

ECBM 4090E: Brain Computer Interface Laboratory

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Further reading

- CSP reading materials in Coursework

So far...

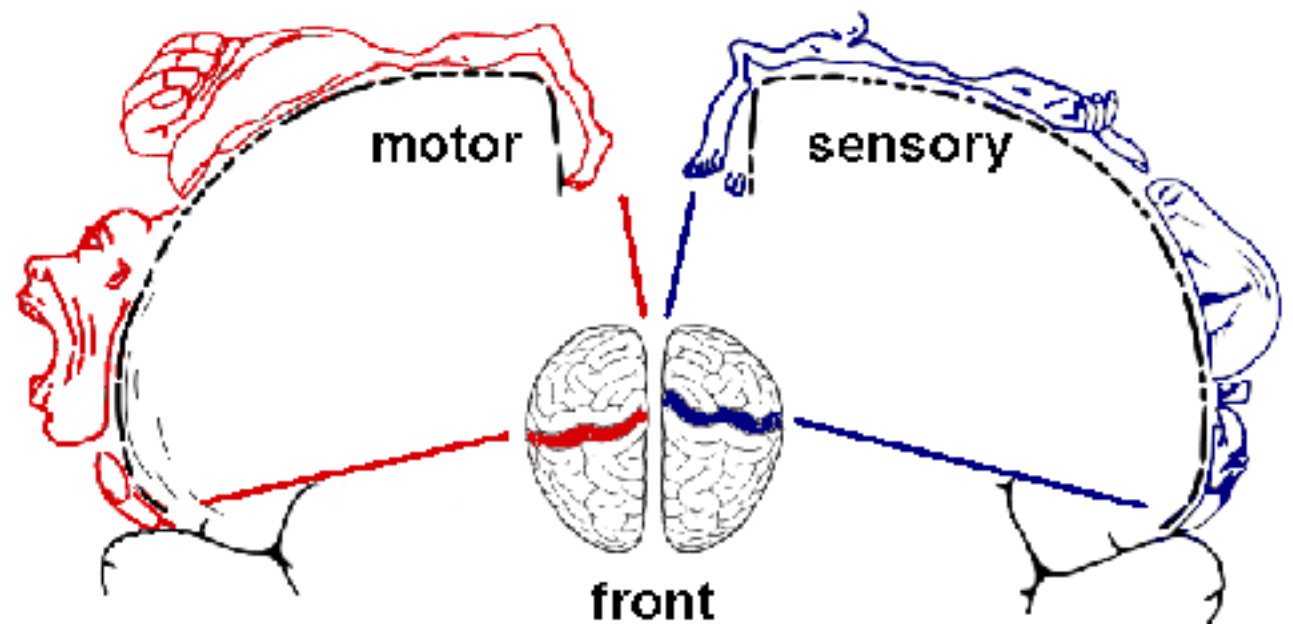
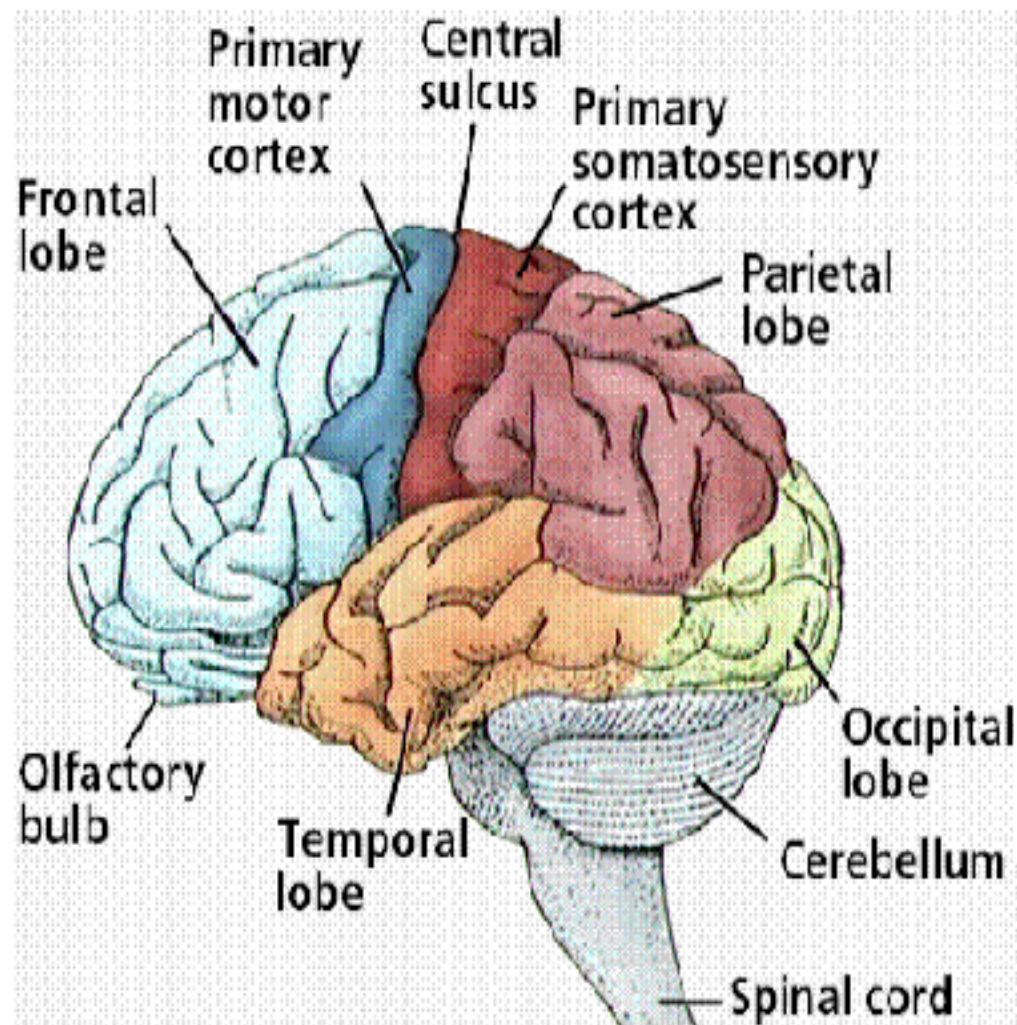
- What is EEG, how is it recorded, artifacts, etc.
- Neurofeedback, control the power in certain frequency bands
- Event Related Potentials: P300 speller, Oddball
- Auditory Steady State Responses (ASSR), attentional modulations
- Steady State Visual Evoked Potentials (SSVEP)

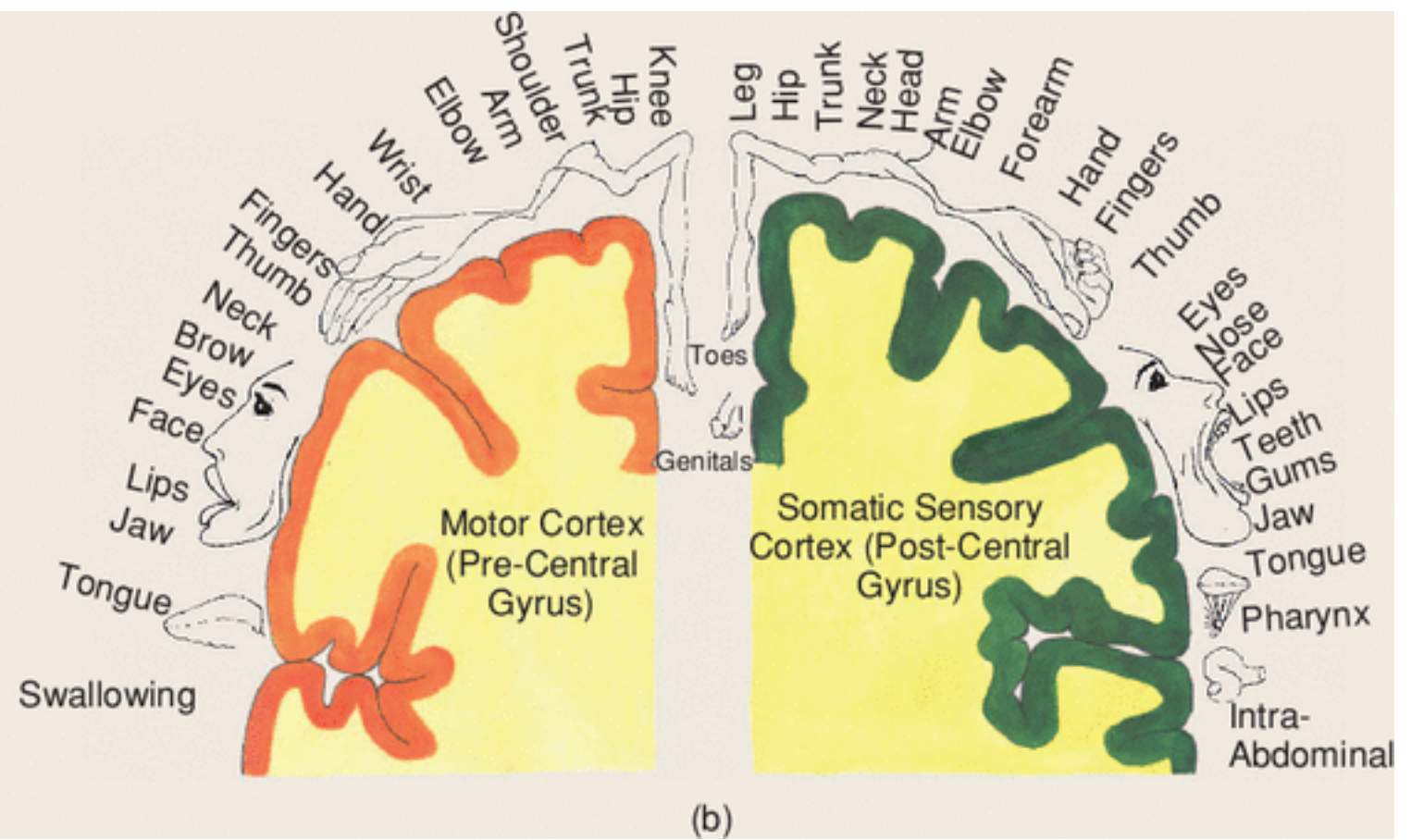
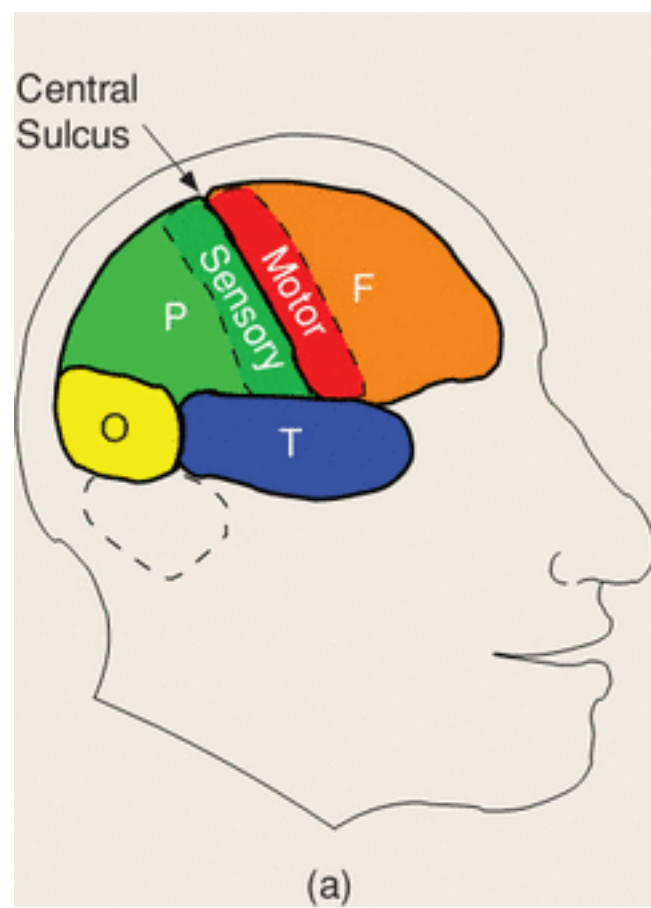
Imagined movement

- No external stimuli
- Requires training, co-adaptation of the brain and BCI
- detecting the desynchronization of Mu Waves

Mu Waves

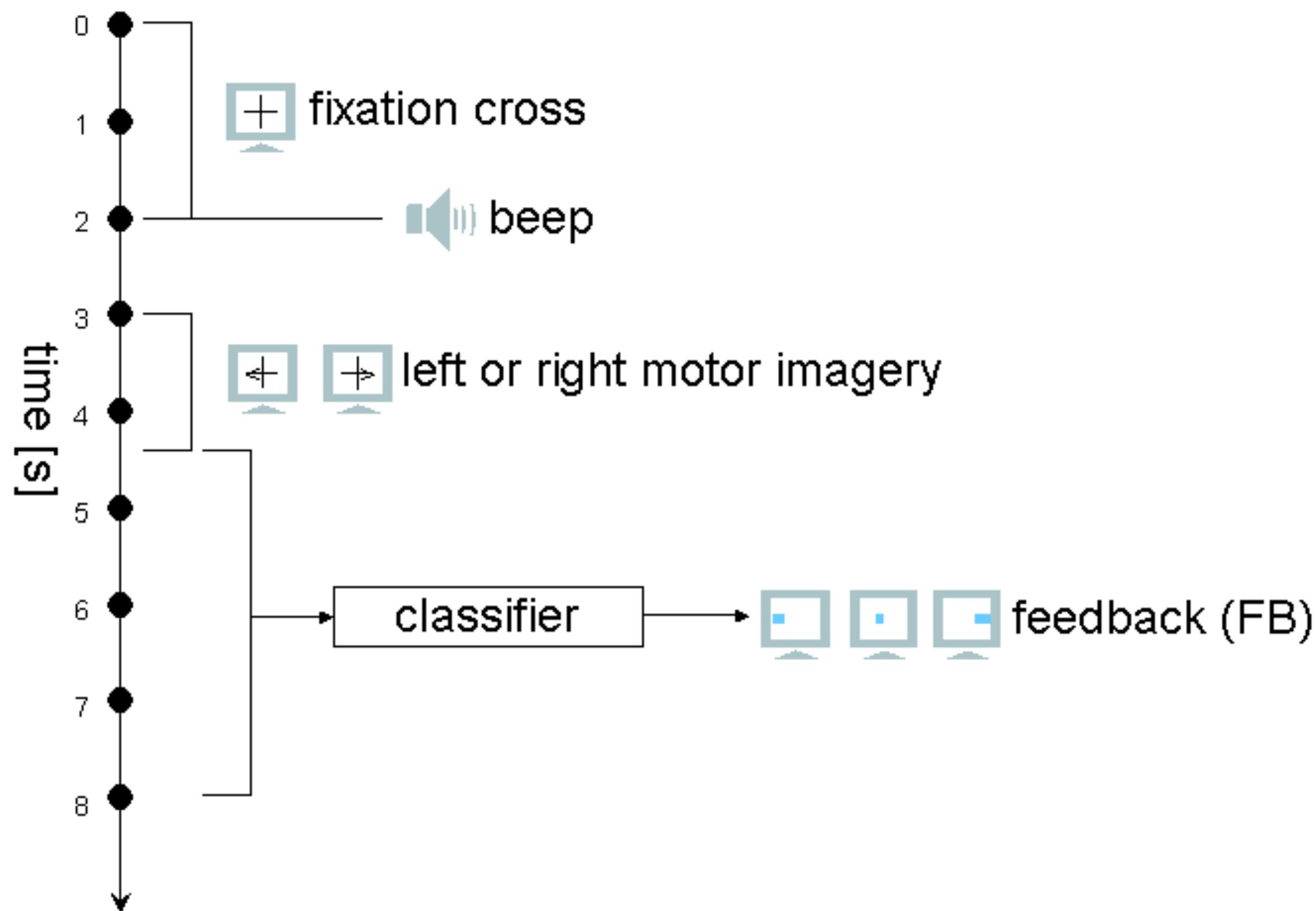
- Synchronized pattern of rhythms that appear when the motor cortex is idle





Mu Waves

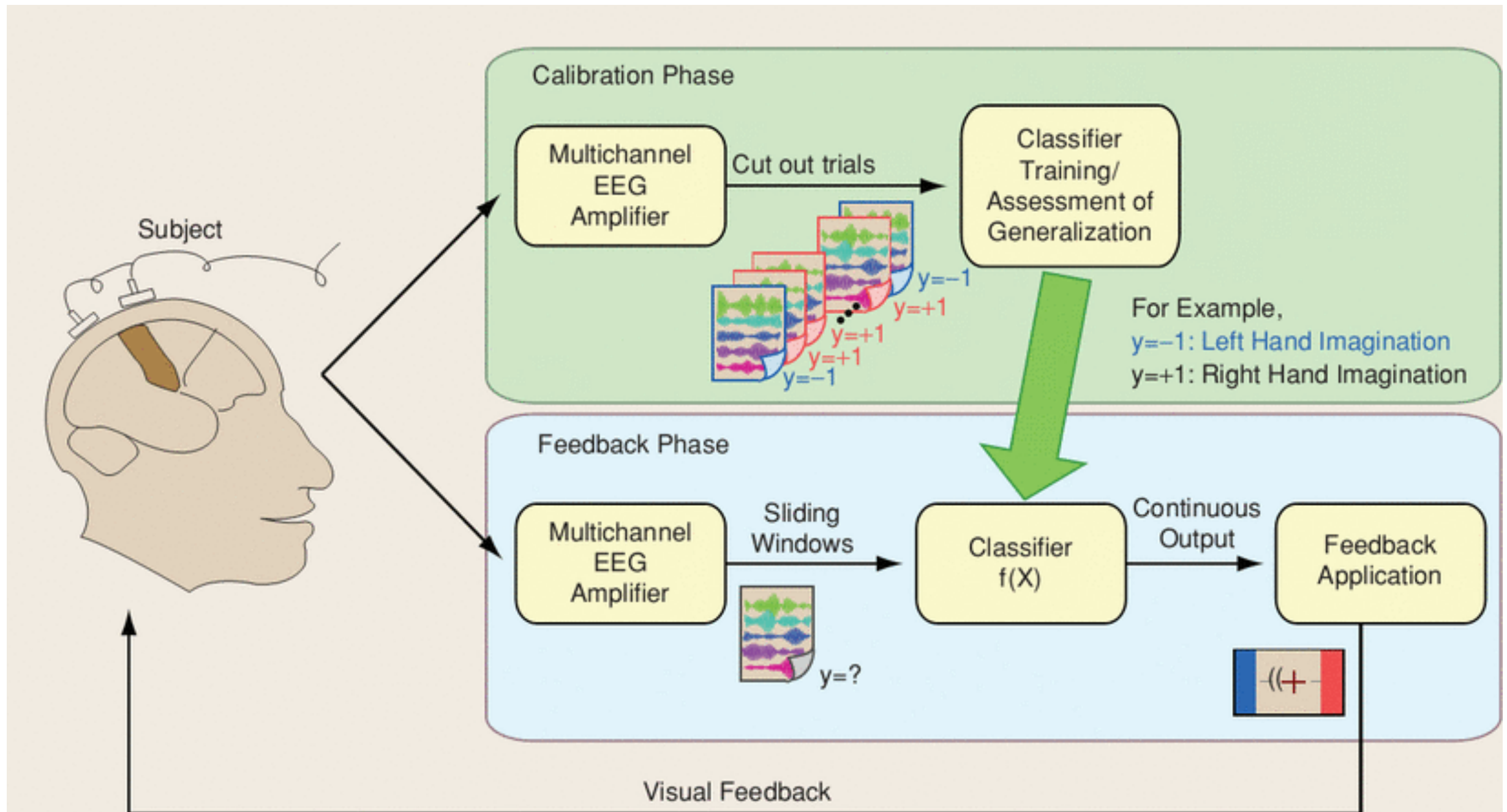
- Mu waves (mu rhythms) are synchronized pattern of electrical activity involving large number of neurons in the part of brain that controls voluntary movement
- Frequency 7.5Hz-12.5Hz, most prominent when body is at rest. When performing or imagining a movement, they are suppressed (desynchronized)
- Spatially specific patterns of neural activation, depending on what part of the body is moving (different for left and right)
- Most common method to detect this spatial pattern is CSP (Common Spatial Patterns): find a linear weighing of the electrodes that maximally separates the two classes



Common Spatial Pattern

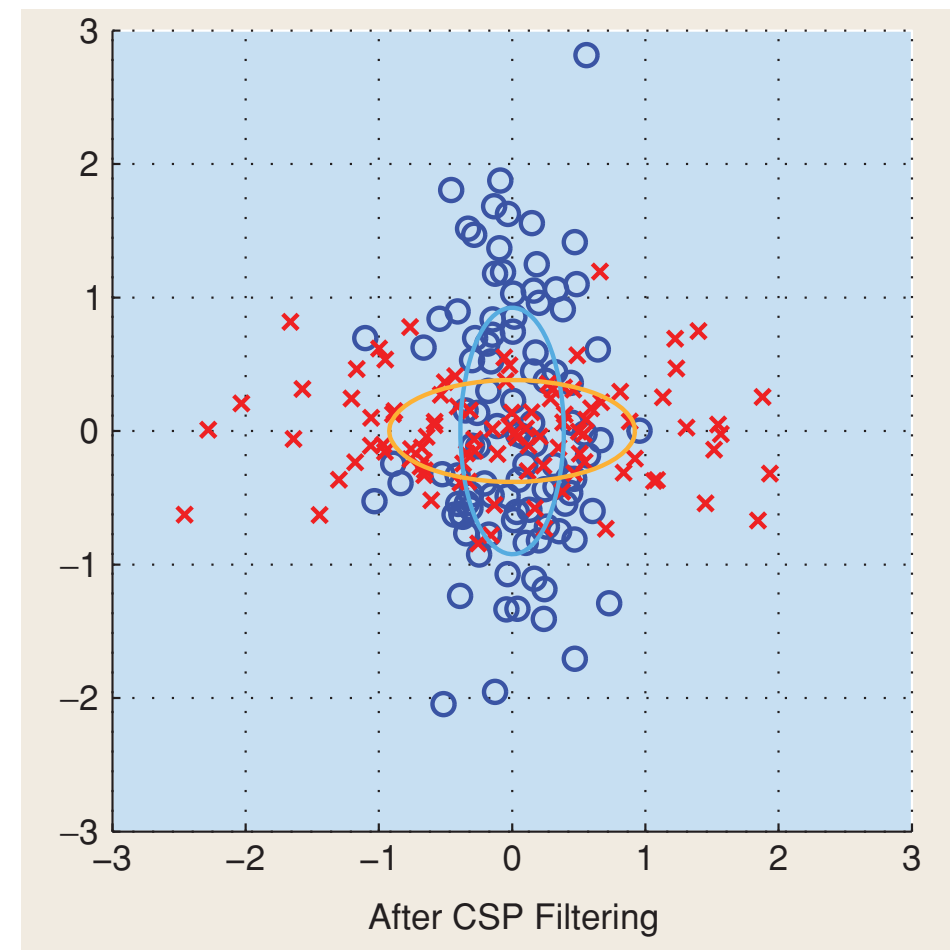
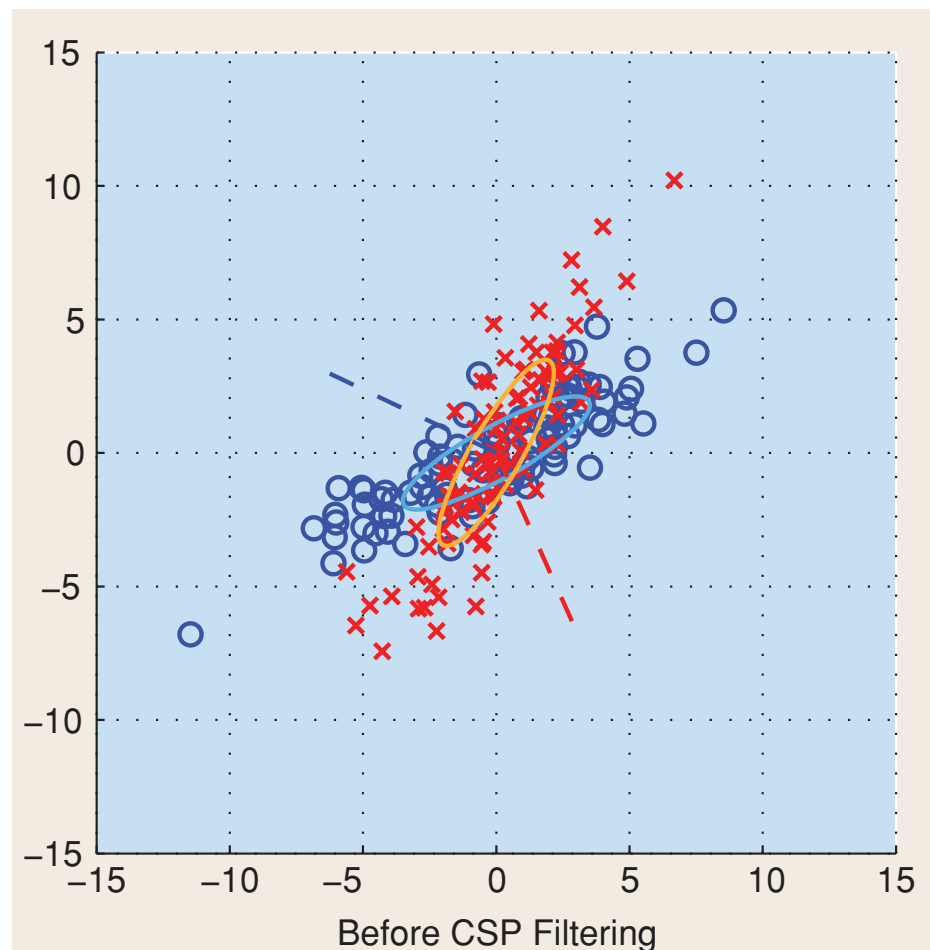
- EEG responses are highly correlated, they reflect combined neural sources
- CSP is a popular spatial filtering methods for oscillatory processes
- Assume that source activity differs between the two classes
- Finds the best weighted combination of electrodes that maximize the separation between the two classes

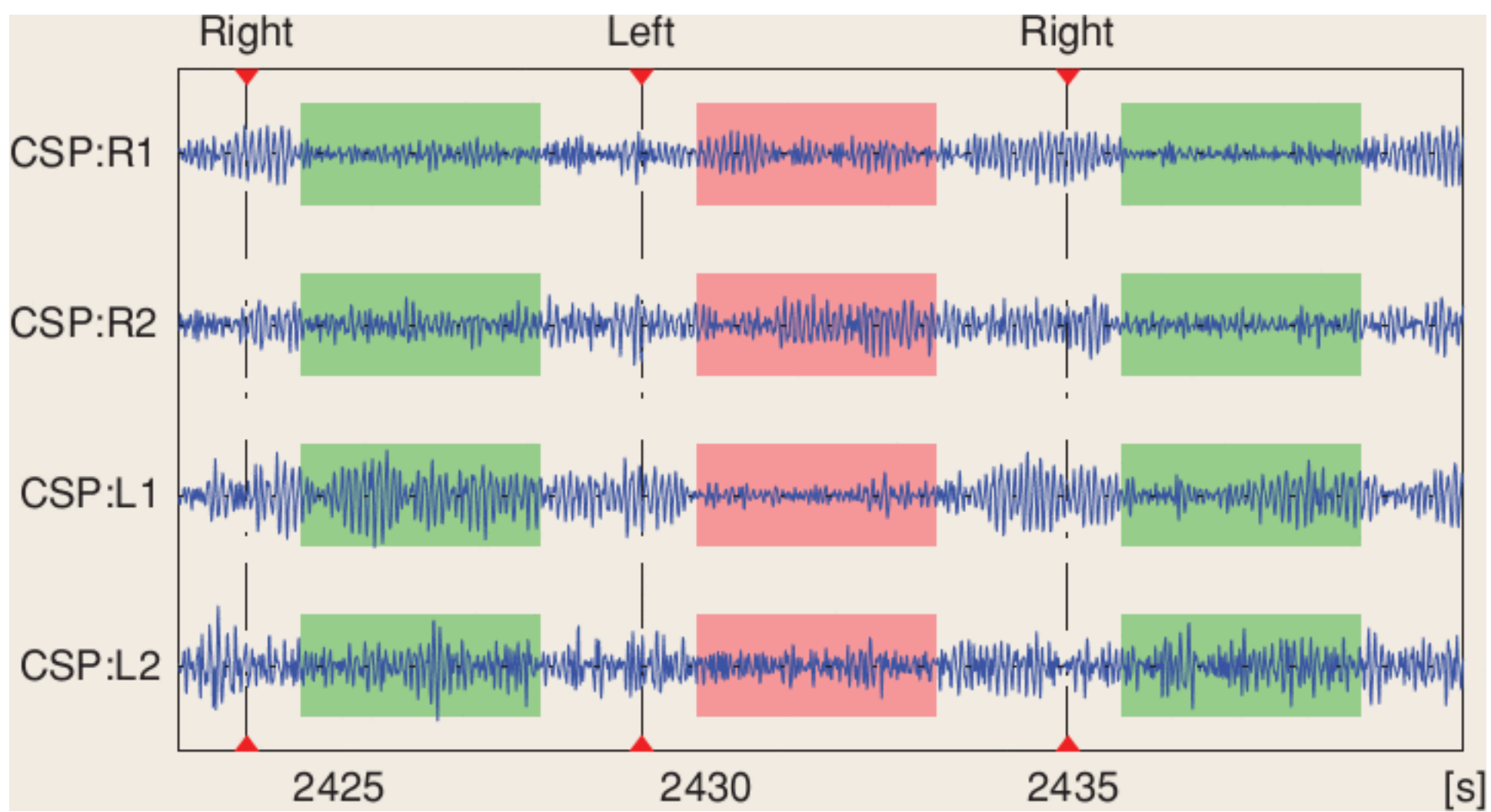
Paradigm

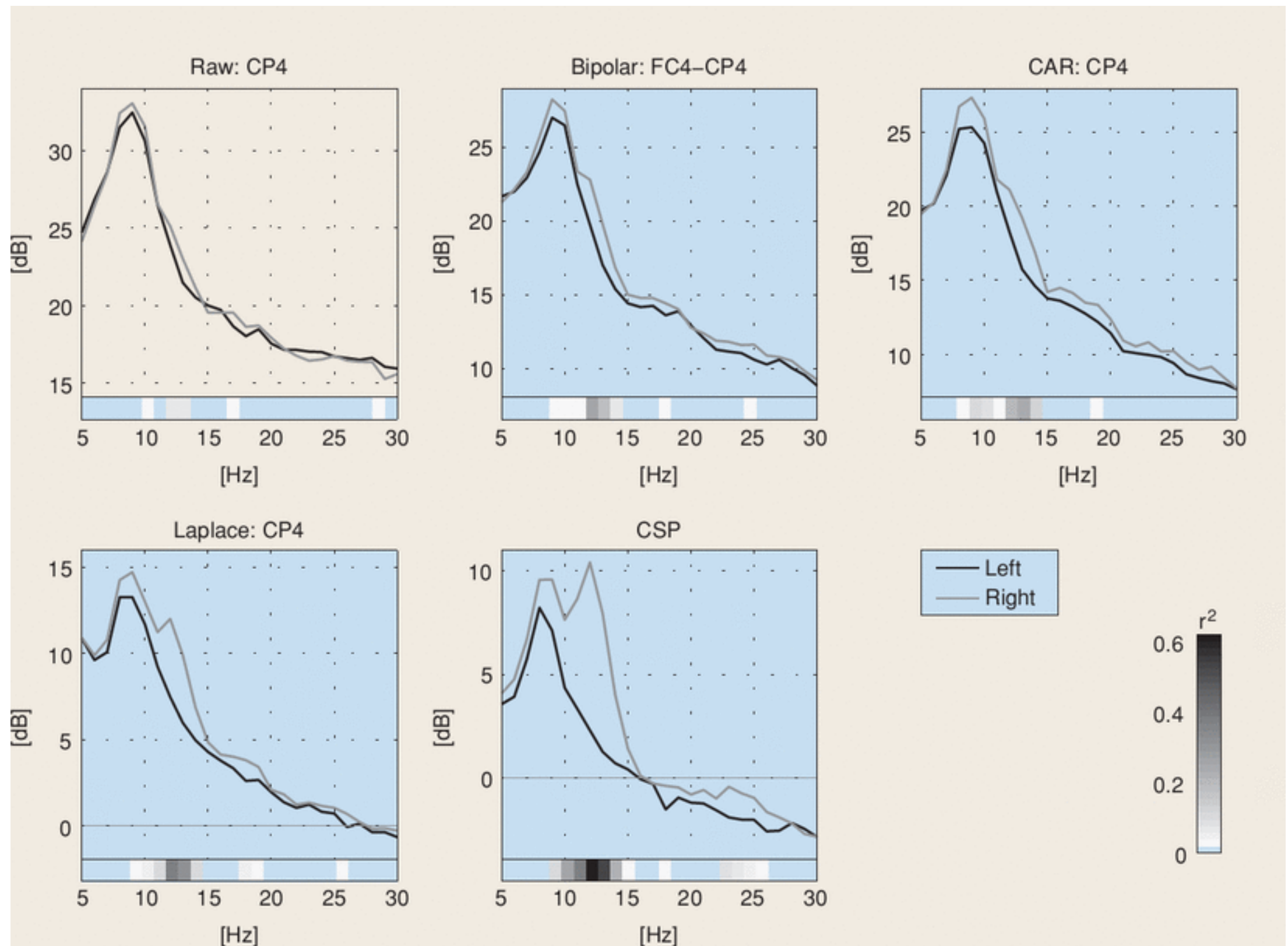


Changing the coordinates of the feature space

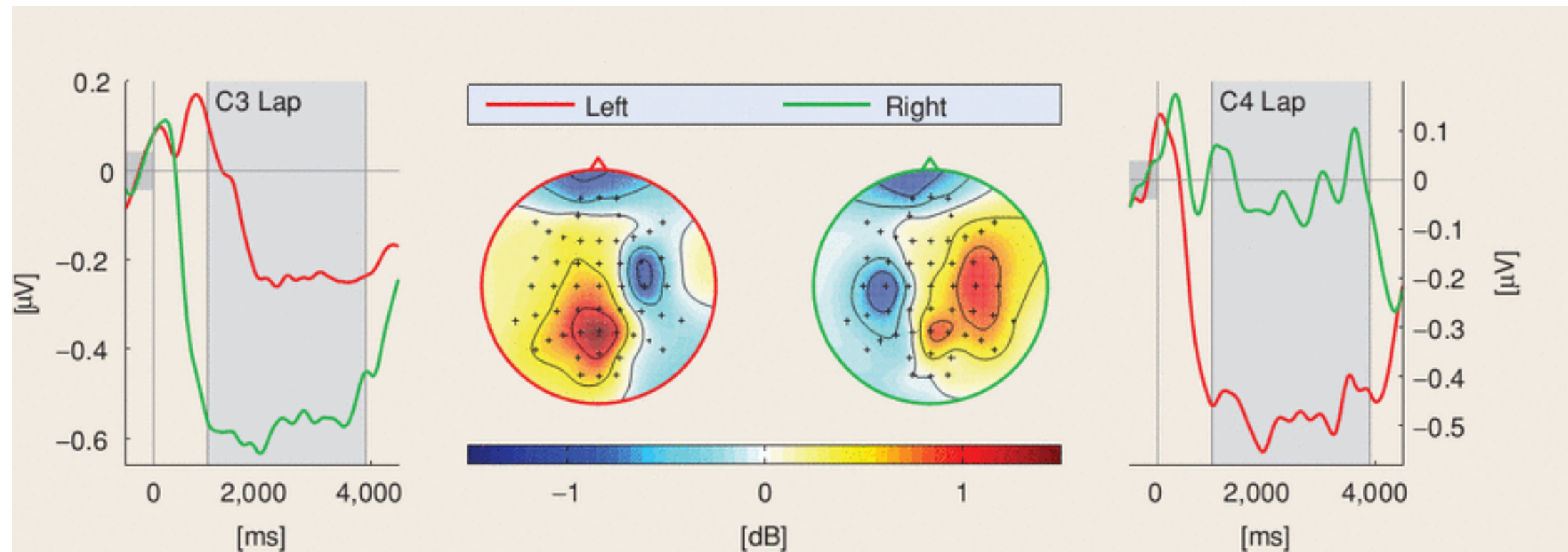
In the transformed space, the variance of each class is maximized in one direction, and minimized in the other







Projected CSP features



Calculating CSP

Covariance of each class:

$$\Sigma^{(c)} = \frac{1}{|\mathcal{I}_c|} \sum_{i \in \mathcal{I}_c} X_i X_i^\top \quad (c \in \{+, -\}),$$

$$\mathcal{I}_c \quad (c \in \{+, -\})$$

Project the data using W :

$$x_{\text{CSP}}(t) = W^\top x(t)$$

Optimization criteria

$$S_d = \Sigma^{(+)} - \Sigma^{(-)} \quad : \text{discriminative activity ,}$$

$$S_c = \Sigma^{(+)} + \Sigma^{(-)} \quad : \text{common activity ,}$$

Project using W that maximizes discrimination
while minimizing common

$$\underset{w \in \mathbb{R}^c}{\text{maximize}} \quad \frac{w^\top S_d w}{w^\top S_c w}$$

Solution: “generalized eigen value problem”

Joint diagonalization of the two covariance matrices

$$\Sigma^{(+)} \boldsymbol{w} = \lambda \Sigma^{(-)} \boldsymbol{w}$$

$$W^{\top} \Sigma^{(+)} W = \Lambda^{(+)},$$

$$W^{\top} \Sigma^{(-)} W = \Lambda^{(-)}, \quad (\Lambda^{(c)} \text{diagonal}),$$

Subject to the following constraint:

$$\Lambda^{(+)} + \Lambda^{(-)} = I$$

In MATLAB: $W = \text{eig}(S1, S1 + S2)$

Where S1 and S2 are the covariance matrices of class 1 and 2

CSP calculation steps

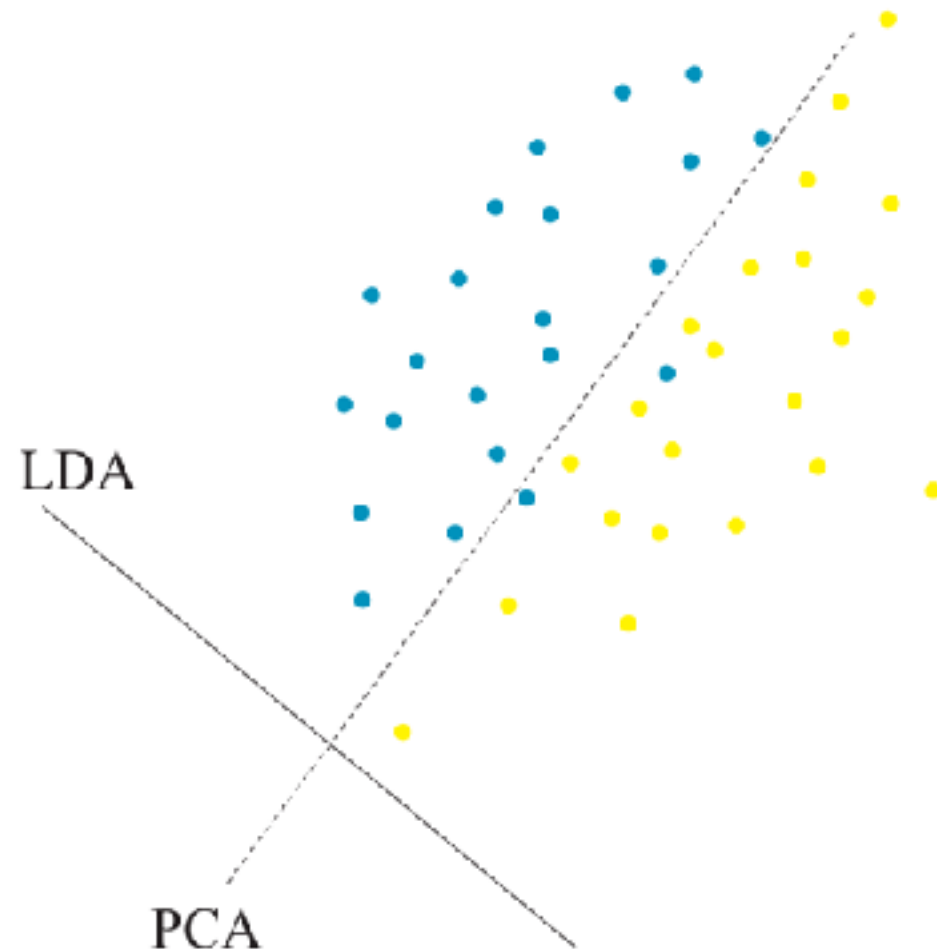
- Separate the trials into + and -
- Find the covariance matrix for each trial, and average over + and - to estimate S_1 and S_2
- Solve the generalized Eigen value problem to find the best projection W
- Choose the first 6 Eigen vectors (columns of W) that correspond to the largest Eigen values: CSP projections
- Project the data (channel x time) using the CSP projections (6 x time)

What next?

- CSP is just a linear transform into a space where the data is more separable
- How do we classify the classes in this space?
- Linear Classification: Linear Discriminant Analysis

Discriminant projections

- Also a dimensionality reduction through projection
- Different than PCA: maximize class discriminability

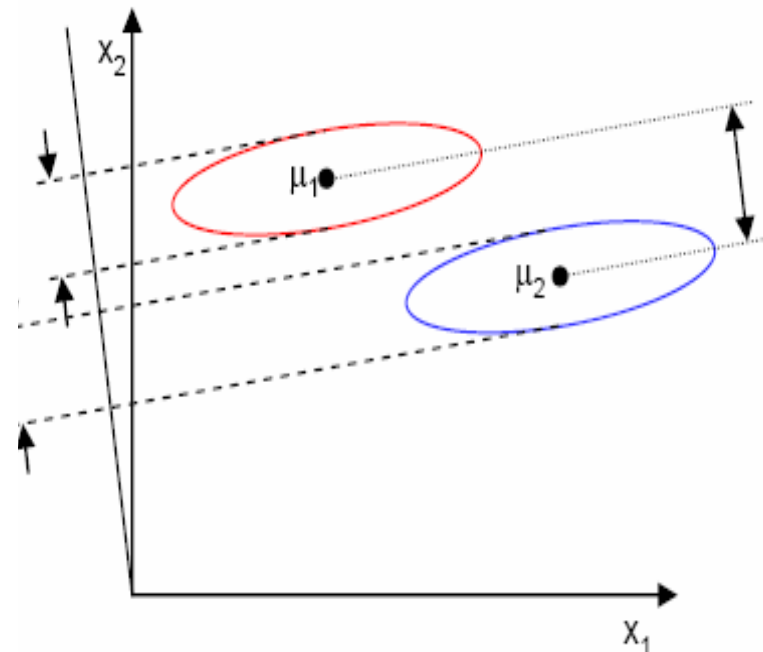


Fisher's solution

Between class
scatter

$$J(w) = \frac{|\tilde{\mu}_1 - \tilde{\mu}_2|^2}{\tilde{s}_1^2 + \tilde{s}_2^2}$$

Within class
scatter



LDA solution

$$S_W^{-1} S_B w = \lambda w \quad \text{where} \quad \lambda = J(w) = \text{scalar}$$

The Eigenvector corresponding to the largest
Eigen value

In Matlab: use “Classify” function