Equity premium and correlated shocks

Siddarth Venkatesh

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Abstract

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1 Model

1.1 The basic problem

I start by examining the basic consumption-saving problem. The individual maximizes their discounted lifetime utility from the consumption stream $\{C_t\}_{t=0}^T$, where $T = \infty$ in the infinite-horizon model

$$\max_{\{C_t\}_{t=0}^T} \mathbb{E}_t \sum_{t=0}^{T} \beta^t u(C_t) \tag{1}$$

subject to the period-wise constraints

$$C_t + A_{t+1} = W_t + Y_t$$

where A_{t+1} is the total level of investment in assets incoming in period $t+1$, W_t the total monetary wealth at the beginning of the period, and Y_t is the current-period income. Income can be modelled as centered around an "expected" permanent income

$$Y_t = \zeta_t P_t \tag{2}$$

where ζ_t is a transitory mean-one shock. Permanent income, P_t, itself grows according to

$$P_t = \Gamma_t P_{t-1} \eta_t \tag{3}$$

where Γ_t is the predictable component of the growth of permanent income, and η_t is a mean-one shock. Further, individuals can also invest, in a perfectly divisible manner, between a risk-free (bond) and a risky (equity) asset. If the consumer chooses to hold κ_{t+1} share of the savings in the risky asset in period t (that is, their portfolio at the start of period $t+1$ contains κ_{t+1} share of the risky asset), wealth in period $t+1$ is determined by

$$W_{t+1} = \overbrace{(\mathsf{R} + \kappa_{t+1}(\Re_{t+1} - \mathsf{R}))}^{\mathcal{R}_{t+1}} A_{t+1} \tag{4}$$

$$\Re_{t+1} = \Re \nu_{t+1} \tag{5}$$

where $\Re_{t+1}$ is the return on the investments made in the risky asset in period t, ν_{t+1} is a mean-one shock, and $\mathcal{R}_{t+1}$ is the effective rate of return on assets stemming from the portfolio optimization decision κ_{t+1}. I impose an artificial borrowing constraint on the consumer that limits their borrowing as a proportion of their permanent income,

$$A_{t+1} \geq \underline{a} P_t \tag{$\underline{a} \leq 0$}$$

We can then use equation (4) rewrite the period budget constraint as

$$W_{t+1} = \mathcal{R}_{t+1}(W_t + Y_t - C_t)$$

Allowing $M_t = W_t + Y_t$ to denote the total current level of monetary resources

$$M_{t+1} = \mathcal{R}_{t+1}(M_t - C_t) + Y_{t+1}$$

Writing the problem from equation (1) in Bellman form with the added assumption that the period utility from consumption assumes a CRRA form,

$$V_t(M_t,\, P_t) = \max_{\{C_t,\, \kappa_{t+1}\}} \frac{C_t^{1-\rho}}{1-\rho} + \beta\, \mathbb{E}_t \left[V_{t+1}(\mathcal{R}_{t+1}(M_t - C_t) + Y_{t+1},\, P_{t+1}) \right] \tag{6}$$

Normalize all period t variables by the permanent income P_t (since A_t is determined in period $t-1$, it is normalized by P_{t-1}), and denote these new variables in lowercase (i.e. $c_t = C_t/P_t$). Letting $\mathcal{G}_{t+1} = \Gamma_{t+1}\eta_{t+1}$,

$$m_{t+1} = \frac{\mathcal{R}_{t+1}}{\mathcal{G}_{t+1}}(m_t - c_t) + \zeta_{t+1}$$

The Bellman formulation then becomes

$$v_t(m_t) = \max_{c_t,\,\kappa_{t+1}} \frac{c^{1-\rho}}{1-\rho} + \beta\,\mathbb{E}\left[(\mathcal{G}_{t+1})^{1-\rho} v_{t+1}\left(\frac{\mathcal{R}_{t+1}}{\mathcal{G}_{t+1}}(m_t - c_t) + \zeta_{t+1}\right)\right] \tag{7}$$

and the consumption Euler equation is then given by[1]

$$c_t^{-\rho} = \beta\,\mathbb{E}_t\left[\mathcal{R}_{t+1}(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right] \tag{8}$$

while the first order condition for κ_{t+1} is given by

$$a_{t+1}\,\mathbb{E}_t\left[(\Re_{t+1} - \mathsf{R})\,(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right] = 0 \tag{9}$$

When optimal $a_{t+1} \neq 0$, then

$$\mathbb{E}_t\left[(\Re_{t+1} - \mathsf{R})(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right] = 0$$

Now see that from equation (8) we get

$$\begin{aligned}
c_t^{-\rho} &= \beta\,\mathbb{E}_t\left[(\mathsf{R} + \kappa_{t+1}(\Re_{t+1} - \mathsf{R}))(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right] \\
&= \beta\left[\mathbb{E}_t\left[\mathsf{R}(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right] + \kappa_{t+1}\,\mathbb{E}_t\left[(\Re_{t+1} - \mathsf{R})(\mathcal{G}_{t+1}c_{t+1})^{-\rho}\right]\right] \\
&= \beta\mathsf{R}\Gamma_{t+1}^{-\rho}\,\mathbb{E}_t\left[(\eta_{t+1}c_{t+1})^{-\rho}\right] && \text{(from (9))}
\end{aligned}$$

I have hitherto remained silent on the exact distribution followed by the shocks to income and wealth. First, I assume that shocks are independent across time periods. Then, I model the shocks to be jointly lognormally distributed. In particular,

$$\log(\eta_t,\, \nu_t,\, \zeta_t) \sim \mathcal{N}(\mu,\, \Sigma)$$

[1] Use the following steps to show that under optimality, $u'(c_t) = v'(m_t)$

$$\begin{aligned}
u'(c_t) &= \beta\,\mathbb{E}\left[\mathcal{G}_{t+1}^{-\rho}\mathcal{R}_{t+1}v'(m_{t+1})\right] \\
v'(m_t) &= v_m(m_t,\, c_t(m_t)) + \frac{\partial c_t}{\partial m_t} v_c(m_t,\, c_t(m_t)) \\
&= v_m(m_t,\, c_t(m_t)) + \frac{\partial c_t}{\partial m_t}\left[u'(c_t) - \beta\,\mathbb{E}\left[\mathcal{G}_{t+1}^{-\rho}\mathcal{R}_{t+1}v'(m_{t+1}]\right]\right. \\
&= v_m(m_t,\, c_t(m_t)) \\
&= \beta\,\mathbb{E}\left[\mathcal{G}_{t+1}^{-\rho}\mathcal{R}_{t+1}v'(m_{t+1})\right]
\end{aligned}$$

Then using the same conditions under optimality for period $t+1$, replace $v'(m_{t+1})$ with $u'(c_{t+1})$.

where

$$\Sigma = \begin{bmatrix} \sigma_\eta^2 & \omega_{\eta,\nu} & \omega_{\eta,\zeta} \\ \omega_{\nu,\eta} & \sigma_\nu^2 & \omega_{\eta,\nu} \\ \omega_{\zeta,\eta} & \omega_{\nu,\eta} & \sigma_\zeta^2 \end{bmatrix}$$

$$\mu = \begin{bmatrix} -\sigma_\eta^2/2 \\ -\sigma_\nu^2/2 \\ -\sigma_\zeta^2/2 \end{bmatrix}$$

The marginal distributions of each of the shocks ensure that $\mathbb{E}_t[\eta] = \mathbb{E}_t[\nu] = \mathbb{E}_t[\zeta] = 1$. Meanwhile, $\omega_{x,y}$ captures the covariance of any two variables x and y among the three.

1.2 Solving the model

1.2.1 Discretizing the joint distribution

The first part of solving the model is to address the problem of efficiently computing expectations of the marginal utilities of consumption decisions in future periods. Under the current formulation, the shocks to income and returns are drawn independently in each time period, which means that it is enough to discretize these shocks using their single-period distribution. I use an equiprobable approximation of a truncated version (at 3 standard deviations) of these lognormal variables.

Here are the steps involved in discretizing the distribution:

1. Choose a suitable truncation of the distribution in each dimension by choosing an interval $[p_{min}, p_{max}] \subseteq [0, 1]$
2. Divide the interval given by $[\Phi^{-1}(p_{min}), \Phi^{-1}(p_{max})]$ into n intervals of $\frac{p_{max}-p_{min}}{n}$ probability each, $I = \left\{ \left[\Phi^{-1}\left(\frac{(i-1)p_{max}+(n-i+1)p_{min}}{n}\right), \Phi^{-1}\left(\frac{ip_{max}+(n-i)p_{min}}{n}\right) \right] \right\}_{i=1}^{n}$
3. Decompose the covariance matrix Σ using the Cholesky decomposition and obtain a matrix L such that $LL^T = \Sigma$
4. Then construct the random variables $Y = \mu + LZ$, where $Z \sim \mathcal{N}(0, I)$, to get $Y \sim \mathcal{N}(\mu, \Sigma)$
5. Construct the set I^3 and , and compute the conditional expectation of the vector of shocks $X = \exp(Y)$ in each set of I^3, yielding the set of equiprobable atoms $S = \left\{ (\eta, \nu, \zeta)_j \right\}_{j=1}^{n^3}$

Computing expectations of functions of these shocks can now be reduced to the following operation

$$\mathbb{E}\left[g(\eta,\, \nu,\, \zeta)\right] = n^{-3} \sum_{j=1}^{n^3} g(\eta_j,\, \nu_j,\, \zeta_j)$$

1.2.2 Computing optimal decisions

FINITE-HORIZON

I solve the model using a sequential application of the endogenous grid method, dividing a period into two subperiods, the first stage involving a consumption decision (c), and the second involving the portfolio optimization problem (κ).

Construct a grid of assets $\mathcal{A} = [\underline{a} = a_1 < a_2 < ... < a_k = \overline{a}]$. To solve the problem pertaining to any period t, observe from equation (9) that whenever $a_i \neq 0$, the optimal share of risky assets is given by the choice of $\hat{\kappa}_{t+1}(a_i) \in [0,\, 1]$ such that

$$n^{-3}\Gamma_{t+1}^{-\rho} \sum_{j=1}^{n^3} (\Re\nu_i - \mathsf{R})(\eta_j c_{t+1}(m_{ij}))^{-\rho} = 0 \tag{10}$$

where

$$m_{ij} = \frac{\mathsf{R} + \hat{\kappa}_{t+1}(a_i)(\Re\nu_j - \mathsf{R})}{\Gamma_{t+1}\eta_j} a_i + \zeta_j$$

The problem then becomes a root-finding operation pertaining to a function of $\hat{\kappa}$, which, given a policy function c_{t+1}, yields an optimal level of $\hat{\kappa}$ for each a_i. Denote this pair as $(a,\, \hat{\kappa})_i$, and the resulting effective return $\mathsf{R} + \hat{\kappa}_i(\Re\nu_j - \mathsf{R})$ for each value of the shocks as $\mathcal{R}_{ij}$.

For each *end-of-period* outcome $(a,\, \hat{\kappa})_i$, given c_{t+1}, we can use the consumption Euler equation to get

$$[\hat{c}_t(a_i,\, \hat{\kappa}_i)]^{-\rho} = \beta\Gamma_{t+1}^{-\rho} n^{-3} \sum_{j=1}^{n^3} \mathcal{R}_{ij}(\eta_j c_{t+1}(m_{ij}))^{-\rho}$$

where $\hat{c}$ denotes that this yields a "consumed" function. However, as $\hat{\kappa}_i$ is the value that solves equation (10)

$$[\hat{c}_t(a_i,\, \hat{\kappa}_i)]^{-\rho} = \beta\Gamma_{t+1}^{-\rho} \mathsf{R} n^{-3} \sum_{j=1}^{n^3} (\eta_j c_{t+1}(m_{ij}))^{-\rho}$$

The consumed function is then given by

$$\hat{c}_t(a_i,\, \hat{\kappa}_i) = \left[\beta\Gamma_{t+1}^{-\rho}\mathsf{R}n^{-3}\sum_{j=1}^{n^3}(\eta_j c_{t+1}(m_{ij}))^{-\rho}\right]^{-\frac{1}{\rho}}$$

Now we have a vector of c_i corresponding to each $(a,\, \hat{\kappa})_i$. Since $m_t = c_t + a_{t+1}$, we can construct the grid $\mathcal{M}$ with each $m_i \in \mathcal{M}$ given by $m_i = c_i + a_i$, where $c_t(a_i,\, \hat{\kappa}_i)$. We can now rewrite $c_t(m_i) = \hat{c}_t(a_i,\, \hat{\kappa}_i)$ and $\kappa_{t+1}(m_i) = \hat{\kappa}_{t+1}(a_i)$, and interpolate to get the policy functions $(c_t(m),\, \kappa_{t+1}(m)) = g_t(m)$ for period t. Finally, we can use $c_T(m) = m$ as a starting point and recursively obtain the estimated policy functions. To simplify the computation, I solve the model with a time-invariant expected permanent income growth rate, though this assumption is in no way necessary.

Infinite-Horizon

Since it is impossible to store an infinite sequence of values for the predictable growth factors Γ_t, I simplify the infinite-horizon problem by assuming that $\Gamma_t = \Gamma$ for all periods t, i.e. $\{\Gamma_t\}_{t=0}^{\infty}$ is a constant sequence. Then the Bellman equation can be simplified to

$$v(m) = \max_{c,\kappa'} \frac{c^{1-\rho}}{1-\rho} + \beta\Gamma^{1-\rho}\,\mathbb{E}\left[(\eta')^{1-\rho} v\left(\frac{\mathsf{R} + \kappa'(\Re' - \mathsf{R})}{\Gamma\eta'}(m-c) + \zeta'\right)\right]$$

We can use the same process as in the finite-horizon case, where I use the first-order conditions to find the policy functions. The only difference, now, is that instead of using c_{t+1} to find κ_{t+1} and c_t, I now use an initial guess about the time-invariant c, say c_0, to compute c_1 and κ_1. I use this newly computed c_1 to obtain c_2 and κ_2. This process continues, till the policy functions converge to an approximation of $c(m)$ and $\kappa(m)$.

2 Results

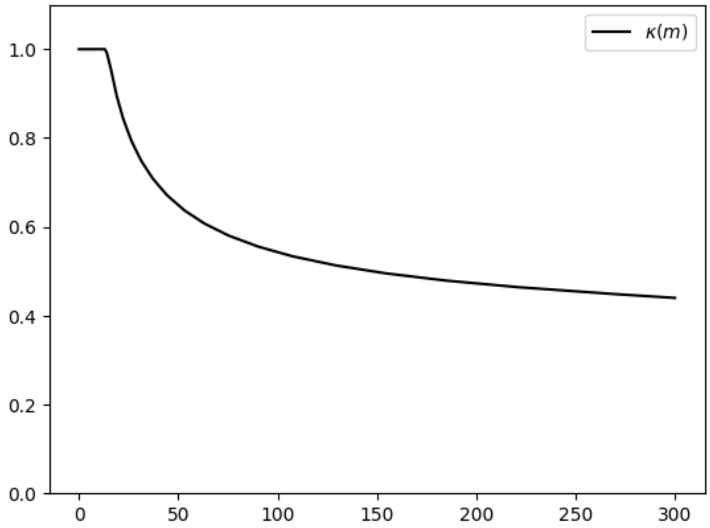
2.1 Predictions from the infinite-horizon model

I first analyze the findings from the infinite-horizon model on three fronts, the optimal portfolio share of the risky-asset for high-wealth individuals, the shape of the optimal portfolio share as a function of normalized monetary resources, and the effect of covariances between the income and asset shocks on the consumption function.

2.1.1 Optimal portfolio share at high-wealth levels

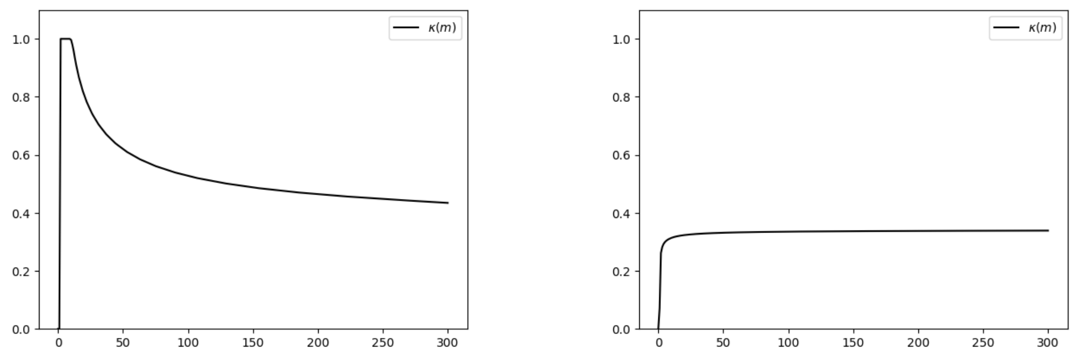
I start by looking at the baseline model with uncorrelated shocks to income and asset returns. Let the equity premium be set at 3 percent for this part of the analysis, and the standard deviation of the log shock to the equity return be set at 15 percent, i.e. $\sigma_\nu = 0.15$. Let all other parameters be as in Table 1. Figure 1 shows that the optimal portfolio allocation $\kappa(m)$ is 1 at low values of m and decreases to an asymptotic value as m tends to infinity.

Figure 1. Optimal portfolio share with uncorrelated shocks



It is well understood that this asymptotic value of κ is positively related to the equity premium and negatively related to σ_ν [ADD GRAPHS IN APPENDIX]. The natural question, then, is the effect of a positive covariance between income shocks and asset returns on optimal asymptotic portfolio share.

Figure 2. Optimal portfolio share under high correlation (> 0.8) between income shocks and asset returns



(a) Transitory shock ($\omega_{\nu,\zeta} = 0.025$) **(b)** Permanent shock ($\omega_{\eta,\nu} = 0.008$)

Figure 2 shows how optimal portfolio share responds to a correlation of greater than 0.8

between one of the income shocks and asset returns. Figure 2a depicts the case when the transitory shock is correlated with asset returns. Since the optimal portfolio allocation is unaffected for all but the lowest values of m (see section 2.1.2), the asymptotic level of κ remains the same as in the model with uncorrelated shocks. However, as Figure 2b depicts, even with a completely different optimal portfolio allocation rule, for high levels of wealth, the optimal portfolio share tends to similar values as in the case with no correlations.

The explanation for this behavior can be found by examining the optimal portfolio share condition, given by:

$$\mathbb{E}_t \left[(\Re_{t+1} - \mathsf{R})(\mathcal{G}_{t+1} c(m_{t+1}))^{-\rho} \right] = 0$$

where

$$m_{t+1} = \frac{\mathcal{R}_{t+1}}{\mathcal{G}_{t+1}}(m_t - c_t) + \zeta_{t+1}$$

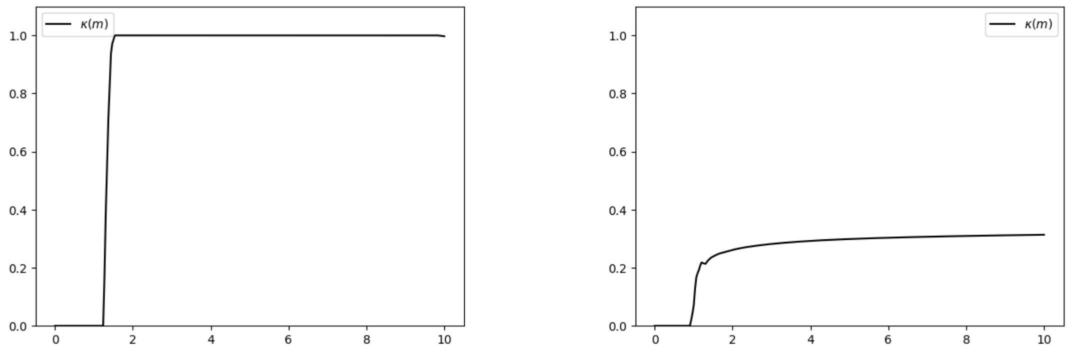
Unquestionably, the covariances between the shocks affect the realized values of m_{t+1}, and therefore the realizations of future consumption given by $c(m_{t+1})$. However, for high values of m_t, given the nature of the consumption function and the low MPC out of wealth (see section 2.1.3), the variability of the amounts consumed in the future is fairly low. Moreover, consider levels of wealth sufficiently high such that $c(m)$ is large. Naturally, $c_{t+1}^{-\rho}$ is bound to be an extremely small quantity, implying that optimal values of κ are likely to grow increasingly similar under the models with and without correlations in shocks as m grows larger.

2.1.2 Optimal portfolio share at low wealth

While optimal portfolio shares at large wealth levels are not affected much by the correlation between shocks, we can observe a stark change in the share of the risky asset among individuals whose wealths are less than twice of their permanent incomes.

Figure 3 shows that agents with normalized wealth less than 2 (somewhere between 1 and 2 to be precise), invest all their savings in the risk-free asset, irrespective of whether asset returns are correlated with transitory or permanent income shocks. At low levels of savings, notice that ζ_{t+1} comprises the major component of m_{t+1}, and high covariance between ν_{t+1} and ζ_{t+1} implies that low values of ζ_{t+1} go with low values of ν_{t+1}. Since the marginal utility of future consumption is high at low values of m_{t+1}, which coincides with low values of $\Re_{t+1}$, greater weight is placed on instances with low asset returns when taking the expectations in equation (9). This lowers the optimal portfolio share of the risky asset, in this case to 0. The other situation is when ν is correlated with η. Given the equation of m_{t+1}, this actually reduces the variability in $c(m_{t+1})$. However,

Figure 3. Individuals with low wealth invest all their savings in the risk-free asset



(a) Transitory shock **(b)** Permanent shock

the positive correlation between ν and η implies that when $\Re_{t+1} - \mathsf{R}$ is negative, $\mathcal{G}_{t+1}$ is low, implying that $\mathcal{G}_{t+1}^{-\rho}$ is higher. Thus, the instances of negative return are weighted higher in the excess return equation. Supposing that ν and η are perfectly correlated, if $\kappa 1$, then $m_{t+1} - \zeta_{t+1}$ becomes a constant, and the higher weight accorded to instances with negative return implies that the expectation becomes negative. On the other hand, if $\kappa = 0$, negative values of $\Re_{t+1} - \mathsf{R}$ are coupled with low values of $\mathcal{G}_{t+1}$ and therefore higher m_{t+1}, implying that the lower marginal utility of normalized consumption under negative excess returns makes $\kappa = 0$ closer to optimality.

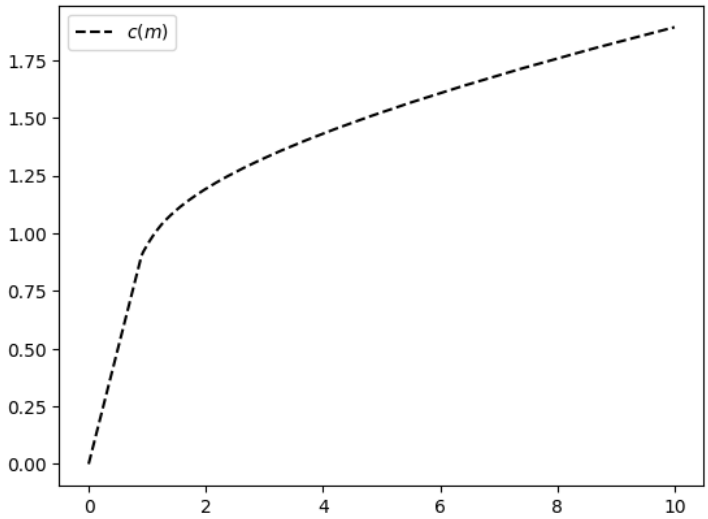
The second aspect is how the optimal portfolio share looks for slightly higher levels of wealth. Upto $m \approx 1.5$, agents consume almost all of their monetary resources and save next to nothing (see section 2.1.3). As such, the high MPC out of consumption causes variability in future monetary resources to translate into variability in future consumption at an almost one-to-one level. After a certain threshold, however, the MPC sharply falls, and the concavity of the consumption function ensures that it continues to fall. Moreover, due to the diminishing marginal utility of consumption and the very low magnitude of the marginal-marginal-utility of consumption, variability in m_{t+1} translates to very little variability in $c(m_{t+1})^{-\rho}$. The analysis of the finite horizon model in section 2.2 shows that the marginal utility channel accounts for most of this effect as opposed to the MPC.

In that light, see that the argument for why optimal κ is low when permanent income shocks and returns are correlated does not crucially depend on the value of a_{t+1} being extremely low, and the optimal κ is always lower than the asymptotic value. However, when looking at the case with correlation between ζ and ν, note that the only channel through which there is any effect is the marginal utility of consumption, or $c(m_{t+1})^{-\rho}$. By the discussion above, this has less of an effect at higher values of m_{t+1}, and the optimal portfolio allocation rule resembles the one in the model with uncorrelated shocks.

2.1.3 Optimal consumption policy

While the properties of the consumption function in the buffer stock model are well understood, I reiterate some important features to augment that arguments provided above. Figure 4 shows the optimal consumption function.

Figure 4. Optimal consumption function in the buffer-stock model



Under the artificial no-borrowing constraint, the $c(m) \leq m$, which implies that c is both defined only for $m \geq 0$ and has a kink at the point where the borrowing constraint begins to bind. For this region, the MPC is 1, implying that for low values of m, as remarked earlier, the total savings rate is extremely low. However, the MPC sharply falls beyond this point. Appendix [ADD ONE] shows that the optimal consumption policy in the models with correlated shocks remains mostly unchanged. The low MPC for high values of m thus leads to the ineffectiveness of transitory shocks in affecting optimal portfolio choice.

2.2 Next-to-last period in finite-horizon

Due to the convergence properties of the consumption function, we know that the optimal consumption rule in the finite-horizon model for periods sufficiently away from the last period closely approximate the infinite horizon consumption rule. As such, if the next period's consumption is similar to the infinite-horizon consumption, the optimal portfolio allocation rule should also be similar to the infinite-horizon rule. On the other end of this discussion is the period that is next to last.

The consumer in the last period knows that their optimization problem in the last period boils down to maximizing utility from current-period consumption, which implies that $c_T(m_T) = m_T$. The first useful feature of this is that it provides us with a consumption function for which we have an analytical expression, which allows us to rewrite the optimal portfolio allocation condition as:

$$\mathbb{E}_{T-1}\left[(\Re_T - \mathsf{R})(\mathcal{R}_T a_T + \mathcal{G}_T \zeta_T)^{-\rho}\right]$$

While the coincidence of negative values of $\Re_T - \mathsf{R}$ and small values of $\mathcal{G}_T$ still holds true, the MPC out of total monetary resources is a constant 1. As a result, the only channel through which the portfolio choice problem differs at high m_{T-1} as opposed to low is the marginal utility of consumption. Figure [ADD APPENDIX] shows that the optimal portfolio allocation is nearly identical to that in the infinite horizon problem, showing that the low MPC implied by the consumption function in the infinite-horizon problem plays a negligible role in affecting the portfolio choice of the wealthy.

2.3 Calibrating to U.S. Data

While the previous sections distil the primary insights from the model with an artificial parameterization of asset returns, both in terms of the equity premium and the variability of the returns from equity, I now look at how the model responds to being calibrated to parameters documented in the literature about U.S. data. Since the primary determinant of optimal portfolio allocation in the model that is of interest to us is the correlation between permanent income shocks and shocks to the return on the risky asset, I vary this parameter while holding the others constant at documented values. To begin, Mehra and Prescott (1985) estimate that the historical real rate of return on equity in the U.S. is 7.67 percent, while the return on a relatively risk-free securities over the same period was 1.31 percent.[2] Furthermore, the Sharpe ratio for these assets was calculated to be 0.37. Since ν is a mean-one lognormal, we know that:

$$\sigma_\nu^2 = \log\left(\left(\frac{\Re - \mathsf{R}}{0.37\Re}\right)^2 + 1\right)$$

I follow Carroll et al. (2015) and set $\sigma_\eta^2 = \frac{0.04}{11}$ and $\sigma_\zeta^2 = 0.04$. Data on per capita income in the U.S. reveals that real income grew at an average rate of 2.16 percent. I also set $\beta = 0.93$ and $\omega_{\zeta,\eta} = \omega_{\zeta,\nu} = 0$. I then solve the model using a baseline of $\rho = 4$ for different values of $\omega_{\eta,\nu}$. The full choice of parameters is then given in Table 1.

[2]The original data was later updated till 2005, which forms the source of these estimates. See Mehra (2006) for details.

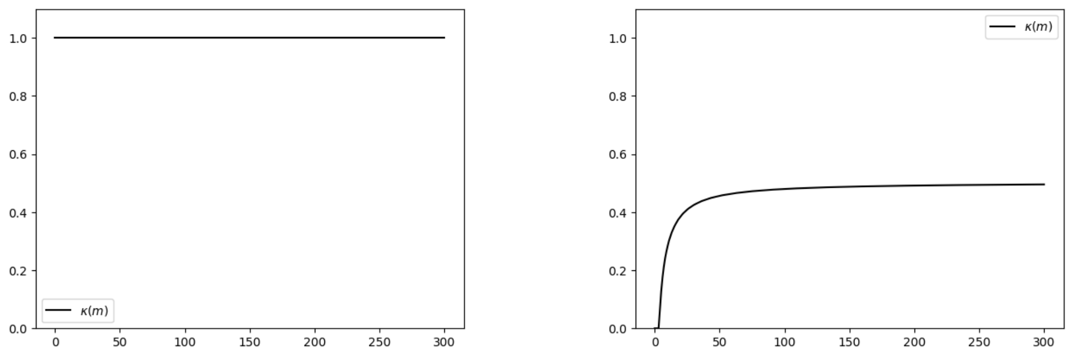
Table 1. Parameters used to solve the model

Parameter	Value	Source
ρ	4	
β	0.93	
Γ	1.0216	U.S. Data
$\mathfrak{R}$	1.0767	Mehra (2006)
R	1.0131	Mehra (2006)
σ_ν^2	0.011	Mehra (2006)
σ_η^2	$0.01 \times \frac{4}{11}$	Carroll et al. (2015)
σ_ζ^2	0.04	Carroll et al. (2015)

2.3.1 Preliminary findings

The first thing we can see under this new parameterization is that even with highly collinear shocks ($\text{corr}(\log \nu, \log \eta) \approx 1$), the optimal portfolio share is 1. This is because despite the high covariance, the excess return of more than 6 percent and the relatively low volatility, with a standard deviation of under 10.5 percent for the logged shock to returns, makes it difficult to justify holding the risk-free asset. In fact, Figure 5 shows that the equity share of portfolio falls to realistic levels only when ρ is as large as 12. This number is close to the benchmark by Schreindorfer (2020), whose model incorporates disappointment averse preferences and has agents exhibit levels of relative risk aversion of close to 10.

Figure 5. U.S. figures requires extremely high CRRA to explain the equity premium



(a) $\rho = 4$ **(b)** $\rho = 12$

One of the key issues with the

A Appendix

A.1 Approximating the excess return equation

We want to approximate the excess return equation

$$\mathbb{E}_t\left[(\Re_{t+1} - \mathsf{R})(\eta_{t+1}c_{t+1})^{-\rho}\right] = 0$$

Define the function $g : \mathbb{R}^3_{++} \to \mathbb{R}$ as

$$g(\eta,\, \nu,\, \zeta) = (\Re\nu - \mathsf{R})\left(\eta c\left(\frac{\mathsf{R} + \kappa(\Re\nu - \mathsf{R})}{\Gamma\eta}(m - c(m)) + \zeta\right)\right)^{-\rho}$$

where $\Re$, R, Γ, m, $c : \mathbb{R}_+ \to \mathbb{R}_+$ are treated as known. Let m' be defined as

$$m' = \frac{\mathsf{R} + \kappa(\Re\nu - \mathsf{R})}{\Gamma\eta}(m - c(m)) + \zeta$$

Observe that the partial derivatives of this function are

$$\begin{aligned}
\frac{\partial g(\eta,\, \nu,\, \zeta)}{\partial \eta} &= -\rho(\Re\nu - \mathsf{R})\left(\eta c\left(m'\right)\right)^{-\rho-1}\left(c(m') - c'(m')(m' - \zeta)\right)\\
\frac{\partial g(\eta,\, \nu,\, \zeta)}{\partial \nu} &= \Re(\eta c(m')) - \rho(\Re\nu - \mathsf{R})(\eta c(m))^{-\rho-1}\left(c'(m')\frac{\kappa\Re}{\Gamma}(m - c(m))\right)\\
\frac{\partial g(\eta,\, \nu,\, \zeta)}{\partial \zeta} &= -\rho(\Re\nu - \mathsf{R})(\eta c(m'))^{-\rho-1}(\eta c'(m'))
\end{aligned}$$

References

Carroll, Christopher D., Jiri Slacalek, and Kiichi Tokuoka. 2015. "Buffer-stock saving in a Krusell–Smith world." *Economics Letters* 132 (July): 97–100.

Mehra, Rajnish. 2006. "The Equity Premium Puzzle: A Review." *Foundations and Trends® in Finance* 2 (1): 1–81.

Mehra, Rajnish, and Edward C. Prescott. 1985. "The equity premium: A puzzle." *Journal of Monetary Economics* 15 (2): 145–161.

Schreindorfer, David. 2020. "Macroeconomic Tail Risks and Asset Prices." *The Review of Financial Studies* 33 (8): 3541–3582.