Numerical Methods, PHY 312/618

Assignment-1

1. Determine the number of terms necessary to approximate $cosx$ to 8 significant figures using the Maclaurin series approximation

$$cos(x) = 1 - x^2 + \frac{x^4}{4!} + \frac{x^6}{6!} + ...$$ (0.1)

   Calculate the approximation using a value of $x = 0.3\pi$. Write a program to determine your result.

2. Find the root of the equation

$$f(x) = e^{-x/2}(4 - x) - 2$$ (0.2)

   Use Newton-Rapson method to find the root. Choose the inital guess of a) 2 and b) 6.

3.

4. Apply the Newton-Raphson method to the function $f(x) = \tanh(x^2 - 9)$ to evaluate its known real root at $x = 3$. Use an initial guess of $x_0 = 3.2$ and take a minimum of four iterations. (b) Did the method exhibit convergence onto its real root? Sketch the plot with the results for each iteration shown.

5. The function $f(x) = x^3 - 2x^2 - 4x + 8$ has a double root at $x = 2$. Use (a) the standard Newton-Raphson to solve for the root at $x = 2$. Discuss the rate of convergence using an initial guess of $x0 = 1.2$.

6. The volume V of liquid in a hollow horizontal cylinder of radius r and length L is related to the depth of the liquid h by,

$$V = \left[-r^2 cos^{-1} \left(\frac{r - h}{r} \right) \right]$$ (0.3)

   Determine h given $r = 2m, L = m$, and $V = 8m^3$.

7. Write a code to estimate matrix multiplication of two matrices $[A]$ and $[B]$. Where ,

$$A = \begin{pmatrix} 4 & 3 & 1 \\ 1 & 1 & 6 \\ 5 & 10 & 1 \end{pmatrix} ; B = \begin{pmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{pmatrix}$$

8. Given the equations,

$$2x_1 - 6x_2 - x_3 = -38; \; -3x_1 - 2x_2 + 7x_3 = -34; \; -8x_1 + x_2 - 2x_3 = -20$$ (0.4)

   Solve by Gauss elimination.