmatrix}\quad \underline{a}_2=\begin{pmatrix}2\\1\end{pmatrix}\quad \underline{b}=\begin{pmatrix}1\\1\end{pmatrix}$$

$$\begin{pmatrix}1\\2\end{pmatrix}x_1+\begin{pmatrix}2\\1\end{pmatrix}x_2=\begin{pmatrix}1\\1\end{pmatrix}$$

$$\frac{1}{3}\begin{pmatrix}1\\2\end{pmatrix}+\frac{1}{3}\begin{pmatrix}2\\1\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$$

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