ECE 1505 Convex Optimization
Prof. Wei Yu

Homework Set #1

1. Consider the function $f(x) = -\sum_{i=1}^{m} \log(b_i - a_i^T x)$, where $x \in \mathbf{R}^n$, $b_i \in \mathbf{R}$ and $a_i \in \mathbf{R}^n$. Compute ∇f and $\nabla^2 f$. Write down the first three terms of the Taylor series expansion of $f(x)$ around some x_0.
2. Problem 2.5 of Boyd and Vandenberghe
3. Problem 2.14(a) of Boyd and Vandenberghe
4. Problem 3.14 of Boyd and Vandenberghe
5. Problem 3.16(a-c) of Boyd and Vandenberghe
6. Problem 3.32(a) of Boyd and Vandenberghe
7. Consider the function $f(x, y) = x^2 + y^2 + \beta xy + x + 2y$. Find (x^*, y^*) for which $\nabla f = 0$. Express your answer as a function of β. For which values of β is the (x^*, y^*) a global minimum of $f(x, y)$?
8. In this problem, we are given a set of data points (x_i, y_i), $i = 1 \cdots 100$. We wish to fit a quadratic model, $y_i = ax_i^2 + bx_i + c + n_i$, to the data. Here, (a, b, c) are the parameters to be determined and n_i is the unknown observation noise. The (x_i, y_i) points are contained in a file `hw1data.mat` available on the course webpage. You may load the data to `MATLAB` using the command `load hw1data` and view them using `scatter(x,y,'+')`. Please use the same data set and find the maximum likelihood estimate of (a, b, c) assuming n_i's are i.i.d., and

   (a) $n_i \sim \mathcal{N}(0, 1)$;

   (b) n_i is always positive and $n_i \sim e^{-z}$ for $z \geq 0$.

   Please plot the data and the models on the same `MATLAB` figure and submit the figure as a part of your solution.

   (`MATLAB` has built-in functions to solve many optimization problems. For example, `linprog` solves a linear programming problem, `quadprog` solves a quadratic programming problem. You may use `help linprog` to get more details. Hint: part (a) has an analytic solution.)