inequalities in and around quantum info

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Abstract

Your abstract.

1 Definitions

- Trace norm $||O||_1 = \mathrm{Tr}[\sqrt{O^\dagger O}]$
- Operator norm $||O|| = \sup\{\sqrt{\langle\psi|O^\dagger O|\psi\rangle} \mid \langle\psi|\psi\rangle = 1\}$
- The Schatten p-norm $||O||_p := [\mathrm{Tr}((O^\dagger O)^{p/2})]^{1/p}$. $p = 1$ corresponds to the trace norm and $p = \infty$ is the operator norm.

1. *Entropy* $S(\rho) := -\mathrm{Tr}(\rho \log \rho)$
2. *Relative Entropy* $S(\rho||\sigma) := \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$
3. *Mutual Information* $I(A, B) := S(\rho_{AB}||\rho_A \otimes \rho_B)$

2 Matrix inequalities

- $||X||_1||Y||_1 \geq \mathrm{Tr}[XY]$

3 Distance between states

- $S(\rho||\sigma) \geq ||\rho - \sigma||_1^2/2$

4 General Statements

1. **Mutual information upper bounds correlations**. If I define a correlation function as

$$C(M_A, M_B) := \langle M_A \otimes M_B\rangle_{\rho_{AB}} - \langle M_A\rangle_{\rho_A}\langle M_B\rangle_{\rho_B}$$

then, it follows that

$$I(A : B) \geq \frac{C(M_A, M_B)^2}{2||M_A||_1^2||M_B||_1^2}$$