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ON THE MEASUREMENT OF THE OVERALL DEGREE OF INCOME STRATIFICATION BETWEEN GROUPS

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This paper proposes a new class of indices that measure overall stratification between groups in a population and can be decomposed as population-weighted averages of pairwise indices. The indices capture not only the extent to which groups form well-defined strata in the income distribution but also the scale of the resultant differences in equally distributed equivalent incomes between them, where these two factors play the same role as identification and alienation respectively in the measurement of polarization. The properties of the class as a whole are investigated as well as those of selected members of it: zeroth and first power indices may be interpreted as measuring the overall incidence and depth of stratification respectively, while second and higher power indices members are directly sensitive to the severity of stratification between groups. An illustrative application provides an empirical analysis of global income stratification by regions in 1993.

JEL Codes: D31, D63

Keywords: between-group inequality, income stratification, polarization

1. Introduction

The concept of stratification is deeply embedded within sociology, most notably in relation to the analysis of social class, but has only been of relatively recent concern within the economics literature. Thus Yitzhaki and Lerman (1991) in their seminal article quote a definition by the sociologist Lasswell (1965, p. 10): "In its general meaning a stratum is a horizontal layer, usually thought of as between, above or below other such layers or strata. Stratification is the process of forming observable layers, or the state of being comprised of layers." Key to this definition is the idea that stratification, unlike segregation, implies a hierarchical ordering of groups according to some metric that in many economic settings may be used to also quantify the scale of the resultant differences in outcomes between groups. For example, occupational segregation in a labor market context will only lead to stratification in the earnings distribution if one group is crowded into lower paid occupations, with the resultant scale of economic disadvantage due to employment discrimination depending not only on the degree of segregation but also on the size of occupational pay differentials. Conversely, direct wage discrimination may not lead to significant stratification if groups are distributed equally among higher and lower paid occupations.

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The main contribution of this paper is to propose a class of stratification indices that depend in general on both the extent to which groups form well-defined layers or strata in the distribution of some economic outcome and the scale of between-group differences in those outcomes, since both are necessary consequences of the process of stratification.[1] Our approach is based on the measurement of stratification in terms of the impact on between-group inequality (see Monti and Santoro, 2011; Milanovic and Yitzhaki, 2002), yielding indices that may be used to judge not only whether the overall level of stratification is higher in one population than another but can also be decomposed to yield unique estimates of the contribution of each pair of groups to overall stratification. In contrast, Yitzhaki and Lerman (1991) measure stratification in terms of the impact of overlapping on inequality within groups, proposing a set of group-specific indices that capture each group's stratification with respect to the rest of the population but fail to provide a measure of the overall degree of stratification between groups in the population. The closely related group-specific indices in Yitzhaki (1994) are decomposable as weighted sums of pairwise indices that measure the degree of overlapping of one group by another, but the asymmetry of the overlapping concept makes intepretation of the indices problematic in terms of evaluating the overall degree of stratification. For example, in a population composed of only two groups then the first group can form a distinct stratum even if the second does not, where this will be the case if all first group incomes are concentrated at a point in the support of the second group distribution.

The proposed class of indices are specified as population-weighted averages of the degree of stratification between all pairs of groups in the population of interest. Pairwise stratification is defined in turn as the product of an "identification index" and an "alienation function," where the terminology is borrowed from the analogous literature on polarization (see Duclos *et al.*, 2004; Esteban and Ray, 1994). The identification index captures the extent to which two groups constitute distinct strata in their combined income distribution and is defined as the difference in the probabilities that a randomly selected member of the more affluent group has a higher rather than a lower income than a randomly chosen member of the less affluent group. The alienation function is specified as a power function of the absolute difference in equally distributed equivalent (ede) incomes between the two groups, with this being set equal to one by definition if the value of the power or exponent is set equal to zero.

As with Foster-Greer-Thorbecke (FGT) poverty measures (Foster *et al.*, 1984), the choice of power determines the interpretation of the resultant indices. In particular, zeroth power indices provide "headcount" measures that reflect the odds that the richer of a randomly chosen pair of individuals will come from the more affluent group of which they are members. First power indices provide "stratification gap" measures that further take into account the depth of stratification as measured by the absolute differences in ede incomes between groups. Stratification gap indices have a simple graphical representation using familiar tools from stochastic dominance analysis, reducing to twice the between-group

[1]For expositional purposes we refer to "income stratification" though the measures are equally applicable to consumption, wealth or earnings.

absolute Gini coefficient in welfare levels if there is no overlapping of group income distributions. Finally, second and higher integer power indices measure alienation as convex functions of pairwise ede income gaps and are therefore also sensitive to the distribution of ede income gaps over pairs of groups.

The paper is organized as follows. The next section introduces the proposed class of stratification indices and discusses both the general properties of the class and the specific attributes of headcount, stratification gap and higher power indices. Section 3 provides an empirical illustration based on the Milanovic and Yitzhaki (2002) analysis of world inequality in 1993 by regions. The final section summarizes the contribution and offers some suggestions for further research.

2. Definition and Properties of the Class of Stratification Indices

We consider a population divided into $K \geq 2$ mutually exclusive and exhaustive groups that are ordered by ede income (as defined in (4) below) from the least to the most affluent group. The population and population share of group k $(k=1,\ldots K)$ are given as n_k and $p_k = n_k/N$ respectively, where $N = \sum_{k=1}^{K} n_k$ is the total size of the population. Let Y_k denote the income variable of group k with cumulative distribution function $F_k(y) = P(Y_k \leq y)$, density function $f_k(y)$ and inverse distribution or quantile function $F_k^{-1}(q)$.[2] The expected value, income share and ede income of group k are given as μ_k, s_k and θ_k respectively. The population distribution function is written as $F_u(y) = P(Y_u \leq y) = \sum_{k=1}^{K} p_k F_k(y)$, where $Y_u = Y_1 \cup Y_2 \ldots \cup Y_K$ is the union of all groups, with expected value $\mu_u = \sum_{k=1}^{K} p_k \mu_k$. The ranking of group k incomes in the group l and population distributions are given as $F_l(F_k^{-1}(q))$ and $F_u(F_k^{-1}(q))$ respectively, with corresponding mean (fractional) ranks $\bar{F}_{kl}$ and $\bar{F}_{ku}$. $\bar{F}_{kl} = \mathrm{P}(Y_k > Y_l)$ is the probability that the income of a random member of group k is greater than that of a random member of group l, where this is equal to the probability of transvariation (Gini, 1916; see Montanari, 2004) if group l is more affluent than group k. If two or more groups have identical ede incomes then they are ranked such that $\mathrm{P}(Y_l > Y_k) > 0.5 > \mathrm{P}(Y_k > Y_l)$ for all relevant pairwise comparisons, where this secondary criterion for ranking distributions will generate a transitive ordering if the probability relationship between the sub-set of groups exhibits mutual rank transitivity (see De Baets *et al.*, 2010).[3] Finally if the two distributions cannot be ranked on the basis of either criteria (e.g. if the two income distributions are identical) then the various indices to be considered below are invariant to the ordering of the groups, which is therefore chosen arbitrarily.

[2] $F_k(y)$ is assumed to be strictly increasing and continuous for notational convenience, implying that the probability of a randomly chosen member of group k having the same income as a randomly selected member of group l will have measure zero. The treatment of ties is discussed in the next subsection.

[3] Note that $\bar{F}_{kk} = P(Y_k > Y_k) = 0.5$ by definition. The need for the transitivity condition arises iff there are more than two groups with the same ede income given that $\mathrm{P}(Y_l > Y_k) > 0.5$ and $\mathrm{P}(Y_m > Y_l) > 0.5$ does not necessarily imply $\mathrm{P}(Y_m > Y_k) > 0.5$. The empirical significance of the issue is likely to be limited but the condition can always be checked should the need arise. Ranking groups in ascending order of average ranks in the overall distribution may not be sufficient to order groups that are distinguishable on a pairwise basis since $\bar{F}_{ku} = \bar{F}_{lu}$ does not imply $\mathrm{P}(Y_k > Y_l) = 0.5$.

2.1. *The measurement of pairwise stratification*

Pairwise stratification $S_{kl}(\upsilon, \alpha)$ between groups k and l is taken to depend in general on both the extent to which the two groups occupy well-defined strata in their combined income distribution and the scale of the between-group difference in ede incomes. Specifically, we define $S_{kl}(\upsilon, \alpha)$ as the product of an identification index I_{kl} and an alienation function $A_{kl}(\upsilon, \alpha)$:

$$S_{kl}(\upsilon, \alpha) = I_{kl} A_{kl}(\upsilon, \alpha); \quad k, l = 1, \ldots .K \tag{1}$$

where the specification and interpretation of I_{kl} and $A_{kl}(\upsilon, \alpha)$ are discussed in turn below.

The pairwise identification index I_{kl} in (1) is defined as:

$$\begin{aligned} I_{kl} &= \operatorname{sgn}(l-k)(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l)) \\ &= \operatorname{sgn}(l-k)((\mathrm{P}(Y_l > Y_k) + 0.5\mathrm{P}(Y_l = Y_k)) - (\mathrm{P}(Y_k > Y_l) + 0.5\mathrm{P}(Y_l = Y_k))) \\ &= \operatorname{sgn}(l-k)(1 - 2(\mathrm{P}(Y_k > Y_l) + 0.5\mathrm{P}(Y_l = Y_k))) \end{aligned} \tag{2}$$

$$\text{where}\ \operatorname{sgn}(l-k) = \begin{cases} 1 & \textit{if}\ l-k > 0 \\ 0 & \textit{if}\ l-k = 0 \\ -1 & \textit{if}\ l-k < 0 \end{cases} \tag{2a}$$

I_{kl} is thus equal to the signed difference in the probabilities that a randomly chosen member of group l will be better rather than worse off than a randomly selected member of group k, such that $I_{kk} = I_{ll} = 0$ by definition and the use of the sign function ensures that $I_{kl} = I_{lk}$ for all $k \neq l$. I_{kl} is defined for both continuous and discrete income distributions with the second line of (2) making explicit the treatment of ties in the case that $\mathrm{P}(Y_k = Y_l) \neq 0$, where this will be a real issue if the income variable is categorical rather than continuous. I_{kl} can always be computed from individual income data by the simple enumeration of cases with $n_k n_l$ comparisons between members of the two groups in total. If the income variable is continuous then $I_{kl} = (1 - 2\bar{F}_{kl})$ from the final line of (2), where $\bar{F}_{kl}$ can be calculated as the average fractional rank of group k incomes in the group income l distribution.

The index I_{kl} is equal to both the "economic distance ratio" D_0 in Dagum (1980) and the first-order "discrimination index" Δ_1 in Le Breton *et al.* (2008) if group l is the more affluent of the two groups, and has also been identified in this case with the "loss of between-group inequality due to overlapping" (Monti and Santoro, 2011). Its interpretation as an identification or classification index follows from the observation that if individuals from the two groups are randomly matched with each other then I_{kl} will reflect the success with which group identity can be correctly determined by assuming that the better off individual within each pair will be from the more rather than less affluent group (see Montanari, 2004). I_{kl} will take its maximum value of one if group identity can be determined with certainty by this rule, which will only be the case if the poorest member of the more affluent group is better off than any member of the less affluent group: not only will everyone from the more affluent group be among the richest people in the two groups but also all the richest people will be from the more affluent group. Conversely, I_{kl} will equal zero if the income distributions of the two groups are identical such that the pairwise identification rule is entirely uninformative of group identity: the richer person in any pair is equally likely to be from one group as the other if the two groups are indistinguishable in terms of incomes. I_{kl} can also be negative, which will be the case if the richer individual is more likely to be from the less rather than the more affluent group, taking its minimum value when all but one person in the more affluent group is worse off than everybody in the less affluent group.

The alienation function $A_{kl}(\upsilon, \alpha)$ in (1) is defined as:

$$A_{kl}(\upsilon, \alpha) = |\theta_l(\alpha) - \theta_k(\alpha)|^{\upsilon}; \quad \textit{for } \upsilon > 0; \quad 0 \leq \alpha \leq 1 \tag{3a}$$

$$A_{kl}(\upsilon, \alpha) = 1; \quad \textit{for } \upsilon = 0; \quad 0 \leq \alpha \leq 1 \tag{3b}$$

where the absolute difference in ede incomes $|\theta_l(\alpha) - \theta_k(\alpha)|$ provides a measure of the 'economic distance' between the two groups (Shorrocks, 1982), with Chakravarty and Dutta (1987) showing under certain mild restrictions that only positive multiples of this measure can reflect differences in the average welfare of the two groups. If $\upsilon > 0$ then $A_{kl}(\upsilon, \alpha)$ is given as a power function of $|\theta_l(\alpha) - \theta_k(\alpha)|$ and can thus be interpreted as an indicator of the degree of 'alienation' between the two groups, with the parameter υ reflecting the degree of aversion to differences in average welfare levels between groups. For example, if the difference in average welfare between two groups doubles then there will be 2^{υ} times the level of alienation between them. Alternatively, υ is the elasticity of alienation with respect to the average welfare gap, so that a 1 percent increase in the gap leads to a υ percent increase in between-group alienation. $A_{kl}(\upsilon, \alpha)$ is strictly increasing in $|\theta_l(\alpha) - \theta_k(\alpha)|$ if $\upsilon > 0$, with $A_{kl}(\upsilon, \alpha) = 0$ if ede incomes in the two groups are the same. If $\upsilon = 0$ then $A_{kl}(0, \alpha) = 1$ by definition and the scale of average welfare differences between groups is not of itself a matter of concern.

Ede incomes $\theta_g(\alpha)$ are in turn defined as generalized or α−order means:

$$\theta_g(\alpha) = \mu_g^{\alpha} = \left(\frac{\sum_{i=1}^{n_g} y_{ig}^{\alpha}}{n_g} \right)^{1/\alpha} \quad \textit{for } 0 < \alpha \leq 1; \quad g = k, l \tag{4a}$$

$$\theta_g(\alpha) = \mu_g^{\alpha} = \left(\prod_{i=1}^{n_g} y_{ig} \right)^{1/n_g} \quad \textit{for } \alpha = 0; \quad g = k, l \tag{4b}$$

where α may be interpreted as the Atkinson (1970) inequality aversion parameter. Blackorby *et al.* (1981) strongly advocate the use of generalized means as measures of ede incomes, with Foster and Szekely (2008) showing them to be the only class of representative income indices that satisfy a basic set of properties including subgroup consistency. Thus, if the population sizes of the groups are held constant, overall population ede income must rise when ede income rises in one group and does not fall in the rest. Given (4), a transfer of income from someone in the more affluent group to someone in the less affluent group must increase $|\theta_l(\alpha)-\theta_k(\alpha)|$ and hence the level of alienation if $\upsilon > 0$. In particular, if $\upsilon=1$ and $\alpha=1$ then alienation is, as in Esteban and Ray (1994), equal to the absolute difference in mean incomes.

The parametric class of measures $S_{kl}(\upsilon,\alpha)=I_{kl}A_{kl}(\upsilon,\alpha)$ thus gives analysts and policymakers an instrument to evaluate stratification with varying sensitivity to distributional issues depending on social preferences. In particular, there seems no reason to believe that aversion to individual income inequality and to groupwise alienation will necessarily be the same so α and υ are treated as independent parameters. For example, income differences among men may be acceptable to the extent that these reflect differential rewards for effort, whereas those between men and women might not as gender is a matter of circumstance. $S_{kl}(\upsilon,\alpha)$ is symmetric in that $S_{kl}(\upsilon,\alpha)=S_{lk}(\upsilon,\alpha)$ but it is nevertheless sensitive to the ordering of groups by the chosen measure of ede income $\theta_g(\alpha)$ given the definition of I_{kl} in (2), providing a 'directional' measure in the sense of Dagum (1997).

2.2. *Definition and general properties of the class of stratification indices*

The proposed class of stratification indices $S(\upsilon,\alpha)$ is obtained as a population-weighted average of the pairwise indices $S_{kl}(\upsilon,\alpha)$:

$$S(\upsilon,\alpha)=\sum_{k=1}^{K}\sum_{l=1}^{K}p_k p_l S_{kl}(\upsilon,\alpha)=\sum_{k=1}^{K}\sum_{l=1}^{K}p_k p_l I_{kl}A_{kl}(\upsilon,\alpha) \tag{5}$$

where $p_k p_l$ is the probability that the first of two individuals randomly selected with replacement from the population will be from group k and the second from group l, and which therefore sum to one over all possible combinations.

$S(\upsilon,\alpha)$ will take a value of zero if pairwise stratification between all pairs of groups is zero, although this does not necessarily imply that all groupwise income distributions are identical. The overall level of stratification $S(\upsilon,\alpha)$ is strictly increasing in the pairwise indices $S_{kl}(\upsilon,\alpha)$, which provide unique estimates of the contribution of each distinct pair of groups to overall stratification. Moreover, the pairwise indices may be meaningfully aggregated, given symmetry, to yield estimates $S_k(\upsilon,\alpha)=p_k\sum_{l=1}^{K}p_l S_{kl}(\upsilon,\alpha)$ of the contribution of each group to overall stratification.

$S(\upsilon,\alpha)$ is increasing in pairwise identification I_{kl}, but identification is inherently a characteristic of groups so the impact on I_{kl} of any particular change in individual incomes will inevitably depend on the configuration of groups in the population (see Esteban and Ray (1994) for further discussion). Consider a population consisting of two or more groups with symmetric, unimodal welfare densities with compact supports $f_k(y^\alpha)$ and corresponding ede incomes μ_k^α, where these will correspond to income densities and mean incomes for the sub-class of indices with $\alpha=1$, i.e. $S(\upsilon,1)$. We note that a symmetric, ede income-preserving 'squeeze' in the

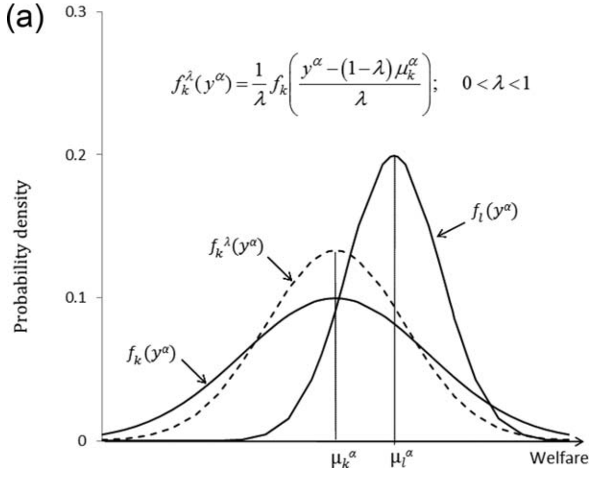

Figure 1a. Illustration of ede income-preserving squeeze of group k welfare distribution

welfare distribution of one group, say from $f_k(y^\alpha)$ to $f_k^\lambda(y^\alpha)$ as shown in Figure 1a, cannot reduce identification and hence stratification. In contrast, a reduction in within-group variation holding between-group differences constant will lead to a fall in inequality according to the Pigou-Dalton transfer principle.

Figure 1b offers a graphical proof of the identification property that makes use of the concept of a "λ-squeeze" defined in Duclos *et al.* (2004),[4] although the argument holds more generally for any symmetric 'squeeze' operator applied to $f_k(y^\alpha)$. Let $f_k^\lambda(y^\alpha)=\{f_k((y^\alpha-(1-\lambda)\mu_k^\alpha)/\lambda)\}/\lambda$ where $0<\lambda<1$, then $f_k^\lambda(y^\alpha)$ has the same mean μ_k^α as $f_k(y^\alpha)$ but is second-order stochastically dominant. Hence, in Figure 1b, $F_k^\lambda(y^\alpha)>F_k(y^\alpha)$ if $y^\alpha>\mu_k^\alpha$ and vice versa, where the absolute difference between the two distribution functions is symmetric about μ_k^α by construction. Given that the contribution of group k to overall stratification is $S_k(\upsilon,\alpha)=p_k\sum_{l=1}^{K}p_lS_{kl}(\upsilon,\alpha)$, $S(\upsilon,\alpha)$ will not fall due to this λ-squeeze if the degree of identification of group k does not fall with respect to either more or less affluent groups. We demonstrate that I_{kl} will not fall if the reference group l is at least as affluent as group k, with extension to the opposite case immediate given symmetry of the welfare distributions.[5] Specifically, for $(\mu_l^\alpha-\mu_k^\alpha)\geq 0$, we need to show that $I_{kl}^\lambda-I_{kl}=2\int_0^\infty[F_k^\lambda(y^\alpha)-F_k(y^\alpha)]f_l(y^\alpha)\,\partial y^\alpha\geq 0$, where $I_{kl}^\lambda-I_{kl}$ is a weighted sum of the distributional differences at each welfare level with weights given by the group l welfare density $f_l(y^\alpha)$. Consider first the limiting case $\mu_l^\alpha=\mu_k^\alpha$ then $I_{kl}^\lambda-I_{kl}=0$ since the weights $f_l(y^\alpha)$ will be symmetric about the common level of ede

[4]Note that Duclos *et al.* (2004) apply the 'λ-squeeze' to so-called 'basic densities' that would be fully identified in our framework even before the application of the operator because they are assumed to have disjoint supports.

[5]Le Breton *et al.* (2008) seek to establish an analogous relationship between second-order stochastic dominance and second-order discrimination (i.e. identification in our terminology) but only manage to show that it will hold if the density of the reference function $f_l(y^\alpha)$ is decreasing over the entire support of $F_k(y^\alpha)$, implying that the group l distribution must be positively skewed with mode of zero.

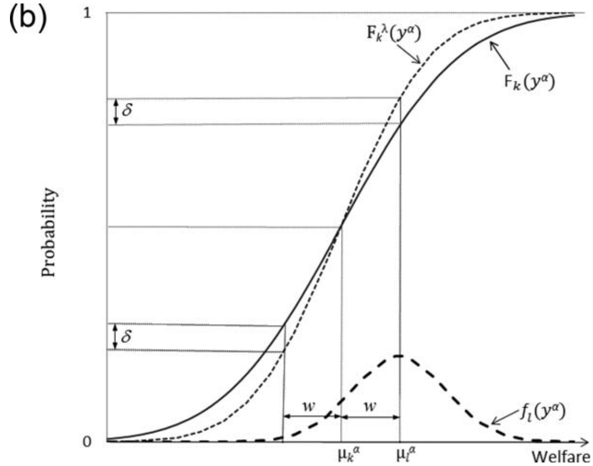

Figure 1b. Graphical proof of the identification property

income, with both $I_{kl}^{\lambda}=0$ and $I_{kl}=0$. For $\mu_l^\alpha > \mu_k^\alpha$, the mode of $f_l(y^\alpha)$ will lie to the right of μ_k^α, as shown in the diagram, and we can proceed as follows. First note that $f_l(y^\alpha)$ is strictly increasing over the range $\mu_k^\alpha \pm (\mu_l^\alpha - \mu_k^\alpha)$ so $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for any pair of points $\mu_k^\alpha \pm w$ with $0 < w \leq (\mu_l^\alpha - \mu_k^\alpha)$. Moreover $f_l(y^\alpha)$ is symmetric about μ_l^α so $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for any pair of points $\mu_k^\alpha \pm w$ for which $w > (\mu_l^\alpha - \mu_k^\alpha)$. Hence we can conclude that $I_{kl}^{\lambda} - I_{kl} > 0$ since $f_l\left(\mu_k^\alpha + w\right) > f_l\left(\mu_k^\alpha - w\right)$ for all possible w.

$S(\upsilon, \alpha)$ is also increasing in alienation between groups $A_{kl}(\upsilon, \alpha)$ if $\upsilon > 0$. Given that $A_{kl}(\upsilon, \alpha)$ is homogeneous of degree υ in the difference in ede incomes, it is apparent that an identification-preserving scalar expansion of all welfare differences about the overall population ede income μ_u^α will unambiguously increase alienation in any population consisting of two or more groups. Figure 2 illustrates this alienation property, which captures the idea that stratification is an increasing function of the scale of between-group differences.

$S(\upsilon, \alpha)$ is invariant to the permutation of groups and to the replication both of the subpopulations within groups (holding the population shares of the groups constant) and of the groups (holding the subpopulations within each group constant). However, stratification is not independent of the partition of the population into groups. For any given set of K groups with fixed income distributions $F_k(y)$, $S(\upsilon, \alpha)$ will be maximized if the population is equally divided between the two groups with the largest pairwise index $S_{kl}(\upsilon, \alpha)$, where this pair will typically consist of the most and least affluent groups in the population although this need not always be the case.

The dominance properties of $S(\upsilon, \alpha)$ are closely analogous to those of the Duclos *et al.* (2004) polarization measures, but identification in our approach depends on the extent to which group membership can be determined from individuals' ranks within the income distribution, rather than being a function of relative frequencies within income classes or at particular levels of income. This

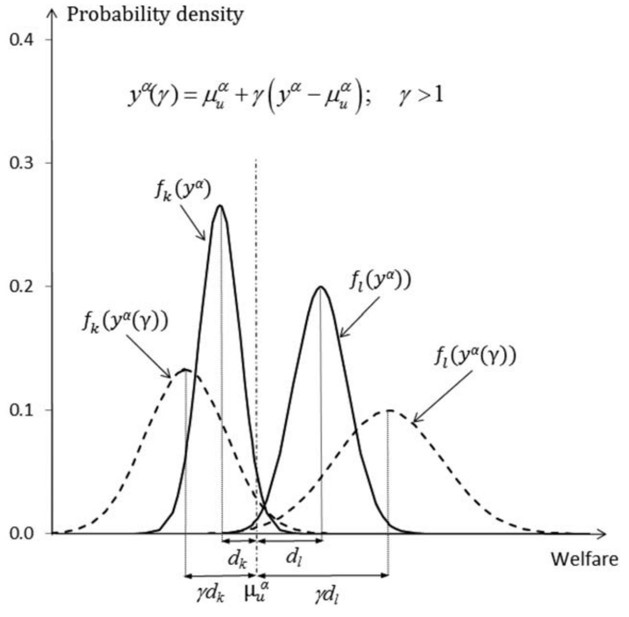

Figure 2. Illustration of alienation axiom

difference fundamentally distinguishes our measurement of income stratification between a set of exogenously classified groups from that of income polarization whether with or without predetermined groups.

2.3. *Properties of headcount stratification indices $S(0, \alpha)$*

The zeroth power member of the class, $S(0, \alpha)$, may be re-written from (5) as:

$$(6) \quad S(0,\alpha) = \sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l I_{kl} = \sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l \,\mathrm{sgn}\,(l-k)(\mathrm{P}(Y_l > Y_k) - \mathrm{P}(Y_k > Y_l));$$

where $S(0, \alpha)$ is written as a function of α since the value of the index is dependent on the ordering of groups by ede income $\theta_g(\alpha)$. $S(0, \alpha)$ is a unit-free measure that is invariant to affine transformations of individual welfare levels.[6]

$S(0, \alpha)$ is a population-weighted average of the pairwise identification indices I_{kl} and may therefore be interpreted as a headcount or incidence measure of stratification. The maximum value of $S(0, \alpha)$ is $\left(1 - \sum_{k=1}^{K} p_k^2\right)$, since $I_{kk} = 0$ by definition for all k. In this case there is perfect stratification in the sense of Lasswell (1965), with members of any particular group restricted to a single interval or range of ranks in the population income distribution that is exclusively occupied by members of their own group. Conversely $S(0, \alpha) = 0$ if group membership is entirely uninformative as a predictor of relative rank in which case $I_{kl} = 0$ for all pairs of groups, though a zero

[6] $S(0, \alpha)$ is not in general invariant to order-preserving transformations of individual welfare levels because these can have an effect on identification through the ordering of groups by ede incomes.

value may also arise in cases in which positive and negative values of the pairwise indices cancel each other out. Negative values of $S(0,\alpha)$ are also possible if, for example, the income distributions of some groups are bimodal although such a result might suggest that one or more of the groups were composite in nature.[7]

Dividing $S(0,\alpha)$ by $\left(1-\sum_{k=1}^{K} p_k^2\right)$ yields a normalized index $\tilde{S}(0,\alpha)$ that is the average of the pairwise identification indices between all distinct groups with a maximum value of one. In particular, $\tilde{S}(0,\alpha)=I_{12}$ if the population is composed of only two groups. Zhou (2012) has independently proposed a stratification measure S_{ZHOU} that is identical to $\tilde{S}(0,\alpha)$ except that groups are ordered by $\bar{F}_{ku}$ alone on the assumption of no ties between groups. Zhou defends his choice of measure on the grounds that it is invariant to all rank-preserving transformations of income but this is achieved by conflating the determination of the hierarchical ordering of groups with the measurement of the degree of identification between them given that $\bar{F}_{ku}=\sum_{l=1}^{K} p_l \bar{F}_{kl}$.[8] In our view these are independent steps with ede incomes providing a more compelling primary criterion for the establishment of the relative economic standing of groups (Chakravarty and Dutta, 1987), with the pairwise comparison of ranks in the case of tied groups.

A small change in the welfare of an individual that leads to a change in the ordering of groups by ede incomes may give rise to a discontinuous change in $S(0,\alpha)$, where this property is similar to the discontinuity of headcount poverty at the poverty line. With only two groups, the reduction in headcount stratification $S(0,\alpha)$ caused by a unit increase in the welfare of one person would be greatest for members of the less affluent group with incomes equal to the modal welfare level in the more affluent group holding the ordering of groups constant. With more than two groups, it is readily apparent that increasing the welfare of the least affluent group, let alone the welfare of the poorest members of that group, may not necessarily have the most impact on headcount stratification: indeed $S(0,\alpha)$ is invariant to changes in the welfare of individuals in the least affluent group whose welfare is less, and remains less, than that of any person in any other group.

2.4. *Properties of stratification gap indices* $S(1,\alpha)$

The first power member of the class, $S(1,\alpha)$, may be re-written from (5) as:

$$
\begin{aligned}
S(1,\alpha) &= \sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l A_{kl}(1,\alpha) I_{kl} \\
&= \sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l |\mu_l^{\alpha}-\mu_k^{\alpha}| \operatorname{sgn}(l-k)(\mathrm{P}(Y_l > Y_k)-\mathrm{P}(Y_k > Y_l)) \qquad (7) \\
&= S(0,\alpha)\bar{D}(\alpha)+\operatorname{cov}(A_{kl}(1,\alpha), I_{kl})
\end{aligned}
$$

[7]For example, consider a population of size $N=14$ consisting of three groups k, l, and m with incomes $Y_k=\{1, 7, 7\}$; $Y_l=\{6, 6, 6, 6\}$; and $Y_m=\{2, 2, 2, 2, 16, 16, 16\}$ such that $\mu_k < \mu_l < \mu_m$ since $\mu_k=5$, $\mu_l=6$ and $\mu_m=8$. Hence $I_{kl}=-1/3$, $I_{km}=5/21$ and $I_{lm}=-1/7$, since $\mathrm{P}(Y_k > Y_l)=2/3$, $\mathrm{P}(Y_k > Y_m)=8/21$. and $\mathrm{P}(Y_l > Y_m)=4/7$, to give $S(0,1)= -3/98$.

[8]Zhou further conjectures that $S_{ZHOU} \geq 0$ but the previous footnote example yields $S_{ZHOU}=\tilde{S}(0,1)= -3/61$ given that $\bar{F}_{ku} < \bar{F}_{lu} < \bar{F}_{mu}$ since $N\bar{F}_{ku}=7\tfrac{1}{3}$, $N\bar{F}_{lu}=7\tfrac{1}{2}$ and $N\bar{F}_{mu}=7\tfrac{4}{7}$.

where $\bar{D}(\alpha)=\sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l|\mu_l^\alpha-\mu_k^\alpha|$ is the population mean ede income gap and $\mathrm{cov}\,(A_{kl}(1,\alpha),I_{kl})=\sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l\left(|\mu_l^\alpha-\mu_k^\alpha|-\bar{D}(\alpha)\right)(I_{kl}-S(0,\alpha))$ is the population covariance between pairwise levels of alienation and identification which will typically be positive. $S(1,\alpha)$ has the same units as income and is invariant to translations of the welfare measure.

$S(1,\alpha)$ reflects not only the incidence but also the depth of stratification and may therefore be interpreted as a 'stratification gap' measure. For example, the lack of overlap between a rich and a poor group will count more towards the 'stratification gap' as measured by $S(1,\alpha)$ than the same lack between two moderately afluent groups: in the limit, two groups with identical ede incomes will not contribute to $S(1,\alpha)$ irrespective of the degree of pairwise identification.

Let $\bar{Y}_g$ be the smoothed income variable obtained by assigning to each individual in the population the mean income of the group to which they belong, with distribution function $F_g(y)$. It follows from Allanson (2014) that $S(1,1)=4\mathrm{cov}\,(\bar{Y}_g,F_u(y))=2\mu_u G_b$, where G_b is the Yitzhaki and Lerman (1991) measure of between-group inequality. Moreover if there is no overlapping of the groupwise distributions of individual incomes $F_k(y)$ then $S(1,1)=4\mathrm{cov}\,(\bar{Y}_g,F_g(y))=\sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l|\mu_l-\mu_k|=\bar{D}(1)=2\mu_u G_B$, where G_B is the conventional between-group Gini index (see, e.g. Mookherjee and Shorrocks, 1982). These correspondences suggest a graphical interpretation of $S(1,1)$ based on the generalized concentration curve for $\bar{Y}_g$ with respect to individual incomes Y_u and the corresponding generalized Lorenz curve, $GC(\bar{Y}_g)$ and $GL(\bar{Y}_g)$ respectively. Figure 3 plots $GC(\bar{Y}_g)$ and $GL(\bar{Y}_g)$ as the cumulative mean smoothed income (i.e. cumulated smoothed income divided by the total population) of the first 100q percent of people when ranked from poorest to richest in the individual and smoothed income distributions respectively, with $S(1,1)$ equal to four times the area A if $GC(\bar{Y}_g)$ lies everywhere below the line of equality and where $GC(\bar{Y}_g)=GL(\bar{Y}_g)$ in the absence of overlapping. More generally, $S(1,\alpha)$ is simply twice the generalized concentration index of the smoothed distribution $\bar{Y}_g^\alpha$ obtained by assigning to each individual in the population the ede income μ_g^α of the group to which they belong, i.e. $S(1,\alpha)=2\mu_u^\alpha G_b^\alpha=4\mathrm{cov}\,(\bar{Y}_g^\alpha,F_u(Y_u))$.

Normalizing each ede income gap by the population-weighted mean ede income gap $\bar{D}(\alpha)$ yields a class of standardized stratification gap measures:

$$\tilde{S}(1,\alpha)=\sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l\left(\frac{A_{kl}(1,\alpha)}{D(\alpha)}\right)I_{kl}=\sum_{k=1}^{K}\sum_{l=1}^{K}\frac{p_k p_l|\mu_l^\alpha-\mu_k^\alpha|}{\sum_{k=1}^{K}\sum_{l=1}^{K} p_k p_l|\mu_l^\alpha-\mu_k^\alpha|}I_{kl}$$

$$\equiv\sum_{k=1}^{K}\sum_{l=1}^{K} w_{kl}I_{kl}; \tag{8}$$

where the weights w_{kl} are non-negative and sum to unity. Thus $\tilde{S}(1,\alpha)$ may be interpreted as a weighted average identification index like $S(0,\alpha)$ but with pairwise weights equal to shares in the total ede income gap $N\bar{D}(\alpha)$ rather than in the population N. Like $S(0,\alpha)$, $\tilde{S}(1,\alpha)$, is invariant to affine transformations of

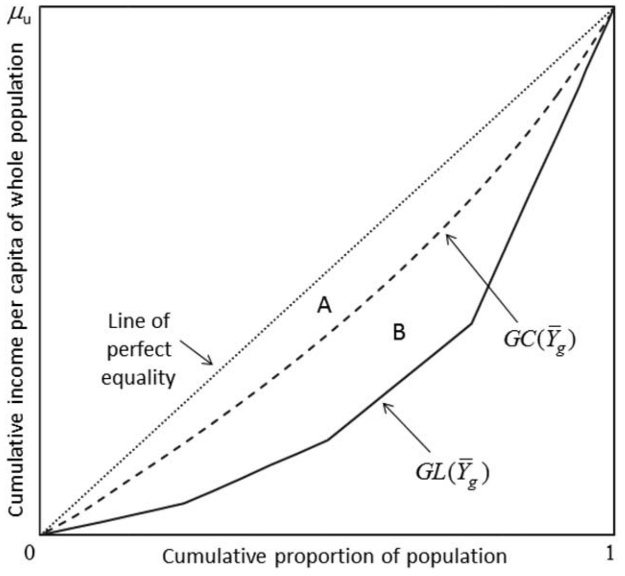

Figure 3. Graphical representation of $\boldsymbol{S}(\mathbf{1},\mathbf{1})$

welfare and also to the replication of population by the replication of groups. Allanson (2014) has previously identified $\tilde{S}(1,1)$ as the ratio of G_b to G_B, generalizing the result in Monti and Santoro (2011) to two or more groups, with Heller and Yitzhaki (2006) interpreting this ratio as a measure of the "quality of identification" achieved in the classification of individual groups by means of some continuous characteristic. Hence $\tilde{S}(1,1)$ is equal to the ratio of area A to (A+B) in Figure 3.

$S(1,\alpha)$ is continuous since $A_{kl}(1,\alpha)$ tends to zero for any pair of groups as the difference in ede incomes between them tends to zero. Increasing the welfare of members of the least affluent group will have the most impact on the mean ede income gap $\bar{D}(\alpha)$ but not necessarily on the stratification gap $S(1,\alpha)$ as this will also depend on the levels of and resultant changes in pairwise identification between groups. For the specific index $S(1,1)$, the minimum cost of eliminating alienation through a policy of group-specific lump sum transfers will be equal to $\sum_{k\neq K} n_k(\mu_K-\mu_k)$ if transfers were perfectly targeted, i.e. the sum over all but the richest group of the product of group size and the mean income gap with the richest group.

2.5. *Properties of $S(\upsilon,\alpha)$ with $\upsilon > 1$*

All indices $S(\upsilon,\alpha)$ with $\upsilon > 1$ have alienation functions that are convex functions of pairwise ede income gaps and are therefore directly sensitive to the distribution of gaps among pairs of groups. For example, consider a population consisting of three equal sized groups with $I_{12}=I_{23}$, i.e. the middle group is equally identified with respect to the two other groups. It then follows from Jensen's inequality that stratification will be minimized if $\mu_2^{\alpha}-\mu_1^{\alpha}=\mu_3^{\alpha}-\mu_2^{\alpha}$, i.e. the ede income of the middle group is also equidistant between those of the two other groups. By implication, stratification will be higher in this population the closer the ede income of the middle group to that of either the most or the least affluent group, holding identification constant.

Thus $S(\upsilon, \alpha)$ reflects not only the incidence and depth but also the severity of stratification if $\upsilon > 1$. In particular, if $\upsilon=2$ then the alienation function is equal to the squared ede income gap and one pair of groups with ede incomes twice as far apart as another pair will contribute four times as much to $S(2, \alpha)$ holding identification equal. Higher values of υ imply greater alienation aversion: in the limit as $\upsilon \rightarrow \infty$ then the value of the index will be dominated by the pairwise stratification between the most and least affluent groups, with the latter group—though not necessarily the poorest members of it—providing the most cost-effective target for an anti-stratification support policy.

3. Empirical Illustration

By way of illustration, this section follows Allanson (2014) in further elaborating the empirical analysis presented in Milanovic and Yitzhaki (2002) of world inequality by regions in 1993.[9] The top panel in Table 1 presents estimates from their Tables 4 and 7 of population shares, p_k; mean incomes, μ_k; and mean rankings in the income distributions of each region, $\bar{F}_{kl}=P(Y_k > Y_l)$, and the world $\bar{F}_{ku}=P(Y_k > Y_u)$. This shows that Africa was the poorest region in per capita terms followed by Asia; Eastern Europe and the Former Soviet Union (EFSU); Latin America and the Caribbean (LAC); and Western Europe/North America/Oceania (WENAO). However the mean rank of Africans in the Asian income distribution was 0.515, implying that an African chosen at random was likely to have been better off than a randomly chosen Asian, and the mean rank of Africans in the world distribution was also higher than that of Asians. Ranks for all other pairs of regions are consistent with the ordering of mean incomes.

The remaining panels show the constituent elements of the stratification indices as defined in (5), with the stratification indices themselves given in Table 2. Note that the population weights $p_k p_l$ reflect the relative frequencies of regional pairs and sum across columns to give the population shares p_k, with the sum of weights not on the leading diagonal $(1-\sum_{k=1}^{K} p_k^2)=0.603$. The pattern of pairwise identification indices I_{kl} and absolute mean income differences $|\mu_l-\mu_k|$ reveals that the regions of the world are broadly divided into three broad layers or strata—with Africa and Asia at the bottom, EFSU and LAC in the middle and WENAO on its own at the top of the world income distribution—where the degree of both identification and alienation between regions in the same layer was much lower than that between regions in different strata. Indeed, there was virtually no stratification of the African and Asian distributions in the bottom stratum

[9]These regions are referred to as “continents” in Milanovic and Yitzhaki (2002) though the correspondence is not exact.

TABLE 1
CONSTITUENT ELEMENTS OF INCOME STRATIFICATION CALCULATIONS

Column (l)	Pop^n share (%)	Mean income ($PPP)	Mean rank in income distribution of: Africa	Asia	EFSU	LAC	WENAO	World
Row (k)								
Africa	10.0	1310.0	0.500	0.515	0.275	0.261	0.049	0.407
Asia	59.5	1594.6	0.485	0.500	0.265	0.247	0.064	0.397
EFSU	7.8	2780.9	0.725	0.735	0.500	0.483	0.136	0.609
LAC	8.4	3639.8	0.739	0.753	0.517	0.500	0.172	0.629
WENAO	14.3	10012.4	0.951	0.936	0.864	0.828	0.500	0.861
World	**100.0**	**3031.8**						**0.500**
			Population weights: $p_k p_l$					Sum
Africa			0.010	0.060	0.008	0.008	0.014	0.100
Asia			0.060	0.354	0.046	0.050	0.085	0.595
EFSU			0.008	0.046	0.006	0.007	0.011	0.078
LAC			0.008	0.050	0.007	0.007	0.012	0.084
WENAO			0.014	0.085	0.011	0.012	0.020	0.143
World								**1.000**
			Pairwise identification indices: I_{kl}					Weighted mean
Africa			0	−0.030	0.450	0.478	0.902	0.186
Asia			−0.030	0	0.470	0.506	0.872	0.201
EFSU			0.450	0.470	0	0.034	0.728	0.431
LAC			0.478	0.506	0.034	0	0.656	0.445
WENAO			0.902	0.872	0.728	0.656	0	0.721
World								**0.312**
			Absolute mean income gaps: $\|\mu_l - \mu_k\|$					Weighted mean
Africa			0	284.6	1470.9	2329.8	8702.4	1722.3
Asia			284.6	0	1186.3	2045.2	8417.8	1494.8
EFSU			1470.9	1186.3	0	858.9	7231.5	1957.5
LAC			2329.8	2045.2	858.9	0	6372.6	2426.4
WENAO			8702.4	8417.8	7231.5	6372.6	0	6980.1
World								**2415.0**

Notes: Top panel.
Source: Milanovic and Yitzhaki (2002) Tables 4 and 7—see also Table 1 for the list of countries in each region (EFSU—Eastern Europe and Former Soviet Union; LAC—Latin America and Caribbean; WENAO—Western Europe, North America and Oceania). Other panels. Author's own calculations.

nor of the EFSU and LAC distributions in the middle layer, with pairwise identification indices close to zero and mean income differences less than $1000. In contrast, the WENAO income distribution was highly stratified from those of every other region, with the relevant pairwise identification indices ranging between 0.656 and 0.902 and all mean income differences greater than twice the mean world income level of $3000. All other cross-pairwise measures were intermediate with the population-weighted mean identification index and mean income gap equal to 0.312 and $2415 respectively: in particular, the pairwise identification of the EFSU and Asian distributions was above average even though the mean income difference between the two regions was not much larger than that between EFSU and LAC.

The top panel of Table 2 reports the headcount index $S(0, 1)$, which is equal to the population-weighted mean identification index reported in Table 1. Thus the

TABLE 2
INCOME STRATIFICATION BETWEEN REGIONS OF THE WORLD

	Africa	Asia	EFSU	LAC	WENAO	Sum	*Share*
Headcount stratification							
Africa	0	−0.0018	0.0035	0.0040	0.0129	0.019	6.0%
Asia	−0.0018	0	0.0218	0.0254	0.0740	0.119	38.2%
EFSU	0.0035	0.0218	0	0.0002	0.0081	0.034	10.8%
LAC	0.0040	0.0254	0.0002	0	0.0079	0.038	12.0%
WENAO	0.0129	0.0740	0.0081	0.0079	0	0.103	33.0%
S(0, 1)						**0.312**	
Stratification gap $PPP							
Africa	0	−0.5	5.2	9.4	112.3	126.4	6.7%
Asia	−0.5	0	25.8	51.9	623.1	700.3	37.4%
EFSU	5.2	25.8	0	0.2	58.6	89.8	4.8%
LAC	9.4	51.9	0.2	0	50.3	111.8	6.0%
WENAO	112.3	623.1	58.6	50.3	0	844.2	45.1%
S(1, 1)						**1872.5**	
Standardised stratification gap							
Africa	0	−0.0002	0.0021	0.0039	0.0465	0.052	6.7%
Asia	−0.0002	0	0.0107	0.0215	0.2580	0.290	37.4%
EFSU	0.0021	0.0107	0	0.0001	0.0242	0.037	4.8%
LAC	0.0039	0.0215	0.0001	0	0.0208	0.046	6.0%
WENAO	0.0465	0.2580	0.0242	0.0208	0	0.350	45.1%
$\tilde{\boldsymbol{S}}(\mathbf{1},\mathbf{1})$						**0.775**	
*Squared stratification gap ($PPP/1000)*^{2}							
Africa	0.0	−0.0001	0.0076	0.0219	0.9774	1.007	7.1%
Asia	−0.0001	0.0	0.0307	0.1061	5.2449	5.382	37.7%
EFSU	0.0076	0.0307	0.0	0.0002	0.4234	0.462	3.2%
LAC	0.0219	0.1061	0.0002	0.0	0.3206	0.449	3.1%
WENAO	0.9774	5.2449	0.4234	0.3206	0.0	6.966	48.8%
S(2, 1)						**14.265**	

Source: Author's own calculations.

difference in the probabilities that the richer of two randomly chosen individuals will come from the richer rather than the poorer region of which they are inhabitants was equal to 0.312, or 0.518=0.312/0.603 conditional on the two individuals being from different regions. It follows from (1) that the population-weighted mean probability of transvariation (between distinct regions) was equal to 0.241=(1 − 0.518)/2, i.e. there was a roughly one in four chance that a randomly chosen individual from a poorer region would be better off than a randomly chosen individual from a richer region. The pairwise decomposition shows that the overall level of identification was mainly driven by the existence of the largely separate WENAO stratum at the top of the world income distribution, with the Asia/WENAO pair alone contribute nearly half (0.474=(0.074 + 0.074)/0.312) of the total value of $S(0, 1)$ as a result of the populousness of the two regions and the low degree of overlap between their income distributions. At the other extreme, the EFSU/LAC and Africa/Asia pairs made a negligible contribution to the total due to the lack of pairwise identification of their income distributions, with the negative value for the latter arising because the probability of transvariation between the two regions, i.e. $P\left(Y_{Africa} > Y_{Asia}\right)$, was greater than 0.5.

The second panel reports the stratification gap index $S(1,1)$, which may loosely be interpreted as a measure of the perceived average difference in mean incomes between regions based on individuals' actual positions in the world income distribution. Thus the stratification gap of \$1873 may be compared to the mean income gap $\bar{D}(1)$ of \$2415 reported in Table 1, with the difference of \$542 reflecting the imperfect identification of regions in the world income distribution. Alternatively, following Milanovic and Yitzhaki (2002), this difference represents twice the loss of absolute between-group inequality due to the overlapping of regional income distributions since $\bar{D}(1) = 2\mu_u G_B$ and $S(1,1)=2\mu_u G_b$. In comparison to $S(0,1)$, WENAO accounts for an even larger share of the total value of the index as a result of the above-average mean income differences between WENAO and every other region in the world. Conversely the shares of the "middle income" regions, EFSU and LAC, fall particularly sharply as a result of their intermediate position in the world income distribution and correspondingly smaller mean income differences with other regions.

The standardized stratification gap index $\tilde{S}(1,1)$ reported in the next panel was 0.775. Like $S(0,1)$, $\tilde{S}(1,1)$ may be interpreted as a weighted average identification index but with total income gap rather than population weights. Given that $\tilde{S}(1,1)=S(0,1)+\mathrm{cov}\,(A_{kl}(1,1)/\bar{D}(1), I_{kl})$, the larger value of $\tilde{S}(1,1)$ reflects the positive correlation between pairwise mean income gaps and identification indices, i.e. region pairs that formed more clearly defined regional strata in their combined income distribution also tended to have had larger differences in mean incomes. The value of the index may also be identified, following Allanson (2014), as the ratio of G_b to G_B, with 0.775=1873/2415. The pairwise decomposition is identical to that given in Allanson (2014) and yields the same relative contributions as those for $S(1,1)$.

The final panel reports the squared stratification gap $S(2,1)$, with the value of 14.3 million implying a root mean squared stratification gap of \$3782. The squared measure puts greater weight on the larger income gaps compared to $S(1,1)$ leading, as expected, to increases in the relative contributions of the regions at the top and bottom of the world income distribution—WENAO, Africa and Asia – at the expense of those in the middle—EFSU and LAC. Higher power indices (i.e. with $\upsilon > 2$) would place increasingly greater weight on the relative contributions of the regions at the top and bottom of the world income distribution, with the pairwise stratification between the poorest and richest regions dominating the value of the index in the limit.

Overall the various indices all portray a broadly similar picture of the pattern of stratification given that the correlation coefficient between the pairwise identification indices and mean income gaps was equal to 0.735. We have argued that stratification necessarily results in both pairwise identification and alienation so this positive correlation is to be expected although the strength of the association will likely differ depending on the specific context. Recalling that a ceteris paribus increase in within-group inequality will (typically) reduce stratification, the combination in some poorer Asian countries, most notably China and India, of high per capita growth rates and the emergence of prosperous middle classes may be expected to have reduced overall levels of both alienation and identification between regions in more recent years.

4. Conclusion

This paper offers a new class of parametric indices that is based on a conceptualization of stratification as a process that results in a hierarchical ordering of groups and therefore seeks to capture not only the extent to which groups form well-defined layers or strata in the income distribution but also the scale of the resultant differences in ede incomes between them. The indices provide measures of the overall degree of stratification between two or more groups in a population, where the dominance properties of the indices are similar to those of the Duclos *et al*. (2004) polarization measures. First the identification property distinguishes stratification from inequality since an ede income-preserving "squeeze" in the welfare distribution of one group cannot reduce identification under certain specified conditions whereas it will lead to a fall in inequality according to the Pigou-Dalton transfer principle. More straightforwardly, an identification-preserving scalar expansion of all welfare differences about the overall population ede income will unambiguously increase alienation between groups. Finally stratification will typically be maximized if the population is equally divided between the most and least affluent groups. However it is important to recognize that stratification is not the same as polarization due to the fundamentally different characterizations of identification employed in the two sets of measures, with an axiomatic derivation of the proposed class of stratification measures remaining a topic for further research. The link between the stratification gap index and the generalized concentration index further suggests that it may be possible to establish welfare foundations for at least some members of the new class of indices.

The proposed class of measures benefit from their ease of interpretation and practical utility. In particular, the headcount and gap indices reflect the incidence and depth of stratification: the former reflects the odds that the richer of any randomly chosen pair of individuals is a member of the more affluent group from which they are drawn, while the latter may be interpreted as a measure of the perceived average difference in ede incomes between groups based on individuals' actual positions in the overall income distribution. Each index is a population-weighted average of pairwise indices so it is possible to judge not only whether the overall level of stratification is higher in one population compared to another but also to estimate the contribution of individual groups to observed levels of overall stratification, with the further potential to identify the characteristics or factors that contribute to stratification. Reporting a range of measures rather than just one enables a fuller characterization of the nature of stratification as shown by the illustrative study of global stratification in this paper. Estimation and inference procedures remain an issue for future work, with the estimation techniques set out in Frick *et al*. (2006) offering one possible approach based on U-statistics. Given suitable procedures, it would be of interest to examine changes in global stratification over time as well as consider applications to a range of other socioeconomic phenomena such as the racial wage hierarchy in South Africa and gender pay differentials in earnings.

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