NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Mathematics, Winter Semester 2019-2020
Mathematics-2 (MA1002D), Tutorial-5

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1. Find the differential equation of all circles which pass through the origin and whose centres are on the x-axis.

2. Find the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$ where A and B are constants.

3. State the existence and uniqueness theorem of first order ordinary differential equations. Check the existence and uniqueness of solution of the following initial value problems. If solution exists, find the unique solution or all solutions.
   (a) $\frac{dy}{dx} = x - y + 1,\ y(1) = 2.$
   (b) $\frac{dy}{dx} = 5y^{\frac{4}{5}},\ y(0) = 0.$
   (c) $\frac{dy}{dx} = 5y^{\frac{4}{5}},\ y(0) = 1.$
   (d) $\frac{dy}{dx} = \frac{2y}{x},\ y(0) = 1.$

4. Solve the following differential equations (Variable-Separable/reducible to Variable-Separable)
   (a) $\frac{dy}{dx} = e^{x+y} + x^2 e^{x^2+y}.$
   (b) $\frac{dy}{dx}\tan y = \sin(x+y) + \sin(x-y).$
   (c) $\frac{dy}{dx} = (4x+y+1)^2.$
   (d) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y).$

5. Solve the following differential equations (Homogenous/Non homogeneous)
   (a) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0.$
   (b) $(x\cos\frac{y}{x} + y\sin\frac{y}{x})y - (y\sin\frac{y}{x} - x\cos\frac{y}{x})\frac{dy}{dx} = 0.$
   (c) $\frac{dy}{dx} = \frac{x+y+4}{x-y-6}.$
   (d) $\frac{dy}{dx} = \frac{2x+3y+4}{6y+4x+5}.$

6. Solve the following differential equations (Linear/Bernoulli/Reducible to linear or Bernoulli)
   (a) $y(\log y)dx + (x - \log y)dy = 0.$
   (b) $(1+y^2)dx = (\tan^{-1} y - x)dy.$
   (c) $\frac{dy}{dx} + x\sin(2y) - x^3\cos^2 y = 0.$
   (d) $\frac{dy}{dx} = y^3\cos(2x) - y\sin(x).$

7. Solve the following differential equations (exact/make it exact with integrating factor)
   (a) $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0.$
   (b) $y\sin(2x)dx = (1 + y^2 + \cos^2 x)dy.$
   (c) $(x^3 + xy^2 + 4y)dx + (y^3 + yx^2 - 4x)dy = 0.$
   (d) $ydx - xdy + (1+x^2)dx + x^2\sin y dy = 0.$

8. Solve the following differential equations (exact/make it exact with integrating factor)
   (a) $(x^2 + y^2 + 2x)dx + 2ydy = 0.$
   (b) $(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0.$
   (c) $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0.$
   (d) $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$

9. Find the orthogonal trajectories of the following family of curves
   (a) $3xy = x^3 - a^3$, a is the parameter.
   (b) $x^2 + y^2 = 2ax$, a is the parameter.
   (c) $r = a(1 + \cos\theta))$, a is the parameter.
   (d) $r^n\sin(n\theta) = a^n$, a is the parameter.

10. Show that the family of parabolas $x^2 = 4a(y+a)$ is self orthogonal.

11. If the temperature of the air is $290K$ and a substance cools from $370K$ to $330K$ in 10 minutes, find when the temperature will be $295K$. (Ans=40 minutes)

12. The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in one hour, find the number of bacteria which will be present at the end of five hours. (Ans: Increases by 3^5 times the initial bacteria).

13. (a) Find the current at any time $t > 0$ in a circuit having in series a constant electromotive force $40V$, a resistor 10Ω and an inductor $0.2H$ given that initial current is zero. (b) Find the current when $E(t) = 150\cos(200t)$. (Ans (a): $4(1 - e^{-50t})$, (b) $\frac{3}{170}(50\cos(200t) + 200\sin(200t)) - \frac{15}{17}e^{-50t}$)

14. A capacitor $C = 0.01F$ in series with a resistor $R = 20$ ohms is charged from a batter $E = 10V$. Assuming that initially the capacitor is completely uncharged, determine the charge $Q(t)$ and the current $i(t)$ at any time t. (Ans: $Q(t) = 0.1(1 - e^{-5t})$, $i(t) = 0.5e^{-5t}$).

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