(C) CONSERVATION OF ENERGY

2.16 PRINCIPLE OF CONSERVATION OF ENERGY

According to the principle of conservation of energy, energy can neither be created nor can it be destroyed. It only changes from one form to another.

In universe, energy occurs in various forms. The sum of all forms of energy in the universe remains constant. When there is a transformation of energy from one form to another, the total energy always remains same *i.e.*, it remains conserved. *If there is only an interchange between the potential energy and kinetic energy, the total mechanical energy (i.e., the sum of kinetic energy K and potential energy U) remains constant i.e., K + U = constant when there are no frictional forces.* This is law of conservation of mechanical energy!

The principle of conservation of energy is one of the fundamental principles of nature.

2.17 THEORETICAL VERIFICATION OF $K + U$ = CONSTANT FOR A FREELY FALLING BODY

As done in class

Let a body of mass m be falling freely under gravity from a height h above the ground (*i.e.*, from position A in Fig. 2.15). As the body falls down, its potential energy changes into the kinetic energy. At each point of motion, the sum of potential energy and kinetic energy remains unchanged. To verify it, let us calculate the sum of kinetic energy K and potential energy U at various positions, say at A (at height h above the ground), at B (when it has fallen through a distance x), and at C (on the ground).

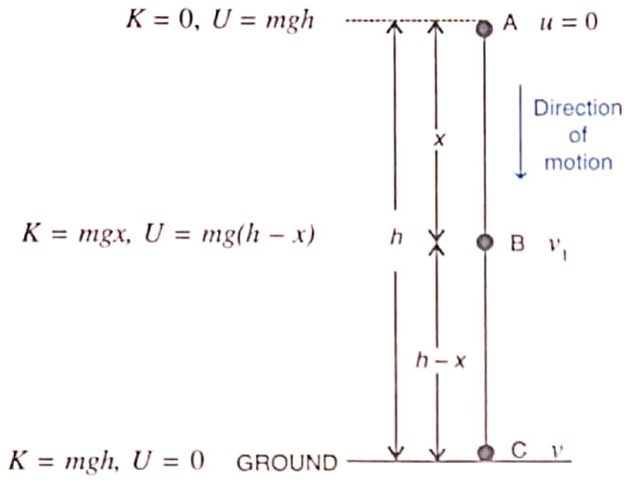

Fig. 2.15 Free fall of a body

At the position A (at height h above the ground) :

Initial velocity of body $= 0$ (since body is at rest at A)

$\therefore$ Kinetic energy $K = 0$

Potential energy $U = mgh$

Hence total energy $= K + U = 0 + mgh = mgh$ ....(i)

At the position B (when it has fallen a distance x) :

Let v_1 be the velocity acquired by the body at B after falling through a distance x. Then $u = 0$, $S = x$, $a = g$

From equation $v^2 = u^2 + 2aS$

$$v_1^2 = 0 + 2gx = 2gx$$

$\therefore$ Kinetic energy $K = \frac{1}{2}mv_1^2$

$$= \frac{1}{2}m \times (2gx) = mgx$$

Now at B, height of body above the ground $= h - x$

$\therefore$ Potential energy $U = mg\,(h - x)$

Hence total energy $= K + U$

$$= mgx + mg(h - x) = mgh \quad .....(ii)$$

At the position C (on the ground) :

Let the velocity acquired by the body on reaching the ground be v. Then $u = 0$, $S = h$, $a = g$

From equation $v^2 = u^2 + 2aS$

$$v^2 = 0 + 2gh$$

or $v^2 = 2gh$

Kinetic energy $K = \frac{1}{2}mv^2$

$$= \frac{1}{2}m \times (2gh) = mgh$$

and potential energy $U = 0$ (at the ground when $h = 0$)

Hence total energy $= K + U = mgh + 0 = mgh$ ..(iii)

Thus from eqns. (i), (ii) and (iii), we note that the total mechanical energy (*i.e.*, the sum of kinetic energy and potential energy) always remains constant at each point of motion and it is equal to the initial potential energy at height h. As the body falls, its potential energy decreases and kinetic energy increases. The potential energy changes into the kinetic energy. Just at the instant when it strikes the ground, whole of the potential energy has changed into the kinetic energy, therefore the kinetic energy of the body on reaching the ground is equal to the initial potential energy at height h.

Similarly, when a body is thrown vertically upwards under gravity, its initial kinetic energy supplied at the instant of throwing up, keeps on decreasing and the potential energy keeps on increasing *by the same amount*. When the body reaches the highest point, whole of its initial kinetic energy has changed into the potential energy and therefore the body momentarily comes to rest. At this instant, the body is still under the influence of the force of gravity, so the body starts falling down and its potential energy begins to change into the kinetic energy.

> ***Note*** *: The initial kinetic energy provided to the body at the ground, so as to reach a certain height h must be equal to the potential energy of the body at that height h* (*i.e.*, equal to mgh). Thus the initial velocity u to be imparted to the body so as to reach a height h is given as :
>
> $$\frac{1}{2}mu^2 = mgh \text{ or } u = \sqrt{2gh} \quad ....(2.18)$$

Fig. 2.16 represents the variation in kinetic energy and potential energy with the height above the ground.

The table below represents the kinetic energy K, potential energy U and total energy E of the body of mass m at various heights above the ground during vertically downward and upward motions under gravity.

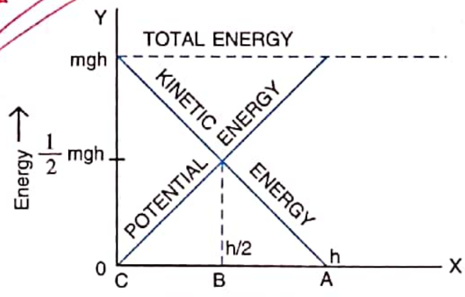

Fig. 2.16 Conservation of mechanical energy in motion under gravity

Table showing the kinetic energy and potential energy of a body in vertical motion.

Motion	Height above the ground	Kinetic energy K	Potential energy U	Total energy $E = K + U$
Downward motion (*i.e.*, free fall)	h (*i.e.*, highest point A)	0	mgh	mgh
	$\frac{1}{2}h$ (*i.e.*, middle point B)	$\frac{1}{2}mgh$	$\frac{1}{2}mgh$	mgh
	0 (*i.e.*, ground C)	mgh	0	mgh
Upward motion	0 (*i.e.*, ground C)	mgh	0	mgh
	$\frac{1}{2}h$ (*i.e.*, middle point B)	$\frac{1}{2}mgh$	$\frac{1}{2}mgh$	mgh
	h (*i.e.*, highest point A)	0	mgh	mgh

It is clear that at the ground (*i.e.*, at point C) $h = 0$, potential energy = 0, kinetic energy = mgh. At the middle point B, $x = \frac{1}{2}h$, potential energy $= \frac{1}{2}mgh$, kinetic energy $= \frac{1}{2}mgh$. At the highest point A (*i.e.*, at height h), potential energy = mgh and kinetic energy = 0 (zero).

> *Note* : In the above calculations, we have ignored the force of friction between the body and air. In fact during the fall, some of the kinetic energy will change into the heat energy due to friction and it will get dissipated in air. At the ground C, the kinetic energy will be less than mgh. The conservation of mechanical energy is therefore strictly valid only in absence of external forces such as friction due to air etc., although the total energy of all kinds is always conserved.

2.17 APPLICATION OF PRINCIPLE OF CONSERVATION OF ENERGY TO A SIMPLE PENDULUM

Fig. 2.17 shows a simple pendulum suspended from a rigid support O. Its resting position is at A. When it is displaced to one side and then released, it swings from one side to the other, reaching equal distance and equal height on either side. Neglecting the force of friction between the bob and the surrounding air, the motion of pendulum can easily be explained by applying the principle of conservation of energy as follows.

Read & understand the article completely

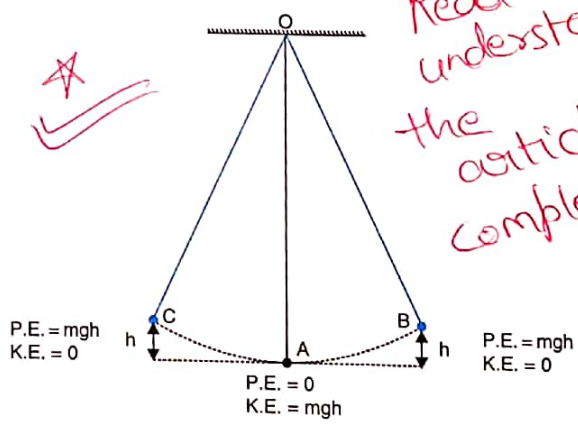

Fig. 2.17 Motion of a simple pendulum

Explanation : Let A be the resting (or mean) position of the bob when it has zero potential energy*. When the bob of the pendulum is displaced to B from its resting position A, the bob gets raised by a vertical height h, so its potential energy increases by mgh if m is the mass of the bob. Now on releasing the bob at B, it moves back from B to A. Its vertical height decreases from h to zero, so its potential energy decreases from mgh to zero and it gets converted into the kinetic energy *i.e.*, $\frac{1}{2}mv^2 = mgh$. At the point B, the bob acquires a velocity $v = \sqrt{2gh}$, so it moves towards C. At the point C, when the bob raises by a vertical height h above the point A, again it acquires the potential energy mgh and its kinetic energy becomes zero. So the bob momentarily comes to rest at the point C. But due to the force of gravity, the bob moves back from C to A.

As the bob swings back from C to A, the potential energy decreases and the kinetic energy increases. At A (mean position), it has its total mechanical energy in the form of kinetic energy and the potential energy is zero, so the bob swings again from A to B to repeat the process.

Thus during the swing, at the extreme positions B and C, the bob has only the potential energy, while at the mean position A, it has only the kinetic energy. At an intermediate position (between A and B or between A and C), the bob has both the kinetic energy and potential energy, but the sum of both (*i.e.*, the total mechanical energy) remains constant throughout the swing. This is strictly true only in vacuum where there is no force of friction due to air.

* In fact, at the mean position, the bob has minimum potential energy. Since we are interested only in the change in potential energy as the bob swings so we may assume it to be zero at the mean position.

EXAMPLES