Abstract. Five quotations by Dirac, Einstein, Gauss, Bohr, Feynman should be known to researchers in statistical causal inference. Dirac emphasizes confounding, indeterminacy, and probability in quantum theory. Einstein considers aspects of causal statistical independence in physical theory. Gauss shows an inspiring fascination with mathematical statistics. Bohr describes the role of conditioning in specification of experimental quantum populations. Feynman discusses the double-slit experiment with comparisons of potential outcome distributions.

Key words and phrases: quantum theory, causality, mathematical statistics.

1. INTRODUCTION

The scientific character and achievements of the quoted statistical physicists are generally known to scientists, so the task is to select the pedagogically central quotations and provide minimal commentary. These five quotations seem especially important in characterizing the relation of physics to causal statistical methodology. Following the philosopher of physics Howard Stein consider "the content of the science we possess" as "a primary object of the philosophy of science" [15, p. 265]. Philosophy of physics studies the content of physics. The content of physics includes statistical considerations through quantum theory [8, 16, 5], so aspects of statistical theory stem from the philosophy of physics. On this view statistical science sits at the intersection of philosophy of physics and applied mathematics. These quotations can provide a helpful entry for statistical scientists to read in philosophy of physics.

2. DIRAC EMPHASIZING CONFOUNDING, INDETERMINACY, AND PROBABILITY

Einstein, in an essay on Maxwell [6], describes Dirac as "to whom in my judgment, we are indebted for the most logically complete account of this theory" perhaps referring to the opening chapter of Dirac's *Principles of Quantum Mechanics*. See also Salmon's discussion of quantum indeterminacy [14].

> At this stage it becomes important to remember that science is concerned only with observable things and that we can observe an object only be letting it interact with some outside influence. An act of observation is thus necessarily accompanied by some disturbance of the object observed. We may define an object to be big when the disturbance accompanying our observation of it may be neglected, and small when the disturbance cannot be neglected. This definition is in close agreement with the common meanings of big and small.
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> It is usually assumed that, by being careful, we may cut down the disturbance accompanying our observation to any desired extent. The concepts of big and small are then purely relative and refer to the gentleness of our means of observation as well as to the object being described. In order to give an absolute meaning to size, such as is required for any theory of the ultimate structure of matter, we have to assume that *there is a limit to the fineness of our powers of observation and the smallness of the accompanying disturbance–a limit which is inherent in the nature of things and can never be surpassed by improved technique or increased skill on the part of the observer*. If the object under observation is such that the unavoidable limiting disturbance is negligible, then the object is big in the absolute sense and we may apply classical mechanics to it. If, on the other hand, the limiting disturbance is not negligible, then the object is small in the absolute sense and we require a new theory for dealing with it.
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> A consequence of the preceding discussion is that we must revise our ideas of causality. Causality applies only to a system which is left undisturbed. If a system is small, we cannot observe it without producing a serious disturbance and hence we cannot expect to find any causal connection between the results of our observations...there is an unavoidable indeterminacy in the calculation of observational results, the theory enabling us to calculate in general only the probability of obtaining a particular result when we make an observation. [Emphasis in original][5, Section 1.1]

3. EINSTEIN CONSIDERING CAUSAL STATISTICAL INDEPENDENCE

There seem aspects of stable unit treatment value [13] and hypothesis testing to Einstein's view of lawlike empirical inquiry, originally presented in a letter to Born. It is widely held that "Bell's theorem addresses the implications and ultimately the tenability, of this picture" [12].

> An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects "are situated in different parts of space". Unless one makes this kind of assumption about the independence of the existence (the "being-thus") of objects which are far apart from one another in space – which stems in the first place from everyday thinking – physical thinking in the familiar sense would not be possible. It is also hard to see any way of formulating and testing the laws of physics unless one makes a clear distinction of this kind. This principle has been carried to extremes in the field theory by localizing the elementary objects on which it is based and which independently of each other, as well as the elementary laws which have been postulated for it, in the infinitely small (four-dimensional) elements of space. The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B; this is known as the "principle of contiguity," which is used consistently in the field theory. If this axiom were to be completely abolished, the idea of the existence of (quasi-) enclosed systems, and thereby the postulation of laws which can be checked empirically in the accepted sense, would become impossible. [7]

4. GAUSS ASTONISHED BY MATHEMATICAL STATISTICS

Gauss made key discoveries in the long-established disciplines of number theory, geometry, algebra, and astronomy so describing the pursuit of the Gauss-Markov theorem as "one of the most important problems in the application of mathematics to the natural sciences" is pedagogically vital for students of statistical methodology, especially since Gauss was known to be an avid reader of Newton's *Mathematical Principles of Natural Philosophy*.

> One of the most important problems in the application of mathematics to the natural sciences is to choose ... the combination that yields values of the unknowns that are least subject to the errors.
>
> In my *Theory of the Motion of Heavenly Bodies* I showed how to calculate most probable values of the unknowns, provided the probability law of the observation errors is known. But in almost all cases this law can only be hypothetical, and for this reason I applied the theory to the most plausible law ... From this supposition came a method which I had already used for some time, especially in astronomical calculations. It is now used by many calculators under the name of the method of least squares.
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> Later Laplace attacked the problem from a different angle and showed that if the number of observations is very large then the method of least squares is to be preferred, whatever the probability law of the errors. But for a modest number of observations, things are as they were, and if one rejects my hypothetical law, the only reason for recommending the method of least squares over other methods is that it lends itself to easy calculation.
>
> I therefore hope that mathematicians will be grateful if in this new treatment of the subject I show that the method of least squares gives the best of all combinations – not approximately, but absolutely, whatever the probability law of the errors and whatever the number of observations. [11, Section 1.17]

5. BOHR DESCRIBING QUANTUM POPULATIONS AS POPULATIONS SPECIFIED BY CONDITIONING ON CLASSICAL EXPERIMENTAL ARRANGEMENTS

Schrödinger in a letter to Bohr states "there must be quite definite and clear grounds, why ... one must interpret observations classically" [17]. Bohr's demand for an "unambiguous" "account of the experimental arrangement" using the "terminology of classical physics" requires theoretical statistics. Each part of a classical "account of the experimental arrangement" is a conditioning value specifying an aspect of an infinite hypothetical population of repeated quantum experiments. Theoretical statistics cannot provide the "unambiguous" account of the experimental arrangement, but can provide Schrödinger's "definite and clear grounds" from which to support the interpretation of quantum observations described with the "unambiguous language" of collected conditioning values specifying the "causal conditions" [9, Section 2] of quantum populations.

> It is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms*. The argument is simply that by the word "experiment" we refer to a situation where we can tell others what we have done and what we have learned and that, therefore, the account

of the experimental arrangement and of the results of the observations must be expressed in unambiguous language with suitable application of the terminology of classical physics. [Emphasis in original] [3, 17]

6. FEYNMAN EXPLAINING THE DOUBLE-SLIT EXPERIMENT BY COMPARISON OF POTENTIAL OUTCOME DISTRIBUTIONS

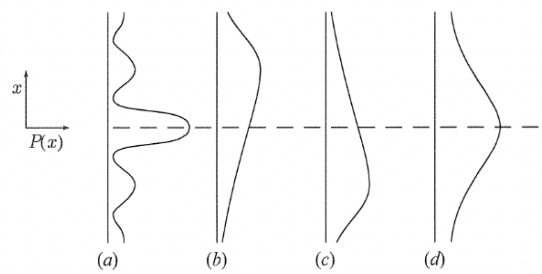

Fig. 1-2 Results of the experiment. Probability of arrival of electrons at x plotted against the position x of the detector. The result of the experiment of Fig. 1-1 is plotted here at (a). If hole 2 is closed, so the electrons can go through just hole 1, the result is (b). For just hole 2 open, the result is (c). If we imagine that each electron goes through one hole or the other, we expect the curve $(d) = (b) + (c)$ when both holes are open. This is considerably different from what we actually get, (a).

FIG 1. From [8, Section 1.1]

The explanation of the double-slit experiment seems to characterize quantum theory in a causal-inferential sense of comparing potential outcomes distributions. Distribution (d) "is considerably different from" distribution (a) solely because of the causal presence of closure. After experimental confirmation of Bell inequalities [10, 1] hidden variables theories are considered not relevant to quantum theory, but prominent quantum theories with hidden variables [2] may relate to statistical notions of missing data [4, Section 1.2]. Expectations over the real-values of diagonalized Hermitean operators may relate to a statistical notion of quantum estimands.

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