Day 1

MODULE II

Trusses

A truss is a system of members that are fastened together at their ends to support stationary & moving loads.

Eg:- Roof truss, bridge truss - plane truss

Plane truss - coplanar system of members

Space truss - electric power transmission system

Gusset plates - intermediate structural elements to which members are fastened

Centerlines should be concurrent

Pin connection or ball & socket joint - idealisation

External loads should be applied at joints - members are long & slender

2-force member

Assumptions to be made for analysis of Plane truss

1. The axes of all bars are lying in a single plane
2. All members are connected at their ends by means of frictionless hinges
3. Self wt. of members can be neglected.
4. The forces acting on the truss are subjected only at joints.

A truss is said to be perfect/rigid when it is non-collapsible when supports are removed.

$m + 3 = 2j$

$m + 3 > 2j$ - over rigid / redundant / SI

$m + 3 < 2j$ - under rigid / deficient / collapsable Truss

Solution of Simple Trusses

→ compute supporting forces – reactions

→ determine forces in members of truss

2 methods < Method of joints / Method of sections

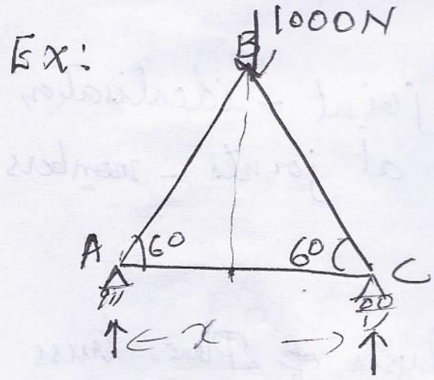
Method of Joints

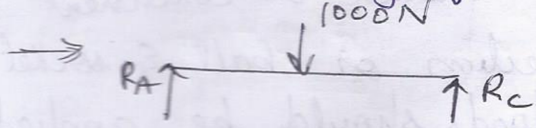
FBD of pins or ball joints – internal forces / external loads

Compressive – arrows point towards the pins

Tension – away from

Ex:



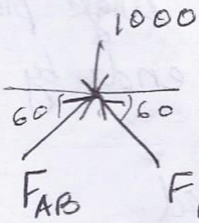


$$R_A + R_C = 1000$$

$$\Sigma M_A = 0 \Rightarrow R_C \times x = 1000\,\frac{x}{2}$$

$$R_C = \underline{500\,N}$$

$$R_A = 500\,N$$



$$F_{AB}\cos 60 = F_{CB}\cos 60$$

$$F_{AB} = F_{CB} \quad \text{—} \ (1)$$

$$F_{AB}\sin 60 + F_{CB}\sin 60 = 1000$$

$$2F_{AB}\frac{\sqrt{3}}{2} = 1000$$

$$F_{AB} = \frac{1000}{\sqrt{3}}$$

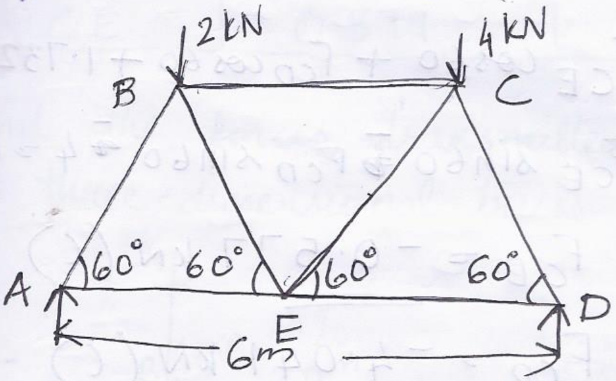
$$F_{AB} = F_{CB} = 577.35\,N$$

$$F_{AB}\cos 60 = F_{AC}$$

$$F_{AC} = 577.35\cos 60$$

$$= 288.67\,N$$

1. Figure shows a Warren girder consisting of 7 members each of 3m length freely supported at its end points. The girder is loaded at B & C as shown. Find the forces in all the members of the girder indicating whether the force is compressive or tensile.

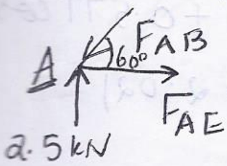


Sol: $R_A + R_D = 2 + 4 = 6\text{ kN}$

$\Sigma M_A = 0 \Rightarrow R_D \times 6 = 2 \times 1.5 + 4 \times 4.5 = 21$

$\underline{R_D = 3.5\text{ kN}}$ $\qquad \underline{R_A = 2.5\text{ kN}}$

Method of Joints

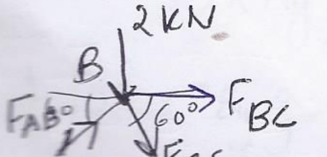


$-F_{AB}\sin 60 + 2.5 = 0 \quad -①$

$\underline{F_{AB} = 2.887\text{ kN (C)}}$

$F_{AE} - F_{AB}\cos 60 = 0$

$\underline{F_{AE} = 1.444\text{ kN (T)}}$



$F_{AB}\cos 60 + F_{BE}\cos 60 + F_{BC} = 0$

~~$F_{BE} = -2.88\text{ kN (C)}$~~

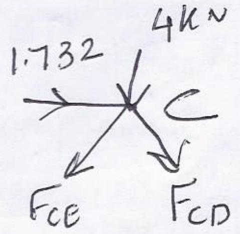
$F_{AB}\sin 60 - F_{BE}\sin 60 - 2 = 0$

$$F_{BE} \sin 60 = 2.887 \sin 60 - 2 = 0.5$$

$$\underline{\underline{F_{BE} = 0.577 \text{ kN } (T)}}$$

$$2.887 \cos 60 + 0.577 \cos 60 + F_{BC} = 0$$

$$\underline{\underline{F_{BC} = -1.732 \text{ kN}}} \quad (C)$$

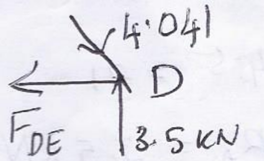


$$-F_{CE} \cos 60 + F_{CD} \cos 60 + 1.732 = 0$$

$$-F_{CE} \sin 60 - F_{CD} \sin 60 - 4 = 0$$

$$\underline{\underline{F_{CE} = -0.577 \text{ kN } (C)}}$$

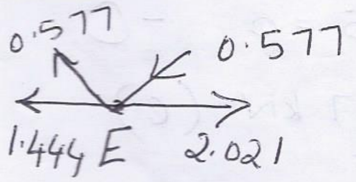
$$\underline{\underline{F_{CD} = -4.041 \text{ kN } (C)}}$$



$$F_{DE} - 4.041 \cos 60 = 0$$

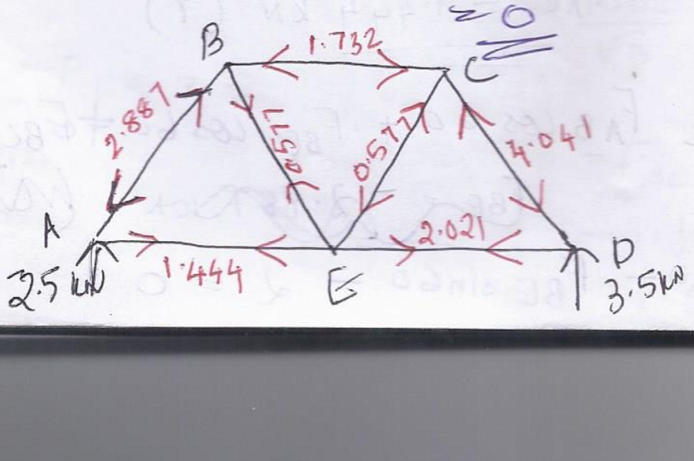
$$\underline{\underline{F_{DE} = 2.021 \text{ kN } (T)}}$$

Check:



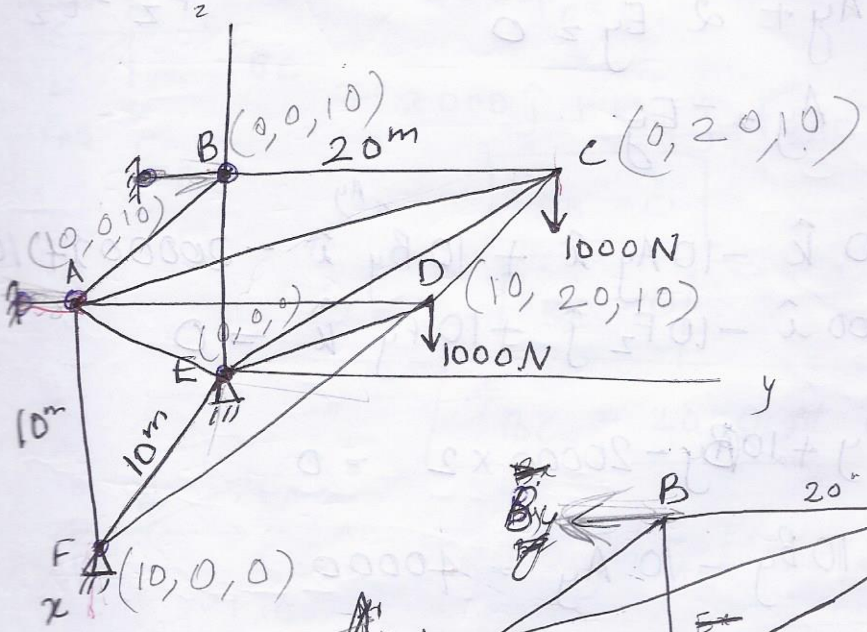
$$\Sigma F_x = 0.577 \cos 60 + 0.577 \cos$$
$$+ 1.444 - 2.021$$
$$= \underline{\underline{0}}$$

$$\Sigma F_y = 0.577 \sin 60 - 0.577 \sin$$
$$= \underline{\underline{0}}$$

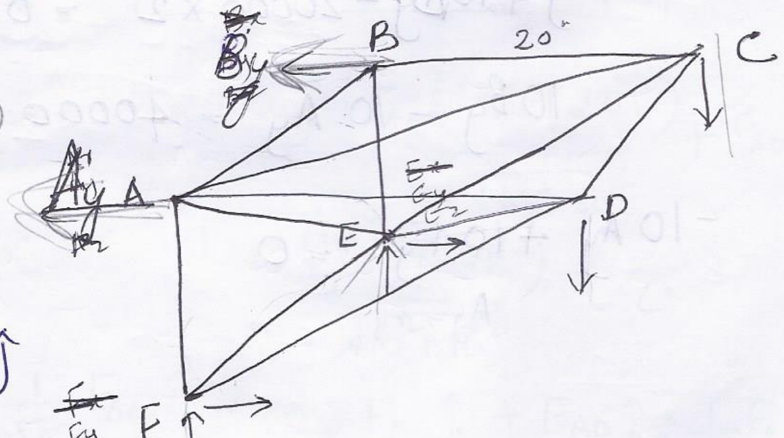


Member	Force (kN)	Nature of force
AB	2.887	C
AE	1.444	T
BC	1.732	C
BE	0.577	T
DE	2.021	T
CD	4.041	C
CE	0.577	C

2. Find the forces transmitted by each member of the three-dimensional truss.



Sol;



$M_E = 0$

$(10\hat{i} - 10\hat{k}) \times -A_y\hat{j}$

$+ 10\hat{k} \times -B_y\hat{j}$

$+ (20\hat{j} + 10\hat{k}) \times -1000\hat{k} + (10\hat{i} + 20\hat{j} + 10\hat{k}) \times -1000\hat{k}$

$+ 10\hat{i} \times (F_z\hat{k} + F_y\hat{j}) = 0$

$M_F = 0$

$10\hat{k} \times -A_y\hat{j} + (-10\hat{i}+10\hat{k}) \times -B_y\hat{j} + (-10\hat{i}+20\hat{j}-10 \ldots$

$+(20\hat{j}+10\hat{k}) \times -1000\hat{k} + -10\hat{i} \times (E_y\hat{j}+E_z\hat{k}) = \ldots$

$\Sigma F_z = 0$

$F_z + E_z - 2000 = 0 \Rightarrow 2F_z = 2000$

$F_z = 1000$

$A_y = B_y$

$F_y = E_y$

$F_z = E_z$

$\Sigma F_y = 0$

$-B_y - A_y + F_y + E_y = 0$

$2A_y + 2E_y = 0$

$A_y = -E_y$

$M_E = 0$

$A_y 10\hat{k} - 10A_y\hat{i} + 10B_y\hat{i} - 20000\hat{i} + 10 \ldots$

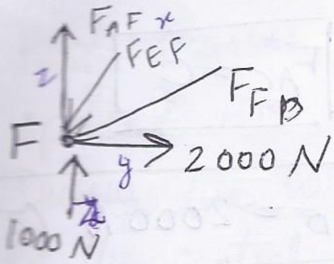
$-20000\hat{i} - 10F_z\hat{j} + 10F_y\hat{k} = 0$

$-10A_y + 10B_y - 20000 \times 2 = 0$

$10B_y - 10A_y = 40000$

$-10A_y + 10F_y = 0$

$A_y = F_y$



$F_R = 0$

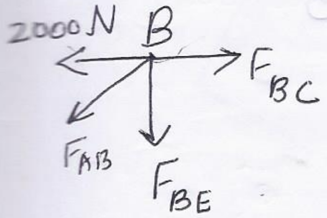
$$F_{AF}\hat{k} + F_{EF}\hat{\imath} + F_{FD}\frac{(-20\hat{\jmath} - 10\hat{k})}{\sqrt{20^2+10^2}} + 2000\hat{\jmath} + 1000\hat{k} = 0$$

$$F_{EF}\hat{\imath} = 0 \Rightarrow \boxed{F_{EF} = 0}$$

$$-0.894 F_{FD} + 2000 = 0 \Rightarrow \boxed{F_{FD} = 2236\ N} \ (C)$$

$$F_{AF} - 0.447 \times 2236 + 1000 = 0$$

$$\boxed{F_{AF} = 0}$$



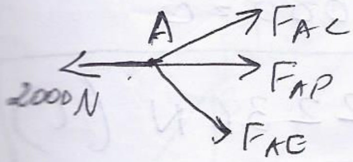
$F_R = 0$

$$-2000\hat{\jmath} + F_{BC}\hat{\jmath} + F_{AB}\hat{\imath} - F_{BE}\hat{k} = 0$$

$$\boxed{F_{AB} = 0}$$

$$\boxed{F_{BE} = 0}$$

$$\boxed{F_{BC} = 2000\ N\ (T)}$$



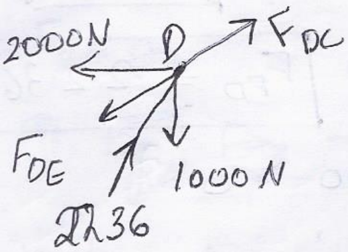
$$-2000\hat{\jmath} + F_{AC}\frac{(-10\hat{\imath} + 20\hat{\jmath})}{\sqrt{10^2+20^2}} + F_{AD}\hat{\jmath} + F_{AE}\frac{(-10\hat{\imath} + 10\hat{k})}{\sqrt{10^2+10^2}} = 0$$

$$-2000\hat{\jmath} - \frac{1}{\sqrt{5}}F_{AC}\hat{\imath} + \frac{2}{\sqrt{5}}F_{AC}\hat{\jmath} + F_{AD}\hat{\jmath} - \frac{1}{\sqrt{2}}F_{AE}\hat{\imath} - \frac{1}{\sqrt{2}}F_{AE}\hat{k} = 0$$

$$\frac{F_{AE}}{\sqrt{2}} = 0 \qquad \boxed{F_{AE} = 0}$$

$-\frac{F_{AC}}{\sqrt{5}} - \frac{F_{AE}}{\sqrt{2}} = 0 \Rightarrow \boxed{F_{AC} = 0}$

$-2000 + F_{AD} = 0$ $\boxed{F_{AD} = 2000\,N\ (T)}$



$-2000\hat{j} - F_{DC}\hat{i} + F_{DE}\frac{(-10\hat{i} - 20\hat{j} - }{\sqrt{10^2+20^2+}}$

$+2236\frac{(20\hat{j}+10\hat{k})}{\sqrt{20^2+10^2}} - 1000\hat{k} = 0$

$-2000 - \frac{\sqrt{6}}{3}F_{DE} + 2000 = 0$

$\boxed{F_{DE} = 0}$

$\boxed{F_{DC} = 0}$

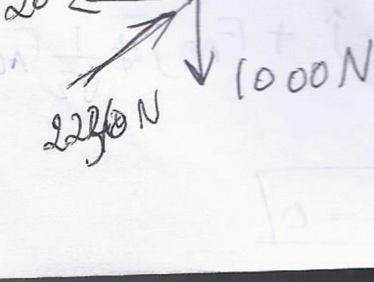
2236 ~~1000~~

$F_{EC}\frac{(20\hat{j}+10\hat{k})}{\sqrt{20^2+10^2}} + 2000\hat{j} + 10$

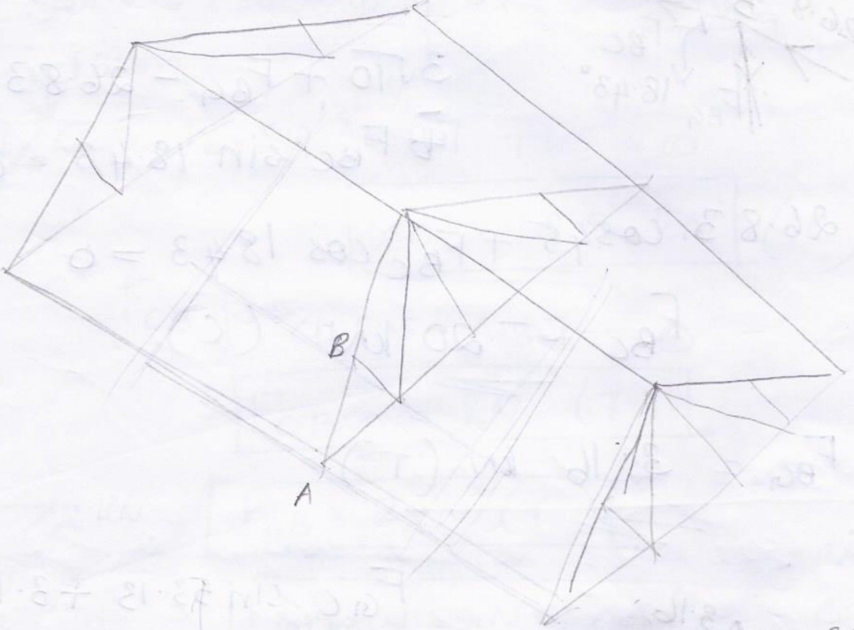
$\frac{2}{\sqrt{5}}F_{EC} + 2000 = 0$

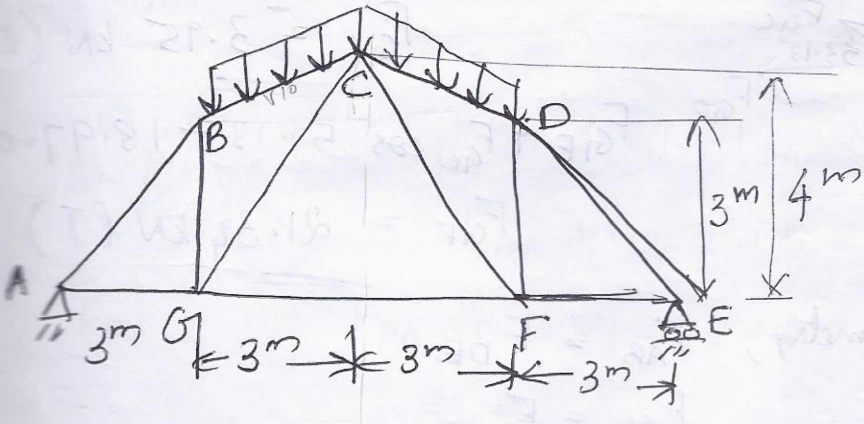
$\boxed{F_{EC} = -2236\,N\ (C)}$

Check in Joint C



6.6 Roof trusses such as the one shown are spaced 6 m apart in a long, rectangular building. During the winter, snow loads of upto 1 kPa accumulate on the central portion of the roof. Find the force in each member for a truss not at the ends of the building.

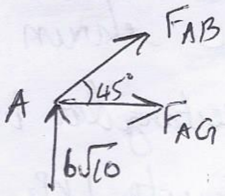




AB = 26.8 C
AG = 19 T
BG = 3.14 T
BC = 20 C
CG = 3.39 C
GF = 21.36 C

Sol: $R_A + R_E = \sqrt{10} \times 6 \times 1 \times 2 = 18.97 \times 2\,kN = 37.95\ kN$

$R_A = R_E = 18.97\ kN$



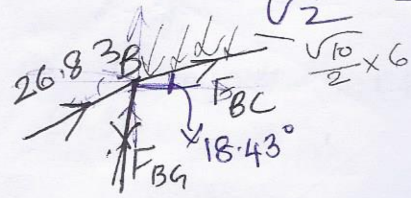
$F_{AB} \cos 45 + F_{AG} = 0$

$F_{AB} \sin 45 + 6\sqrt{10} = 0$

$F_{AB} = -12\sqrt{5}$ kN

$= -26.83$ kN (C)

$F_{AG} = \dfrac{26.83}{\sqrt{2}} = 18.97$ kN (T)



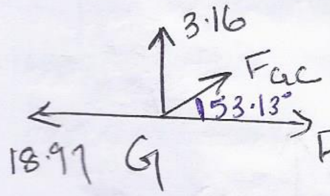
$3\sqrt{10} + F_{BG} - 26.83 \sin 45$

$- F_{BC} \sin 18.43 = 0$

$26.83 \cos 45 + F_{BC} \cos 18.43 = 0$

$F_{BC} = -20$ kN (C)

$F_{BG} = 3.16$ kN (T)



$F_{GC} \sin 53.13 + 3.16 = 0$

$F_{GC} = -3.95$ kN (

$F_{GF} + F_{GC} \cos 53.13 - 18.97 =$

$F_{GF} = 21.34$ kN (T)

By symmetry, $F_{AB} = F_{DE}$

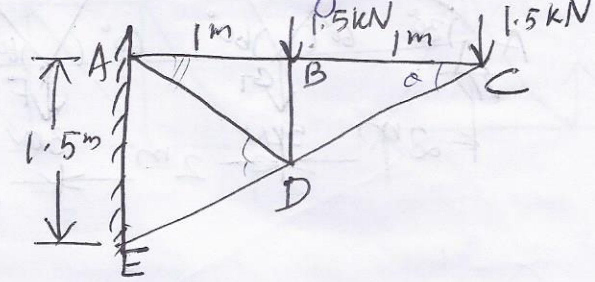
$F_{AG} = F_{EF}$

$F_{BC} = F_{DC}$

$F_{GC} = F_{FC}$

Analysis of Cantilever Trusses

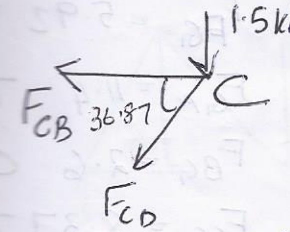
1. Determine the forces in the various members of a pin-jointed frame as shown in Fig.



$\tan\theta = \frac{1.5}{2}$

$\theta = 36.87°$

Sol:

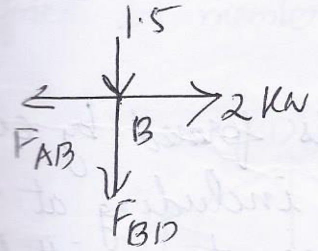


$F_{CD} \overset{3/5}{\sin 36.87} + 1.5 = 0$

$\boxed{F_{CD} = -2.5\ kN}$ (C)

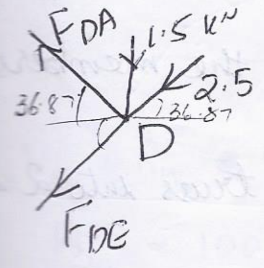
$F_{CB} + F_{CD} \cos 36.87 = 0$

$\boxed{F_{CB} = 2\ kN\ (T)}$



$\boxed{F_{AB} = 2\ kN\ (T)}$

$\boxed{F_{BD} = -1.5\ kN\ (C)}$



$F_{DA} \cos 36.87 + F_{DE} \cos 36.87 + 2.5 \cos 36.87 = 0$

$F_{DA} + F_{DE} + 2.5 = 0$

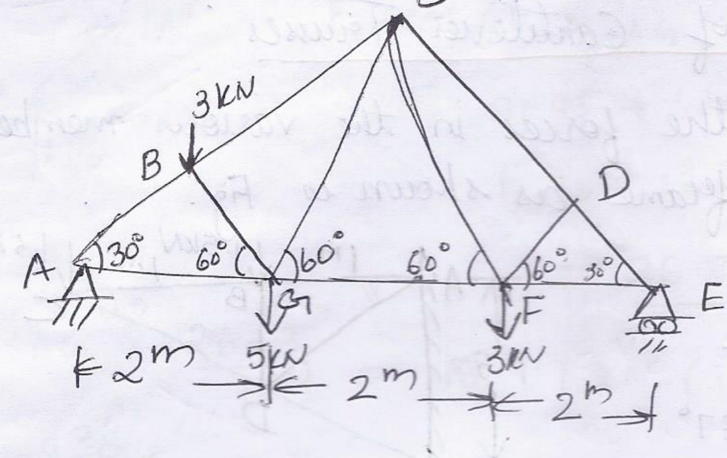
$F_{DA} \sin 36.87 - 1.5 - 2.5 \sin 36.87 - F_{DE} \sin 36.87 = 0$

$\cancel{\frac{3}{5}} F_{DA} - 2.5 - F_{DE} - 1.5 \times \frac{5}{3} = 0$

$F_{DA} - F_{DE} = 5$

$\boxed{F_{DA} = 1.25\ kN\ (T)}$ $\boxed{F_{DE} = -3.75\ kN\ (C)}$

HW



$R_A = 6.583$
$R_E = 4.416$
$F_{AB} = 13.16$
$F_{BC} = 11.66$
$F_{CD} = 8.83$
$F_{DE} = 8.83$
$F_{EF} = 7.65$
$F_{FG} = 5.92$
$F_{GA} = 11.4$
$F_{BG} = 2.6$
$F_{CG} = 8.37$
$F_{FC} = 3.46$ T
$F_{FD} = 0$

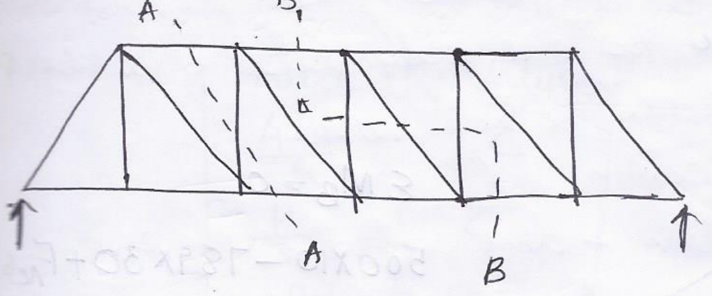
Day 2

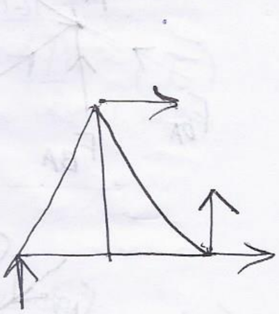
Method of sections

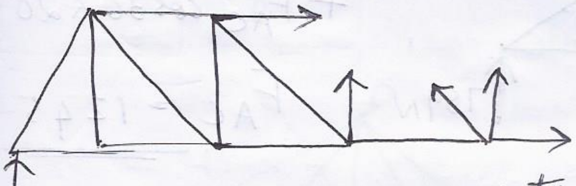
A free body in this method is formed by c[ut]ting away a portion of a truss & including at cut sections the forces that are transmitted across these sections.

1. The section should pass thru the members not thru the joints
2. The section should divide the truss into 2 portions
3. The section should not cut more than 3 m[embers] since only 3 unknown forces can be dete[rmined] using 3 eqns of eqlbm $\Sigma F_x = 0$ $\Sigma F_y = 0$
4. While using moment eqn of eqlbm, any p

on the truss can be taken as the moment centre which may or may not like on the portion under consideration.

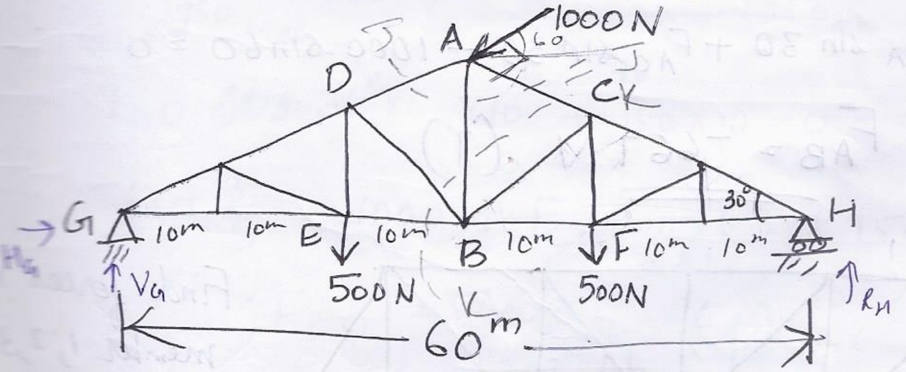






Section cuts

1. A plane truss is shown in Fig for which only the force in member AB is desired.



$\tan 30 = \frac{x}{30}$

Sol: $V_G + R_H = 500 + 500 + 1000 \sin 60 = 1866.025$ N

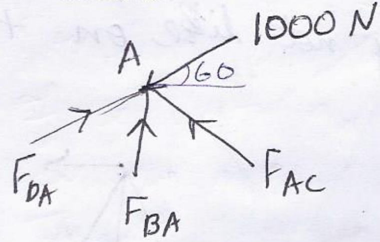
$H_G = 1000 \cos 60 = 500$N

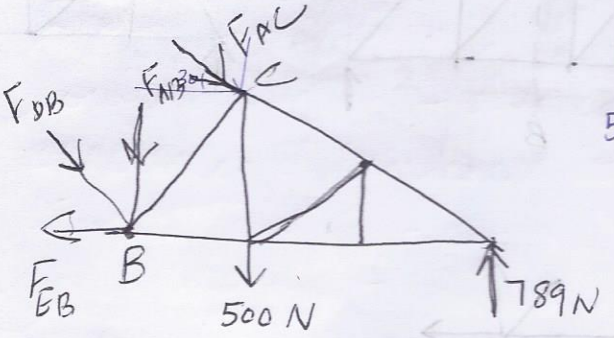
$\Sigma M_G = 0 \Rightarrow R_H \times 60 - 500 \times 40 - 500 \times 20 + 1000 \cos 60 \times 30 \tan 30$
$- 1000 \sin 60 \times 30 = 0$

$R_H = \frac{47320.5}{60} = 788.7$ N $\quad V_G = 1077.35$N

Joint A

3 unknowns





$\Sigma M_B = 0$

$500 \times 10 - 789 \times 30 + F_{AC} \sin$

$+ F_{AC} \cos 30 \times 20 \tan$

$\underline{F_{AC} = 1245\ N}$

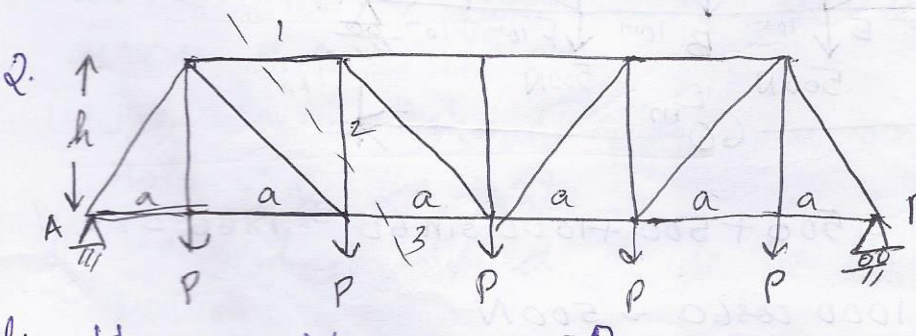
Joint A $\quad \Sigma F_x = 0 \Rightarrow -1000 \cos 60 + F_{DA} \cos 30 - F_{AC} \cos$

$$\underline{F_{DA} = 1822.35\ N}$$

$$F_{AB} + F_{DA} \sin 30 + F_{AC} \sin 30 - 1000 \sin 60 = 0$$

$$\underline{F_{AB} = -667\ N \quad (T)}$$

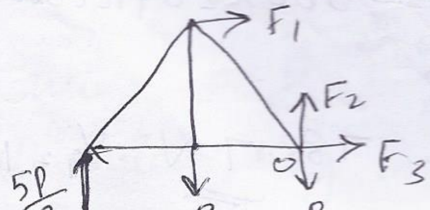
2.



Find for

member

Sol: $\quad H_A = 0 \qquad V_A = R_B = \frac{5P}{2}$



$\Sigma M_O = 0 \Rightarrow -F_1 \times h - \frac{5P}{2} \times 2$

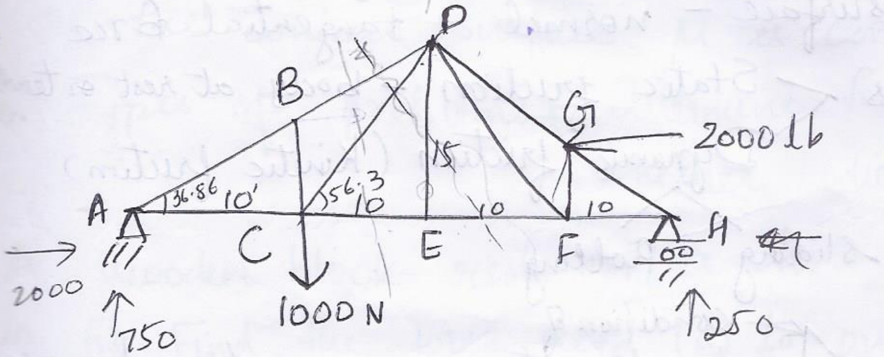
$$\underline{F_1 = \frac{-4Pa}{h}} \quad ($$

$\Sigma F_y = 0 \Rightarrow \frac{5P}{2} - 2P + F_2 = 0 \Rightarrow F_2 = \frac{-P}{2}$ (C)

$\Sigma F_x = 0 \Rightarrow F_1 + F_3 = 0 \Rightarrow F_3 = \frac{4Pa}{h}$ (T)

Members	Force	Nature
1	$\frac{4Pa}{h}$	C
2	$\frac{P}{2}$	C
3	$\frac{4Pa}{h}$	T

6.22



$F_{DE} = 0$

F_{BD} ~~cos~~ $36.86 \times 10 \overset{\tan 36.86}{} + 750 \times 10 = 0 \Rightarrow F_{BD} = -1250\,N$

$F_{BD}\frac{3}{5} - 1000 + F_{CD} \underset{0.832}{\sin 56.3} + 750 = 0$

$F_{CD} =$ ~~1800 N~~ (T)

1202 lb

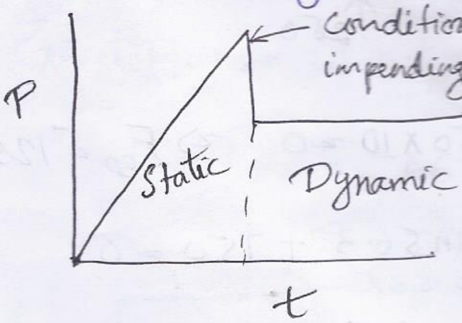
Day 3

Friction

Friction is the force distribution at the s
of contact b/n 2 bodies that prevents or i
sliding motion of one body relative to the ot
Tangential to the contact surface
Energy dissipation
Coulomb friction – dry contact surfaces
microscopic roughness of the surfaces of conta
Smooth surface – support only a normal force
Rough surface – normal + tangential force

2 types < Static friction – body at rest or ten
Dynamic friction (Kinetic friction)

Dynamic friction: Sliding, Rolling



Laws of Coulomb Friction

① Total force of friction is independent of area
② For low relative velocities b/n sliding object
frictional force is independent of velocity. Slidi
frictional force < impending slippage frictional f
③ F ∝ normal force transmitted across the
surface of contact. $f \propto N$ $\boxed{f = \mu N}$ coeff of friction

$\mu_d < \mu_s$

Simple Contact Friction Problems

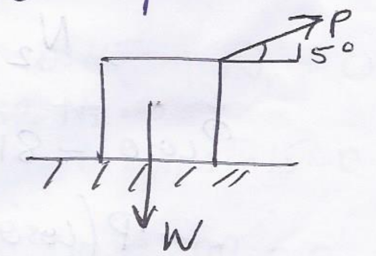
① A plane surface of contact

② An impending or actual motion which is in the same direction for all area elements of contact surface. Thus, there is no impending or actual rotation b/n bodies in contact.

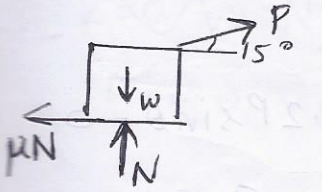
③ The properties of the respective bodies are uniform at the contact surface. μ is constant

2 types of problems ← determine forces or position
← whether bodies will move or not

1. A wooden block rest on a hor. plane as shown in Fig. Find the force reqd (a) to pull the body (b) to push the body. $m = 5\,kg$, $\mu = 0.4$, $g = 9.81\,m/s^2$



Sol:
(a)

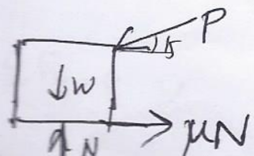


$\Sigma F_x = 0 \Rightarrow P\cos 15 - 0.4N = 0$

$\Sigma F_y = 0 \Rightarrow P\sin 15 + N - 5\times 9.81 = 0$

$\underline{P = 18.34\,N}$ $\quad$ $\underline{N = 44.3\,N}$

(b)

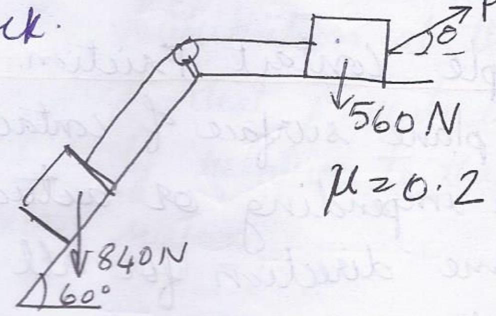


$\Sigma F_x = 0 \Rightarrow -P\cos 15 + 0.4N = 0$

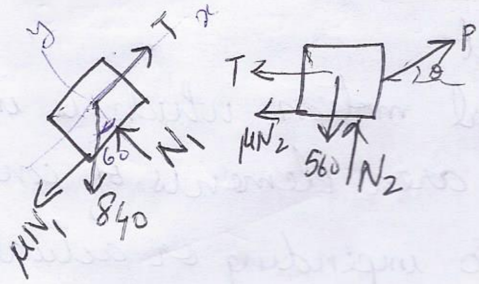
$-P\sin 15 + N - 5\times 9.81 = 0$

$\underline{P = 22.77\,N}$ $\quad$ $\underline{N = 54.94\,N}$

2. Find the least value of P reqd to sta[rt] motion of the block.



Sol:



$\Sigma F_x = 0 \Rightarrow \quad T - 0.2N_1 - 840 \sin 60 = 0$

$\Sigma F_y = 0 \Rightarrow \quad N_1 - 840 \cos 60 = 0$

$N_1 = 420 N$

$T = 811.46 N$

$\Sigma F_x = 0 \Rightarrow \quad P\cos\theta - T - 0.2N_2 = 0$

$\Sigma F_y = 0 \Rightarrow \quad P\sin\theta + N_2 - 560 = 0$

$N_2 = 560 - P\sin\theta$

$P\cos\theta - 811.46 - 112 + 0.2P\sin\theta = 0$

$P(\cos\theta + 0.2\sin\theta) = 923.46$

For P to be least, denominator should be [max]

$\frac{d}{d\theta}(\cos\theta + 0.2\sin\theta) = 0$

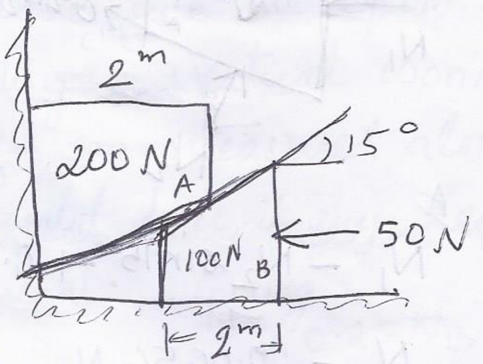
$-\sin\theta + 0.2\cos\theta = 0$

$\tan\theta = 0.2$

$\theta = 11.31°$

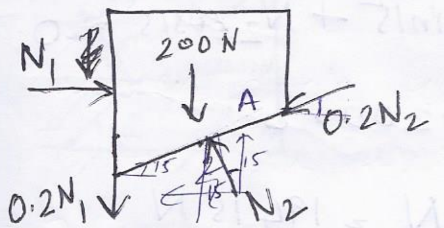
$\therefore P_{least} = 905.53 N$

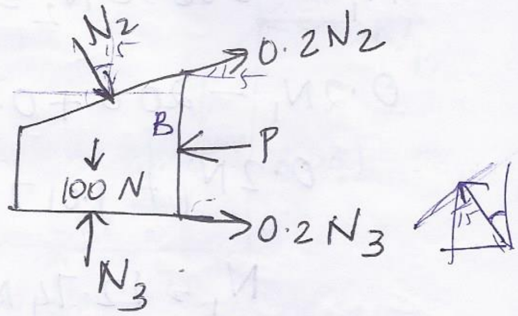
the 3. The coefficient of static friction for all contact surfaces in Fig is 0.2. Does the 50 N force move the block A up, hold it in eqlbm or is it too small to prevent A from coming down & B from moving out?



Sol: Find a force P in place of 50 N causing motion of block B to left & P to right. Compare action of 50 N force.

Block B to left:





A $\Sigma F_x = 0 \Rightarrow -N_2 \sin 15 - 0.2 N_2 \cos 15 + N_1 = 0$

$N_1 - 0.452 N_2 = 0$

$\Sigma F_y = 0 \Rightarrow -200 + N_2 \cos 15 - 0.2 N_2 \sin 15 - 0.2 N_1 = 0$

$-0.2 N_1 + 0.914 N_2 - 200 = 0$

$N_1 = 109.76\ N \qquad N_2 = 242.84\ N$

B $\Sigma F_x = 0 \Rightarrow N_2 \sin 15 + 0.2 N_2 \cos 15 - P + 0.2 N_3 = 0$

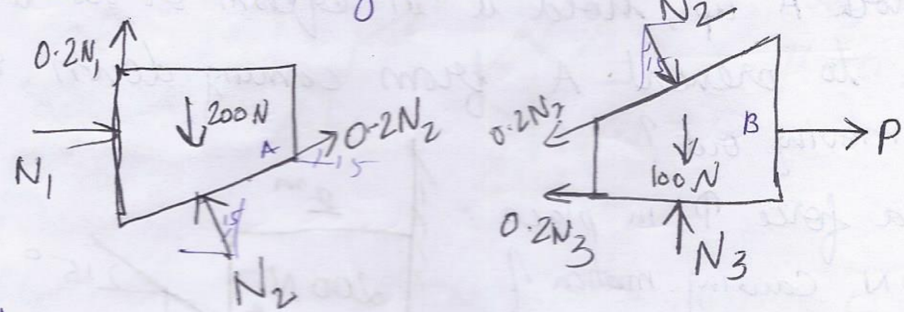
$\boxed{P = 174.16\ N}$

$\Sigma F_y = 0 \Rightarrow N_3 - 100 - N_2 \cos 15 + 0.2 N_2 \sin 15 = 0$

$N_3 = 321.99\ N$

Hence, stipulated force of 50N is insuff to induce a motion of block B to the left

Block B to right



A

$N_1 - N_2 \sin 15 + 0.2N_2 \cos 15 = 0$

$N_1 - 0.0656 N_2 = 0$

$0.2N_1 - 200 + 0.2N_2 \sin 15 + N_2 \cos 15 = 0$

$0.2N_1 + 1.017 N_2 - 200 = 0$

$N_1 = 12.74\,N \qquad N_2 = 194.15\,N$

B

$N_2 \sin 15 + P - 0.2N_3 - 0.2N_2 \cos 15 = 0$

$N_3 - 100 - N_2 \cos 15 - 0.2N_2 \sin 15 = 0$

$N_3 = 297.58\,N$

$P = 46.77\,N$

So we would have to pull to the right to get B to move in this direction.

∴ Blocks are in eqlbm.