

$e ::=$

$false$
 $true$
 i
 s
 $\text{succ } e$
 $\text{pred } e$
 $\text{iszero } e$
 $\text{if } e \text{ then } e \text{ else } e$
 $\text{concat } e \ e$
 $\text{isstring } e$
 $\text{isint } e$
 $\text{isbool } e$

Expressions

Integers
Strings

$t ::=$

$Bool$
 Int
 $String$

[SS-SUCC]	$\frac{e_1 \rightarrow e'_1}{succ\ e \rightarrow e_1}$
[SS-SUCC]	$\frac{i' = i + 1}{succ\ i \rightarrow i'}$
[SS-PRED]	$\frac{e_1 \rightarrow e'_1}{pred\ e \rightarrow e_1}$
[SS-PRED]	$\frac{i' = i - 1}{pred\ i \rightarrow i'}$
[SS-ISZERO]	$\frac{e \rightarrow e'}{iszero\ e \rightarrow e'}$
[SS-ISZERO]	$\frac{}{iszero\ 0 \rightarrow true}$
[SS-ISZERO]	$\frac{i \neq 0}{iszero\ i \rightarrow false}$
[SS-IF]	$\frac{e_1 \rightarrow e'_1}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 \rightarrow \text{if } e'_1 \text{ then } e_2 \text{ else } e_3}$
[SS-IF]	$\frac{}{\text{if true then } e_2 \text{ else } e_3 \rightarrow e_2}$
[SS-IF]	$\frac{}{\text{if false then } e_2 \text{ else } e_3 \rightarrow e_3}$
[SS-CONCAT]	$\frac{e_1 \rightarrow e'_1}{concat\ e_1\ e_2 \rightarrow concat\ e'_1\ e_2}$
[SS-CONCAT]	$\frac{e_2 \rightarrow e'_2}{concat\ s_1\ e_2 \rightarrow concat\ s_1\ e'_2}$
[SS-CONCAT]	$\frac{s_3 = concat\ s_1\ s_2}{concat\ s_1\ s_2 \rightarrow s_3}$

[SS-ISBOOL]	$\frac{e \rightarrow e'}{isbool\ e \rightarrow e'}$
[SS-ISBOOL]	$\overline{isbool\ false \rightarrow true}$
[SS-ISBOOL]	$\overline{isbool\ true \rightarrow true}$
[SS-ISBOOL]	$\overline{isbool\ i \rightarrow false}$
[SS-ISBOOL]	$\overline{isbool\ s \rightarrow false}$
[SS-ISINT]	$\frac{e \rightarrow e'}{isint\ e \rightarrow e'}$
[SS-ISINT]	$\overline{isint\ false \rightarrow false}$
[SS-ISINT]	$\overline{isint\ true \rightarrow false}$
[SS-ISINT]	$\overline{isint\ i \rightarrow true}$
[SS-ISINT]	$\overline{isbool\ s \rightarrow false}$
[SS-ISSTRING]	$\frac{e \rightarrow e'}{isstring\ e \rightarrow e'}$
[SS-ISSTRING]	$\overline{isstring\ false \rightarrow false}$
[SS-ISSTRING]	$\overline{isstring\ true \rightarrow false}$
[SS-ISSTRING]	$\overline{isstring\ i \rightarrow false}$
[SS-ISSTRING]	$\overline{isstring\ s \rightarrow true}$

[TS-SUCC]	$\frac{e: Int}{succ\ e: Int}$
[TS-PRED]	$\frac{e: Int}{pred\ e: Int}$
[TS-ISZERO]	$\frac{e: Int}{iszero\ e: Bool}$
[TS-ISBOOL]	$\frac{e: Bool}{isbool\ e: Bool}$
[TS-ISINT]	$\frac{e: Int}{isint\ e: Bool}$
[TS-ISTRING]	$\frac{e: String}{istring\ e: Bool}$
[TS-IF]	$\frac{e_1: Bool \quad e_2: T \quad e_3: T}{\mathbf{if}\ e_1\ \mathbf{then}\ e_2\ \mathbf{else}\ e_3: T}$
[TS-CONCAT]	$\frac{e_1: String \quad e_2: String}{concat\ e1\ e2: String}$