Buck DC-DC Converter: Mathematical Modeling and Transient State Analyzes

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***Abstract*— In this paper, a new method is proposed for mathematical modeling of buck dc-dc converter in continuous conduction mode (CCM). In this method, firstly, the differential equations of inductor current and capacitor voltage are obtained according to the equivalent circuit of the converter and then using Laplace and Z transforms the differential equations are solved and the relations of inductor current and output voltage are obtained. In this method Laplace transform is used to determine the general relations of inductor current and output voltage and Z-transform is used to determine the initial conditions of current and voltage. After that, the transient state response of inductor current and output voltage of buck dc-dc converter are analyzed using the obtained mathematical model. Then the effects of converter's components and load resistance on the transient response are analyzed. Finally, the results obtained from the theoretical analysis are compared with the simulation results through PSCAD/EMTDC to verify the analysis.**

***Keywords*— Buck dc-dc converter, modeling, Laplace transform, Z-transform.**

I. INTRODUCTION

The dc-dc converters have a wide range of applications in industry, electric machines control, aviation, as interface in distributed generation, portable devices and also power factor correction. They dc-dc converters are regarded as an essential part in all of these applications. Nowadays, considering ever developing trend in power electronics, the role of dc-dc converters is highlighted in different applications; therefore, the study of these converters is very essential and has high significance [1-4].

The study of a system demands an exact modeling of the system. By modeling the dc-dc converters, operation of the different components of the converter circuit can be evaluated. Also, the performance of converter in different operational modes in steady states and transients can be investigated. The analysis of these converters using common modeling methods requires applying numerical solution methods or approximation methods. This reduces the accuracy of the analysis results. In addition, analyzing the converters using these modeling methods requires mathematical operations such as inversing matrices or solving complicated algebraic equations which makes the system slower [5, 6].

The small-signal analysis [7-9], the state space method [10] and the state space mean technique [11, 12] are some of the usual mathematical modeling techniques presented in several papers. The presented methods in these references can be useful if the converter parameters have small changes; otherwise, if the changes are considerable large, these methods cannot offer the required accurate results to analyze the system. Another drawback of these methods is the existence of matrix equations in the converter model which makes the mathematical calculations complicated.

In this paper, a new method is proposed for mathematical modeling of dc-dc converters in CCM. The proposed method is based on Z and Laplace transforms. In this method, Laplace transform is used to obtain the equations of inductor current and output voltage and Z-transform is used as a tool to determine the initial values of inductor current and output voltage at each switching period. One of the benefits of using Z-transform in converter modeling is applying this transform in analyzing transient state of the converter.

The mathematical model proposed in this paper can also be used to analyze dc-dc converters with large variations of parameters. It can also be used as a powerful tool in studying control methods for appropriate control of the convector's switching, to reach efficient operational conditions.

In this paper, by applying the proposed mathematical model, the effects of converter's components and load resistance on the steady state and transient response is analyzed. Eventually the theoretical discussions are verified by simulation results in PSCAD/EMTDC environments.

II. THE PROPOSED MATHEMATICAL MODEL

Fig. 1 shows the circuit configuration of a buck dc-dc converter. In this circuit, the switch S and the diode D are considered to be ideal and R_L is considered to be the equivalent resistance of the inductor.

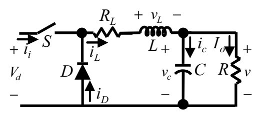


Fig. 1. Buck dc-dc converter

A. Analysis of the Converter in CCM

According to Fig. 1 and applying Kirchhoff's current and voltage laws, it can be written:

$$i_L = C\frac{dv}{dt} + \frac{v}{R} \tag{1}$$

$$L\frac{di_L}{dt} + R_L i_L + v = f(t)V_d \tag{2}$$

Function $f(t)$ in (2) is the converter switching function which is defined, to determine the converter relations during on-state (t_1) and off-state (t_2) time intervals of the switch S, as follows:

$$f(t) = \sum_{n=0}^{\infty}\left[u(t-nT) - u(t-t_1-nT)\right] \tag{3}$$

In (3), n is the number of switching intervals and T is the switching period. In view of the discontinuous behavior of the function $f(t)$, the following variable exchange is performed:

$$t = (n+m)T \qquad for \quad n = 0,1,2,\cdots \quad 0 \le m < 1 \tag{4}$$

By applying the variable exchange (4) in (3), the function $f(t)$ is rewritten as follows:

$$f(m) = \begin{cases} 1 & 0 \le m < D \\ 0 & D \le m < 1 \end{cases} \tag{5}$$

In (5), D is the converter duty cycle which is defined as follows:

$$D = \frac{t_1}{T} \tag{6}$$

Considering (5), it is observed that the interval $[0,1]$ is divided into two intervals $[0,D]$ and $[D,1]$. For the interval $[0,D]$ the switch S will be in on-state and for the interval $[D,1]$, it will be in off-state.

Applying (4) in (1) and (2), and applying the value of $f(m)$ in the obtained equations, the equations which are related to on and off states of the switch S will be equal with:

$$\begin{bmatrix} \frac{di_{L,n}}{dm} \\ \frac{dv_n}{dm} \end{bmatrix} = \begin{bmatrix} -\frac{R_L T}{L} & -\frac{T}{L} \\ \frac{T}{C} & -\frac{T}{RC} \end{bmatrix} \begin{bmatrix} i_{L,n} \\ v_n \end{bmatrix} + \begin{bmatrix} \frac{T}{L} \\ 0 \end{bmatrix} V_d \ for\ 0 \le m < D \tag{7}$$
$$i_{L,n}(0) = i_{L0,n}\ ,\ v_n(0) = v_{0,n}$$

$$\begin{bmatrix} \frac{di_{L,n}}{dm} \\ \frac{dv_n}{dm} \end{bmatrix} = \begin{bmatrix} -\frac{R_L T}{L} & -\frac{T}{L} \\ \frac{T}{C} & -\frac{T}{RC} \end{bmatrix} \begin{bmatrix} i_{L,n} \\ v_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_d \ for\ D \le m < 1 \tag{8}$$
$$i_{L,n}(D) = i_{L1,n}\ ,\ v_n(D) = v_{1,n}$$

In the above equation, $i_{L0,n}$, $v_{0,n}$, $i_{L1,n}$ and $v_{1,n}$ are the initial values of inductor current and output voltage in the interval $[0,D]$ and $[D,1]$, respectively.

B. Solving the Equations of Output Voltage and Inductor Current Using Laplace Transform

In order to solve (7) and (8), Laplace transform can be used. By applying Laplace transform to (7) and (8) the Laplace transforms of output voltage and inductor current will be as:

$$\begin{bmatrix} I_{L,n}(s) \\ V_n(s) \end{bmatrix} = \frac{1}{s^2 + 2\alpha Ts + (\omega^2 + \alpha^2)T^2} \times \begin{bmatrix} s+(\alpha+\gamma) & -\frac{T}{L} \\ \frac{T}{C} & s+(\alpha-\gamma) \end{bmatrix} \begin{bmatrix} i_{L0,n} + \frac{TV_d}{Ls} \\ v_{0,n} \end{bmatrix} \quad for \quad 0 \le m < D \tag{9}$$

$$\begin{bmatrix} I_{L,n}(s) \\ V_n(s) \end{bmatrix} = \frac{1}{s^2 + 2\alpha Ts + (\omega^2 + \alpha^2)T^2} \times \begin{bmatrix} s+(\alpha+\gamma) & -\frac{T}{L} \\ \frac{T}{C} & s+(\alpha-\gamma) \end{bmatrix} \begin{bmatrix} i_{L1,n} \\ v_{1,n} \end{bmatrix} \quad D \le m < 1 \tag{10}$$

In (9) and (10), the parameters α, ω and γ are defined as follows:

$$\alpha = \frac{1}{2}\left(\frac{R_L}{L} + \frac{1}{RC}\right) \tag{11}$$

$$\omega = \sqrt{\frac{1}{LC}\left(\frac{R_L}{R} + 1\right) - \alpha^2} \tag{12}$$

$$\gamma = \frac{1}{2}\left(\frac{1}{RC} - \frac{R_L}{L}\right) \tag{13}$$

By applying the inverse Laplace transform for (9) and (10), the inductor current and output voltage are obtained by (14) and (15) (given in Appendix A)

$$A = \frac{\omega^2 - \alpha\gamma}{\omega(\alpha+\gamma)} \tag{16}$$

$$B = \frac{RV_d}{R_L + R} \tag{17}$$

$$q = \frac{V_d}{R_L + R} \tag{18}$$

$i_{L1,n}$ and $v_{1,n}$ are the initial values of the inductor current and output voltage in the interval $[D,1]$, respectively. To simplify the calculations these values can be written in terms of $i_{L0,n}$ and $v_{0,n}$. Considering the fact that the inductor current and the capacitor voltage are continuous, for every switching interval the following equation is always true.

$$\lim_{m \to (D)^-} i_{L,n}(m) = \lim_{m \to (D)^+} i_{L,n}(m) \tag{19}$$

By substituting the value of $i_{L,n}(m)$ from (14) in (19), the value of $i_{L1,n}$ will be equal to:

$$\begin{aligned} i_{L1,n} &= \frac{V_d}{R_L + R}\left[1 - e^{-\alpha t_1}\cos\omega t_1 + \frac{\omega^2 - \alpha\gamma}{\omega + \alpha\gamma} e^{-\alpha t_1}\sin\omega t_1\right] \\ &+ i_{L0,n} e^{-\alpha t_1}\left[\cos\omega t_1 + \frac{\gamma}{\omega}\sin\omega t_1\right] - \frac{v_{0,n} e^{-\alpha t_1}}{\omega L}\sin\omega t_1 \end{aligned} \tag{20}$$

For the capacitor voltage, the following relation can be written:

$$\lim_{m \to (D)^-} v_n(m) = \lim_{m \to (D)^+} v_n(m) \tag{21}$$

By substituting the value of $v_n(m)$ from (15) in (21), the value of $v_{1,n}$ will be equal to:

$$\begin{aligned} v_{1,n} &= \frac{RV_d}{R_L + R}\left[1 - e^{-\alpha t_1}\cos\omega t_1 - \frac{\alpha}{\omega} e^{-\alpha t_1}\sin\omega t_1\right] + \\ &+ v_{0,n} e^{-\alpha t_1}\left[\cos\omega t_1 - \frac{\gamma}{\omega}\sin\omega t_1\right] + \frac{i_{L0,n} e^{-\alpha t_1}}{\omega C}\sin\omega t_1 \end{aligned} \tag{22}$$

By substituting (20) and (22) in (14) and (15) the equations of the inductor current and the capacitor voltage are obtained in terms of $i_{L0,n}$ and $v_{0,n}$.

C. Determination of the Initial Values of the Inductor Current and the Capacitor Voltage Using Z-Transform

By substituting (20) and (22) in (14) and (15) is considered the only undetermined parameters are $i_{L0,n}$ and $v_{0,n}$, considering the fact that $i_{L0,n}$ and $v_{0,n}$ are a function of n and n is a discrete variable, therefore, in order to determine the values of these parameters Z-transform can be used.
Considering the continuity characteristic of the inductor current and the capacitor voltage, the following is true

$$\lim_{m \to 1^-} i_{L,n}(m) = \lim_{m \to 0^+} i_{L,n+1}(m) \tag{23}$$

$$\lim_{m \to 1^-} v_n(m) = \lim_{m \to 0^+} v_{n+1}(m) \tag{24}$$

By calculating the above mentioned limits for (14) and (15) and considering that $t_2 = T - t_1$, the following equations are true for the initial values of the inductor current and the capacitor voltage at the interval $n+1$.

$$i_{L0,n+1} = i_{L1,n} e^{-\alpha t_2}\left[\cos\omega t_2 + \frac{\gamma}{\omega}\sin\omega t_2\right] - \frac{v_{1,n} e^{-\alpha t_2}}{\omega L}\sin\omega t_2 \tag{25}$$

$$v_{0,n+1} = v_{1,n} e^{-\alpha t_2}\left[\cos\omega t_1 - \frac{\gamma}{\omega}\sin\omega t_2\right] + \frac{i_{L1,n}}{\omega C} e^{-\alpha t_2}\sin\omega t_2 \tag{26}$$

Equations (25) and (26) are two difference equations and in order to solve these equations, the Z-transform can be used. For a discrete function the following equations are always true.

$$\mathrm{Z}\{i_{L0,n}\} = I_{L0}(z) \tag{27}$$

$$\mathrm{Z}\{v_{0,n}\} = V_0(z) \tag{28}$$

$$Z\{i_{L0,n+1}\} = zI_{L0}(z) - zi_{L0,0} \tag{29}$$

$$Z\{v_{0,n+1}\} = zV_0(z) - zv_{0,0} \tag{30}$$

In (29) and (30), $i_{L0,0}$ and $v_{0,0}$ are the initial values of inductor current and the capacitor voltage at $t = 0$. Applying the Z transform in (25) and (26), solving the obtained equations in the Z domain, and considering $i_{L0,0}$ and $v_{0,0}$ equal with zero $I_{L0}(z)$ and $V_{o0}(z)$ are obtained by (31) and (32) (given in Appendix A)
The values of b_1 and b_2 in (31) and (32) are obtained as follows:

$$\begin{aligned} b_1 &= \Big[(1 - e^{-\alpha t_1}\cos\omega t_1 + Ae^{-\alpha t_1}\sin\omega t_1)\times \\ &\left(\cos\omega t_2 + \frac{\gamma}{\omega}\sin\omega t_2\right)\times qe^{-\alpha t_2}\Big] - \\ &\left[\sin\omega t_2 \times\left(1 - e^{-\alpha t_1}\cos\omega t_1 + \frac{\alpha e^{-\alpha t_1}}{\omega}\sin\omega t_1\right)\frac{Be^{-\alpha t_2}}{\omega L}\right] \end{aligned} \tag{33}$$

$$\begin{aligned} b_2 &= \left[\sin\omega t_1 \times (1 - e^{-\alpha t_1}\cos\omega t_1 + Ae^{-\alpha t_1}\sin\omega t_1)\times \frac{qe^{-\alpha t_2}}{\omega C}\right] \\ &+ \Bigg[\left(\cos\omega t_2 + \frac{\gamma}{\omega}\sin\omega t_2\right)\times \\ &\left(1 - e^{-\alpha t_1}\cos\omega t_1 - \frac{\alpha e^{-\alpha t_1}}{\omega}\sin\omega t_1\right)\times Be^{-\alpha t_2}\Bigg] \end{aligned} \tag{34}$$

The initial values of inductor current and output voltage in steady state are obtained as follows:

$$i_{L0,ss}=\frac{\left[1-e^{-\alpha T}\left(\cos\omega T-\frac{\gamma}{\omega}\sin\omega T\right)\right]b_1-\left(\frac{e^{-\alpha T}}{\omega L}\sin\omega T\right)b_2}{1-2e^{-\alpha T}\cos\omega T+e^{-2\alpha T}} \tag{35}$$

$$v_{0,ss}=\frac{\left(\frac{e^{-\alpha T}}{\omega C}\sin\omega T\right)b_1+\left[1-e^{-\alpha T}\left(\cos\omega T+\frac{\gamma}{\omega}\sin\omega T\right)\right]b_2}{1-2e^{-\alpha T}\cos\omega T+e^{-2\alpha T}} \tag{36}$$

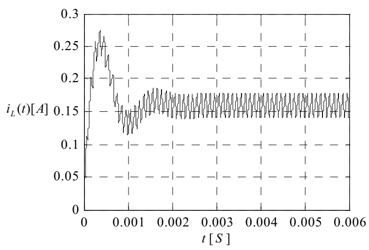
By applying the inversion Z-transform, $i_{L0,n}$ and $v_{0,n}$ are obtained by (37) and (38) (given in Appendix A).

III. THEORETICAL ANALYSES

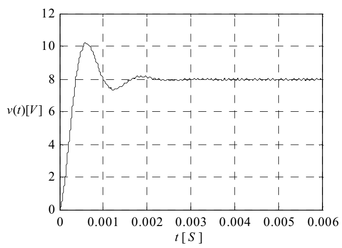
Every system has different responses in its output dependent on the different inputs. Analysis of the performance of the system is based on these responses. Time domain response of each system has two main parts including transient and steady state responses.

A. Time Domain Response of Buck dc-dc Converter

Fig. 2 shows the time domain step responses of the functions $i_L(t)$ and $v(t)$ .To draw these curves, (14) and (15) have been used. These curves have been plotted for $V_d=12V$, $L=7mH$, $D=0.67$, $C=5\mu F$, $R=50\Omega$, $R_L=0.5\Omega$ and $f=10KHz$. In this paper the aim is to propose a mathematical modeling method and to investigate the accuracy of this method and also analysis of the effects of the converter components on electrical parameters of it. Therefore, the range of the chosen values for the converter parameters is in experimental level. In addition, other reason of selecting such numerical values for the electrical quantities of converter is considering a sample of application of these kinds of converters (mines control system, petrochemical, refinery) which referred in [13].



(a)



(b)

Fig. 2. Time domain step response, (a) inductor current, (b) capacitor voltage

B. Analysis of the Transient Response

One of the essential parameters in the analysis of transient response of each system is the time constant of that system. Time constant of a system is obtained by solving the characteristic equation of the system and determining its roots. Also, the type of roots of the characteristic equation in each system determines the damping feature of the system transient response. In the buck dc-dc converter shown in Fig. 1, in order to obtain the time constant of functions $i_L(t)$ and $v(t)$, first the roots of the characteristic equation is calculated in the discrete time domain. Then, by applying the following equation, the roots of the characteristic equations are obtained in the continuous time domain.

$$\lambda_{1,2}=\frac{1}{T}\ln(\rho_{1,2}) \tag{39}$$

In the (39), $\lambda_{1,2}$ are the roots of the characteristic equation in the continuous time domain and $\rho_{1,2}$ are the roots of the characteristic equation in the discrete time domain.

According to (39), the time constant of the equations $i_L(t)$ and $v(t)$ are obtained using (31) and (32). The denominators of (31) and (32) are the characteristic equation of the system in discrete time domain. The roots of these equations are equal to:

$$\rho_{1,2}=e^{-(\alpha\pm j\omega)T} \tag{40}$$

By substituting (40) in (39), the time constant of functions $i_L(t)$ and $v(t)$ will be equal to:

$$\lambda_{1,2}=-\alpha\pm j\omega \tag{41}$$

According to (11), it is observed that the value of α in (54) is a real and positive value. Considering (12) and also the values of inductance, capacitance, the load resistance and the equivalent resistance of inductor, ω can be real or imaginary. In (41), the value of ω determines the type of damping for the transient response of the inductor current and the capacitor voltage. If ω is imaginary, the characteristic equation will contain two real and negative roots and in this case the transient response of converter will be over-damped. In case ω is real, the characteristic equation will have two complex paired roots and in this case the transient response of the converter will be under-damped. According to (12), it is observed that ω is not a function of duty cycle, therefore, the value of D will not affect the type of damping of the inductor current and output voltage transient responses.

Fig. 3 shows the variations in the real parts of the roots of characteristic equation versus the value of inductance and capacitance for different values of the load resistance. As it is observed, for a specific value of load resistance, the absolute of real part of roots of the characteristic equation, will have a small value (i.e. The poles are near of the imaginary axes), for higher values of L and C. Therefore, $i_L(t)$ and $v(t)$ will have slow responses. Also, according to Fig. 3, it is clear that for a specific value of inductance and capacitance by increasing of the load resistance, the absolute of real part of roots increases and this leads to decrease in the response time of the system.

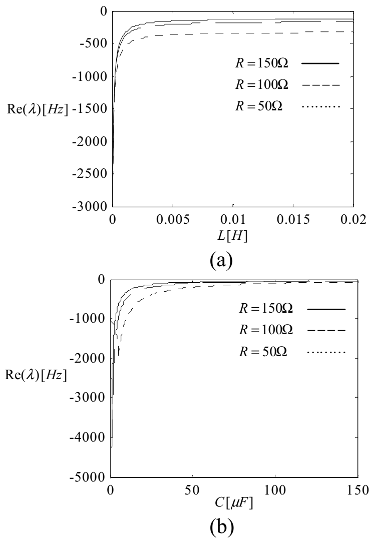


Fig. 3. Variations of the real part of roots of characteristic equations $i_L(t)$ and $v(t)$, versus; (a) inductance; (b) capacitance

IV. SIMULATION RESULTS

To verify the presented theoretical investigations in the previous sections, the buck dc-dc converter shown in Fig. 1, is simulated in PSCAD/EMTDC environment in different operational conditions. In Figs. 4(a) and 4(b), the step responses of the inductor current and output voltage are shown. These results are obtained based on the data used to plot the curves in Fig. 2. As illustrated in the figure, the results obtained from simulation, verifies the curves shown in Fig. 2.

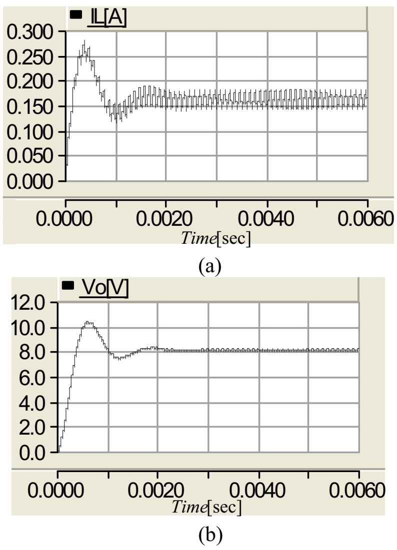


Fig. 4. The step response; (a) inductor current; (b) output voltage

The Figs. 5(a) and 5(b) show the step response of the output voltage for $D = 0.4$, $R_L = 0.5\Omega$, $V_d = 12V$, $f = 10kHz$, the inductance values $5mH$ and $15mH$, the values of the capacitance $35\mu F$ and $50\mu F$, and the load resistance 40Ω and 90Ω, respectively. According to the figure, it is observed that by increasing the values of the inductance and the capacitance, the transient response of the system becomes slower and damping time of system is increased This subject is the confirmer of the presented subjects in the section IV.

Fig. 6 shows the step response of the output voltage for $R_L = 0.5\Omega$, $V_d = 12V$, $f = 10kHz$, $L = 7mH$, $C = 35\mu F$ if $D = 0.4$ and $D = 0.6$. Considering the figure, it can be observed that by increasing the value of the duty cycle in CCM, the damping time of the transient response does not vary. Therefore, it can be concluded that in CCM the type of damping of the transient response is independent of duty cycle. the issue discussed in this section verifies the issues presented in section third

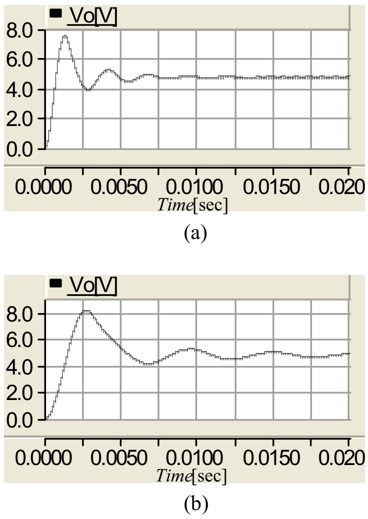


Fig. 5. The step response of the output voltage versus; (a) $R = 40\Omega$, $L = 5mH$ and $C = 35\mu F$; (b) $R = 90\Omega$, $L = 15mH$ and $C = 50\mu F$

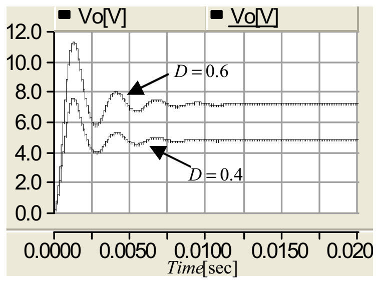


Fig. 6. The step response of the output voltage versus; $D = 0.4$ and $D = 0.6$

V. CONCLUSIONS

In this paper, a mathematical method based on the Laplace and Z transforms has been proposed to model the buck dc-dc converter operating in CCM. In the proposed method, the Laplace transform has been used to obtain the inductor current and output voltage equations and Z-transform is used as a tool to find the initial values of the inductor current and output voltage.

By means of the time domain functions of the inductor current and output voltage which are obtained using the proposed modeling method in this paper, the step response of each of these functions is plotted.

Using the proposed modeling method in this paper, the effects of the converter components on the transient response of inductor current and output voltage were analyzed and it was shown that the Z-transform can be used for the analysis of transients states. The effects of each of the buck dc-dc converter parameters such as inductance, capacitance, load resistance and duty cycle on the transient response of the converter were analyzed and shown that the buck dc-dc converter operating in CCM has slow transient response when the values of inductance and capacitance are higher. Also it was shown that in the buck dc-dc converter, the duty cycle of the converter does not have any effects on the damping type of transient response. Since the transient response of the system has an essential role in the analysis of stability of the system, the proposed method can be used as a powerful tool to analyze the stability and also to design suitable control methods for the dc-dc converters.

APPENDIX A

$$i_{L,n}(m)=\begin{cases} q\times\left[1-e^{-\alpha mT}\cos(\omega mT)+Ae^{-\alpha mT}\sin(\omega mT)\right]+i_{L0,n}e^{-\alpha mT}\left[\cos(\omega mT)+\dfrac{\gamma}{\omega}\sin(\omega mT)\right]-\dfrac{v_{0,n}e^{-\alpha mT}\sin\omega mT}{\omega L} & for\ \ 0\le m<D \\ i_{L1,n}e^{-\alpha(mT-t_1)}\left[\cos(\omega mT-\omega t_1)+\dfrac{\gamma}{\omega}\sin(\omega mT-\omega t_1)\right]-\dfrac{v_{1,n}}{\omega L}e^{-\alpha(mT-t_1)}\sin(\omega mT-\omega t_1) & for\ \ D\le m<1 \end{cases} \tag{14}$$

$$v_n(m)=\begin{cases} B\times\left[1-e^{-\alpha mT}\cos(\omega mT)-\dfrac{\alpha}{\omega}e^{-\alpha mT}\sin(\omega mT)\right]+v_{0,n}e^{-\alpha mT}\left[\cos(\omega mT)-\dfrac{\gamma}{\omega}\sin(\omega mT)\right]+\dfrac{i_{L0,n}e^{-\alpha mT}\sin(\omega mT)}{\omega C} & for\ 0\le m<D \\ v_{1,n}e^{-\alpha(mT-t_1)}\left[\cos(\omega mT-\omega t_1)-\dfrac{\gamma}{\omega}\sin(\omega mT-\omega t_1)\right]+\dfrac{i_{L1,n}}{\omega C}e^{-\alpha(mT-t_1)}\sin(\omega mT-\omega t_1) & for\ \ D\le m<1 \end{cases} \tag{15}$$

$$I_{L0}(z)=\frac{z}{z^2-2ze^{-\alpha T}\cos\omega T+e^{-2\alpha T}}\times\left\{\left\{\left[z-e^{-\alpha T}\left(\cos\omega T-\frac{\gamma}{\omega}\sin\omega T\right)\right]b_1-\left[\frac{e^{-\alpha T}}{\omega L}\sin\omega T\right]b_2\right\}\frac{1}{z-1}\right\} \tag{31}$$

$$V_0(z)=\frac{z}{z^2-2ze^{-\alpha T}\cos\omega T+e^{-2\alpha T}}\times\left\{\left\{\left[z-e^{-\alpha T}\left(\cos\omega T+\frac{\gamma}{\omega}\sin\omega T\right)\right]b_2+\left[\frac{e^{-\alpha T}}{\omega C}\sin\omega T\right]b_1\right\}\frac{1}{z-1}\right\} \tag{32}$$

$$i_{L0,n}=i_{L0,ss}+i_{L0,ss}\cos\omega nTe^{-\alpha nT}+\left(\frac{i_{L0,ss}e^{-\alpha T}\cos\omega T-i_{L0,ss}+b_1}{e^{-\alpha T}}\right)\frac{e^{-\alpha nT}\sin\omega nT}{\sin\omega T} \tag{37}$$

$$v_{0,n}=v_{0,ss}+v_{0,ss}\cos\omega nTe^{-\alpha nT}+\left(\frac{v_{0,ss}e^{-\alpha T}\cos\omega T-v_{0,ss}+b_2}{e^{-\alpha T}}\right)\frac{e^{-\alpha nT}\sin\omega nT}{\sin\omega T} \tag{38}$$

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