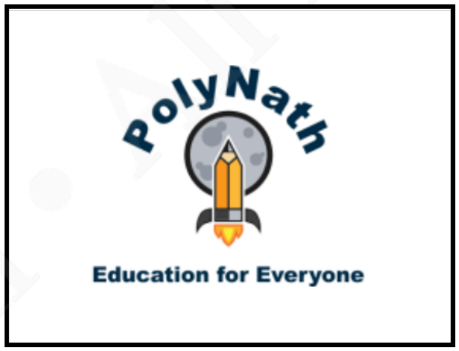
OCR MEI Exercises

(Year 2)

Department of Mathematics



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Author

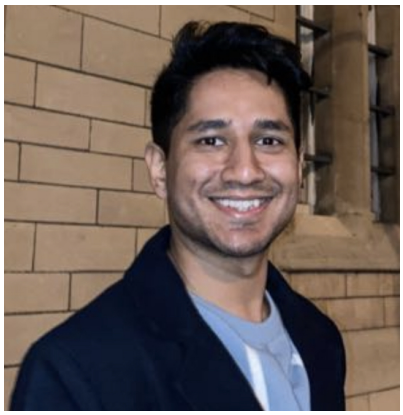
Siddartha Nath is the founder of *PolyNath* and the author of a plethora of top-selling books (*Quantitative Finance, MAT, NLP, TMUA/CTMUA*).

He has an undergraduate First-Class Honours BSc Mathematics with Statistics from Imperial College London (ICL) and a postgraduate Distinction MSc Computational Statistics and Machine Learning from University College London (UCL). He has taken his education and amassed over 2 years of industry experience, having worked as a Data Scientist for PayPal and DataSpartan.

In terms of academic teaching, he is an extremely dedicated and supportive tutor, having taught for over 5 years at public and private schools, for GCSE, A-Levels and University preparation. He possesses a wealth of knowledge in all UK-based Mathematics and Computer Science admission tests, with the following accomplishments:

- Achieved a perfect score of 9.0/9.0, in the Cambridge TMUA, resulting in a top 10% ranking.
- Achieved a commendable score of 63/100, in the Oxford MAT, resulting in a top 25% ranking.
- Achieved a high score of $1, 1, 1$ in the Cambridge STEP I, II and III.

Outside of academia, he enjoys music, dance, watching sports and creating content.



Preface

This document includes a breadth of questions within Chapter 1 - Proof (OCR MEI Year 2).

Happy Learning!

Chapter 1

Proof

Question 1

"Let p be a prime number such that $2 < p < 25$. Prove, by exhaustion, that for all such p, $(p-1)(p+1)$ is divisible by 8."

Question 2

"Prove that an integer is divisible by 5 if it is the sum of 5 consecutive integers."

Question 3

"Is it true that n^2 is odd only if n is odd?"

Question 4

1. *"Find a counter example to disprove the conjecture that curves of the form $y = \frac{a}{x^2} + b$ do not cross the x-axis."*
2. *"Find a counter example to disprove the conjecture that an asymptote cannot be crossed by a curve by considering rational functions of the form $f(x) = \frac{ax^2+bx}{x^n}$ where $n > 2$."*

Question 5

1. *"Prove by contradiction that for any integer $n > 1$, n and $n+1$ do not have a prime factor in common."*

2. *"Explain why this implies that* $n(n+1)$ *must have at least* 2 *distinct prime factors."*
3. *"What can you conclude about the number of distinct prime factors which* $n(n+1)(n(n+1)+1)$ *has."*

BONUS

"If the circle $(x-a)^2+(y-b)^2=r^2$ *and the line* $y=mx+c$ *do not meet, prove that*

$$m^2\left(r^2-a^2\right)+2am(b-c)+2bc-b^2-c^2+r^2<0."$$