NATIONAL INSTITUTE OF TECHNOLOGY CALICUT
Department of Mathematics

Winter Semester 2019-20
MA1002D MATHEMATICS II

Tutorial I

1. Solve the following systems by the Gauss elimination or indicate the non existence of solutions

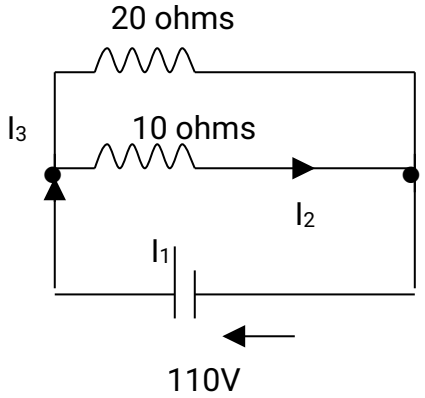
(a)
$$x + y - z = 9$$
$$8y + 6z = -6$$
$$-2x + 4y - 6z = 40$$

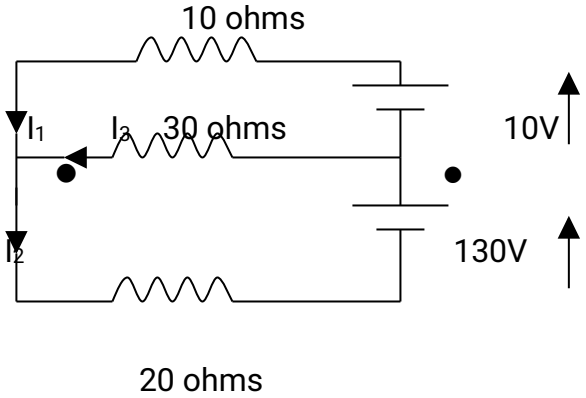
(b)
$$4y + 3z = 8$$
$$2x - z = 2$$
$$3x + 2y = 5$$

(c)
$$10x + 4y - 2z = -4$$
$$-3w - 17x + y + 2z = 2$$
$$w + x + y = 6$$
$$8w - 34x + 16y - 10z = 4$$

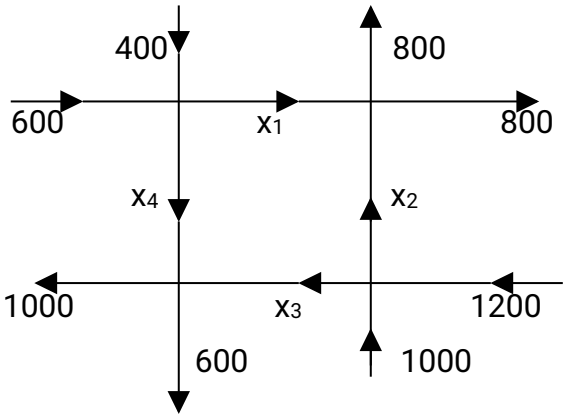
2. Find the current in the following networks

a)





3. Applying the analog of Kirchhoff's current law, find the traffic flow (cars per hour) in the net of one-way streets (in the direction indicated by the arrows) shown in the figure. Is the solution unique.



4. (a) Solve x+2y+4z=3, 3x+8y+14z=13, 2x+6y+13z=4, using LU decomposition/factorization.
(b) Using the decomposition in 4(a) solve x+2y+4z=17, 3x+8y+14z=61, 2x+6y+13z=53.

5. Do the following matrices have LU decomposition

(a) $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 & 7 \\ -2 & 6 & 1 \\ 0 & 3 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 1 & 3 & 4 \end{bmatrix}$

6. Find the rank of the following matrices using row echelon form

(a) $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 8 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 4 & 5 \end{bmatrix}$ (c) $\begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 2 \\ 4 & 5 & 5 \\ 5 & 8 & 1 \\ -1 & -2 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & -3 & -2 & -3 \\ 1 & 3 & -2 & 0 & -4 \\ 3 & 8 & -7 & -2 & -11 \\ 2 & 10 & 9 & -10 & -3 \end{bmatrix}$ (f) $\begin{bmatrix} m & n & p \\ n & m & p \end{bmatrix}$ $m^2 \neq n^2$

7. Find the values of k for which the matrix $\begin{bmatrix} 4 & 4 & -3 & 1 \\ 1 & 1 & -1 & 0 \\ k & 2 & 2 & 2 \\ 9 & 9 & k & 3 \end{bmatrix}$ has rank 3.

8. Let A = $\begin{bmatrix} 3 & -6 & 2 & -1 \\ -2 & 4 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$. For which Y=($y_1$, y_2 , y_3 , y_4) does the system of equations AX = Y have a solution?

9. Find all values of k for which the given linear system is consistent.
$x - 3y + 2z = 1, 2x - 2y = k^2, 3x - 5y + z = 0, -2x + 8y + 4z = 49$

10. Show that the equations $3x + 4y + 5z = a$, $4x + 5y + 6z = b$, $5x + 6y + 7z = c$ do not have a solution unless $a + c = 2b$.

11. For what values of a and b do the equations $x + 2y + 3z = 6$, $x+ 3y + 5z = 9$, $2x + 5y + az = b$ have (a) no solutions (b) a unique solution (c) more than one solution.