Programming Assignment to solve 1-D Transient Heat Transfer Problem using Finite Difference Numerical Method

Submitted for Core Course MM204

For the requirements of the MEMS B.Tech. Program

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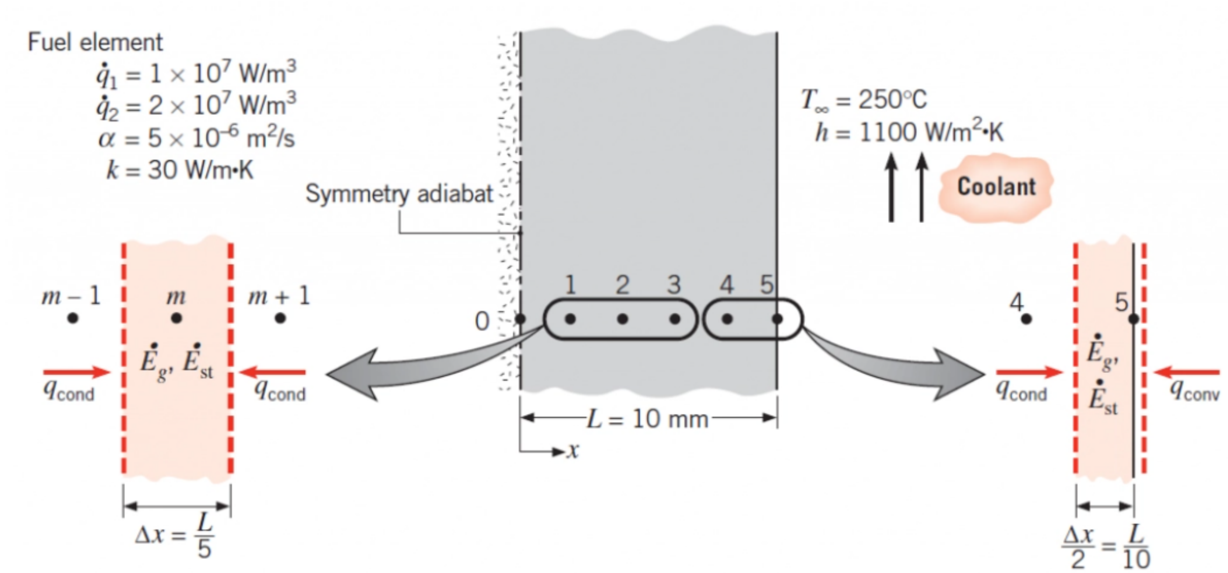
PROBLEM STATEMENT

A fuel element of a nuclear reactor is in the shape of a plane wall of thickness 2L = 20 mm and is convectively cooled at both surfaces, with h = 1100W/m^2.K and T_∞ = 250°C. At normal operating power, heat is generated uniformly within the element at a volumetric rate of $q_1 = 10^7$ W/m^3. A departure from the steady-state conditions associated with normal operation will occur if there is a change in the generation rate. Consider a sudden change to , and use the explicit finite-difference method to determine the fuel element temperature distribution after 1.5 s. The fuel element thermal properties are k = 30 W/m.K and α = 5 x 10^-6 m^2/s



Known: Conditions associated with heat generation in a rectangular fuel element with surface cooling.

Find: Temperature distribution 1.5 s after a change in operating power.

Assumptions:

1. One-dimensional conduction in x.

2. Uniform generation.

3. Constant properties.

PROBLEM SOLVING APPROACH

ANALYSIS:

A numerical solution will be obtained using a space increment of $\Delta x = 2mm$. Since there is symmetry about the midplane, the nodal network yields 6 unknown nodal temperatures. Using the energy balance method, Equation 5.84, an explicit finite-difference equation may be derived for any interior node m.

HEAT TRANSFER EQUATION FOR INTERMEDIATE NODES

$$kA\frac{T_{m-1}^p - T_m^p}{\Delta x} + kA\frac{T_{m+1}^p - T_m^p}{\Delta x} + \dot{q}A\Delta x = \rho A\Delta x c\frac{T_m^{p+1} - T_m^p}{\Delta t}$$

Solving for T_m^{p+1} and rearranging,

$$T_m^{p+1} = Fo\left[T_{m-1}^p + T_{m+1}^p + \frac{\dot{q}(\Delta x)^2}{k}\right] + (1 - 2Fo)T_m^p \quad (1)$$

HEAT TRANSFER EQUATION FOR SURFACE NODE

Accounting for the symmetry about x = 0, this equation may also be used for the m = 0 node, with $T_{-1}^p = T_1^p$. Applying energy conservation to a control volume about node 5,

$$T_5^{p+1} = 2Fo\left[T_4^p + BiT_\infty + \frac{\dot{q}(\Delta x)^2}{2k}\right] + (1 - 2Fo - 2Bi\ \ Fo)T_5^p \quad (2)$$

STABILITY CRITERION

Since the most restrictive stability criterion is associated with Equation 2, we select Fo from the requirement that

$$Fo(1 + Bi) \le \frac{1}{2}$$

Hence, with

$$Bi = \frac{h\,\Delta x}{k} = \frac{1100\ \mathrm{W/m^2\cdot K}\,(0.002\ \mathrm{m})}{30\ \mathrm{W/m\cdot K}} = 0.0733$$

it follows that

$$Fo \leq 0.466$$

or

$$\Delta t = \frac{Fo(\Delta x)^2}{\alpha} \leq \frac{0.466(2\times 10^{-3}\ \mathrm{m})^2}{5\times 10^{-6}\ \mathrm{m^2/s}} \leq 0.373\ \mathrm{s}$$

To be well within the stability limit, we select $\Delta t = 0.3$ s, which corresponds to

$$Fo = \frac{5\times 10^{-6}\ \mathrm{m^2/s}(0.3\ \mathrm{s})}{(2\times 10^{-3}\ \mathrm{m})^2} = 0.375$$

Substituting numerical values, including $\dot{q} = \dot{q}_2 = 2\times 10^7 W/m^3$, the finite-difference equations become

$$T_0^{p+1} = 0.375(2T_1^p + 2.67) + 0.250T_0^p$$

$$T_m^{p+1} = 0.375(T_{m-1}^p + T_{m+1}^p + 2.67) + 0.250T_m^p \quad m = 1, 2, 3, 4$$

$$T_5^{p+1} = 0.750(T_4^p + 19.67) + 0.195T_5^p$$

INITIAL CONDITION (t=0)

Using equations derived for plane walls at steady state with heat generation at the center (here q = q1).

$$T_s = T_\infty + \frac{\dot{q}L}{h}$$

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$$

Initial temperatures for the nodal points are shown in the first row of the accompanying table.

RESULTS

Using the finite-difference equations, the nodal temperatures may be sequentially calculated with a time increment of 0.3 s until the desired final time is reached.

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The temperature distribution at t = 1.5 sec is in the row with index 5.0
Moreover the temperature distribution after large amount of time is in the last row
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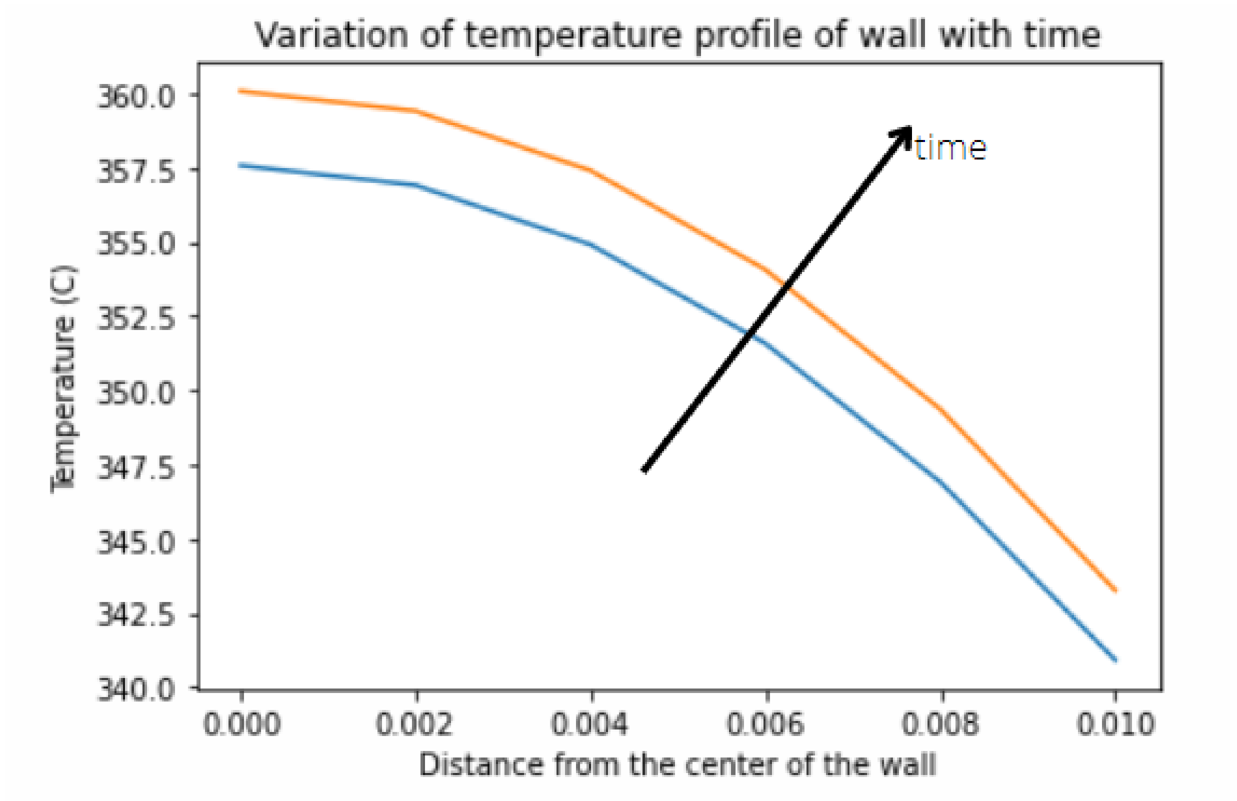
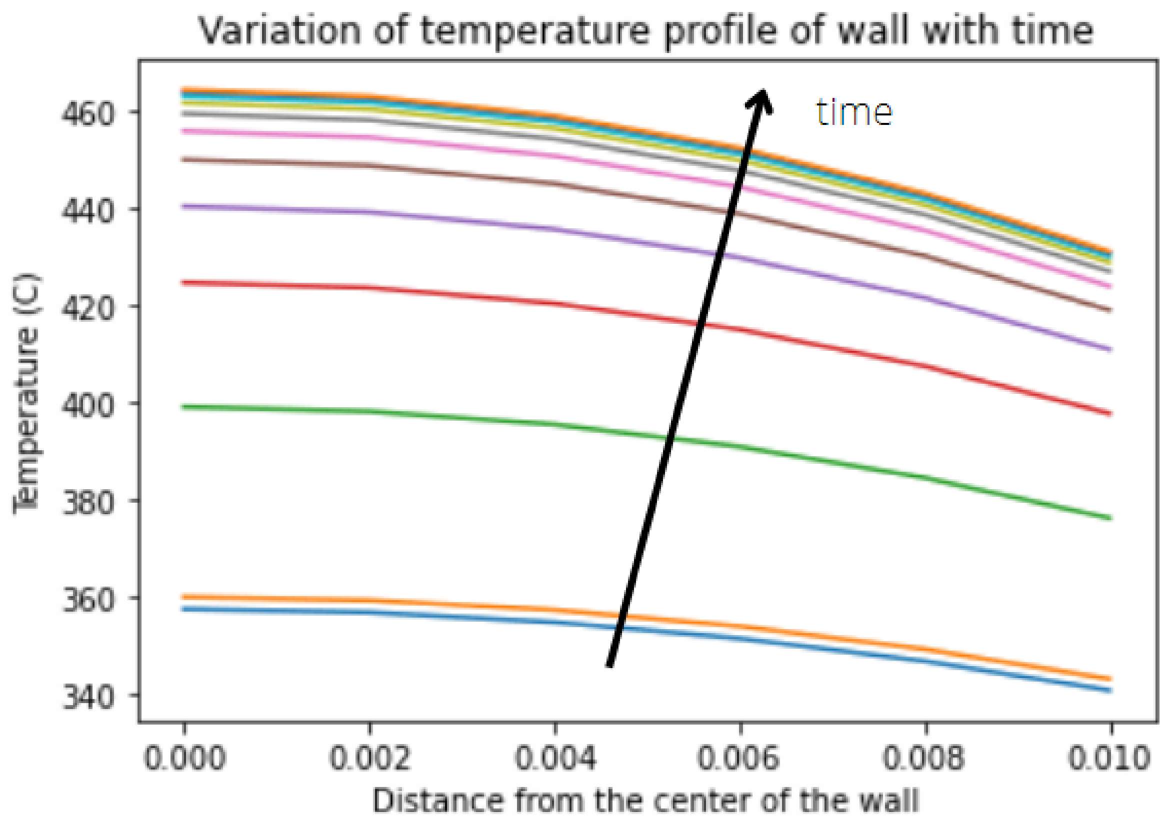
	0	1	2	3	4	5
0	357.575758	356.909091	354.909091	351.575758	346.909091	340.909091
1	358.075758	357.409091	355.409091	352.075758	347.409091	341.409091
2	358.575758	357.909091	355.909091	352.575758	347.909091	341.881591
3	359.075758	358.409091	356.409091	353.075758	348.398778	342.348728
4	359.575758	358.909091	356.909091	353.571890	348.883877	342.807086
5	360.075758	359.409091	357.407641	354.065336	349.363085	343.260289





Comments:

1. It is evident that, at 1.5 s, the wall is in the early stages of the transient process and that many additional calculations would have to be made to reach steady-state conditions with the finite-difference solution. The computation time could be reduced slightly by using the maximum allowable time increment $(\Delta t = 0.373\ s)$, but with some loss of accuracy. In the interest of maximizing accuracy, the time interval should be reduced until the computed results become independent of further reductions in Δt.

Extending the finite-difference solutions to t = 400 s, the temperature histories computed for the midplane (0) and surface (5) nodes are shown in the figure below.

It is evident that the new equilibrium condition is reached within approximately 250 s of the step change in operating power.