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Class - X
Board - CISCE

Subject - Maths
Chapter - Ratio and Proportion and A-P
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Rohiet
25/30

Q1> A.P = 8, 13, 18, 23, ...
$a = 8$
$d = 13 - 8 = 5$
Let the n^{th} term = 402
$\therefore a + (n-1)d = 402$
$\Rightarrow 8 + (n-1)5 = 402$
$\Rightarrow 8 + 5n - 5 = 402$
$\Rightarrow 5n = 402 - 3$
$\Rightarrow n = \frac{399}{5}$

Since n is a fraction, 402 is not the term of the A.P

1

Q2> A.P = 254, ..., 14, 9, 4
$a = 254$
$d = \cancel{14 - 9 =}\ 9 - 14 = -5$
$\therefore t_{10} = a + (n-1)d$
$= 254 + (10-1)(-5)$
$= 254 - 45$
$\therefore \boxed{t_{10} = 209}$
The tenth term from the end is 209.

1

$254 - 45 = 209$

$24 \times 7 = 168$

$168 + 46 = 214$

Q3> A.P = 4, 11, 18, 25, ...
$a = 4$
$d = 11 - 4 = 7$
let the n^{th} term = 42 + t_{25}
$\Rightarrow t_n = 42 + a + (n-1)d$
$= 42 + 4 + (25-1) 7$
$= 42 + 4 + 168$
$\therefore \boxed{t_n = 214}$

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The 214th term is 42 more than it's 25th term.

Q4> Given:-

$t_2 + t_7 = 30$

$\Rightarrow a+(n-1)d + a+(n-1)d = 30$

$\Rightarrow 2a + (2-1)d + (7-1)d = 30$

$\Rightarrow 2a + d + 6d = 30$

$\Rightarrow 2a + 7d = 30$ — ①

$t_{15} = 2t_8 - 1$

$\Rightarrow a+(n-1)d = 2[a+(n-1)d] - 1$

$\Rightarrow a + (15-1)d = 2[a+(8-1)d] - 1$

$\Rightarrow a + 14d = 2[a+7d] - 1$

$\Rightarrow a + \cancel{14d} = 2a + \cancel{14d} - 1$

$\Rightarrow 1 = 2a - a$

$\therefore a = 1$

Substituting the value of a in ①

$2 + 7d = 30$

$\Rightarrow 7d = 30 - 2$

$\Rightarrow d = \frac{\cancel{28}\,4}{\cancel{7}\,1}$

$\therefore$ ~~d=4~~ $d = 4$

$\therefore$ $\boxed{A.P = 1, \cancel{1+4}\ 5, 9, \ldots\ldots}$

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Q5>

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Q5> Given:-

$S_{\cancel{n}7} = 49$

$S_{17} = 289$

$\therefore S_7 = 49$

$\Rightarrow \frac{n}{2} \times [2a + (n-1)d] = 49$

$\Rightarrow \frac{\cancel{7}^{7}}{2} \times [2a + (7-1)d] = 49$

$\Rightarrow \frac{\cancel{n}^{7}}{2} \times [2a + 6d] = 49$

$\Rightarrow 2a + 6d = \frac{\cancel{49}^{7} \times 2}{\cancel{7}_{1}}$

$\Rightarrow 2a + 6d = 14$

Dividing both sides by 2

$a + 3d = 7$ — (1)

$S_{17} = 289$

$\Rightarrow \frac{n}{2} \times [2a + (n-1)d] = 289$

$\Rightarrow \frac{17}{2} \times [2a + (17-1)d] = 289$

$\Rightarrow 2a + 16d = \frac{\cancel{289}^{17} \times 2}{\cancel{17}_{1}}$

$\Rightarrow 2a + 16d = 34$

Dividing both sides by 2

$a + 8d = 17$ — (2)

Subtracting (1) from (2)

$$\begin{array}{r} \cancel{a} + 8d = 17 \\ \cancel{a} + 3d = 7 \\ - \quad - \quad - \\ \hline 5d = 10 \end{array}$$

119

$\begin{array}{r} 17 \\ \times 7 \\ \hline 119 \end{array}$

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4

$\therefore\ d = 2$

Substituting the value of d in 1

$a + 6 = 7$

$\therefore\ a = 1$

The $S_n = \frac{n}{2} \times [2a + (n-1)d]$

$= \frac{n}{2} \times [2 + (n-1)(2)]$

$= \frac{n}{2} \times [\cancel{2} + 2n - \cancel{2}]$

$= \frac{n}{2} \times 2n$

$\therefore\ \boxed{S_n = n^2}$

2

Q6> $t_4 = 11$

$\therefore\ a + (n-1)d = 11$

$\Rightarrow a + (4-1)d = 11$

$\Rightarrow a + 3d = 11$ — 1

$t_8 = (2 \times t_4) + 5$

$\Rightarrow a + (n-1)d = [2 \times a + (n-1)d] + 5$

$\Rightarrow a + (8-1)d = [2 \times 11) + 5$

$\Rightarrow a + 7d = 22 + 5$

$\Rightarrow a + 7d = 27$ — 2

Subtracting 1 from 2

$\cancel{a} + 7d = 27$

$\cancel{a} + 3d = 11$

$-\quad - \quad -$

$4d = 16$

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$\therefore d = 4$

Substituting the value of d in 1

$a + 12 = 11$

$\therefore a = -1$

$\therefore \boxed{A.P = -1, 3, 7, \ldots}$

$S_{50} = \frac{n}{2} \times [2a + (n-1)d]$

$= \frac{50}{2} \times [2(-1) + (50-1)(4)]$

$= 25 \times [-2 + 196]$

$= 25 \times 194$

$\therefore \boxed{S_{50} = 48\cancel{10}}$

$1\frac{1}{2}$

Q 7) $A.P = 43, 39, 35, \ldots$

$a = 43$

$d = 39 - 43 = -4$

Let S_n be the sum of 252

$\Rightarrow S_n = 252$

$\Rightarrow \frac{n}{2} \times [2a + (n-1)d] = 252$

$\Rightarrow \frac{n}{2} \times [2(43) + (n-1)(-4)] = 252$

$\Rightarrow n \times [86 - 4n + 4] = 252 \times 2$

$\Rightarrow 90n - 4n^2 = 504$

$\Rightarrow 4n^2 - 90n + 504 = 0$

Dividing both sides by 2

$\Rightarrow 2n^2 - 45n + 252 = 0$

$\Rightarrow 2n^2 - 21n - 24n + 252 = 0$

$\Rightarrow$ ~~2n(n 2n(n-12)-21(n n(2n-21)-24~~

$2 \mid 504$, $2 \mid 252$, $2 \mid 126$, $3 \mid 63$, 21

3

$49 \times 4 = 196$

236

$194 \times 25 = 4810$

$43 - 39 = 4$

$252 \times 2 = 504$

$252 \times 2 = 504$

$504n^2$

$-21n - 24n$

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$2n(n-12)-21(n-12)=0$

$\Rightarrow (2n-21)(n-12)=0$

$\Rightarrow 2n \neq 21$ and $n=12$

n ~~must be~~

a cannot

be a

fraction

3

$\therefore \boxed{n=12 \text{ terms}}$

Q8> $t_1=-4=a$

$l=29$

$S=150$

$\therefore$ ~~$\frac{n}{2}$~~ $l=29$

$\Rightarrow a+(n-1)d=29$

$\Rightarrow -4+dn-d=29$

$\Rightarrow d(n-1)=33$

$\Rightarrow dn-d=33$

$\Rightarrow dn=33+a$

$\frac{1}{2}$

$S=150$

$\therefore$ ~~S~~ $\frac{n}{2}\times[2a+(n-1)d]=150$

$\Rightarrow \frac{n}{2}\times[-8+(n-1)d]=150$

$\Rightarrow n\times[-8+(n-1)d]=300$

$\Rightarrow -8n+dn^2-dn=300$

$\Rightarrow -8n+dn^2-33-d=300$

?

Q9) Let the number to be added $= x$

$\therefore 16+x : 7+x :: 79+x : 43+x$ (They are in proportion)

$\frac{16+x}{7+x} = \frac{79+x}{43+x}$

$\Rightarrow (16+x)(43+x) = (79+x)(7+x)$

$\Rightarrow 688+16x+43x+\cancel{x^2} = 553+79x+7x+\cancel{x^2}$

$\Rightarrow 688-553 = 86x - 59x$

$\Rightarrow 135 = 27x$

$\therefore x = \frac{\cancel{135}\,5}{\cancel{27}\,1} = 5$

2

The no. to be added is 5

$$\begin{array}{r} 43 \\ \times 16 \\ \hline 258 \\ 43\times \\ \hline 688 \end{array} \quad \begin{array}{r} 79 \\ \times 7 \\ \hline 553 \end{array} \quad \begin{array}{r} 688 \\ -553 \\ \hline 135 \end{array} \quad \begin{array}{r} 786 \\ -59 \\ \hline 27 \end{array}$$

Q10) Since x, y, z are in continued proportion

$y^2 = xz$

To prove

$\frac{x^2-y^2+z^2}{x^{-2}-y^{-2}+z^{-2}} = y^4$

LHS

$\frac{x^2-y^2+z^2}{\frac{1}{x^2} - \frac{1}{y^2} + \frac{1}{z^2}}$

$= \frac{x^2-xz+z^2}{\frac{y^2z^2 - x^2z^2 + x^2y^2}{x^2y^2z^2}}$

$= \frac{x^2-\cancel{xz}+y^2+z^2}{\frac{y^2z^2-(y^2)^2+x^2y^2}{x^2y^2z^2}}$

$\frac{x^2-xz+z^2}{\frac{z^2-xz+x^2}{x^2z^2}}$

$\therefore x^2z^2$

$= (xz)^2$

$= (y^2)^2$

$= y^4$

LHS = RHS

Hence proved

3

(8)

Q11> $\dfrac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}} = \dfrac{7}{3}$

By componendo and dividendo

$\dfrac{\sqrt{x+5}+\sqrt{x-16}+\sqrt{x+5}-\sqrt{x-16}}{\sqrt{x+5}+\sqrt{x-16}-\sqrt{x+5}+\sqrt{x-16}} = \dfrac{7+3}{7-3}$

$\dfrac{\cancel{2}\sqrt{x+5}}{\cancel{2}\sqrt{x-16}} = \dfrac{\cancel{10}\,5}{\cancel{4}\,2}$

Squaring both sides

$\dfrac{x+5}{x-16} = \dfrac{25}{4}$

• By componendo and dividendo

$\dfrac{x+5+x-16}{x+5-x+16} = \dfrac{25+4}{25-4}$

$\dfrac{2x-11}{\cancel{21}} = \dfrac{29}{\cancel{21}}$

$\therefore$ ~~$+2x$~~ $2x-11=29$

$\Rightarrow 2x = 29+11$

$\therefore x = 40/2$

$\therefore \boxed{x=20}$

3

Q12> $(a^2+b^2)(x^2+y^2) = (ax+by)^2$

$\Rightarrow a^2x^2+a^2y^2+b^2x^2+b^2y^2 = a^2x^2+b^2y^2+2axby$

$\Rightarrow a^2y^2+b^2x^2-2axby = 0$

$\Rightarrow (ay-bx)^2=0$

$\Rightarrow ay-bx=0$

$\therefore ay=bx$

$\therefore \dfrac{a}{x} = \dfrac{b}{y}$; Hence proved

3

Q13> a, b, c are in continued proportion

$\therefore \frac{a}{b} = \frac{b}{c} = k$

$\Rightarrow a = bk$ and $b = ck$

$\therefore a = (ck)k$

$\therefore a = ck^2$ and $b = ck$

To prove

$\frac{a^2+ab+b^2}{b^2+bc+c^2} = \frac{a}{c}$

$\Rightarrow$ LHS

$\frac{a^2+ab+b^2}{b^2+bc+c^2}$

$= \frac{(ck^2)^2+(ck^2)(ck)+(ck)^2}{(ck)^2+(ck)c+c^2}$

$= \frac{c^2k^4+c^2k^3+c^2k^2}{c^2k^2+c^2k+c^2}$

$= \frac{\cancel{c^2}k^2(\cancel{k^2+k+1})}{\cancel{c^2}(\cancel{k^2+k+1})} \quad \frac{\cancel{c^2}k^2(\cancel{k^2+k+1})}{\cancel{c^2}(\cancel{k^2+k+1})}$

$= \frac{k^2(\cancel{k^2+k+1})}{\cancel{(k^2+k+1)}} \quad k^2$

RHS

$\frac{a}{c}$

$= \frac{\cancel{c}k^2}{\cancel{c}}$

3

LHS = RHS

Hence proved

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