

# Design of Robust Control System for delivery drones

1<sup>st</sup> Ainun Kusnul Khotimah

Department of Mathematics  
Institut Teknologi Sepuluh Nopember  
Surabaya, Indonesia  
ainun1123@gmail.com

2<sup>nd</sup> Mardlijah

Department of Mathematics  
Institut Teknologi Sepuluh Nopember  
Surabaya, Indonesia  
mardlijah@matematika.its.ac.id

3<sup>rd</sup> Ari Santoso

Department of Electrical Engineering  
Institut Teknologi Sepuluh Nopember  
Surabaya, Indonesia  
santoso@ee.its.ac.id

**Abstract**—At this time, people prefer to shop online rather than shop directly to the store. Online shopping can be done through websites or e-commerce sites that provide buying and selling sites. Goods that are traded will be sent using a freight forwarder. This resulted in a buildup of consignment goods, so that goods that were important/emergency in nature were delayed for delivery. One of the efforts that can be made to overcome this problem is to make shipments by air. Modeling Quadcopter is done to overcome these problems. The modeling of Quadcopter is based on rotational, translational and angular kinematics. In its implementation, drones will experience interference, both interference from the system or interference from outside, for example, wind disturbance. Therefore, it is necessary to design a control system to maintain flight stability. In this research, a robust h-infinity control system will be applied. Where, the H-infinity control system is designed to ensure the stability and state of the system when disturbances occur, both internal and external disturbances. From the simulation results, it was found that the H-infinity control system was able to stabilize the drone's motion during disturbances.

**Index Terms**—Drone, Control System, H-infinity, Delivery System, Robust Control

## I. INTRODUCTION

At this time, the development of science and technology in the world has been very rapid. This affects the occurrence of changes in space and time in various aspects of human life, one of which is to facilitate humans in various activities and work and save time. The technology that has made a big contribution to changing human life is information and communication technology. This technology is used to exchange information and can reduce distance and time. People who are separated by long distances can communicate using mobile phones or PCs. In addition, this information technology has begun to be used by the community for buying and selling transactions. Thus, the people of area A do not need to go far to area B just to buy goods there. People can use social media and marketplaces to buy and sell them. At this time, technological developments have penetrated the payment and shopping sector. The increasing number of collaborations between financial companies and online sales

companies makes it easier for people to shop online [1]. where, payment by the buyer is done by transfer, and delivery of goods is done by courier. This resulted in the number of goods on the expedition increasing drastically. so that goods that are urgent can be delayed until their arrival. Even though some of the goods that consumers buy are perishable and medical goods. So a solution is needed to overcome this problem.

Drones are one of the unmanned aircraft that are of interest to various groups because they have many advantages, namely a simple structure, good maneuverability, and the fact that they do not require a large take-off area. The drone moves with the help of a multirotor. Some of the multirotors will rotate clockwise, and some will rotate counterclockwise [2] [3]. Specifically, the use of drones has grown rapidly, both in the digital world and in the delivery of goods. The delivery of goods using drones is one solution that can be used to overcome this problem. Goods delivery drones can only be used to deliver goods by air within a short distance (one city). This is because the drone uses a remote control, so it has a short flying radius. In addition, the delivery of goods by drone is still being developed in developed countries.

A drone has six degrees of freedom (6 dof), which are divided into two types of movement: translational motion and rotational motion. Rotational motion is a motion centered around the mass of the drone. This movement consists of roll, pitch, and yaw. whereas translational motion is movement caused by rotational motion. Movement in the drone is divided into two categories: longitudinal motion and lateral motion. longitudinal motion occurs during takeoff, takeoff, and landing. while lateral motion is the movement of the drone from one point to another in the horizontal direction, namely on the x-axis and y-axis.

Several control strategies and methods have been proposed in previous literature. In 2013, Ting, Lixin, and Junqiang looked into the creation of an H-infinity control system for a

Identify applicable funding agency here. If none, delete this.

tailless folding-wing morphing aircraft. This study established the H-infinity control system's capability to ensure adequate dynamic performance for a deforming aircraft during the process of overall wing shape transition. Based on the uncertainty of the aerodynamic parameters in the nonlinear model, the robustness of the flight control system to the wing folding process is confirmed [4]. In 2018, Kurak and Hodziq designed a Linear Quadratic Gaussian (LQG) control system for controlling drone motion that was resistant to natural disturbances [5]. In 2019, Mung and Hong designed a PID control system for quadcopter altitude control [2]. To stabilize the position and dynamics of a quadcopter in 2019, Fessi developed a Gaussian quadratic linear control system. The weight matrices Q and R for this study were produced through optimization utilizing the PSO approach [6]. In 2021, Guo and his friends modeled the hexacopter's equations of motion and continued with the design of an adaptive trajectory linearization control system to stabilize the drone's motion in the air during the manipulation process [7]. In 2023, researchers in [8] conducted research related to a robust H-infinity fuzzy static output feedback (RHF-SOF) control with norm-bounded strategy to enhance the path-following performance for the four-wheel-independent-driven electric vehicles (FWID-EVs). From [8] it is found that RHF-SOF shows advantages in achieving path following performance and can prevent actuator instability. To ensure steady movement in the air, a drone controller must be designed. By doing this, the drone's flight stability is ensured. The stability of the drone's movement is ensured by this controller, in addition to the drone's movement being stable and unhindered. This controller must be able to maintain the stability of the drone's movement in the face of external disturbances such variations in wind, air pressure, etc. The drone will encounter numerous turbulences in the air, which will affect its movement. A reliable H-infinity control system is thus constructed in this study. The rotational and translational motions that the system conducts are subject to disturbances and uncertainties that this control system is intended to eliminate. Because it can deal with system unpredictability, the reliable H-infinity control system was chosen. Data for state variables and the values of system parameters represent this uncertainty. Estimating can be used to get data for unknown state variables. Due to its potential to lower drone production costs, this is extremely profitable. To represent system assumptions and uncertainties that cannot be represented, the H-infinity control system can also incorporate output noise and system noise.

## II. MATHEMATICAL MODEL OF THE DRONE'S EQUATIONS OF MOTION

Kinematics and dynamics analysis is used to create the quadcopter's dynamic system. The dynamic equation of the system, which is a hybrid system made up of linear equations from the Earth-frame and angular equations from

the Body-frame, is required for the modeling process. Figure 1 displays the quadcopter's system coordinates in both Earth-frame and body-frame.

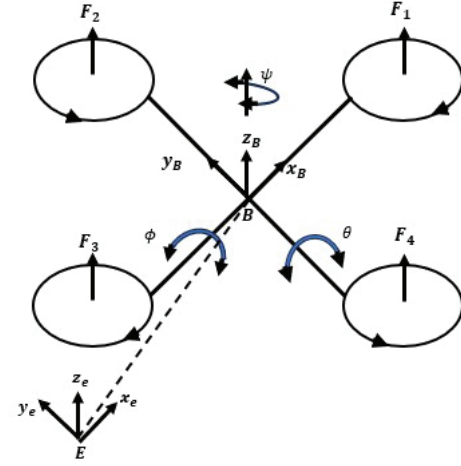


Fig. 1. Quadcopter Model Coordinate.

The derivation of quadcopter motion equations is closely related to the coordinate frame and reference frame chosen to describe the dynamics of drone motion. The angular orientation of the aircraft is described by the three angles (roll, pitch, yaw) that govern the flight of the quadcopter, called Euler angles. Applying these Euler angles will produce a rotation matrix which is used to obtain the velocity vector. The next stage, the Newton - Euler equations are prepared to obtain the equations of translational and rotational motion. After all stages have been carried out, the drone motion equation model is obtained which is shown in equations (1)-(12).

$$\dot{x} = u(C\psi C\theta) + v(S\phi S\theta C\psi - C\phi S\psi) + w(C\phi S\theta C\psi + S\phi S\psi) \quad (1)$$

$$\dot{y} = u(C\theta S\psi) + v(S\phi S\theta S\psi + C\theta C\psi) + w(C\phi S\theta S\psi - S\theta S\psi) \quad (2)$$

$$\dot{z} = v(S\phi C\theta) - uS\theta + wC\phi C\theta \quad (3)$$

$$\dot{u} = vr - qw - gS\theta + \frac{u_1}{m + M} \quad (4)$$

$$\dot{v} = pw - ur + gC\theta S\phi + \frac{u_1}{m + M} \quad (5)$$

$$\dot{w} = uq - pv + gC\theta S\phi + \frac{u_1}{m + M} - \frac{F_b}{m + M} \quad (6)$$

$$\dot{\phi} = p + qS\phi\theta + rC\phi\theta \quad (7)$$

$$\dot{\theta} = qc\phi - rS\phi \quad (8)$$

$$\dot{\psi} = q\left(\frac{S\phi}{C\theta}\right) + r\left(\frac{C\phi}{C\theta}\right) \quad (9)$$

$$\dot{p} = \frac{qrI_y - qrI_z}{I_x} + \frac{u_2}{m + M} \quad (10)$$

$$\dot{q} = \frac{prI_z - prI_x}{I_y} + \frac{u_3}{m + M} \quad (11)$$

$$\dot{r} = \frac{pqI_x - pqI_y}{I_z} + \frac{u_4}{m + M} \quad (12)$$

where

$x$  = x-position (m)

$y$  = y-position (m)

$z$  = z-position (m)

$\phi$  = roll angle (rad)

$\theta$  = pitch angle (rad)

$\psi$  = yaw angle (rad)

$m$  = mass of quadcopter (kg)

$M$  = mass of package (Kg)

$g$  = gravity acceleration (m.s<sup>-2</sup>)

$I_x$  = moment of inertia x-axis (N.m.s<sup>2</sup>)

$I_y$  = moment of inertia y-axis (N.m.s<sup>2</sup>)

$I_z$  = moment of inertia z-axis (N.m.s<sup>2</sup>)

$\Omega$  = total of propeller rotational speed (rad.s<sup>-1</sup>)

$u_i$  = the force acting on the propeller

Input values for  $u_1, u_2, u_3, u_4$  are given as follows [9], [10], [11]:

$$u_1 = b(F_1^2 + F_2^2 + F_3^2 + F_4^2)$$

$$u_2 = lb(-F_2^2 + F_4^2)$$

$$u_3 = lb(-F_1^2 + F_3^2)$$

$$u_4 = d(-F_1^2 + F_2^2 - F_3^2 + F_4^2)$$

$$\Omega = -F_1 + F_2 - F_3 + F_4$$

where  $F_i$  = rotational speed of propeller  $i$  ( $i = 1,2,3,4$ ) (rad.s<sup>-1</sup>)

$l$  = quadcopter arm length (m)

$b$  = thrust force (N.s<sup>2</sup>)

$d$  = drag force (N.m.s<sup>2</sup>)

Equations (1) to (12) are non-linear systems, so first linearization is required. Therefore, in this study, the linearization of the system is performed by the Jacobian method [12]. so that the resulting linearization is displayed in the state space system as follows:

$$\dot{x} = Ax + Bu \quad (13)$$

$$y = Cx + Du \quad (14)$$

where,

$$x = [x \ y \ z \ u \ v \ w \ \phi \ \theta \ \psi \ p \ q \ r]^T$$

$$u = [u_1 \ u_2 \ u_3 \ u_4]^T$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ m^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & I_x^{-1} & 0 & 0 \\ 0 & 0 & I_y^{-1} & 0 \\ 0 & 0 & 0 & I_z^{-1} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The equation of state in (13) is the one formed when the drone flies through the air. From this equation we get that the drone can transport cargo and fly at a predetermined height. However, when implementing it, a control system is required to maintain the stability of the drone flying in the air. so in the next step the H infinity control system is designed to maintain the drone's flight stability.

### III. ROBUST H-INFINITY CONTROL

Two feedback control systems may be combined to form the h-infinity control system. We design a quadratic linear control system in this post. Finding the weight matrix and then computing the  $K_1$  gain value are the first steps in LQ control design. A new state system is built for the LPV control architecture using the  $K_1$  gain values that were obtained. The  $K_2$  gain value will be obtained for use in LPV design. When dealing with disturbances or noise from outside the system, such as wind disturbances, the LPV in this case serves as a system amplifier [4]. The operation

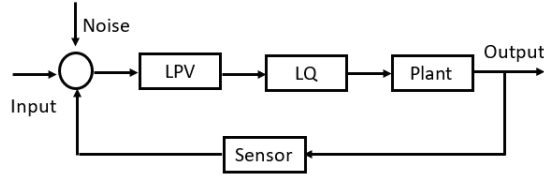


Fig. 2. The Flow Diagram of H-Infinity Control System

diagram of the H infinity control system is shown in Figure 2.

#### A. Linear Quadratic Optimal Control

This step will examine the LQ Gain Regulator with stable features in closed loop. Applying the system's control law yields the LQ gain value as follows [13]:

$$u_{LQ} = -K_1 y \quad (15)$$

where,

$$K_1 = B^T P_c C^T \quad (16)$$

With  $P_c$  is a solution of riccati algebra as follows:

$$A^T P_c + P_c A + Q_c - P_c B R_c^{-1} B^T P_c = 0 \quad (17)$$

Prior to attempting to solve the Riccati Algebra, it is necessary to ascertain the values of  $Q_c$  and  $R_c$ .  $Q_c$  is the weight matrix of the system variables, which is symmetric and positive semi-definite, and  $R_c$  is the weight matrix of the system input values, which is symmetric and a positive definite matrix. The values for  $Q_c$  and  $R_c$  were discovered by trial and error. The  $Q_c$  value increases as the  $R_c$  value decreases, resulting in a higher  $K_1$  gain value.

#### B. Linear Parameter Varying

At this point, a close-loop system will be used to calculate the LPV control's feedback gain value using the inner-loop feedback gained from the LQ controller's output [4] [8].

$$\dot{x} = A_c(\theta(t)) + B_1(\theta(t))w + B(\theta(t))u \quad (18)$$

$$z = Hx \quad (19)$$

$$y = Cx \quad (20)$$

With  $B_1$  is the matrix of disturbances that occur in the system. while for the matrix  $A_c$  obtained as follows:

$$A_c = A(\theta(t)) - B(\theta(t))K_{LQ}C \quad (21)$$

The LPV gain value is obtained by applying the control law of a system as follows [4]:

$$u_{LPV} = -K_2 y \quad (22)$$

system  $P(\theta)$  can be written as

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} A_c(\theta) & B_1(\theta) & B(\theta) \\ H(\theta) & D_{11}(\theta) & D_{12}(\theta) \\ C(\theta) & D_{21}(\theta) & D_{22}(\theta) \end{bmatrix} \quad (23)$$

The search for the  $K_2$  gain value is carried out using the Riccati Algebraic equation as follows:

$$A_c^T P_\infty + P_\infty A_c - P_\infty B K R_k^{-1} B_k^T P_\infty + Q_k = 0 \quad (24)$$

Furthermore, the two control methods will be designed in a system to control the stability of drone flight in the air.

#### IV. RESULT AND DISCUSSION

In this research, an H-infinity control system was designed by implementing double control. First, the design of the LQ control system was carried out, followed by the design of the LPV control system. The two control systems are arranged in series. The design of this control system is focused on improving the roll and pitch angle output of the system. The system is given disturbances in the form of wind disturbances, where these disturbances affect the system output. the system will take longer or have difficulty reaching the desired setpoint. The simulation in this research was carried out in 3 output experiments, namely for output angles of 0.1 rad, 0.2 rad. The additional load is assumed to be solid and is used as the additional mass of the drone. Apart from that, the system output will be shown when the system is not controlled, when it is controlled with LQ and when it is controlled with the H-infinity control system.

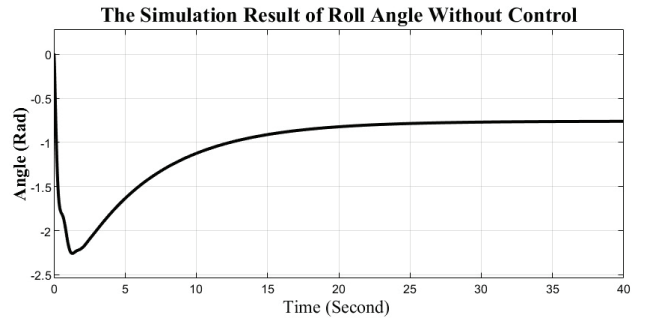


Fig. 3. The simulation result system without control

The simulation results shown in Figure 3 are the roll angle output without a control system. Figure 3 shows that the system cannot achieve the desired output angle, which is 0.1 rad. So this supports the design of the control system.

Figure 4 is the roll angle output with a setpoint of 0.1 rad. Before being disturbed, the system has a smoother output compared to after being disturbed when using either the LQ or LPV control system. The disturbance caused causes oscillations in the system output. LQ is able to reach the given setpoint more quickly than LPV, however LPV has a faster stable time compared to LQ. This can also be seen in

Figure 5 which is the system output with a setpoint of 0.2 rad.

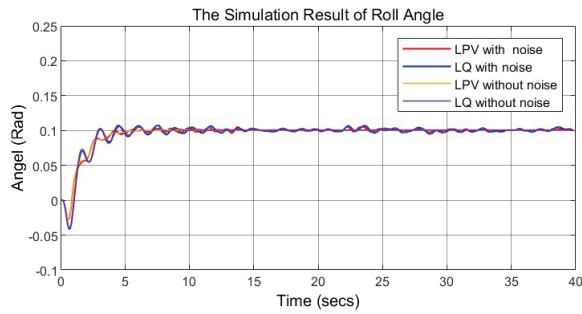


Fig. 4. The simulation result of roll angle with setpoint 0.1 rad

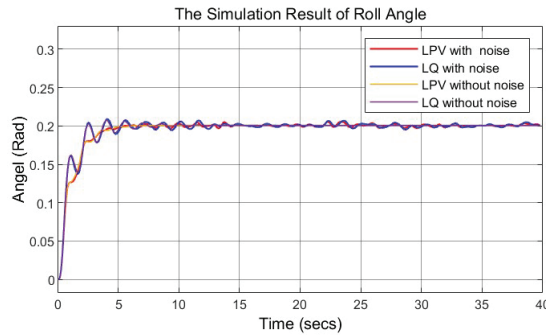


Fig. 5. The simulation result of roll angle with setpoint 0.2 rad

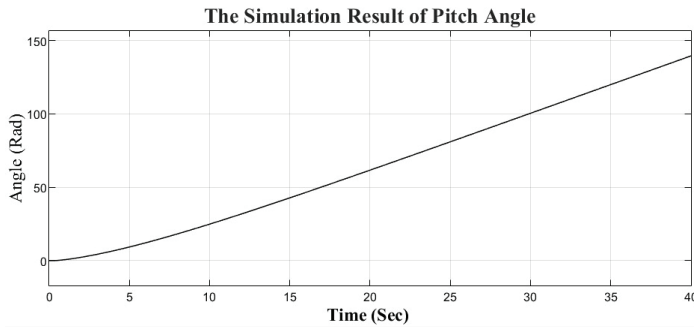


Fig. 6. The simulation result of roll angle with setpoint 0.2 rad

The simulation results in Figure 6 show the pitch angle output of the system without being controlled. In Figure 6, it can be seen that the output results show that the system looks unstable and is moving away from the given setpoint. So the control system design was carried out, the results of which can be seen in Figure 7. In Figure 7, it can be seen that the output of the pitch angle when controlled with LPV or LQ has a similar output whether it is disturbed or not. The graph shows that the results are not good, but the resulting error value is less than 10 percent. Next, Figure 8 is the output of the pitch angle using a setpoint of 0.1 rad. Figure 8 shows that the pitch angle output results when controlled with LQ or with the addition of LPV have similar results.

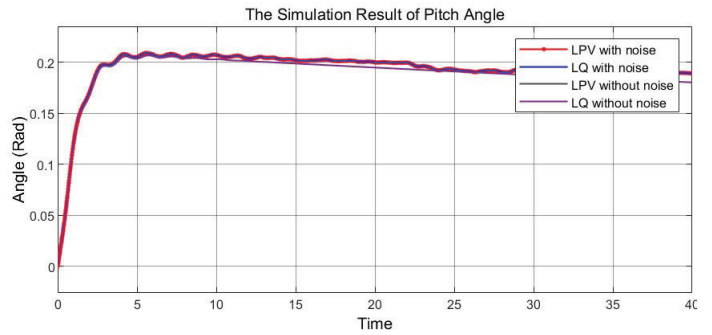


Fig. 7. The simulation result of roll angle with setpoint 0.2 rad

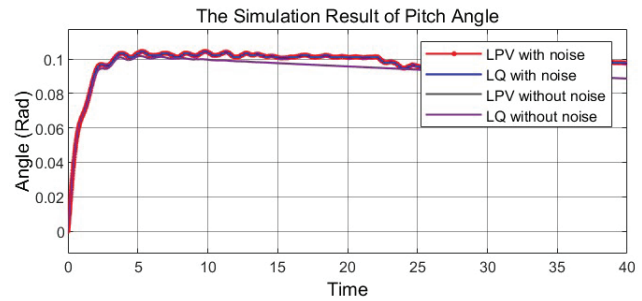


Fig. 8. The simulation result of pitch angle with setpoint 0.1 rad

The resulting error value is also very small whether there is interference or not. This shows that the system is able to follow the given output. System respon comparison in detail are shown in table 1.

The comparison of system responses in table 1 is a comparison of systems with an output of 0.1 rad in the presence of external disturbances. From the table it can be seen that the overall system response is better when the system is double controlled with an LPV control system rather than using LQ alone.

TABLE I  
SYSTEM RESPON COMPARISON

Response Time	LPV		LQ	
	Roll Angle	Pitch Angle	Roll Angle	Pitch Angle
Rise Time (sec)	0	0	0	0
Settling Time (sec)	5.1	5.1	7.5	5.1
Max Overshot (rad)	0.1032	0.1083	0.1069	0.1083
Peak Time (sec)	5	2.1	3.1	2.1

## V. CONCLUSION

Based on observations, when the system is not controlled, the system output results are unstable and/or unable to follow the desired output. So the control system design is carried out. When the LQ control system is designed, the system



output, be it roll angle or pitch angle, will follow the given set point. When a disturbance is given to the system, the LQ output becomes increasingly oscillating. So additional control is given, namely LPV. The LPV control system is able to overcome the oscillations that occurred previously and stabilize the system more quickly. This shows that the H-infinity control system can make the system more resistant to external interference, so that the system output is able to follow the desired setpoint.

#### ACKNOWLEDGMENT

This work was supported by Penelitian Dasar Unggulan Perguruan Tinggi (PDUPT) 2023, Direktorat Riset dan Pengabdian Masyarakat (DRPM) DIKTI Indonesia with title: “Pemodelan dan Perancangan Sistem Kendali pada Persamaan gerak Drone Menggunakan Metode Robust Control”, No: 1203/PKS/ITS/2023.

#### REFERENCES

- [1] K. Hick, "HostGator," 16 January 2019. [Online]. Available: <https://www.hostgator.com/blog/online-payment-methods-e-commerce/>. [Accessed 07 September 2023].
- [2] Xuan-Mung, N., & Hong, S. Improved Altitude Control Algorithm for Quadcopter unmanned Aerial vehicles. *Applied Sciences* 9, 1-15.(2019).
- [3] Caceres, C., Amaya, D., Rosario, J., Simulation, Model and Control of a Quadcopter AR Drone 2.0, (2016) *International Review of Mechanical Engineering (IREME)*, 10 (3), pp. 197-202
- [4] Yue T, Wang LX, and Ai JQ., Gain self-scheduled  $H_{\infty}$  Control for Morphing Aircraft in The Wing Transition Process Based on an LPV Model, *Chinese Journal of Aeronautics*, 2013,26(4): 909-917
- [5] Kurak, S., & Hodzic, M. Control and Estimation of a Quadcopter Dynamical Model. *Periodical of Engineering and Natural sciences*, 63-75.(2018).
- [6] R. Fessi, *Int. J. Automation and Control* 13(5), 569–594, (2019)
- [7] Guo, P., Xu, K., Deng, H., Liu, H., & Ding, X. Modeling and control of a hexacopter with a passive manipulator for aerial manipulation. *Complex Intell. Syst.*, 3051-3065.(2021).
- [8] Liu, T., Wang, X., Zhao, J., Wong, P.K., Wang, Y. Robust H-infinity Output Feedback Control for Path Following of FWID-EVs with Actuator Saturation. *International Journal Fuzzy System* 25(4), 1674 - 1688, (2023).
- [9] Abbasi E. and Mahjoob M.J. “Controlling of Quadrotor UAV Using a Fuzzy System for Tuning the PID Gains in Hovering Mode” (pdf). University of Tehran, Iran.(2011).
- [10] Zhang X., Li X. and Lu Y. A Survey of Modelling and Identification of Quadrotor Robot. *Hindawi Publishing Corporation*.(2014).
- [11] Musa S. Techniques for Quadcopter Modelling & Design: A Review. *Journal of Unmanned System Technology*. Vol:5. No:3.(2017).
- [12] G. J. Olsder, *Mathematical Systems Theory*, Delft, The Netherlands : Delft University Press , 2003.
- [13] D. S. Naidu, *Optimal Control Systems*, Pocatello: CRC Press, 2018.