1.

7 letter 3 v's

3 v's - placement doesn't matter

3 different cases

$\binom{4}{4} = 1$

1 v's v n s a l

$\binom{4}{3} = 4$

2 v's v v n s l + 2

$\binom{4}{2} = 6$

3 v's v v v n s

11 diff strings

1 v = 5! = 120

2 v = $\frac{5!}{2!} \cdot 4 = 240$ = 480

3 v = $\frac{5!}{3!} \cdot 6 = 120$

2

$\binom{21}{16} + \binom{?}{15}$

$6 + 16 - 1 \rightarrow \binom{21}{16} \rightarrow \binom{6+15-1}{15} = \binom{20}{15}$

52

13 diff values

4 of each

$13C \cdot 4C \cdot 4C \cdot 44C$

$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{44}{1} = 13 \cdot 6 \cdot 6 \cdot 44$

= 123552

$\binom{n+k-1}{k}$

3. n = 16, 6 couples

Case 1: finding couple n+5 k sets

$\binom{n+k-1}{k-1} = \binom{16+6-1}{6-1} = \binom{21}{5} = 20349$

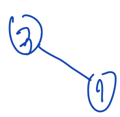
Case 2: finding couple n+1 sets

n = 15, 6 couples

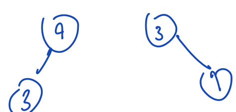
$\binom{n+k-1}{k-1} = \binom{15+6-1}{6-1} = \binom{20}{5} = 15504$

20349 + 15504 = 35,853 ways

4.

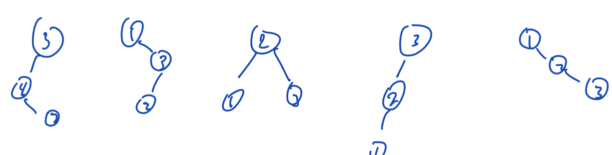


2 nodes = 2



3 nodes = 5

# of subtrees given nodes is $\frac{(2n)!}{(n+1)!\, n!}$



$\frac{(2 \cdot 3)!}{6!\,5!}$ $2C_1 \cdot 42C_1 \cdot 5C_1 = 420$ ← correct answer

5. $(10C_1 \cdot 8) + (10C_2 \cdot 7)$

4 nurses

2 cases; 3 nurses

1 8 1 1
2 7 2 1
2 6 2 1
4 6 2 2
5 5 4 1
6 5 3 2
7 4 4 2
8 4 3 3

4 nurses

1 7 1 1 1
2 6 2 1 1
3 5 3 1 1
4 5 2 2 1
5 4 4 1 1
6 4 3 2 1
7 3 3 2 2
8 3 1 3 3
9 2 2 2 4

17 diff. combos