NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
MA1002D MATHEMATICS II
Winter Semester 2019-2020
Tutorial on Beta-Gamma functions.

1. Evaluate (a) Γ (8) (b) Γ (-5.5)

2. Using Beta, Gamma function, evaluate the following

   (a) $\int_0^\infty e^{-x^3}\, dx$

   (b) $\int_{-\infty}^\infty e^{-k^2x^2}\, dx$

   (c) $\int_0^1 y^{q-1}\ (\log y)^{p-1} dy (p>0)$

   (d) $\int_0^\infty e^{-kx}\ x^{p-1} dx \quad (k>0)$

   (e) $\int_0^\infty \frac{x^c}{c^x} dx.$

3. Prove that $\int_0^1 x^m (\log x)^n\, dx \quad = \quad \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive integer and m>-1.

4. Using Gamma function evaluate, $\int_0^1 x^4 \left[\log\left[\frac{1}{x}\right]\right]^3 dx$.

5. Using Gamma function evaluate $\int_{-\infty}^\infty e^{-x^2-2axx}\ dx \quad = \int_{-\infty}^\infty e^{-\left[(x+a)^2-a^2\right]} dx = e^{a^2} \quad \int_{-\infty}^\infty e^{-(x+a)^2}\ dx.$

6. Evaluate in terms of Gamma function

   (a) $\beta(4.5,3.5)$

   (b) $\beta(4,5)$

7. Prove that $\int_0^1 \frac{xdx}{\sqrt{1-x^5}} = \frac{1}{5}\beta\left[\frac{2}{5},\frac{1}{2}\right].$

8. Using Beta, Gamma functions evaluate $\int_0^3 x^3\ (3-x)^7 dx.$

9. Using Beta, Gamma functions evaluate $\int_0^1 x^m\ (1-x^n)^p\ dx.$

****