PICT, PUNE

L5 Batch ; 32350
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Assignment 4

Q.1 Given:

Parallel plate capacitor

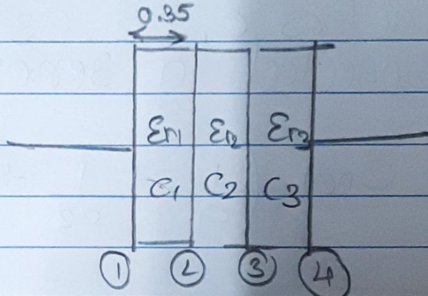
Plate dimensions: $3cm \times 7cm$

Three sheets of dielectric - 0.35 mm thick

$\varepsilon_{r_1} = 1.2$

$\varepsilon_{r_2} = 2.7$

$\varepsilon_{r_3} = 4.6$



Solution:

For parallel plate capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r S}{d} \quad \text{—— (A)}$$

$S = (3 \times 7)\, cm^2$

$d = 0.35\, mm$

Considering the numbering of surfaces shown in the figure,

C_1 = Capacitance of surface 1 & 2

$$\therefore C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} S_1}{d_1} = \frac{8.854 \times 10^{-12} \times 1.2 \times (3 \times 7 \times 10^{-4})}{0.35 \times 10^{-3}}$$

$C_1 = 63.7488\, pF$ —— (B)

Similarly,

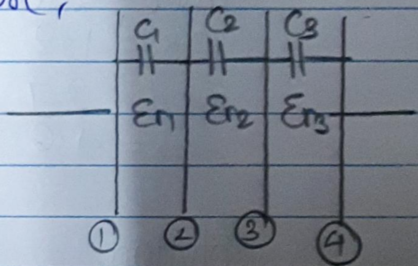
$$C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} S_2}{d_2} = \frac{8.854 \times 10^{-12} \times 2.7 \times 3 \times 7 \times 10^{-4}}{0.35 \times 10^{-3}}$$

$C_2 = 143.4348\, pF$ —— (C)

$$C_3 = \frac{\varepsilon_0 \varepsilon_{r3} S_3}{d_3} = \frac{8.854\times10^{-12}\times 4.6\times 3\times 7\times10^{-4}}{0.35\times10^{-3}}$$

$C_3 = 244.3704\ pF$ —— (D)

As observed from the figure, Capacitors c_1, c_2, c_3 are connected in series



$$\therefore C_{net} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$$\boxed{C_{net} = 37.3824\ pF}$$

∴ Net capacitance of the parallel plate capacitor is = 37.3824 pF

Q.2 Given

$\varepsilon_{r1} = 2.8$

$\varepsilon_{r2} = 1.3$

Planar interface $S = 5x + 12y - 9z = 2$

Origin lies in region 1

$\bar{E}_1 = 12\bar{a_x} - 5\bar{a_y} + 27\bar{a_z}$ V/m

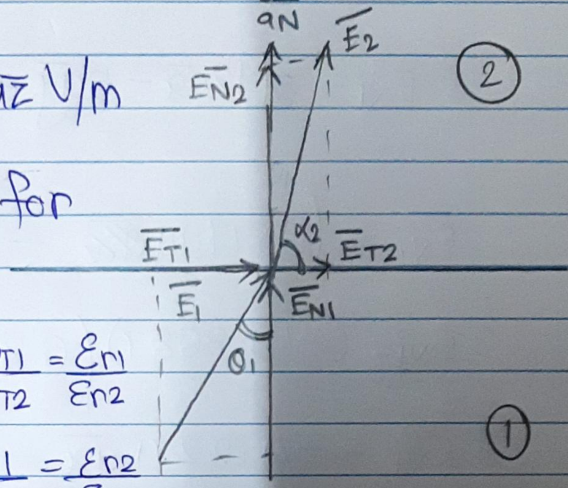
To find:

i) $\bar{E}_{N1}$, ii) $\bar{E}_{T1}$ iii) $\bar{E}_{T2}$ iv) $\bar{E}_{N2}$ v) $\bar{E}_2$

vi) $\bar{P}_2$ vii) Q_1 viii) Q_2 ix) Energy densities

Solution:

$\overline{E_1} = 12\bar{a_x} - 5\bar{a_y} + 27\bar{a_z}$ V/m



- Boundary Conditions for Dielectric - Dielectric

Tangential: $E_{T1} = E_{T2}$; $\frac{D_{T1}}{D_{T2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$

Normal: $D_{N1} = D_{N2}$; $\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$

- Seperating the normal and tangential components of the electric field $\overline{E_1}$

$\overline{a_N} = \pm \nabla S$ where $S = 5x + 12y - 9z - 2 = 0$

$\overline{a_N} = \nabla(5x + 12y - 9z - 2)$

$= \frac{\partial}{\partial x}(5x + 12y - 9z - 2)\bar{a_x} + \frac{\partial}{\partial y}(5x + 12y - 9z - 2)\bar{a_y} +$

$\frac{\partial}{\partial z}(5x + 12y - 9z - 2)\bar{a_z}$

$\overline{a_N} = \frac{5\bar{a_x} + 12\bar{a_y} - 9\bar{a_z}}{\cancel{189}}$

$\overline{a_N} = \frac{5\bar{a_x} + 12\bar{a_y} - 9\bar{a_z}}{\sqrt{25 + 144 + 81}} = \frac{5\bar{a_x} + 12\bar{a_y} - 9\bar{a_z}}{15.81138}$

$\overline{a_N} = 0.3162\,\bar{a_x} + 0.3589\,\bar{a_y} - 0.5692\,\bar{a_z}$ — A

$\therefore \overline{E_{N1}} = (\overline{E_1} \cdot \overline{a_N})\, \overline{a_N}$ — vector projection

$\therefore \overline{E_{N1}} = (12\overline{a_x} - 5\overline{a_y} + 27\overline{a_z}) \cdot (0.3162\overline{a_x} + 0.3589\overline{a_y} - 0.56921\overline{a_z})\, \overline{a_N}$

$\overline{E_{N1}} = (3.7944 - 1.7945 - 15.3686)(0.3162\overline{a_x} + 0.3589\overline{a_y}$

$E_{N1} = (-13.3687)(0.3162\overline{a_x} + 0.3589\overline{a_y} - 0.56921\overline{a_z})$

$\overline{E_{N1}} = -4.2271\overline{a_x} - 4.7980\overline{a_y} + 7.60959\overline{a_z}$

$$\therefore \boxed{\overline{E_{N1}} = -4.2271\overline{a_x} - 4.7980\overline{a_y} + 7.609\overline{a_z}} \quad \text{— (i)}$$

$\overline{E_{T1}} = \overline{E_1} - \overline{E_{N1}}$

$\therefore \overline{E_{T1}} = (12\overline{a_x} - 5\overline{a_y} + 27\overline{a_z}) - (-4.227\overline{a_x} - 4.7980\overline{a_y} + 7.6\overline{a_z})$

$$\boxed{E_{T1} = 16.227\overline{a_x} - 0.202\overline{a_y} + 19.391\overline{a_z}} \quad \text{— (ii)}$$

From the boundary conditions, $E_{T2} = E_{T1}$

$\therefore \overline{E_{T2}} = 16.227\overline{a_x} - 0.202\overline{a_y} + 19.391\overline{a_z}$ — (iii)

From boundary conditions,

$$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

$$\therefore \overline{E_{N2}} = \frac{\overline{E_{N1}} \cdot \varepsilon_{r1}}{\varepsilon_{r2}}$$

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$$\boxed{\overline{E_{N2}} = -9.10432\,\overline{a_x} - 10.3339\,\overline{a_y} + 16.3882\,\overline{a_z}} \quad \text{—(IV)}$$

$\therefore \overline{E_2} = \overline{E_{N2}} + \overline{E_{T2}}$

Substituting eqⁿ (3) & (4)

$$\overline{E_2} = \left(-9.10432\,\overline{a_x} - 10.3339\,\overline{a_y} + 16.3882\,\overline{a_z}\right) + \left(16.227\,\overline{a_x} - 0.202\,\overline{a_y} + 19.391\,\overline{a_z}\right)$$

$$\boxed{\overline{E_2} = 7.123\,\overline{a_x} - 10.5359\,\overline{a_y} + 35.7792\,\overline{a_z}} \quad \text{—(V)}$$

Polarization: $\overline{P_2}$

$\overline{P_2} = \chi_{e2}\,\varepsilon_0\,\overline{E_2}$

$\chi_{e2} = \varepsilon_{r2} - 1 \quad = 1.3 - 1 = 0.3$

$$\overline{P_2} = (0.3)(8.854\times10^{-12})\left(7.123\,\overline{a_x} - 10.5359\,\overline{a_y} + 35.7792\,\overline{a_z}\right)$$

$$\overline{P_2} = 18.92011\,\overline{a_x} - 27.9854\,\overline{a_y} + 95.0367 \text{ pC/m}^3 \quad \text{—(VI)}$$

Angle θ_1

$$\theta_1 = \tan^{-1}\left(\frac{E_{T1}}{E_{N1}}\right) = \tan^{-1}\left(\frac{24.480}{9.9083}\right)$$

$$\boxed{\theta_1 = 59.45^\circ} \quad \text{—(V)}$$

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$$\alpha_2 = \tan^{-1}\left(\frac{E_{T2}}{E_{N2}}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{E_{T2}}{E_{N2}}\right) = \boxed{57.4329} \text{ — (vi)}$$

Energy Density:

$$W_1 = \frac{1}{2} \varepsilon_0 \varepsilon_r |E_1|^2$$

$$W_1 = 11.1312 \text{ nJ/m}^3 \text{ — (viii)}$$

$$W_2 = \frac{1}{2} \varepsilon_0 \varepsilon_{r2} |E_2|^2$$

$$W_2 = 16.8001 \text{ nJ/m}^3 \text{ — (ix)}$$

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Q.3 Given:

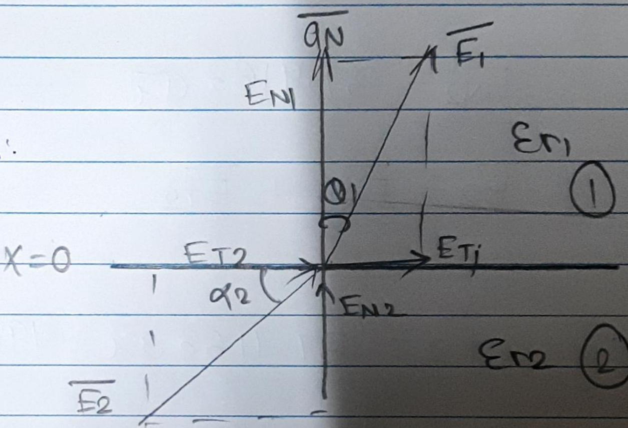
Plane $x=0$: $\overline{a_N} = \overline{a_x}$

$x>0$ $\varepsilon_{r1} = 3.5$

$x<0$ $\varepsilon_{r2} = 1.2$

$\overline{E_1} = 13\overline{a_x} - 19\overline{a_y} + 7\overline{a_z}$ V/m

Solution:



Boundary Conditions:

$E_{T1} = E_{T2}$

$\frac{D_{T1}}{D_{T2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$

$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$

$E_{N1} = (\overline{E_1} \cdot \overline{a_x})\,\overline{a_x}$

$\boxed{\overline{E_{N1}} = 13\overline{a_x} \quad \text{— (1)}}$

$\overline{E_{T1}} = \overline{E_1} - \overline{E_{N1}}$

$\boxed{\overline{E_{T1}} = -19\overline{a_y} + 7\overline{a_z} \quad \text{—— (2)}}$

From boundary conditions,

$\overline{E_{T1}} = \overline{E_{T2}}$

$\therefore \boxed{\overline{E_{T2}} = -19\overline{a_y} + 7\overline{a_z} \quad \text{—— (3)}}$

From boundary condition,

$$\overline{E_{N2}} = \overline{E_{N1}} \cdot \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\overline{E_{N2}} = (13\overline{a_x})\left(\frac{3.5}{1.2}\right)$$

$$\boxed{\overline{E_{N2}} = 37.9166\,\overline{a_x}} \quad \text{— (4)}$$

$$\overline{E_2} = \overline{E_{N2}} + \overline{E_{T2}}$$

$$\boxed{\therefore \overline{E_2} = 37.9166\,\overline{a_x} - 19\overline{a_y} + 7\overline{a_z}} \quad \text{— (5)}$$

Polarization

$$\overline{P_2} = \chi_{e2}\, \varepsilon_0 \overline{E_2}$$

$$\chi_{e2} = \varepsilon_{r2} - 1 = 1.2 - 1 = 0.2$$

$$\overline{P_2} = (0.2)(8.854 \times 10^{-12})(37.9166\,\overline{a_x} - 19\overline{a_y} + 7\overline{a_z})$$

$$\boxed{\overline{P_2} = 67.1428\,\overline{a_x} - 33.6452\,\overline{a_y} + 12.3956\,\overline{a_z}\ \mathrm{pC/m^2}} \quad \text{— (6)}$$

vii) Angle θ_1

$$\theta_1 = \tan^{-1}\left(\frac{E_{T1}}{E_{N1}}\right) = \tan^{-1}\left(\frac{20.248}{13}\right) = 57.2979° \quad \text{— (7)}$$

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viii) Angle α_2

$$\alpha_2 = \tan^{-1}\left(\frac{E_{N2}}{E_{T2}}\right) = \tan^{-1}\left(\frac{37.9168}{20.248}\right)$$

$$\boxed{\alpha_2 = 61.8967^\circ} \text{ --- } (8)$$

Energy density:

$$W_1 = \frac{1}{2}\varepsilon_0 \varepsilon_{r1} |E_1|^2$$

$$W_1 = \frac{1}{2} \times 8.854 \times 10^{-12} \times 3.5 \times 579$$

$$\boxed{W_1 = 8.9713 \ nJ/m^3} \text{ --- } (9)$$

$$W_2 = \frac{1}{2}\varepsilon_0 \varepsilon_{r2} |E_2|^2$$

$$W_2 = \frac{1}{2} \times 8.854 \times 10^{-12} \times 1.2 \times \cancel{4}1847.163$$

$$\boxed{W_2 = 9.8128 \ nJ/m^3} \text{ --- } (10)$$

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Q.1 Given: $E_y = 14\,V/m$ $\therefore$

$$\overline{P} = \frac{1}{12\pi}(4\overline{a_x} - 2\overline{a_y} + 13\overline{a_z})\,nC/m^2$$

To find:

a) χ_e b) $\overline{E}$ c) $\overline{D}$

Solution:

$$\overline{P} = \chi_e \varepsilon_0 \overline{E}$$

Let $\overline{E} = E_x\overline{a_x} + E_y\overline{a_y} + E_z\overline{a_z}$ — (A)

$$\therefore \overline{P} = \chi_e(8.854\times10^{-12})(E_x\overline{a_x} + E_y\overline{a_y} + E_z\overline{a_z})$$

Considering only y component of $\overline{P}$ and RHS,

$$\frac{10^{-9}(-2)\cancel{\overline{a_y}}}{12\pi} = \chi_e(8.854\times10^{-12})E_y\overline{a_y}$$

$$\therefore \frac{-2}{12\pi}\times10^{-9} = \chi_e\times8.854\times10^{-12}\times14$$

$$\therefore \boxed{\chi_e = -0.427} \quad \text{— (1)}$$

$\overline{E} = ?$ We know $\overline{P} = \chi_e\varepsilon_0\overline{E}$

$$\therefore \overline{E} = \frac{\overline{P}}{\chi_e\varepsilon_0} = \frac{\frac{1}{12\pi}(4\overline{a_x} - 2\overline{a_y} + 13\overline{a_z})\times10^{-9}}{-0.427\times8.854\times10^{-12}}$$

$$\boxed{\overline{E} = -28.0847\overline{a_x} + 14\overline{a_y} - 91.210\overline{a_z}} \quad \text{— (2)}$$

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$\bar{D} = \varepsilon_0 E\ (\chi_e + 1)$

$\therefore \bar{D} = (8.854 \times 10^{-12}) \left(-28.084\bar{a_x} + 14\bar{a_y} - 91.2\bar{a_z}\right)(0.573)$

$$\boxed{\bar{D} = -142.053\bar{a_x} + 71.0267\bar{a_y} - 462.688\bar{a_z}\ pC/m^2}$$

3

Q.2 Given: $\bar{J} = 4r^2\cos\theta\ \bar{a_r} + r^3\sin\theta\ \bar{a_\theta}\ \ A/m^2$

Surface: $\theta = 47°$, $0 < \phi < 2\pi$, $0 < r < 5m$

To find: Current crossing the surface

Solution:

$I = \oint \bar{J} \cdot \bar{ds}$

$= \int_{r=0}^{5} \int_{\phi=0}^{2\pi} (4r^2\cos\theta\ \bar{a_r} + r^3\sin\theta\ \bar{a_\theta}) \cdot (r^2\sin\theta\ dr\,d\phi)\ \bar{a_\theta}$

$= \iint r^3\sin\theta \cdot r^2\sin\theta\ dr\,d\phi$

$= \iint r^5\sin^2\theta\ dr\,d\phi$

$= \left[\frac{r^6}{6}\right]_0^5 \cdot \left[\ \cancel{\oint \frac{1 - \cos 2\theta}{2}\,d\theta}\ \right] \sin^2(47)\ [2\pi]$

$$\boxed{I = 8751.9246\ A}$$

1

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Q.3 Given: $\varepsilon = 2.4\varepsilon_0$ $\therefore \varepsilon_r = 2.4$

$V = 350 z^3 \, V$

To find: $\bar{D}$ and ρ_v

$\bar{P}$ and ρ_{pv}

Solution:

$$\bar{E} = -\nabla V = -\frac{\partial}{\partial z}(350 z^3)\,\bar{a_z}$$

$$\bar{E} \; -1050 z^2 \,\bar{a_z} \; V/m \quad \text{—(1)}$$

$$\therefore \bar{D} = \varepsilon \bar{E}$$

$$\bar{D} = -2.4 \times 8.854 \times 10^{-12} \times 1050 z^2 \,\bar{a_z}$$

$$\boxed{\bar{D} = -22.31208 z^2 \,\bar{a_z} \; nC/m^2} \quad \text{—(2)}$$

$$\rho_v = \nabla \cdot \bar{D}$$

$$\rho_v = \frac{\partial}{\partial z}(-22.31208 z^2) \; nC/m^3$$

$$\rho_v = (-22.31208 \times 2) z \; nC/m^3$$

$$\boxed{\rho_v = -44.62416 z \; nC/m^3} \quad \text{—(3)}$$

$$\bar{P} = \varepsilon_0 (\chi_e) \bar{E}$$

$\varepsilon_r = 2.4$ $\therefore$ $\chi_e = \varepsilon_r - 1 = 1.4$

$$\bar{P} = (1.4)(8.854 \times 10^{-12})(-1050 z^2 \,\bar{a_z})$$

$$\boxed{\bar{P} = -13.01538 z^2 \,\bar{a_z} \; nC/m^2} \quad \text{—(4)}$$

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$\rho_{PV} = -\nabla \cdot \bar{P}$

$\rho_{SV} = \frac{-\partial}{\partial z}(-13.01538 \times z^2)$ nC/m3

$\rho_{PV} = (13.01538 \times 2) z$ nc/m3

$\boxed{\rho_{PV} = 26.03076 z \, nC/m^3}$ — (5)

Q.4 Given: Parallel plate capacitor

a) $S = 0.12 m^2$ $d = 80 \mu m$ $V_0 = 12V$ $U = 1 \mu J$

b) Energy density = $100 J/m^3$ $V_0 = 200V$, $d = 45 \mu m$

~~Solution~~: To find: ε_r,

Solution:

a) For parallel plate capacitor:

$C = \frac{\varepsilon_0 \varepsilon_r S}{d}$ Energy stored $= \frac{1}{2} C V^2 = U$

$\therefore C = \frac{2U}{V^2}$

$\therefore \frac{2U}{V^2} = \frac{\varepsilon_0 \varepsilon_r S}{d}$

Substituting the values,

$\varepsilon_r = \frac{2 \times 1 \times 10^{-6} \times 80 \times 10^{-6}}{144 \times 8.854 \times 10^{-12} \times 0.12} = 1.04577$

$\boxed{\varepsilon_r = 1.04577}$ — (a)

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b)

Energy density $= u = \frac{\text{Energy Stored}}{\text{Volume}}$

Volume $= S \cdot d$

Energy stored $= \frac{1}{2}CV^2$

$\therefore u = \frac{\frac{1}{2}CV^2}{S \cdot d}$

$\therefore \frac{C}{S} = \frac{2u \cdot d}{V^2} = \frac{2 \times 100 \times 45 \times 10^{-6}}{(200)^2}$

$\therefore \frac{C}{S} = 225 \; nF/m^2$ —— (1)

For parallel plate capacitor, $C = \frac{\varepsilon_0 \varepsilon_r S}{d}$

$\therefore \frac{C}{S} = \frac{\varepsilon_0 \varepsilon_r}{d}$

$\therefore \frac{8.854 \times 10^{-12} \times \varepsilon_r}{45 \times 10^{-6}} = 225 \times 10^{-9}$ — from (1)

$\boxed{\therefore \varepsilon_r = 1.1435}$ —— (B)

Q.5 Given:

$L = 1.5 \; ft = 0.4572 \, m$

Inner diameter: $2a = 0.1045$ inch

inner ~~radius~~ $2a = \frac{2.654 \times 10^{-3}}{2} m$

inner radius $= a = 1.32715 \times 10^{-3} m$

Outer diameter = 0.73 inch

$\therefore b =$ outer radius $= b = 9.271 \times 10^{-3}$

$\varepsilon_r = 2.8$

Solution:

Capacitance of co-axial cable: $C = \dfrac{2\pi\varepsilon_0\varepsilon_r L}{\ln(b/a)}$

$$\therefore C = \frac{2 \times \pi \times 8.854 \times 10^{-12} \times 2.8 \times 0.4572}{\ln\left(9.271/1.327\right)}$$

$$\boxed{C = 36.6348\,pF} \quad \text{—} (1)$$

Q.6 Given:

Region 1: $\rho \leq 2.8\,cm$ $\quad \varepsilon_{r1} = 3.7$

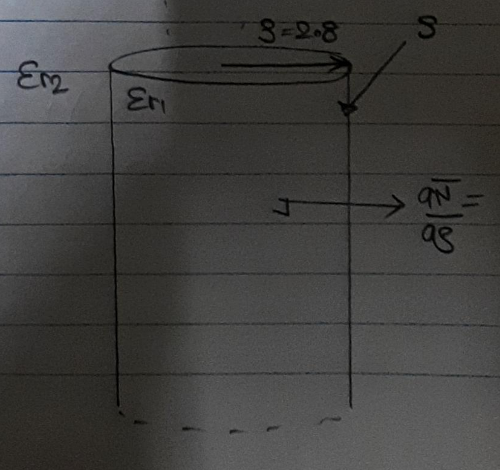
Region 2: $\rho \geq 2.8\,cm$ $\quad \varepsilon_{r2} = 2.8$

$\bar{D_2} = 4\bar{a_\rho} - 13\bar{a_\phi} + 6\bar{a_z}\ nC/m^2$

To find: $\bar{E_1}$ & $\bar{D_1}$, $\bar{P_2}$, Energy density

Solution:

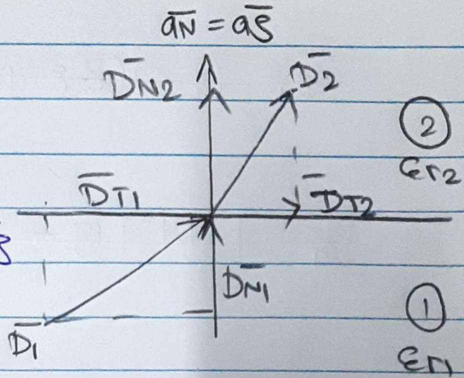
For dielectric dielectric boundary,



$E_{T1} = E_{T2}$, $\quad \dfrac{D_{T1}}{D_{T2}} = \dfrac{\varepsilon_{r1}}{\varepsilon_{r2}}$

$D_{N1} = D_{N2}$, $\quad \dfrac{E_{N1}}{E_{N2}} = \dfrac{\varepsilon_{r2}}{\varepsilon_{r1}}$

Seperating the normal & tangential components,



$\bar{D}_{N2} = \bar{D}_2 \cdot \bar{a}_N = (\bar{D}_2 \cdot \bar{a}_\rho)\,\bar{a}_\rho$

$\bar{D}_{N2} = 4\bar{a}_\rho \ nC/m2$

$\therefore D_{T2} = \bar{D}_2 - D_{N2}$ $\quad D_{T2} = -13\bar{a}_\phi + 6\bar{a}_z \ nC/m^2$

From boundary conditions,

$$\boxed{D_{N1} = D_{N2} = 4\bar{a}_\rho \ nC/m2} \quad — (1)$$

$$D_{T1} = D_{T2} \cdot \frac{\varepsilon_{r1}}{\varepsilon_{r2}} = (-13\bar{a}_\phi + 6\bar{a}_z) \cdot \left(\frac{3.7}{2.8}\right)$$

$$\boxed{D_{T1} = -17.178\bar{a}_\phi + 7.928\bar{a}_z \ nC/m^2} \quad — (2)$$

$\therefore \bar{D}_1$ from (1) & (2): $\quad \bar{D}_1 = \bar{D}_{N1} + \bar{D}_{T1}$

$$\therefore \boxed{\bar{D}_1 = 4\bar{a}_\rho - 17.178\bar{a}_\phi + 7.928\bar{a}_z \ nC/m2} \quad — (a)$$

$$\bar{E}_1 = \frac{\bar{D}_1}{\varepsilon} = \frac{(4\bar{a}_\rho - 17.178\bar{a}_\phi + 7.928\bar{a}_z) \times 10^{-9}}{8.854 \times 10^{-12} \times 3.7}$$

$$\boxed{\bar{E}_1 = 122.100\,\bar{a}_\rho \cancel{+} - 524.3621\,\bar{a}_\phi + 241.759\,\bar{a}_z \ V/m} \quad — (a)$$

$\bar{P}_2 = \chi_{e2}\,\varepsilon_0\,\bar{E}_2$

$\chi_{e2} = \varepsilon_{r2} - 1 = 1.8$

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~~$\therefore \overline{P_{2}} = (1.8)(8.854\times10^{-12})(122.100\overline{a_S} - 524.36\overline{a_\phi} + 241.75\overline{a_Z})$~~

~~$\overline{P_{2}} = 1.9459\overline{a_S} - 8.3568\overline{a_\phi} + 3.8528\overline{a_Z}$ nC/m2~~ — (b)

~~Energy Density - Region 1~~:

$$\overline{P_2} = 1.8 \times 8.854\times10^{-12} \times \left(\frac{4\overline{a_S} - 13\overline{a_\phi} + 6\overline{a_Z}}{2.8 \times 8.854\times10^{-12}} \right)$$

$$\boxed{\overline{P_2} = 2.5771\overline{a_S} - 8.3571\overline{a_\phi} + 3.8571\overline{a_Z} \text{ nC/m}^2}$$

Energy Density: Region 2

$$\frac{dW_2}{dV} = \frac{1}{2\varepsilon}|\overline{D_2}|^2 = \frac{1}{2\times8.854\times10^{-12}} \cdot (14.86606)^2$$

$$\boxed{\frac{dW_2}{dV} = 12.48023 \text{ pJ/m3}}$$ — (c)

Energy Density: Region 1

$$\frac{dW_1}{dV} = \frac{1}{2\varepsilon}|\overline{D_1}| = \frac{1}{2\times8.854\times10^{-12}} \times 373.9368$$

$$\boxed{\frac{dW_1}{dV} = 21.1168 \text{ pJ/m3}}$$ — (c)

Q.7 Given:

Surface $x = 0$ $\quad \bar{a}_N = \bar{a}_x$

Region 1: $\varepsilon_{r1} = 5.4$ $\quad x > 0$

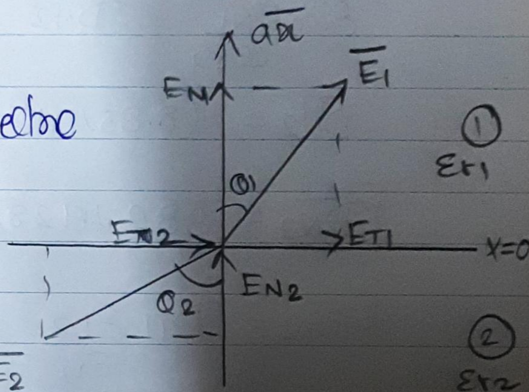
Region 2: $\varepsilon_{r2} = 3.5$ $\quad x < 0$

$\bar{E}_1 = 8\bar{a}_x - 12\bar{a}_y + 13\bar{a}_z$ V/m

To find: E_{N1}, E_{T1}, Angle θ, D_{N2}, D_{T2}, D_2, P_2, θ_2

Solution:

Boundary conditions for Dielectric-Dielectric boundary:-



$E_{T1} = E_{T2}$

$$\frac{D_{T1}}{D_{T2}} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \quad ; \quad D_{N1} = D_{N2}$$

Seperating the normal and tangential components:

$\bar{E}_{N1} = (E_1 \cdot \bar{a}_x)\,\bar{a}_x = 8\bar{a}_x$

$\boxed{\bar{E}_{N1} = 8\bar{a}_x \text{ V/m}}$ — (a)

$\bar{E}_{T1} = \bar{E}_1 - \bar{E}_{N1} = -12\bar{a}_y + 13\bar{a}_z$

$\boxed{\bar{E}_{T1} = -12\bar{a}_y + 13\bar{a}_z \text{ V/m}}$ — (b)

$\theta_1 = \tan^{-1}\left(\frac{E_{T1}}{E_{N1}}\right) = \tan^{-1}\left(\frac{17.691}{8}\right)$

$\boxed{\theta_1 = 65.6681^\circ}$ —— (c)

From the boundary Conditions,

$D_{N1} = D_{N2}$

$\varepsilon_0 \varepsilon_r E_{N1} = D_{N2}$

$\bar{D}_{N2} = (8.854\times10^{-12} \times 5.4)(8\bar{a}_x)$

$\boxed{\bar{D}_{N2} = 382.4928\,\bar{a}_x\ pC/m^2}$ —— (d)

$\bar{D}_{T2} = D_{T1}\frac{\varepsilon_{r2}}{\varepsilon_{r1}} = \varepsilon_0 \cancel{\varepsilon_{r1}}\,\bar{E}_{T1}\frac{\varepsilon_{r2}}{\cancel{\varepsilon_{r1}}}$

$\bar{D}_{T2} = 8.854\times10^{-12} \times 3.5 \times (-12\bar{a}_y + 13\bar{a}_z)$

$\boxed{\bar{D}_{N2} = -371.868\bar{a}_y + 402.857\bar{a}_z\ \ pC/m^2}$ —— (e)

$\therefore \bar{D}_2 = \bar{D}_{N2} + \bar{D}_{T2}$

$\boxed{\therefore \bar{D}_2 = 382.4928\bar{a}_x - 371.868\bar{a}_y + 402.857\bar{a}_z\ pC/m^2}$

Angle θ_2: $\theta_2 = \tan^{-1}\left(\frac{E_{T2}}{E_{N2}}\right) = \tan^{-1}\left(\frac{D_{T2}}{D_{N2}}\right)$ — (f)

$\theta_2 = \tan^{-1}\left(\frac{548.246}{382.4928}\right)$

$\boxed{\theta_2 = 55.098^\circ}$ —— (b)