Spectral Analysis

The fourier series & fourier transform are two mathematical tools which are useful for analysis of signal & design of LTI systems.
They basically involve in the decomposition of signals in terms of sinusoidal or complex exponential part so that the corresponding signal is said to be representing the frequency domain.

Fourier series is mainly used for periodic signal in which the periodic signal can be broken into sine & Cosine waves. It gives a discrete spectrum.
Fourier Transform is used for both periodic & non periodic signal. It gives continious spectrum.

Fourier Series

(i) Trigonometric Fourier Series

Any periodic signal can be written as.

$$\boxed{v(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{2\pi nt}{T_0} + \sum_{n=1}^{\infty} B_n \sin \frac{2\pi nt}{T_0}}$$

$T_0 \rightarrow$ Time period.
A_0, B_n & A_n is called fourier coefficient.

$$A_0 = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} v(t)\, dt$$

$$A_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} v(t) \cdot \cos \frac{2\pi nt}{T_0}\, dt$$

$$B_n = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} v(t) \sin \frac{2\pi nt}{T_0}\, dt$$

Standard Trigonometric Fourier Series

From trigonometric FS we got

$$V(t) = A_0 + \sum_{n=1}^{\infty} A_n \cos\frac{2\pi nt}{T_0} + \sum_{n=1}^{\infty} B_n \sin\frac{2\pi nt}{T_0}$$

Let → $A_0 = C_0$

$A_n = C_n \cos\phi_n$

$B_n = C_n \sin\phi_n$

$$V(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\phi_n \cos\frac{2\pi nt}{T_0} + \sum_{n=1}^{\infty} C_n \sin\phi_n \sin\frac{2\pi nt}{T_0}$$

$$\boxed{V(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi nt}{T_0} - \phi_n\right)}$$

$$\boxed{\begin{aligned} C_n &= \sqrt{A_n^2 + B_n^2} \\ \phi_n &= \tan^{-1}\frac{B_n}{A_n} \\ C_0 &= A_0 \end{aligned}}$$

Exponetial Fourier Series

From the standard trigonometric form we know

$$V(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi nt}{T_0} - \phi_n\right)$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$V(t) = C_0 + \sum_{n=1}^{\infty} C_n \left[\frac{e^{j\left(\frac{2\pi nt}{T_0} - \phi_n\right)} + e^{-j\left(\frac{2\pi nt}{T_0} - \phi_n\right)}}{2}\right]$$

$$= C_0 + \sum_{n=1}^{\infty} \frac{C_n}{2}\left[e^{j\frac{2\pi nt}{T_0}} \cdot e^{-j\phi_n} + e^{-j\frac{2\pi nt}{T_0}} \cdot e^{j\phi_n}\right]$$

Let $C_0 = V_0$

$V_n = \frac{C_n}{2} e^{-j\phi_n}$

$V_{-n} = \frac{C_n}{2} e^{j\phi_n}$

$$V(t) = V_0 + \sum_{n=1}^{\infty} V_n e^{j\frac{2\pi nt}{T_0}} + \sum_{n=1}^{\infty} V_{-n} e^{-j\frac{2\pi nt}{T_0}}$$

$$V(t) = V_0 + \sum_{n=1}^{\infty} V_n e^{j\frac{2\pi nt}{T_0}} + \sum_{n=-1}^{-\infty} V_n e^{j\frac{2\pi nt}{T_0}}$$

$$\boxed{V(t) = V_n e^{j\frac{2\pi nt}{T_0}}}$$

$$\boxed{V_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} V(t)\, e^{-j\frac{2\pi nt}{T_0}}\, dt}$$

Relation betn Fourier Coefficients

$A_0 = C_0 = V_0$

$$V_n = \frac{C_n}{2} e^{-j\phi_n} = \frac{C_n}{2}(\cos\phi_n - j\sin\phi_n)$$

$$= \frac{C_n\cos\phi_n - j\, C_n\sin\phi_2}{2}$$

$$V_n = \frac{1}{2}(A_n - jB_n)$$

$$V_{-n} = \frac{C_n}{2} e^{j\phi_n} = \frac{C_n}{2}(\cos\phi_n + j\sin\phi_n) = \frac{C_n\cos\phi_n + jC_n\sin\phi_n}{2}$$

$$\boxed{V_{-n} = \frac{A_n + jB_n}{2}}$$

$$|V_n| = |V_{-n}| = \frac{C_n}{2} = \sqrt{\frac{A_n^2 + B_n^2}{2}}$$

V_n & V_{-n} are Complex Conjugate.

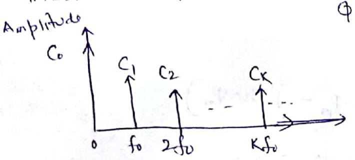
Fourier Spectrum → The standard trigonometric fourier series indicate that any periodic signal $v(t)$ can be expressed as sum of sinusoids of freq^n 0 (dc), f_0, $2f_0$, -- nf_0 whose amplitudes are $C_0, C_1, C_2 \cdots C_n$ & whose phases are $0, \phi_1, \phi_2, \cdots \phi_n$.

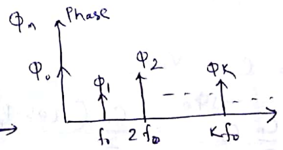
Thus the periodic signal $V(t)$ in time domain can be represented in frequency domain with components of freq^n $0, f_0, 2f_0 \cdots$ is called spectrum.

i.e we can plot the amplitude C_n Vs f_n (amplitude spectrum)

& ϕ_n Vs f_n (Phase Spectrum)

These two plots are called freq^n spectra of $V(t)$ ~~domain iden~~.





In fourier series representation C_n & ϕ_n are the spectral amplitude & phase of the spectral component of $C_n \cos(2\pi n f_0 t - \phi_n)$ at freq^n nf_0.

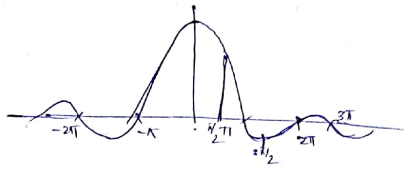
These spectrum is called discrete spectrum because both amplitude & phase have non-zero values only for discrete frequencies that are integer multiple of fundamental frequency (f_0).

Some Special Functions

① Sampling function

$$S_a(x) = \frac{\sin x}{x}$$

$x \to 0$, $S_a(x) = \lim_{x \to 0} \frac{\sin x}{x} = 1$



L. Hospital rule.

$$S_a(x) = \lim_{x \to 0} \frac{\sin x}{x} = \frac{d(\sin x)/dx}{\frac{d(x)}{dx}} = \left.\frac{\cos x}{1}\right|_{x=0} = 1$$

$x \to n\pi$ $\quad S_a(x) = 0$

$x = (2n+1)\frac{\pi}{2}$ $\Rightarrow$ $x = \frac{\pi}{2}$ $\quad S_a(x) = 2/\pi$

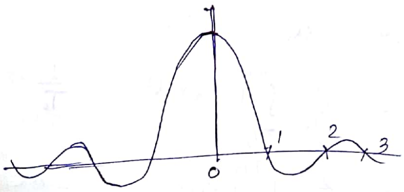
$x = \frac{3\pi}{2}$ $\quad S_a(x) = -\frac{2\pi}{3}$

Sinc function

$$\text{Sinc}(x) = \frac{\sin \pi x}{\pi x}$$

$$\text{Sinc}(x) = S_a(\pi x)$$

$x \to 0$ $\quad \lim_{x \to 0} \frac{\sin \pi x}{\pi x} = 1$

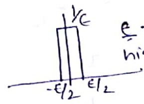


$x \to \pm n$ $\quad \sin \pi x = 0$

Comparing both $\text{sinc}(x)$ & $S_a(x)$ → ~~So~~

In case of sampling f^n the first null point occurs at $x = \pi$ in case of sinc the first null position is at $x = 1$. So sinc is narrow compared to sampling function.

Unit Impulse function



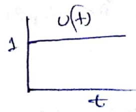
$\epsilon \to 0 \Rightarrow \frac{1}{\epsilon} \to \infty$ hight is large

$\delta(t) = 0 \quad t \neq 0$

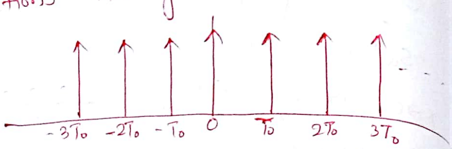
$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$

$\delta(t) = 0$ everywhere except $t = 0$

Unit Step Function, $U(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



Q Find the fourier Series for a train of impulse functions having amplitude I & time Period T_0.



Ans Function $v(t)$ can be represented as

$$v(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

The trigonometric fourier series for $v(t)$ is given by

$$v(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos\left(\frac{2\pi n t}{T_0} - \phi_n\right)$$

Compute a_0, a_n, b_n

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} I\,\delta(t)\,dt = \frac{I}{T_0}$$ ($I \to$ amplitude of impulse)

$$a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} I\,\delta(t)\cos\frac{2\pi n t}{T_0}\,dt = \frac{2I}{T_0}$$

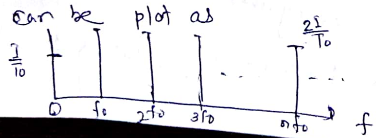
($\because$ $\delta(t)$ only exist at $t=0$ & when $t \neq 0$ $\delta(t)$ doesn't exist)

$$b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} I\,\delta(t)\sin\frac{2\pi n t}{T_0}\,dt = 0$$

$$C_0 = a_0 = \frac{I}{T_0} \qquad C_n = \sqrt{a_n^2 + b_n^2} = \frac{2I}{T_0} \qquad \phi_n = \tan^{-1}\frac{B_n}{A_n} = 0$$

$$v(t) = \frac{I}{T_0} + \frac{2I}{T_0}\sum_{n=1}^{\infty}\cos\frac{2\pi n t}{T_0}$$

The Spectrum can be plot as



The same problem can be solved by exponential form

$$v(t) = \sum_{-\infty}^{\infty} V_n e^{\frac{j2\pi n t}{T_0}}$$

$$V_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\, e^{-\frac{j2\pi n t}{T_0}}\,dt$$

$$V_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} I\,\delta(t)\, e^{-\frac{j2\pi n t}{T_0}}\,dt$$

or

$$V_n = \frac{1}{T_0}\int_{T_0/2}^{3T_0/2} I\,\delta(t - T_0)\, e^{-\frac{j2\pi n t}{T_0}}\,dt$$

$$V_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} I\,\delta(t)\, e^{-j\frac{2\pi n t}{T_0}}\,dt$$

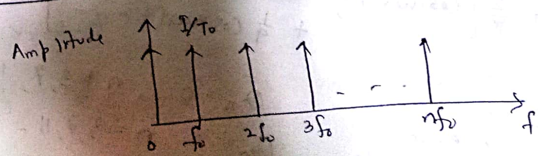
$\delta(t)$ exist only at $t = 0$

$$\Rightarrow V_n = \frac{I}{T_0}$$

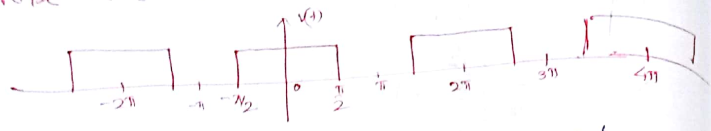
$$v(t) = \sum_{n=-\infty}^{\infty} V_n e^{\frac{j2\pi n t}{T_0}}$$

$$= \sum_{n=-\infty}^{\infty} \frac{I}{T_0} e^{\frac{j2\pi n t}{T_0}}$$

Plot of Spectrum



Q. Find the Fourier Series for the periodic ... Pulse wave $v(t)$ & Plot its spectrum.



Sol^n

$$v(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi n t}{T_0} + \sum_{n=1}^{\infty} b_n \sin \frac{2\pi n t}{T_0}$$

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\, dt$$

$v(t) = 1$ for $-\pi/2$ to $\pi/2$

$T_0 = 2\pi$

then $v(t) = 1$ for $-T_0/4$ to $T_0/4$

$$a_0 = \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} 1 \cdot dt = \frac{1}{2}$$

$$a_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} \cos \frac{2\pi n t}{T_0}\, dt = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$a_n = \begin{cases} 0 & n \text{ even} \\ \frac{2}{n\pi} & n = 1, 5, 9 \ldots \\ -\frac{2}{n\pi} & n = 3, 7, 11, 15 \ldots \end{cases}$$

$$b_n = \frac{2}{T_0}\int_{-T_0/4}^{T_0/4} \sin \frac{2\pi n t}{T_0}\, dt = 0$$

$$v(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \frac{1}{5}\cos 5\omega_0 t - \frac{1}{7}\cos 7\omega_0 t \ldots\right)$$

$$v(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos \omega_0 t + \frac{1}{3}\cos(3\omega_0 t - \pi) + \frac{1}{5}\cos 5\omega_0 t + \frac{1}{7}\cos(7\omega_0 t - \pi) + \frac{1}{9}\cos 9\omega_0 t + \ldots\right]$$

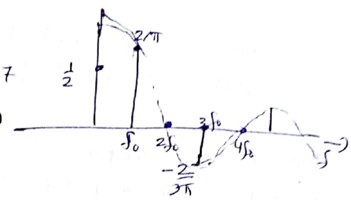
$C_0 = \frac{1}{2}$ $\qquad \theta_n = 0 \quad n \neq 3, 7, 11 \ldots$

$$C_n = \begin{cases} 0 & n \text{ even} \\ \frac{2}{\pi n} & n \text{ is odd} \end{cases} \qquad \theta_n = \pi \quad n = 3, 7, 11 \ldots$$

OR

$$C_n = \sqrt{a_n^2 + b_n^2} = a_n$$

$$C_n = \begin{cases} \frac{2}{n\pi} & n \text{ is odd } 1, 5, 7 \\ 0 & n \text{ is even} \\ -\frac{2}{n\pi} & n = 3, 7, 11 \end{cases}$$



In exponetial form

$$v(t) = \sum_{n=-\infty}^{\infty} V_n e^{\frac{j2\pi n t}{T_0}}$$

$$V_n = \frac{1}{T_0}\int_{T_0} v(t)\, e^{-\frac{j2\pi n t}{T_0}} dt$$

$$= \frac{1}{T_0}\int_{-T_0/4}^{T_0/4} e^{-\frac{j2\pi n t}{T_0}} dt$$

$$= \frac{1}{T_0}\frac{T_0}{-j2\pi n}\left[e^{-\frac{j2\pi n}{T_0}\cdot T_0/4} - e^{\frac{j2\pi n}{T_0}\cdot T_0/4}\right]$$

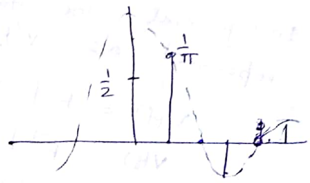
$$= \frac{1}{\pi n}\left[\frac{-e^{-j\frac{n\pi}{2}} + e^{j\frac{n\pi}{2}}}{2j}\right]$$

$$V_n = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

$$V_n = \frac{1}{2}\frac{\sin\left(\frac{n\pi}{2}\right)}{\left(\frac{n\pi}{2}\right)}$$

$V_0 = \frac{1}{2}$ at $n = 0$

$V_1 = \frac{1}{\pi}$, $V_2 = 0$, $V_3 = -\frac{1}{3\pi}$ ...



Fourier Transform

From the exponential fourier series we know that

$$V(t) = \sum_{n=-\infty}^{\infty} V_n e^{+\frac{j2\pi nt}{T_0}}, \quad V_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v(t)\, e^{-\frac{j2\pi nt}{T_0}} dt$$

The above eq^n represent the sum of spectral components having finite amplitude V_n & separated by the freq^n interval $f_0 = \frac{1}{T_0}$.

As $T_0 \to \infty$, $f_0 \to 0$ i.e the separation between the spectral component becomes very small.

So for a non-periodic signal $(T_0 \to \infty)$ the spectrum becomes a continious spectrum.

Hence if $v(t)$ is fourier transformable then it must satisfy the dirichilet's principle according to which

(i) $v(t)$ must be single valued function.
(ii) $v(t)$ must have finite number of discontinuity
(iii) It must have finite no. of maxima & minima.
(iv) It must be absolutely integrable

i.e $\int_{-\infty}^{\infty} |v(t)|\, dt < \infty$

In general the fourier transform of $v(t)$ is represent as $V(f)$

$$V(f) = F[\,v(t)\,]$$
$$v(t) = F^{-1}[\,V(f)\,]$$
$$v(t) \rightleftharpoons V(f)$$

It is called fourier Transform Pair.

In general the F.T. is a complex function of freq^n

i.e $V(f) = |V(f)|\, e^{j\theta_f}$

$|V(f)| \to$ amplitude spectrum, $\theta_f \to$ Phase spectrum.

For a real value function $v(t)$ the F.T is always complex conjugate. i.e $V(-f) = V^*(f)$

It can also be shown that the amplitude spectrum is an even function of the freq^n & the phase spectrum is an odd f^n of frequency.

If $v(t) \rightleftharpoons V(f)$

$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi ft}\, dt$$

$$v(t) = \int_{-\infty}^{\infty} V(f)\, e^{j2\pi ft}\, df$$

Conjugate Symmetry Property

If $v(t)$ is a real function then $V(f)$ & $V(-f)$ are complex conjugates

i.e $V(-f) = V^*(f)$

So $|V(-f)| = |V(f)|$
$\theta_v(-f) = -\theta_v(f)$

Amplitude spectrum even symmetry.
Phase " odd "

Q(i) Find the F.T of an impulse function ?
or Draw the spectrum of an impulse ?

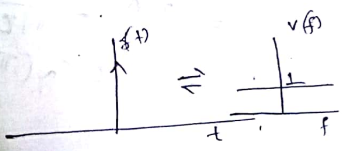
$$V(f) = \int_{-\infty}^{\infty} v(t)\cdot e^{-j2\pi ft}\, dt \qquad \int_{-\infty}^{\infty}\delta(t) = 1 \quad t=0; \quad \delta(t) = 0 \quad t \neq 0$$

$$= \int_{-\infty}^{\infty} \delta(t)\, e^{-j2\pi ft}\, dt$$

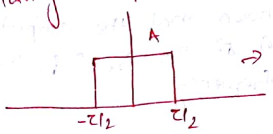
$$= 1$$

$V(f) = 1$

$\delta(t) \rightleftharpoons 1$



Q Find the F.T of a gate function pulse having amplitude A & duration τ.



→ Time Limited signal.

$$v(t) = \begin{cases} 0 & -\infty < t < -\tau/2 \\ A & -\tau/2 < t < \tau/2 \\ 0 & \frac{\tau}{2} < t < \infty \text{ or } t > \tau/2 \end{cases}$$

$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A\, e^{-j2\pi f t} \cdot dt$$

$$= A \left[\frac{e^{-j2\pi f t}}{-j2\pi f} \right]_{-\tau/2}^{\tau/2}$$

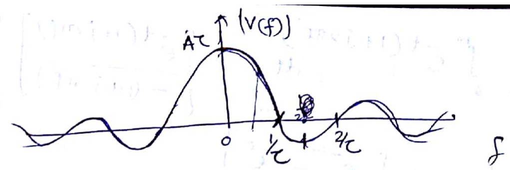
$$= \frac{-A}{j2\pi f} \left[e^{-j2\pi f \tau/2} - e^{j2\pi f \tau/2} \right]$$

$$= \frac{A}{j2\pi f} \left[e^{j\pi f\tau} - e^{-j\pi f \tau} \right]$$

$$= \frac{A}{\pi f} \sin(\pi f \tau)$$

$$= \tau A \frac{\sin(\pi f \tau)}{\pi f \tau}$$

$$= A\tau\, Sa(\pi f \tau)$$



From the above it is clear that the bandwidth of the ~~sinc fo~~ rectangular pulse is ideally infinite. Since the maxm power is concentrated in the major lobe hence the null to null BW is $\frac{1}{\tau}$.

It is also clear that the rectangular pulse is time limited but in freqn domain it is having ∞ Bandwidth.

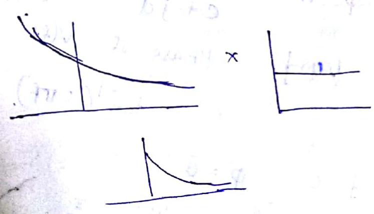
$$\boxed{A\, rect\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\, sinc(f\tau)}$$

Find the F.T of (i) decaying exponential pulse? (ii) Rising exponential pulse?

(i) Since exponential function is from $-\infty$ to ∞ so $\int_{-\infty}^{\infty} v(t) < \infty$ is not satisfied.

So we multiply $u(t)$.

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases}$$



So $e^{-t} \cdot u(t) = \begin{cases} e^{-t} & t > 0 \\ 0 & t < 0 \end{cases}$

$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t}\, dt$$

$$\int^{\infty} e^{-t} \cdot e^{-j2\pi f t}\, dt$$

$$V(f) = \int_0^{\infty} e^{-t(1+j2\pi f)}\,dt = \left[\frac{e^{-t(1+j2\pi f)}}{-(1+j2\pi f)}\right]_0^{\infty}$$

$$= \frac{1}{-(1+j2\pi f)}\left[e^{-\infty} - e^{0}\right]$$

$$= \frac{1}{1+j2\pi f}$$

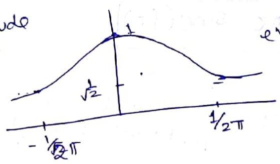
$$|V(f)| = \frac{1}{\sqrt{1+(2\pi f)^2}}$$

$f = 0$ $\quad |V(f)| = 1$

$f = \frac{1}{2\pi}$ $\quad |V(f)| = \frac{1}{\sqrt{2}}$

$f = -\frac{1}{2\pi}$ $\quad |V(f)| = \frac{1}{\sqrt{2}}$

Magnitude plot



even f^n.

Phase Plot.

$$\phi \text{ of } \frac{a+jb}{c+jd} = \frac{\Sigma\phi_N}{\Sigma\phi_D} = \tan^{-1}\left(\frac{b}{a}\right) - \tan^{-1}\left(\frac{d}{c}\right)$$

$\therefore$ Phase of $V(f) = \tan^{-1}\left(\frac{0}{1}\right) - \tan^{-1}\left(\frac{2\pi f}{1}\right)$

$$= -\tan^{-1}(2\pi f)$$

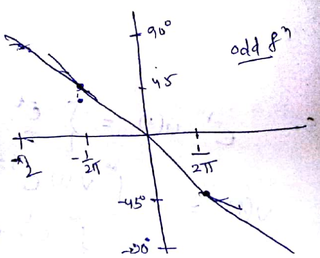
$f = 0$ $\quad \phi = 0$

$f = \frac{1}{2\pi}$ $\quad \phi = -45^\circ = -\frac{\pi}{4}$

$f = -\frac{1}{2\pi}$ $\quad \phi = \frac{\pi}{4}$

$f = \infty \Rightarrow \phi = -\pi/2$

$f = -\infty$ $\quad \phi = \pi/2$

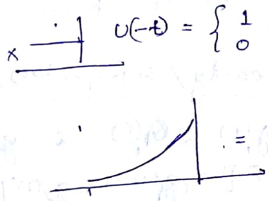


odd f^n

(ii) $e^{\alpha t}$

$$u(-t) = \begin{cases} 1 & t<0 \\ 0 & t>0 \end{cases}$$

So multiply $u(-t)$.



$\therefore e^{t}u(-t) = e^{t}$ ~~$t<0$~~ ~~$= 0\ t>0$~~

$$e^{t}u(-t) = e^{t} \quad -\infty < t < 0$$
$$= 0 \quad \text{elsewhere or } t \geq 0$$

$$V(f) = \int_{-\infty}^{\infty} v(t)\,e^{-j2\pi f t}\,dt$$

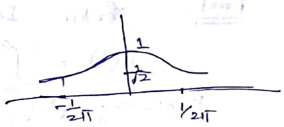
$$= \int_{-\infty}^{0} e^{t} e^{-j2\pi f t}\,dt = \int_{-\infty}^{0} e^{t(1-j2\pi f)}\,dt$$

$$V(f) = \left.\frac{e^{t(1-j2\pi f)}}{(1-j2\pi f)}\right]_{-\infty}^{0}$$

$$e^{0} - e^{-\infty}$$

$$= \frac{1}{(1-j2\pi f)}$$

$$= \frac{1}{1-j2\pi f}$$



Magnitude Spectrum $|V(f)| = \frac{1}{\sqrt{1+(2\pi f)^2}}$

Phase Spectrum

$$\phi = -\tan^{-1}(-2\pi f) = \tan^{-1}(2\pi f)$$

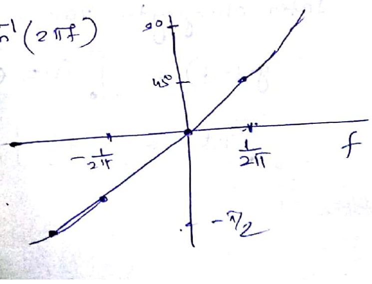
$f = 0$ $\quad \phi = 0$

$f = \frac{1}{2\pi}$ $\quad \phi = \frac{\pi}{4}$

$f = -\frac{1}{2\pi}$ $\quad \phi = -\frac{\pi}{4}$

$f = \infty$ $\quad \phi = \frac{\pi}{2}$

$f = -\infty$ $\quad \phi = -\pi/2$



Properties of Fourier Transform

① Linearity / superposition .

If $g_1(t) \rightleftharpoons G_1(f)$ & $g_2(t) \rightleftharpoons G_2(f)$

then $F[a_1 g_1(t) + b g_2(t)] = a G_1(f) + b G_2(f)$

(2) Time Scaling Property

If $g(t) \rightleftharpoons G(f)$

then $\boxed{g(at) \rightleftharpoons \frac{1}{|a|} G\left(\frac{f}{a}\right)}$

(3) Duality Property

If $g(t) \rightleftharpoons G(f)$ then

$\boxed{G(t) \rightleftharpoons g(-f)}$

Proof From Inverse fourier transform

$$g(t) = \int_{-\infty}^{\infty} G(f)\, e^{j2\pi f t}\, df$$

Replacing 't' by '-t'

$$g(-t) = \int G(f) e^{-j2\pi f t}\, df$$

Inter changing to variable t & f

$$g(-f) = \int_{-\infty}^{\infty} G(t)\, e^{-j2\pi f t}\, dt$$

$$\boxed{G(t) \rightleftharpoons g(-f)}$$

Q Find out the F.T of 1 using duality Property ?

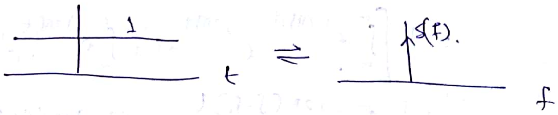
$g(t) \rightleftharpoons G(f)$

$G(t) \rightleftharpoons g(f)$

$g(t) \Rightarrow \delta(t) \rightleftharpoons 1 \Rightarrow G(f)$

$G(t) = 1$ $\qquad$ $\delta(-f) = \delta(f)$.

$\Rightarrow g(-f)$



Q A Sinc $\quad g(t) = A\, rect\left(\frac{t}{\tau}\right)$

$G(f) = A\tau\, Sinc(f\tau)$

By duality

$A\tau\ sinc(t\tau) \rightleftharpoons A\ rect\left(\frac{-f}{\tau}\right)$

$A = 1 \quad \tau = 1$

$$\boxed{Sinc(t) \rightleftharpoons rect(f).}$$

Time Shifting Property

If $g(t) \rightleftharpoons G(f)$ then

$g(t - t_0) \rightleftharpoons G(f)\, e^{-j2\pi f t_0}$

t_0 = shift in time domain

$g(t + t_0) \rightleftharpoons G(f)\, e^{j2\pi f t_0}$

Frequency shifting Property

If $g(t) \rightleftharpoons G(f)$

$g(t)\, e^{j2\pi f_0 t} \rightleftharpoons G(f - f_0)$

$g(t)\, e^{-j2\pi f_0 t} \rightleftharpoons G(f + f_0)$

$f_0 \rightarrow$ shift in freq^n domain

Q Draw the Spectrum of $\cos\omega_0 t$ & $\sin\omega_0 t$?

F.T of $\cos\omega_0 t$ is $\cos 2\pi f_0 t$

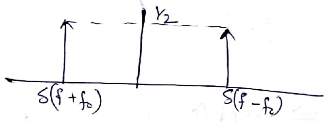
$$V(f) = \int_{-\infty}^{\infty} v(t)\, e^{-j2\pi f t}\, dt$$

$$F[\cos(2\pi f_0 t)] = F\left[\frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)\right].$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{j2\pi f_0 t} \cdot e^{-j2\pi f t}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi f_0 t} \cdot e^{-j2\pi f t}\, dt\right]$$

$$= \frac{1}{2}\left[\int_{-\infty}^{\infty} e^{-j2\pi (f-f_0) t}\, dt + \int_{-\infty}^{\infty} e^{-j2\pi (f+f_0)}\, dt\right]$$

$$= \frac{1}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$



$$F\left[\frac{1}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)\right]$$

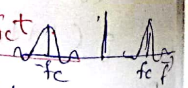
$$\Rightarrow \frac{1}{2} F\left(1 \cdot e^{j2\pi f_0 t} + 1 \cdot e^{-j2\pi f_0 t}\right)$$

$\Rightarrow$ $v(t) = 1$

$$v(t) \cdot e^{j2\pi f_0 t} \rightleftharpoons V(f-f_0)$$

$$1\, e^{j2\pi f_0 t} \rightleftharpoons \delta(f-f_0)$$

Find the F.T of $g(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)\cos 2\pi f_c t$



$$\cos 2\pi f_c t = \frac{1}{2}\left[e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right]$$

$$g(t) = \frac{1}{2} A\,\mathrm{rect}\left(\frac{t}{\tau}\right) e^{j2\pi f_c t} + \frac{1}{2} A\,\mathrm{rect}\, e^{-j2\pi f_c t}$$

$$F[g(t)] = \frac{1}{2} F\left[\frac{1}{2} A\,\mathrm{rect}\left(\frac{t}{\tau}\right) e^{j2\pi f_c t}\right] + \frac{1}{2} F\left[\frac{1}{2} A\,\mathrm{rect}\left(\frac{t}{\tau}\right) e^{-j2\pi f_c t}\right]$$

$$A\tau\,\mathrm{sinc}[(f-f_c)\tau] + A\tau\,\mathrm{sinc}[(f+f_c)\tau]$$

Amplitude Modulation (AM).

Modulation shift the frequency spectrum of a baseband signal. Communication so that uses modulation is known as carrier communication. In this mode one of the basic parameters (Amplitude, Phase, freqⁿ) of a sinusoidal carrier of high frequency ω_c is varied in proportion to the base band signal $m(t)$.

Amplitude Modulation → It is defined as the process in which the amplitude of carrier is varried about a mean value in proportion to the baseband or message signal. The freqⁿ & phase of the carrier remain constant.

Advantages of Modulation →

(i) Multiplexing
(ii) To design an antenna having practical height.
(iii) Narrow banding.

AM equations

Let $m(t)$ = message signal. $m(t) \rightleftharpoons M(f)$

$c(t) = A_c \cos 2\pi f_c t$ - Carrier signal.

In AM. $\Rightarrow S_{AM}(t) = A_i(t) \cos 2\pi f_c t$ A_i → instantaneous amplitude

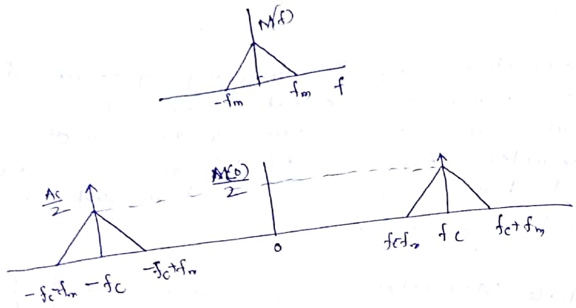
$$S_{AM}(t) = A_i(t) \cos 2\pi f_c t$$

$$= [A_c + K_a m(t)] \cos 2\pi f_c t$$

K_a → amplitude sensitivity, If K_a is not ~~maintained~~ given $K_a = 1$.

$$S_{AM}(t) = A_c \cos 2\pi f_c t + K_a m(t) \cos 2\pi f_c t$$

$$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{K_a}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



Transmission BW

$$B_T = (f_c + f_m) - (f_c - f_m) = 2f_m$$

Thus the bw of amplitude modulated wave is twice the maximum freq^n in the baseband signal.

Single tone Sinusoidal Modulation (Derive the eq^n & Draw the spectrum).

$$m(t) = A_m \cos 2\pi f_m t$$

$$S_{AM}(t) = \left(A_c + K_a A_m \cos 2\pi f_m t\right) \cos 2\pi f_c t$$

$$= A_c \left[1 + \left(K_a \frac{A_m}{A_c} \cos 2\pi f_m t\right)\right] \cos 2\pi f_c t$$

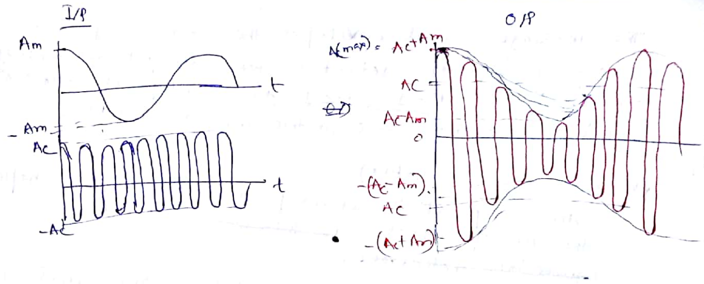
$$S_{AM}(t) = A_c \left[\left(1 + \mu \cos 2\pi f_m t\right)\right] \cos 2\pi f_c t$$

$\mu = m_f = K_a \frac{A_m}{A_c}$ = Modulation Index or depth of modulation

In general $K_a = 1$ So $\boxed{\mu = \frac{A_m}{A_c}}$

$$\cancel{A_i(t) =}\ S_{AM}(t) = A(t) \cos 2\pi f_c t$$

$$A_i(t) = A_c\left(1 + \mu \cos 2\pi f_m t\right).$$



~~When $\mu = 0$~~ When $\mu < 1$

~~$A_c(t) = A_c$~~

$$A_i(max) = A_c(1+\mu). = \left(A_c\left(1 + \frac{A_m}{A_c}\right)\right) = A_c + A_m$$

$$A_i(min) = A_c(1-\mu) = A_c\left(1 - \frac{A_m}{A_c}\right) = A_c - A_m$$

in +ve direction since max cos = 1, min cos = -1

In -ve direction $A_i(max) = -(A_c + A_m)$

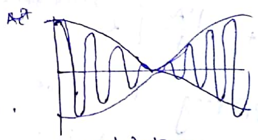
$A_i(min) = -(A_c - A_m)$

~~When $\mu = 0$ $A_c(t) = A_c$~~

~~When $\mu = 1$~~ ~~Critical Modulation~~

$$\boxed{\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}} \Rightarrow$$

~~$\mu = A_{max}$~~ $\boxed{\frac{A_{max}}{A_{min}} = \frac{1+\mu}{1-\mu}}$



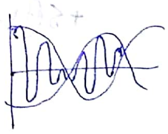
If $\mu = 1$ $A_m = A_c$ Critical modulation.

If $\mu > 1$ $\frac{A_m}{A_c} > 1$ = over modulation

In over modulation —

$$A_i(max) = A_c(1+\mu) \geq 2A_c$$

$$A_i(min) = A_c(1-\mu) < 0$$



Let

The message signal envelopes the carrier signal. Undergoing envelope detection we can get the message signal. For $\mu = 1$ & $\mu < 1$ the envelope of carrier signal is exactly same as the message signal.

For $\mu > 1$ → over modulation
the shape of the envelop doesn't follow the shape the message signal. ~~for this case we can follow square law detection~~.

Q. Draw the Spectrum of Singletone AM Signal?

$m(t) = A_m \cos 2\pi f_m t$

$c(t) = A_c \cos 2\pi f_c t$

$S_{AM}(t) = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$

$= \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \frac{\mu A_c}{2}\left[\underbrace{\cos 2\pi(f_m + f_c)t}_{\text{Upper Sideband}} + \underbrace{\cos 2\pi(f_m - f_c)t}_{\text{Lowe Sideband}}\right]$

The above is called DSBFC or Full AM

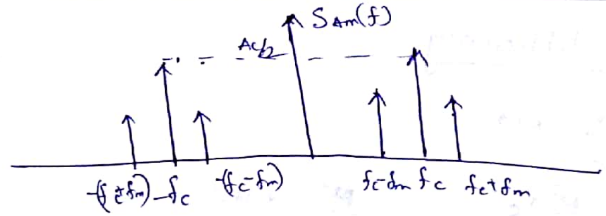
$S_{AM}(t) = \frac{A_c}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right) + \frac{\mu A_c}{4}\left[e^{j2\pi(f_c + f_m)t} + e^{-j2\pi(f_c + f_m)t}\right]$

$+ \frac{\mu A_c}{4}\left[e^{j2\pi(f_c - f_m)t} + e^{-j2\pi(f_c - f_m)t}\right]$

Applying F.T on bothside

$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{\mu A_c}{4}\left[\delta(f - (f_c + f_m))\right.$

$\left. + \delta(f + (f_c + f_m))\right] + \frac{\mu A_c}{4}\left[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))\right]$



$BW = 2f_m$

Power Contained in AM Signal

The AM Signal is given by.

$S_{AM}(t) = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$

$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{\mu A_c}{2} \cos 2\pi(f_c + f_m)t + \frac{\mu A_c}{2} \cos 2\pi(f_c - f_m)t$

$P_T = P_{carrier} + P_{USB} + P_{LSB}$

$P_T = \left(\frac{A_c}{\sqrt{2}}\right)^2 + \left(\frac{\mu A_c}{2\sqrt{2}}\right)^2 + \left(\frac{\mu A_c}{2\sqrt{2}}\right)^2$ where $R = 1\Omega$, $P = \frac{V^2}{R}$

$= \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{4} + \frac{\mu^2}{4}\right)$

$= \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right) = \frac{A_c^2(2 + \mu^2)}{4}$

$\boxed{P_T = \frac{A_c^2}{2}\left(1 + \frac{\mu^2}{2}\right)} \Rightarrow P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$

$P_{USB} = P_{LSB} = \frac{\mu^2 A_c^2}{8}$

$P_{SB} = P_{USB} + P_{LSB} = \frac{\mu^2 A_c^2}{4}$

$P_T = P_c + \frac{\mu^2}{2} P_c = P_c + P_{SB}$

$P_{SB} = \frac{\mu^2}{2} P_c \Rightarrow \boxed{\frac{P_{SB}}{P_c} = \frac{\mu^2}{2}}$

Transmission Efficiency

$$\eta_t = \frac{P_{SB}}{P_T} \times 100$$

$$= \frac{\frac{\mu^2}{2} P_c}{P_c\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2} \times 100$$

$\mu_{max} = 1 \qquad \eta_t = \frac{1}{3} \times 100 = 33.33\%$

The rest power 66.66% is used in transmission of carrier.

Q What is the value of the A_{max} & A_{min} on the oscilloscope when a $V_{pp} = 1KV$ signal is modulated to 50%.

The V_{pp} of carrier is 1KV. $\qquad V_m = 0.5\,KV$

$\mu = 50\% = 0.5$

$A_{max} = A_c(1+\mu)$

$= 0.5 \times 1.5 \cancel{\times 0.5KV} = 0.75KV$

$A_{min} = A_c(1-\mu)$

$= 0.5 \times 0.5 = 0.25\,KV$

Q 2 Sine waves of amplitude A_1 & A_2 are multiplied then what is the amplitude of the Spectral Component?

$V_1(t) = A_1 \sin \omega_1 t \qquad V_2 = A_2 \sin \omega_2 t$

$$V_1(t) \cdot V_2(t) = \frac{A_1A_2}{2}\left[\cos(\omega_1-\omega_2)t - \cos(\omega_1+\omega_2)t\right]$$

$$V(f) = F[V_1(t) \cdot V_2(t)] = F\left[\frac{A_1A_2}{2}\left[\cos(\omega_1-\omega_2)t - \cos(\omega_1+\omega_2)t\right]\right]$$

$$V(f) = F\left[\frac{A_1A_2}{2}\left[\cos 2\pi(f_1-f_2)t - \cos 2\pi(f_1+f_2)t\right]\right]$$

$$= \frac{A_1A_2}{2}\left[\frac{\delta(f-(f_1-f_2)) + \delta(f+(f_1-f_2))}{2}\right] - \left[\frac{\delta(f-(f_1+f_2)) + \delta(f+(f_1+f_2))}{2}\right]$$

$$= \frac{A_1A_2}{4}\left[\delta[f-(f_1+f_2)] - \delta(f_1-f_2) + \delta(f_1-f_2) - \delta(f_1+f_2)\right]$$

Q AM equation is given by ~~$S_{AM}(t) = 7\cos 4800\pi t + 20\cos(5000\pi t)$~~

$S_{AM}(t) = 7\cos 4800\pi t + 20\cos 5000\pi t + 7\cos 5200\pi t$.

Find μ & transmission ~~efficiency~~ freq^n.

$$S_{AM}(t) = 7\left[\cos(4800\pi t) + \cos(5200\pi t)\right] + 20\cos(5000\pi t)$$

$$= \frac{7}{2}\left[2\cos\left[\left(\frac{4800+5200}{2}\right)\pi t\right] \cdot \cos\left[\left(\frac{-400}{2}\right)\pi t\right]\right] + 20\cos(5000\pi t)$$

$$= 14\left[\cos(5000\pi t) \cdot \cos 200\pi t + 20\cos 5000\pi t\right.$$

$$= 20\left[1 + \frac{14}{20}\cos 200\pi t\right]\cos 5000\pi t$$

$$= A_c\left[1 + \mu\cos 2\pi f_m t\right]\cos 2\pi f_c t$$

$\mu = \frac{14}{20} = 0.7$

$A_c = 20 \qquad f_m = 100\,Hz \qquad f_c = 2.5\,KHz$

Q An audio signal given by $15\sin(3000\pi t)$ is amplitude modulated by carrier signal $60\sin(2\pi 10^5 t)$. Determine μ. Draw the modulated wave. Draw the spectrum & find out the freq^n components present after modulation.

Ans $m(t) = 15 \cos 3000\pi t$

Carrier $- c(t) = 60 \cos 2\pi \times 10^5 t$

Modulated wave —

$S_{AM}(t) = A_c(1 + \mu \cos 2\pi f_m t) \cos 2\pi f_c t$

$\mu = \frac{A_m}{A_c} = \frac{15}{60} = \frac{1}{4} = 0.25$

$\boxed{\mu = 0.25}$

$f_c = 10^5 Hz = 100 KHz$

$f_m = \frac{3 \times 10^3}{2} Hz = 1500 Hz$

$S_{AM}(t) = 60\left(1 + 0.25 \cos(2\pi\, 3 \times 10^3 t)\right] \cos 2\pi \times 10^5 t$

$= 60 \cos(2\pi \times 10^5 t) + 15 \cos 3\pi \times 10^3 t \cdot \cos 2\pi \times 10^5 t$

After modulation.

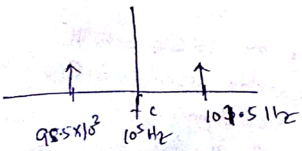
$(60 + 15 \cos \cdot (\pi\, 3000 t)) \cos 2\pi \times 10^5 t$

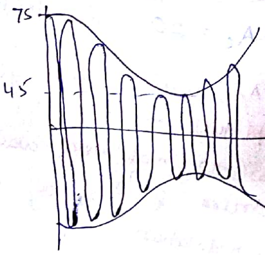
$A_c = 60$

$A_{max} = A_c(1+\mu) = 60(1.25) = 75$

$A_{min} = A_c(1-\mu) = 60 \times 0.75 = 45$

$f_c + f_m = 10^5 \times 1.5 \times 10^3 = 101.5 \times 10^3 Hz$





Q. A 100 KW Carrier modulated the message signal with $\mu = 0.35$. (i) Find the sideband Power. (ii) Powers transmitted if $\mu = 0.45$.

(i) $P_c = 100 KW$

$\frac{P_{SB}}{P_c} = \frac{\mu^2}{2}$

$P_T = P_c\left(1 + \frac{\mu^2}{2}\right)$

$= P_c + \frac{P_c \mu^2}{2}$

$\underbrace{P_c}_{c} \quad \underbrace{\frac{P_c\mu^2}{2}}_{SB}$

$P_{SB} = \frac{\mu^2}{2} \times P_c$

$= \frac{(0.35)^2}{2} \times 100 KW = \underline{6.125 KW}$

$P_{USB} = P_{LSB} = \frac{P_{SB}}{2} = 3.062 KW$

(ii) $P_T = P_c + \frac{\mu^2}{2} P_c$

$= 100 KW + \frac{(0.45)^2}{2} \times 100 KW$

$= \underline{110.10 KW}$

Q. A 10 KW Radio broad Casting Transmitter is modulated by a 1 KHz tone signal. (i) Calculate Power in Carrier & P_{SB} when the $\mu = 0.1$. (ii) What is the freq^n that will be present in the modulated wave if the transmitter is broadcasting on a 41.69 m antenna.

(i) $P_T = 10 KW$

$\mu = 0.1$

$P_T = P_c + \frac{1}{2}\mu^2 P_c$

$= P_c\left[1 + \frac{(0.1)^2}{2}\right]$

$\Rightarrow 10 KW = 1.005 P_c$

$\Rightarrow \boxed{P_c = 9.950 KW}$

$P_{SB} = \frac{\mu^2}{2} P_c$

$= 0.04975 KW = 49.75 W$

(ii) $f_m = 1 KHz$

f_c is freq^n of modulated wave

$f_c = \frac{c}{\lambda} \quad (\because \frac{\lambda}{2} = 41.69 \Rightarrow \lambda = 41.69 \times 2$

$f_c = \frac{3 \times 10^8}{41.69 \times 2} = \frac{7.19 \times 10^6}{2}$

$f_c = 3.6 MHz$

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Q. A transmitter radiates 140 KW power with modulation & 100 KW without modulation. Find the % of modulation. Find the power in the side band?

$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

$140 = 100\left[1+\frac{\mu^2}{2}\right]$

$\Rightarrow 1.4 = 1+\frac{\mu^2}{2}$

$\Rightarrow \mu = \sqrt{0.8}$

$P_{SB} = \frac{\mu^2}{2} P_c = \frac{0.8}{2} \times 100 = 40\ KW$

$P_{USB} = P_{LSB} = 20W$

Q. A 400 watt carrier is modulated to depth of 75%. Calculate the total power in modulated wave assuming the modulated signal to be sinusoidal.

$P_T = P_c\left(1+\frac{\mu^2}{2}\right)$

$= 400\left(1+\frac{(0.75)^2}{2}\right) = 512.5$ watt

Multitone Modulation

It is a technique in which more than one message signal modulate a single carrier.

If $m_1(t)$ & $m_2(t)$ are two message signals & $C(t) = A_c \cos 2\pi f_c t$, then the eqn of Am is given by

$S_{Am}(t) = [A_c + K_a m_1(t) + K_a m_2(t)] \cos 2\pi f_c t$

If $m_1(t) = A_{m_1} \cos 2\pi f_{m_1} t$

$m_2(t) = A_{m_2} \cos 2\pi f_{m_2} t$

$S_{Am}(t) = A_c\left[1 + K_a \frac{A_{m_1}}{A_c} \cos 2\pi f_{m_1} t + K_a \frac{A_{m_2}}{A_c} \cos 2\pi f_{m_2} t\right] \cos 2\pi f_c t$

$= \cancel{A_c \cos 2\pi f_c t + K_a}$

$= A_c\left[1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t\right] \cos 2\pi f_c t$

$S_{Am}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu_1}{2}\left[\cos 2\pi (f_c + f_{m_1})t + \cos 2\pi (f_c - f_{m_1})t\right]$

$+ \frac{A_c \mu_2}{2}\left[\cos 2\pi (f_c + f_{m_2})t + \cos 2\pi (f_c - f_{m_2})t\right]$

Carrier Power $P_c \Rightarrow A_c \cos 2\pi f_c t$

$\left.\begin{array}{lll} P_{USB} & m_1(t) \rightarrow & \frac{\mu_1 A_c}{2} \cos 2\pi (f_c + f_{m_1})t \\ P_{LSB} & m_1(t) \rightarrow & \frac{\mu_1 A_c}{2} \cos 2\pi (f_c - f_{m_1})t \end{array}\right\} PSB_1$

$\left.\begin{array}{lll} P_{USB} & m_2(t) = & \frac{\mu_2 A_c}{2} \cos[2\pi (f_c + f_{m_2})t] \\ P_{LSB} & m_2(t) = & \frac{\mu_2 A_c}{2} \cos[2\pi ((f_c - f_{m_2})t)] \end{array}\right\} PSB_2$

$$P_T = P_c + P_{SB_1} + P_{SB_2}$$

$$= \underbrace{\frac{A_c^2}{2}}_{P_c} + \underbrace{\left(\frac{\mu_1 A_c}{2\sqrt{2}}\right)^2 + \left(\frac{\mu_1 A_c}{2\sqrt{2}}\right)^2}_{P_{SB_1}} + \underbrace{\left(\frac{\mu_2 A_c}{2\sqrt{2}}\right)^2 + \left(\frac{\mu_2 A_c}{2\sqrt{2}}\right)^2}_{P_{SB_2}}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu_1^2}{4} + \frac{\mu_1^2}{4} + \frac{\mu_2^2}{4} + \frac{\mu_2^2}{4}\right]$$

$$P_T = \frac{A_c^2}{2}\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right]$$

$$\mu_T = \sqrt{\mu_1^2 + \mu_2^2} \Rightarrow \text{effective modulation}$$

$$\mu_T = \sqrt{\frac{A_{m_1}^2}{A_c^2} + \frac{A_{m_2}^2}{A_c^2}} = \frac{\sqrt{A_{m_1}^2 + A_{m_2}^2}}{A_c}$$

For multitone effective modulation

$$\boxed{\mu_T^2 = \mu_1^2 + \mu_2^2 + \cdots + \mu_n^2 + \cdots}$$

$$\boxed{P_T = P_c\left(1 + \frac{\mu_T^2}{2}\right)}$$

In multitone, BW = 2 × highest freq^n of the respective side band

Draw the Spectrum of a multitone modulated signal ?

$$S_{AM}(t) = [A_c + K_a A_{m_1}\cos 2\pi f_{m_1}t + K_a A_{m_2}\cos 2\pi f_{m_2}t]\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + K_a A_{m_1}\cos 2\pi f_{m_1}t\cdot\cos 2\pi f_c t + K_a A_{m_2}\cos 2\pi f_{m_2}t\cos 2\pi f_c t$$

$$= A_c\cos 2\pi f_c t + \frac{K_a A_{m_1}}{2}\left[\cos 2\pi(f_c + f_{m_1})t + \cos 2\pi(f_c - f_{m_1})t\right]$$

$$+ \frac{K_a A_{m_2}}{2}\left[\cos 2\pi(f_c + f_{m_2})t + \cos 2\pi(f_c - f_{m_2})t\right]$$

$$S_{AM}(t) = A_c\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right]$$

$$+ \frac{K_a A_{m_1}}{4}\left[e^{j2\pi(f_c+f_{m_1})t} + e^{-j2\pi(f_c+f_{m_1})t}\right]$$

$$+ \frac{K_a A_{m_1}}{4}\left[e^{j2\pi(f_c-f_{m_1})t} + e^{-j2\pi(f_c-f_{m_1})t}\right]$$

$$+ \frac{K_a A_{m_2}}{4}\left[e^{j2\pi(f_c+f_{m_2})t} + e^{-j2\pi(f_c+f_{m_2})t}\right]$$

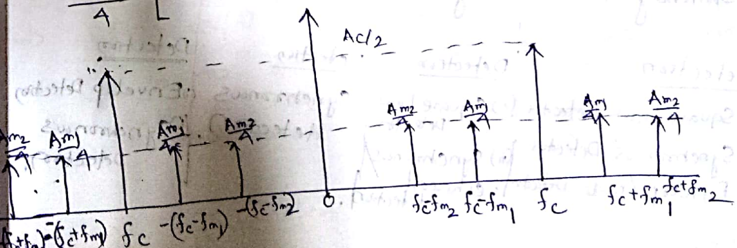
$$+ \frac{K_a A_{m_2}}{4}\left[e^{j2\pi(f_c-f_{m_2})t} + e^{-j2\pi(f_c-f_{m_2})t}\right]$$

$$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{K_a A_{m_1}}{4}\left[\delta(f-(f_c+f_{m_1})) + \delta(f+(f_c+f_{m_1}))\right]$$

$$+ \frac{K_a A_{m_1}}{4}\left[\delta(f-(f_c-f_{m_1})) + \delta(f+(f_c-f_{m_1}))\right]$$

$$+ \frac{K_a A_{m_2}}{4}\left[\delta(f-(f_c+f_{m_2})) + \delta(f+(f_c+f_{m_2}))\right]$$

$$+ \frac{K_a A_{m_2}}{4}\left[\delta(f-(f_c-f_{m_2})) + \delta(f+(f_c-f_{m_2}))\right]$$



Assuming $\underline{f_{m_2} > f_{m_1}}$

Q. A 300 W carrier is simultaneously modulated by 2 message signals having $\mu_1 = 60\%$ & $\mu_2 = 50\%$. Find the power in side band?

$P_c = 300\text{ W}$ $\qquad P_c = \frac{A_c^2}{2}$.

$P_{SB_1} = 2\times\left(\frac{\mu_1 A_c}{2\sqrt{2}}\right)^2$

$= \frac{\mu_1^2}{2} P_c = \frac{(0.6)^2}{2}\times 300 = 54\text{ W}$

$P_{SB_2} = \frac{\mu_2^2}{2} P_c = \frac{(0.5)^2}{2}\times 300 = 37.5\text{ W}$

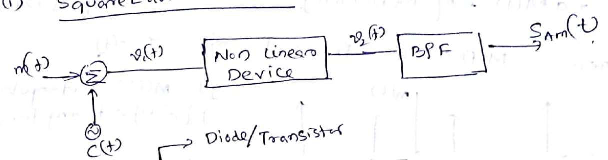
$P_{SB} = 91.5\text{ W}$

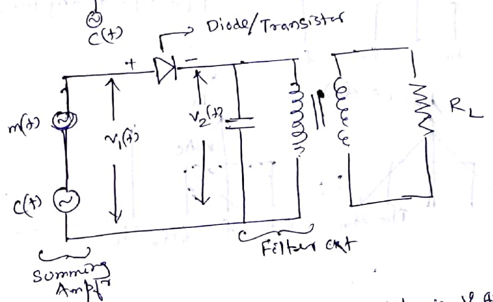
Different types of AM

DSBFC/AM	DSBSC	SSB	NSB
Generation	Generation	Generation	Generation
① Square Law Modulator ② Switching Modulator.	① Balanced Modulator (ii) Ring Modulator	① Filter Method (ii) Phasing method	(i) Filter Method.
Detection	Detection	Detection	Detection
(i) Square Law Detector (ii) Synchronous Detector (iii) Envelope/Diode Detector	(i) Square Law Detector (ii) Synchronous/coherent detection.	Synchronous detection.	(i) Envelop Detection (ii) Synchronous Detection.

Generation of DSBFC/AM.

(i) Square Law Modulator





The output of the summing ckt is $v_1(t)$

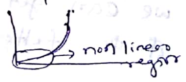
⇒ Summing amplifier will give the sum of base band & carrier. It may be OPAMP (< 1 MHz) or Transistor (> 1 MHz).

$v_1(t) = m(t) + A_c \cos 2\pi f_c t$

$\because v_2(t) = ax + bx^2$

⇒ $v_2(t) = a v_1(t) + b v_1^2(t)$.

$v_2(t)$ is the o/p of an non linear device. ~~To operate~~ The diode in non linear region is required for this operation, The transistor can be used.



$$v_2(t) = a\left[(m(t) + A_c \cos 2\pi f_c t)\right] + b\left[(m(t) + A_c\cos 2\pi f_c t)^2\right]$$

$$= a\,m(t) + a A_c \cos 2\pi f_c t + b\, m^2(t) + bA_c^2 \cos^2 2\pi f_c t + 2bA_c m(t)\cos 2\pi f_c t$$

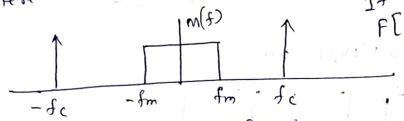
$$= aA_c\left[1 + \frac{2b}{a} m(t)\right]\cos 2\pi f_c t + a\,m(t) + b\,m^2(t) + bA_c^2\cos^2 2\pi f_c t,$$

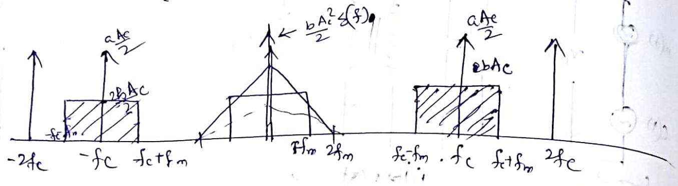
$\left(\cos^2 2\pi f_c t = \frac{1 + \cos 2\pi f_c}{2}\right)$

$$V_2(f) = \frac{aA_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{2bA_c}{2}\left[M(f-f_c) + M(f+f_c)\right]$$
$$+ aM(f) + b\left[M(f)\cdot M(f)\right] + \frac{bA_c^2}{2}\delta(f) + \frac{bA_c^2}{2}\left[\delta(f-2f_c) + \delta(f+2f_c)\right]$$

Where $M(f)$ is the fourier transform of $m(t)$

If $M(f)$ is rectangular
$F[m^2(t)] \Rightarrow$ triangular





$$V_2(t) = a\left[m(t) + A_c\cos 2\pi f_c t\right] + b\left[m^2(t) + \frac{A_c^2}{2}\left[1+\cos 2(2\pi f_c t)\right] + 2A_c m(t)\cos 2\pi f_c t\right]$$

Taking a BPF having center freq^n $f = f_c$ & $BW = 2f_m$ we can get the desired Am equation i.e. O/P of the filter.

$$\Rightarrow \quad V_0(t) = aA_c\left[1 + \frac{2b}{a}m(t)\right]\cos 2\pi f_c t$$

$$S_{Am}(t) = \left[A_c + K_a m(t)\right]\cos 2\pi f_c t$$

Here $\mu = \frac{2b}{a}$ for $\mu < 1$ $\quad \frac{2b}{a} < 1$

To avoid the over lapping of the spectrum

$$2f_m \leq f_c - f_m$$
$$f_c \geq 3f_m$$

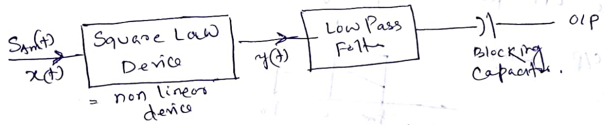
$$f_c = \frac{1}{2\pi\sqrt{LC}}$$

$$BW = 2f_m.$$

$$Q = \frac{f}{BW} = \frac{f_c}{BW}$$

Square Law Demodulator

Demodulation is a process in which we have to recovers the message signal from modulated waveform at the receiver. It is down ward frequency translation Process.



The response of squre law device.

$$y(t) = ax(t) + bx^2(t)$$

$$x(t) = S_{Am}(t) = A_c\left[1 + \mu m(t)\right]\cos 2\pi f_c t$$

let $A_c = \mu = 1$

$$x(t) = S_{Am}(t) = \left[1 + m(t)\right]\cos 2\pi f_c t$$

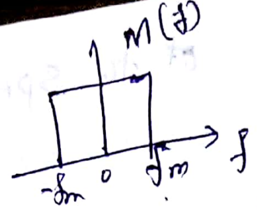
$$y(t) = a\left[1+m(t)\right]\cos 2\pi f_c t + b\left[1+m(t)\right]^2\cos^2 2\pi f_c t$$

$$= a\left[1+m(t)\right]\cos 2\pi f_c t + b\left[1+m^2(t)+2m(t)\right]\cos^2 2\pi f_c t$$

$$y(t) = a\cos 2\pi f_c t + am(t)\cos 2\pi f_c t + b\cos^2 2\pi f_c t + bm^2(t)\cos^2 2\pi f_c t + b\,2m(t)\cos^2 2\pi f_c t$$

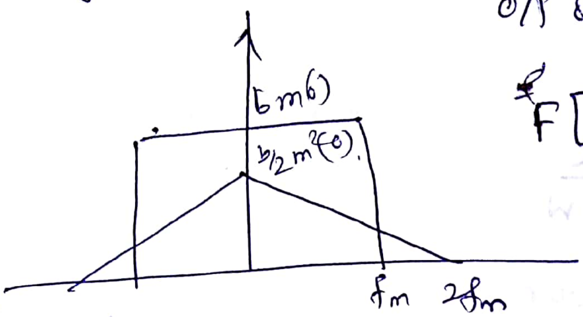
$$\cos^2 2\pi f_c t = \frac{1+\cos 4\pi f_c t}{2}$$

$$y(t) = a\cos 2\pi f_c t + am(t)\cos 2\pi f_c t + \frac{b}{2}(1+\cos 4\pi f_c t) + \frac{bm^2(t)}{2} + \frac{bm^2(t)}{2}\cos 4\pi f_c t + \frac{2b\,m(t)}{2} + bm(t)\cos 4\pi f_c t$$

If $m(t) \rightleftharpoons$ 

Then considering in O/P those signal whose freq^n range in range of $-f_m$ to f_m will pass through the LPF.

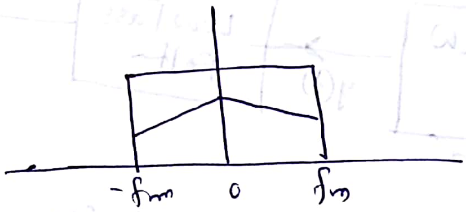
O/P of LPF $= \frac{b}{2} + \frac{b}{2} \times 2m(t) + \ldots$



$F[m^2(t)] = M(f) * M(f)$ having BW $2f_m$. ~~This~~

Blocking capacitor blocks all d.c quantitys. So $\frac{b}{2}\delta$ can't pass through the capacitor.

Since LPF passes only signals from $-f_m$ to f_m,

O/P will be 

The $\frac{b}{2}m^2(t)$ signal can't be separate out. But its amplitude can be suppressed.

$$\frac{b}{2}m^2(t) < b\,m(t)$$
$$\boxed{m(t) < 2}$$

The output of the squre law demodulator is passed through a LPF having BW $= f_m$ which contains the term $\frac{b}{2}\delta(f)$, $b\,m(0)$ & $\frac{b}{2}m^2(0)$ out of which only $b\,m(0)$ is desired. Hence by using a blocking capacitor we can avoid the dc term i.e $b/2$ since the capacitor behaves as open ckt for dc. But the effect of $\frac{b}{2}m^2(t)$ can't be avoided by using LPF. But if we assume that if $\frac{b}{2}m^2(t) < b\,m(t) \Rightarrow$

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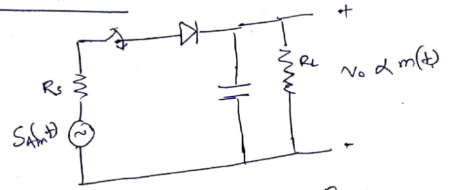
$m(t) \ll 2$ ie the message signal whose amplitude is very very small corresponding $\mu \ll 1$, we can apply the above method. Otherwise we go for envelop detection or synchronous detection.

$O/P = 2\cos w_c t + 0.2\cos w_m t \cos w_c t$

$S_{Am}(t) = \left[2\left(1 + \frac{0.2}{2}\cos w_m t\right)\cos w_c t\right]$

$\mu = 0.1 \qquad A_c = 2$

Envelop Detector



[The current can flow through C or R_L at $t=0$. $(X_c = \frac{1}{2\pi f c})$
It flows through 'C' when $X_c = 0$
i.e. Capacitor is short ckt.
So when no time the capacitor will reach the input voltage.]

Charging time Constant = $R_s C$

Discharging time Constant = $R_L C$.

The envelop detector consists of a diode, a resistor & a capacitor. When the input or the incoming modulating signal is applied accross the diode & for the positive half cycle of the input the diode becomes forward bias. The capacitor is charged to the maximum value of i/p signal. [At t=0 c behaves as short ckt]. When the input signal falls below the capacitor voltage the diode becomes reverse bias hence the capacitor discharges through the load $(R = R_L)$. This dischange process continue untill the next positive half cycle when the input signal becomes greater than the voltage accross the capacitor & the diode conduct again. The process of charging & discharging continues to take place for the total modulated ~~output~~ i/p and the discharging path of the capacitor which is taken accross load resistance R_L follows the shape of the message signal which is present in the envelop of the modulated waveform.

For better recovery the charging time constant $(R_s C)$ & discharging time constant $R_L C$.

$$\boxed{R_s C < R_L C}$$

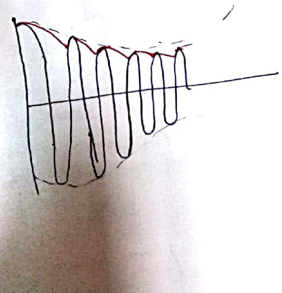
The charging time constant must be shorting short compared to carrier time period.

$$R_s C << \frac{1}{f_c}$$

So that the capacitor charges rapidly & thereby follow the applied voltage when the diode is conducting. On the other hand, the discharging time constant $(R_L C)$ must be long enough to ensure that the capacitor discharge slowly through the resistance (R_L) between the positive peaks of the waveform, but not so long that the capacitor voltage will not discharge at the maximum rate of change of modulating wave.

$$\boxed{\frac{1}{f_c} << R_L C << \frac{1}{f_m}}$$

The result is that the capacitor voltage which is the detected output is very nearly same as the envelop of the AM wave. So the detected output some ripples may present which may be farely removed with the help of low pass filter.



Therefore,

$$\begin{aligned}
x_1(t) &= a_0 + a_1 \cdot x(t) + a_2 \cdot x^2(t) \\
&= a_0 + a_1 \cdot [m(t) + A_c \cos(2\pi f_c t)] + a_2 \cdot [m(t) + A_c \cos(2\pi f_c t)]^2 \\
&= a_0 + a_1 m(t) + a_1 A_c \cos(2\pi f_c t) \\
&\quad + a_2 m^2(t) + a_2 A_c^2 \cos^2(2\pi f_c t) + a_2 2m(t) A_c \cos(2\pi f_c t) \\
&= a_1 A_c \cos(2\pi f_c t) + a_2 2m(t) A_c \cos(2\pi f_c t) \\
&\quad + a_0 + a_1 m(t) + a_2 m^2(t) + a_2 A_c^2 \cos^2(2\pi f_c t)
\end{aligned}$$

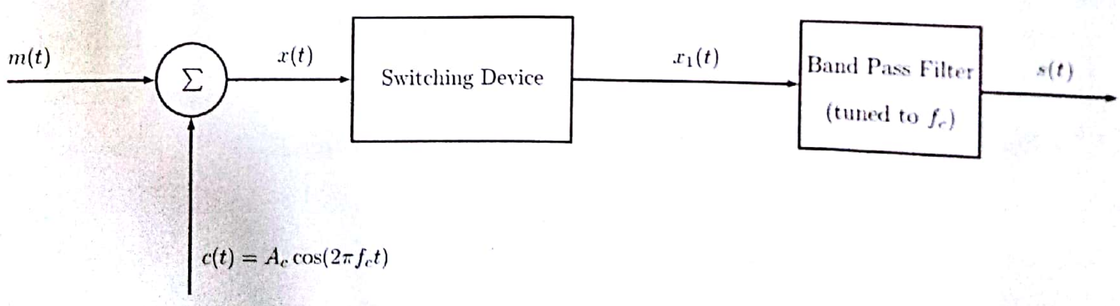
If the maximum frequency component of the message signal is B, then the Bandpass filter is designed to allow only the frequencies in the range $f_c - B$ to $f_c + B$. Hence, the output of the BPF is

$$\begin{aligned}
s(t) &= a_1 A_c \cos(2\pi f_c t) + a_2 2m(t) A_c \cos(2\pi f_c t) \\
&= a_1 A_c \left[1 + \frac{2a_2}{a_1} \cdot m(t)\right] \cos(2\pi f_c t) \\
&= A[1 + k_a m(t)] \cos(2\pi f_c t)
\end{aligned}$$

which is the required AM wave, where $k_a = \dfrac{2a_2}{a_1}$

1.3 Switching Modulator

The block diagram of a square law modulator is shown below.



The message signal $m(t)$ is added with the carrier signal $c(t)$ and applied to a switching device (e.g., ideal diode). The resulting signal is then passed through a Band-Pass Filter to get the required AM wave.

It is assumed that the switching operation is dependent only on $c(t) = A_c \cos(2\pi f_c t)$. For this to be true, we must have

$$|m(t)| \ll A_c$$

i.e., the amplitude of the message signal should not effect the switching operation. The switch will be in ON-state when $c(t) > 0$ and will be in OFF-state when $c(t) < 0$. Hence, the output $x_1(t)$ can be represented as

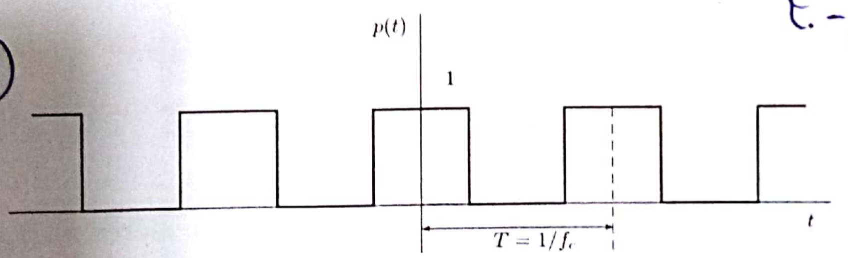
$$x_1(t) = \begin{cases} x(t) & = x(t).1 \quad \text{for } c(t) > 0 \\ 0 & = x(t).0 \quad \text{for } c(t) < 0 \end{cases}$$

As the switching process depends only on the carrier, the response of the switch can be represented as a pulse train, $p(t)$ of frequency f_c as shown below. Representing $p(t)$ by its Fourier series, we have $p(t) = \frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \ldots$

$a_n = \begin{cases} 0 & n = \text{even} \\ \frac{2}{n\pi} & n = 1, 5, 9. \\ -\frac{2}{n\pi} & n = 3, 7, \ldots \end{cases}$

$a_0 = \frac{1}{2}$

$a_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right)$



Therefore,

$$\begin{aligned} x_1(t) &= x(t) \cdot p(t) \\ &= [m(t) + A_c \cos(2\pi f_c t)] \cdot \left[\frac{1}{2} + \frac{2}{\pi}\cos(2\pi f_c t) + \ldots\right] \\ &= \frac{1}{2}m(t) + \frac{2}{\pi}m(t)\cos(2\pi f_c t) + \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2A_c}{\pi}\cos^2(2\pi f_c t) + \ldots \end{aligned}$$

If the maximum frequency component of the message signal is B, then the Bandpass filter is designed to allow only the frequencies in the range $f_c - B$ to $f_c + B$. Hence, the output of the BPF is

$$
\begin{aligned}
s(t) &= \frac{A_c}{2}\cos(2\pi f_c t) + \frac{2}{\pi} m(t) A_c \cos(2\pi f_c t) \\
&= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} \cdot m(t)\right]\cos(2\pi f_c t) \\
&= A[1 + k_a m(t)]\cos(2\pi f_c t)
\end{aligned}
$$

which is the required AM wave, where $k_a = \dfrac{4}{\pi A_c}$

27/09/12

Generation of DSBSC

In case of DSBFC, we transmitting d carrier along w/d d carrier. But d carrier does not convey ne info at d receiver side. Hence if we can suppress d carrier v can save d P_c, so dt d transmission of wl ↑.

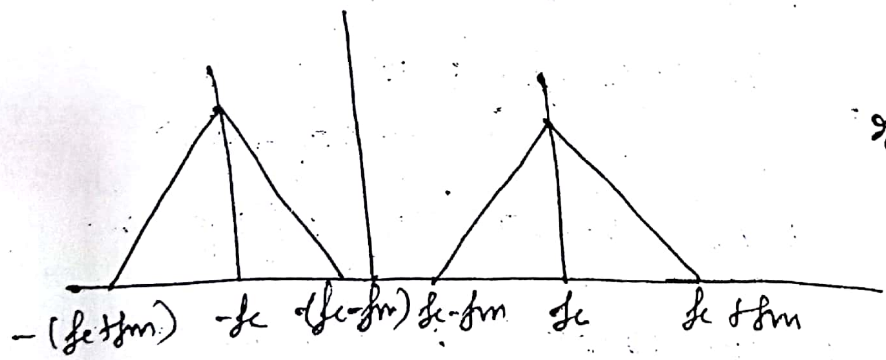
Mathematically,

d DSBFC signal => $s(t) = (A_c + k_a m(t))\cos 2\pi f_c t$

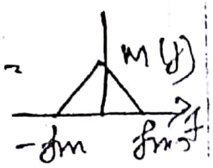
$$= \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier}} + \underbrace{k_a m(t)\cos 2\pi f_c t}_{SB}.$$

If $m(t) \Leftrightarrow M(f)$

$$S(f) = \frac{A_c}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right] + \frac{k_a}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



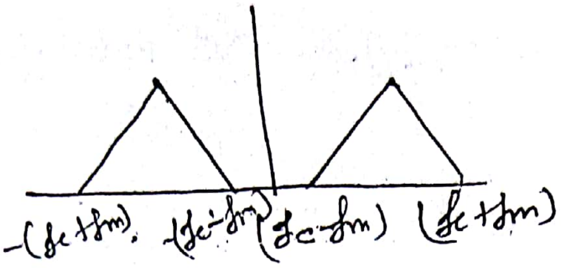
If M(f) =



Hence, by suppressing d carrier, we will get DSBSC.

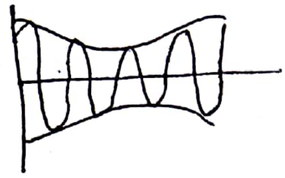
$$\boxed{s(t) = A_c k_a m(t) \cos 2\pi f_c t}$$

Scanned by CamScanner



$\Leftarrow$ DSB SC.

DSBFC $\quad s(t) = (A_c + k_a m(t))\cos \omega_c t$

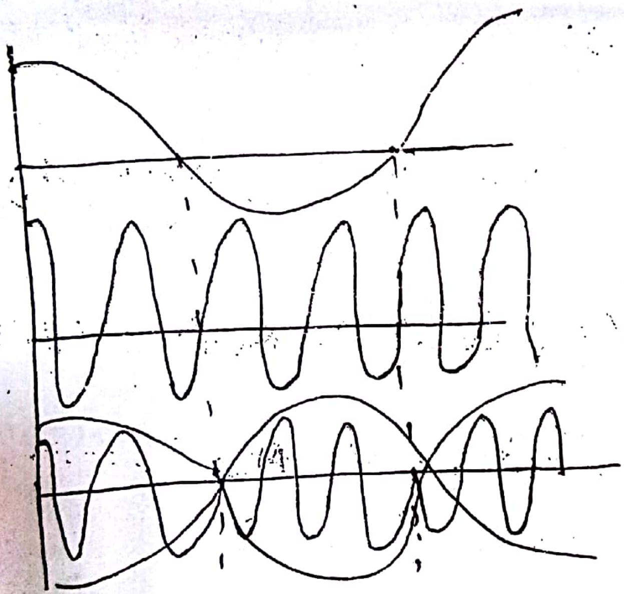


DSBSC $\quad m(t)=0 \quad s(t)=0 \quad k_a m(t)$

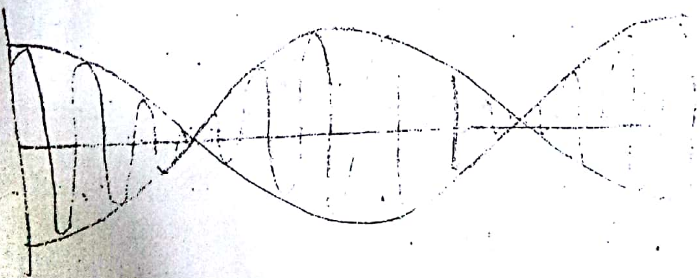
$m(t) = +ve \quad s(t) = A_c \cos\omega_c t \Rightarrow m(t) = 1 \quad s(t) = A_c \cos \omega_c t$

$m(t) = -1 \quad s(t) = -A_c k_a \cos\omega_c t$

$\qquad = A_c \cos(\omega_c t + \pi)$



phase change of π occurs when signal goes from +ve to -ve or -ve to +ve.



$P_T = P_c + P_{SB}$

$P_c = 0.$

$\boxed{P_T = P_{SB}} \qquad \boxed{P_{SB} = \frac{\mu^2}{2} P_c}$

% of power saved $= \frac{P_c}{P_T} \times 100\% = \frac{P_c}{(1+\frac{\mu^2}{2})P_c} \times 100$

$\mu = 1 \Rightarrow \quad = 66.66\%$

$$\boxed{\eta = \frac{P_{SB}}{P_T} \times 100}$$

$$\boxed{= 100\%}$$

Q) Draw the spectrum for the DSBSC signal for a single tone modn.

Ans

$$m(t) = A_m \cos 2\pi f_m t.$$

$$s(t) = A_c k_a m(t) \cos \omega_c t$$

$$= A_c A_m \cos \omega_m \times \cos \omega_c t$$

$$= \frac{A_c A_m}{2} \left(\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right)$$

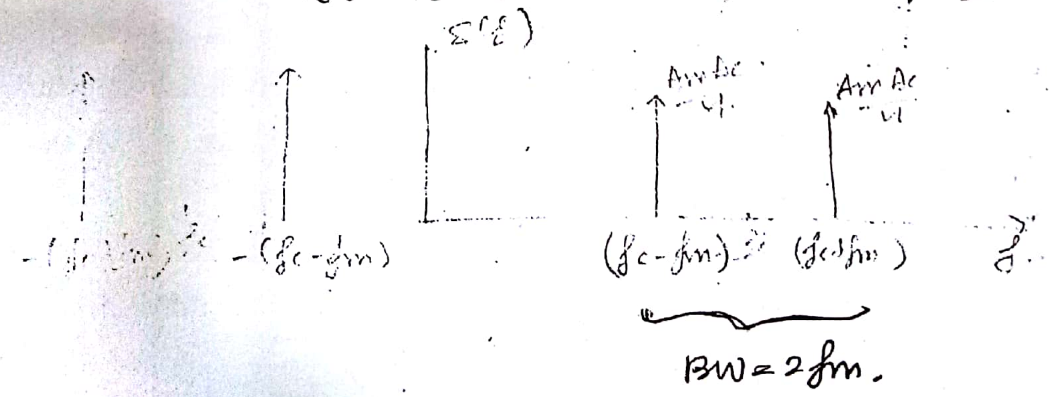
$S(f) \Rightarrow$

$$= \frac{A_c A_m}{4} \left(e^{j(\omega_c + \omega_m)t} + e^{-j(\omega_c + \omega_m)t} \right)$$

$$+ \frac{A_c A_m}{4} \left(e^{j(\omega_c - \omega_m)t} + e^{-j(\omega_c - \omega_m)t} \right)$$

$$S(f) = \frac{A_c A_m}{4} \Big[M(f + (f_c + f_m)) + M(f - (f_c + f_m))$$

$$+ M(f - (f_c - f_m)) + M(f + (f_c - f_m)) \Big]$$

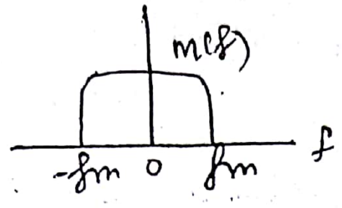


BW DSBFC = DSBFC

28/09/12

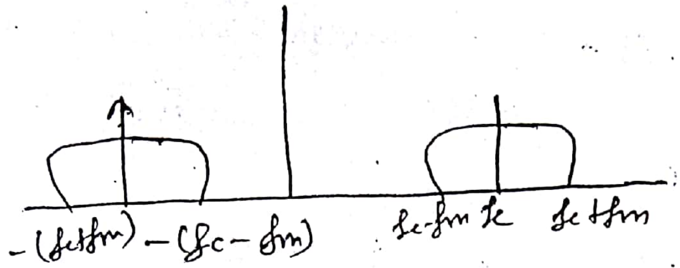
(i) Balanced Modulator. (Generation)

Frequency Translation is defined as shifting a msg signal of low freq to high freq by using a carrier signal so that it can be efficiently transmitted by suitable any antenna.



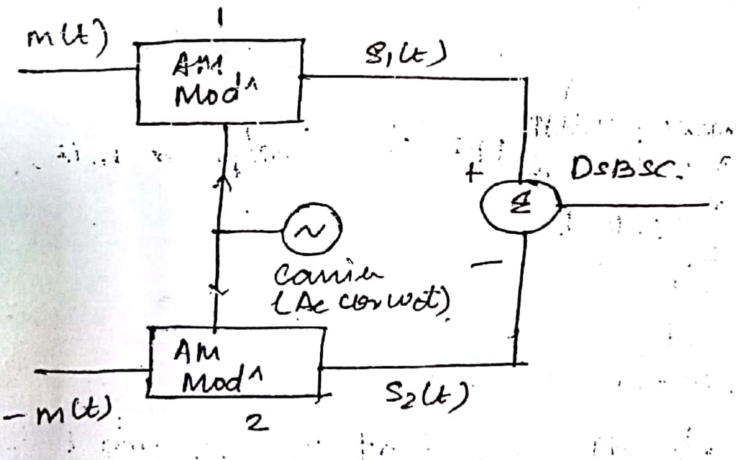
$m(t) \Leftrightarrow M(f)$

$m(t)\cos\omega_c t \Rightarrow \frac{1}{2}M(f-f_c) + \frac{1}{2}M(f+f_c)$



Generation of DSBSC

Balance Modulator



To generate d DSBSC signal, v take d help of AM mod^n which may b a square law modulator or switching mod^n whose o/p is DSBFC. Hence, d o/p of Mod^n 1

$$s_1(t) = (A_c + k_a m(t))\cos\omega_c t$$

$$s_2(t) = (A_c - k_a m(t))\cos\omega_c t$$

$-m(t)$ is generated fm $m(t)$ by using a 90° phase shifter if it is sinusoidal or Hilbert Transform for other waves.

o/p of adder ckt

$$s(t) = s_1(t) - s_2(t)$$

$$= (A_c + k_a m(t))\cos w_c t - (A_c - k_a m(t))\cos w_c t$$

$$= 2 k_a m(t) \cos w_c t$$

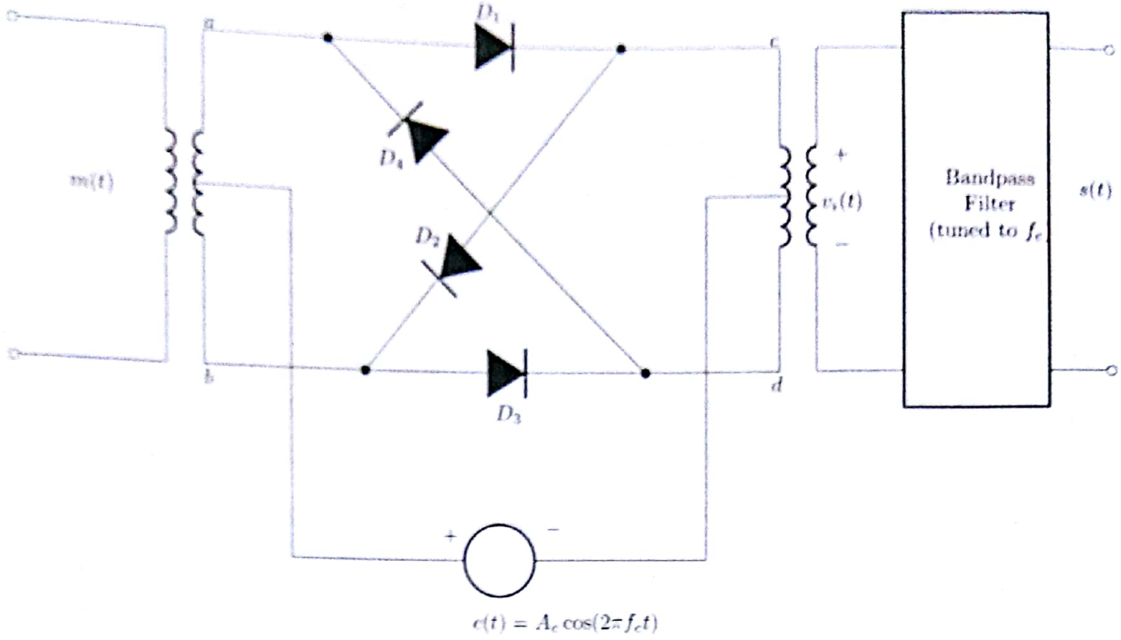
$$\boxed{s(t) = k_a' m(t) \cos w_c t.}$$

2.3 Ring Modulator

Ring modulator is a double balanced modulator and it operates on the principle of switching modulator. A ring modulator circuit using diodes is shown below.

The amplitude of $m(t)$ is such that the switching operation depends only on the carrier wave $c(t)$.

During the positive half-cycles of the carrier, diodes D_1 and D_3 are forward-biased, and D_2 and D_4 are reversed-biased. Hence, terminal a is connected to c, and terminal b is connected to d. Hence, the output is proportional to $m(t)$.

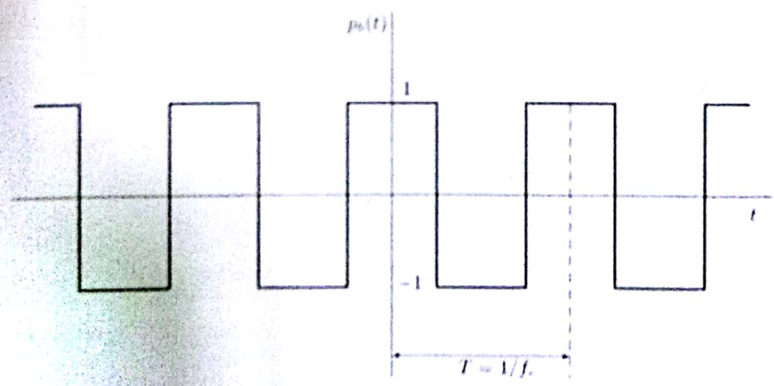


During the negative half-cycles of the carrier, diodes D_1 and D_3 are open, and D_2 and D_4 are conducting, thus connecting terminal a to d, and terminal b to c. Hence, the output is proportional to $-m(t)$.

Therefore, the output $v_i(t)$ can be represented as

$$v_i(t) = \begin{cases} m(t) & = m(t).1 \qquad \text{for } c(t) > 0 \\ -m(t) & = m(t).(-1) \qquad \text{for } c(t) < 0 \end{cases}$$

As the switching process depends only on the carrier, the response of the switch can be represented as a bipolar pulse train $p_b(t)$ of frequency f_c as shown below. Representing $p_b(t)$ by its Fourier series, we have $p_b(t) = \frac{4}{\pi}\cos(2\pi f_c t) + \ldots$



$p(t) = \frac{4}{\pi}\left[\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t - \ldots\right]$

Therefore,

$$\begin{aligned} v_i(t) &= m(t) \cdot p_b(t) \\ &= m(t) \cdot \left[\frac{4}{\pi} \cos(2\pi f_c t) + \dots \right] \\ &= \frac{4}{\pi} m(t) \cos(2\pi f_c t) + \dots \end{aligned}$$

If the maximum frequency component of the message signal is B, then the Bandpass filter is designed to allow only the frequencies in the range $f_c - B$ to $f_c + B$. Hence, the output of the BPF is

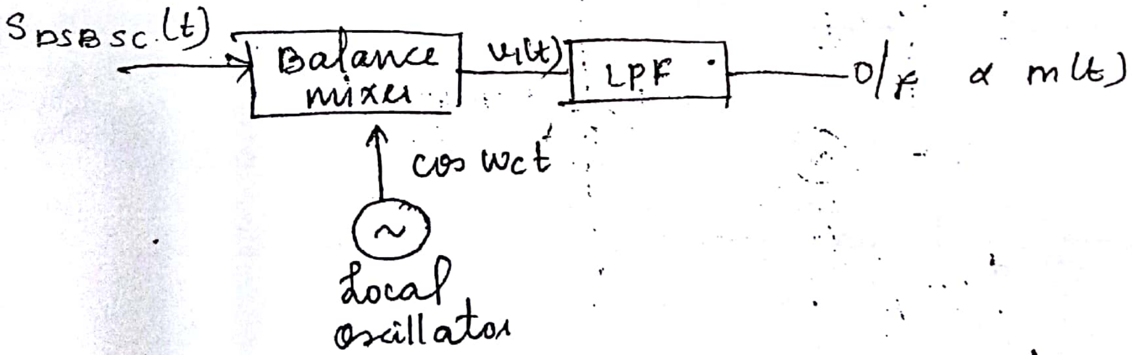
$$s(t) = \frac{4}{\pi} m(t) \cos(2\pi f_c t)$$

which is the required DSB-SC signal.

Detection of DSBSC

1) Synchronous / Coherent Detection

In DSBSC signal, since v are transmitting only d SB, hence 2 recover d msg at d receiver v need d help of d carrier i.e being used at d transmitter side. Since d carrier is absent, hence v hav 2 generate a carrier signal of same phase n same freq which is synchronous to dt of d transmitter carrier. ∴ its called synchronous/coherent detection.



The local oscillator dt is used at receiver side is used 2 generate d carrier signal of same phase n same freq as dt of d Tx side.

The o/p of balance mixer multiplier

$$v_1(t) = S_{DSBSC}(t) \cos w_c t$$

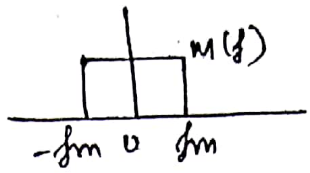
$$= m(t) \cos w_c t \cdot \cos w_c(t)$$

$$= m(t) \cos^2 w_c t$$

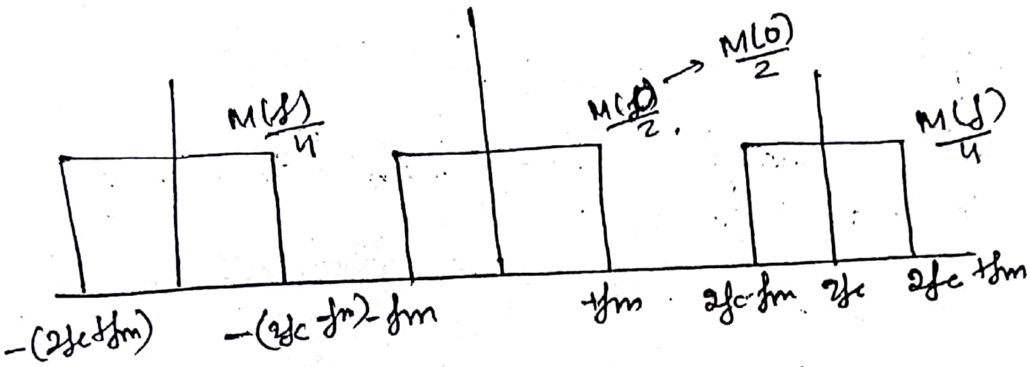
$$= \frac{m(t)}{2} [1 + \cos 2w_c t]$$

$$= \frac{m(t)}{2} + \frac{m(t)}{2} \cos 2w_c t$$

$m(t) \Leftrightarrow M(f)$



$$V_1(f) = \frac{1}{2} M(f) + \frac{1}{4}\left[M(f - 2f_c) + M(f + 2f_c)\right]$$



The o/p of LPF $= \frac{1}{2} m(t)$
$\propto m(t)$

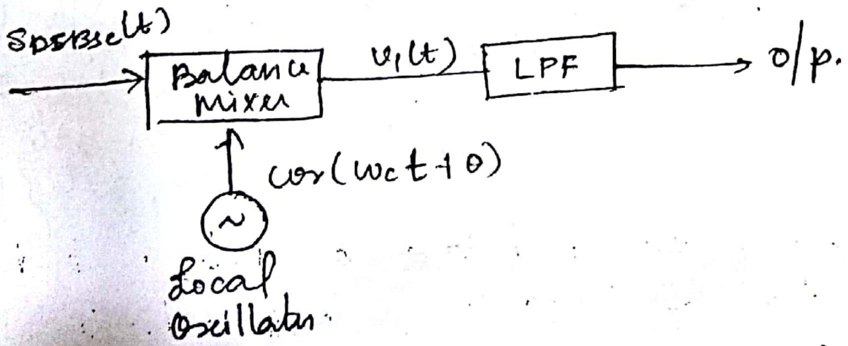
Constraints of Synchronous Detection

(i) Quadrature /Null Effect

(ii) Beating Effect

Quadrature Effect

It occurs if there is a phase diff of θ b/w the incoming carrier and the local oscillator carrier.



$$v_1(t) = m(t)\cos w_c t \cos(w_c t + \theta)$$
$$= \frac{m(t)}{2}\left[\cos(w_c t + w_c t + \theta) + \cos(-\theta)\right]$$
$$= \frac{m(t)}{2}\left[\cos(2w_c t + \theta) + \cos\theta\right]$$

dt

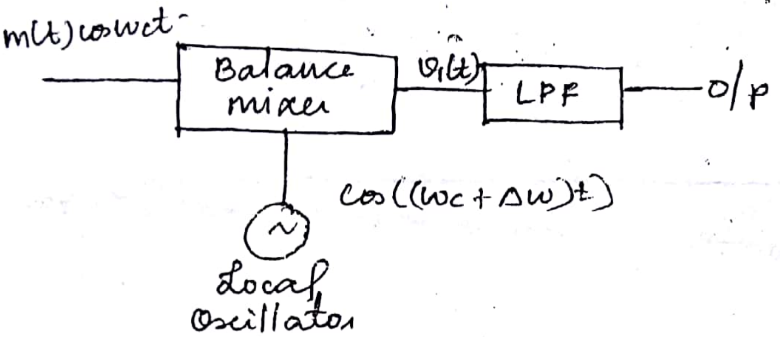
o/p of LPF

$$o/p = \frac{m(t)}{2} \cos\theta$$

If $\theta = \pi/2 \Rightarrow o/p = 0 \Rightarrow$ Null Effect
as we obtain no signal.

Beating Effect

It occurs if dr is a change in freq b/w d incoming carrier and d local oscillator carrier.



$$v_1(t) = m(t) \cos\omega_c t \cos((\omega_c + \Delta\omega)t)$$

$$= \frac{m(t)}{2} \left[\cos(2\omega_c t + \Delta\omega t) + \cos(\Delta\omega t) \right]$$

$$= \frac{m(t)}{2} \left[\cos(2\omega_c t + \Delta\omega t) + \cos(-\Delta\omega t) \right]$$

$$= \frac{m(t)}{2} \left[\cos[(2\omega_c t) + \Delta\omega t] + \cos(\Delta\omega t) \right]$$

o/p of LPF

$$= \frac{m(t)}{2} \cos(\Delta\omega t)$$

EL(3) Present 19.1.17

155, 159, 161, 166, 168, 171, 174, 176, 177, 180, 184, 190, 192, 193, 195, 196, 197, 198, 199, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 218, 221, 224, 226, 608

01/10/12

If dr is a change in fc of d local oscillator wrt d incoming freq d o/p of d LPF $\propto \cos(\Delta\omega t)$, due to which d voice signal or received signal wl not b audible properly.
OR dr may b some drift in d freq components of d original voice signal.

18/1/17 EL3 155, 160, 163, 165, 167, 170, 174, 177, 176, 180, 184, 187, 188, 190, 192, 193, 195, 197, 199, 202, 204, 203, 201, 209, 210, 211, 213, 221, 226, 224, 609, 608