

ME598/494 Homework 1

1. Solve the following problem using [Python SciPy.optimize](#). Please attach your code and results. Specify your initial guesses of the solution. If you change your initial guess, do you find different solutions? (30 points)

$$\begin{aligned} \text{minimize:} \quad & (x_1 - x_2)^2 + (x_2 + x_3 - 2)^2 + (x_4 - 1)^2 + (x_5 - 1)^2 \\ \text{subject to:} \quad & x_1 + 3x_2 = 0 \\ & x_3 + x_4 - 2x_5 = 0 \\ & x_2 - x_5 = 0 \\ & -10 \leq x_i \leq 10, \quad i = 1, \dots, 5 \end{aligned}$$

Note: Please learn how to use **break points** to debug. You can use Python on [Google Colab](#).

2. Let x and $b \in \mathbb{R}^n$ be vectors and $A \in \mathbb{R}^{n \times n}$ be a square matrix. Define $f : \mathbb{R}^n \rightarrow \mathbb{R}$ as $f(x) = b^T x + x^T A x$. (50 points)
 - (a) What is the gradient and Hessian of $f(x)$ with respect to x ?
 - (b) Derive the first and second order Taylor's approximations of $f(x)$ at $x = 0$. Are these approximations exact?
 - (c) What are the necessary and sufficient conditions for A to be positive definite?
 - (d) What are the necessary and sufficient conditions for A to have full rank?
 - (e) If there exists $y \in \mathbb{R}^n$ and $y \neq 0$ such that $A^T y = 0$, then what are the conditions on b for $Ax = b$ to have a solution for x ?
3. Due to the recent inflation, let's reconsider the [Stigler diet](#) problem proposed by Nobel laureate George Stigler after the second World War: Consider that the grocery store offers N types of food of your interest, and each food contains the same M types of nutrition. Denote a_{ij} as the quantity of nutrition type j of food type i for $i = 1, 2, \dots, N$ and $j = 1, 2, \dots, M$, c_i the unit price of food type i , and b_j the necessary

quantity of nutrition type j for a month. Formulate an optimization problem to determine the minimum grocery cost to satisfy the nutrition needs. **(20 points)**