MA 106 D1-T3
Recap-0

Siddhant Midha

23-03-2022

Table of Contents

Matrices

- Rectangular Arrays

Matrices

- Rectangular Arrays
- $A = [a_{ij}], B = [b_{ij}]$

Matrices

- Rectangular Arrays
- $A = [a_{ij}], B = [b_{ij}]$
- Sum

$$A + B := [a_{ij} + b_{ij}]$$

Matrices

- Rectangular Arrays
- $A = [a_{ij}], B = [b_{ij}]$
- Sum

$$A + B := [a_{ij} + b_{ij}]$$

- Scalar Multiplication

$$\alpha A := [\alpha a_{ij}]$$

Matrices

- Rectangular Arrays
- $A = [a_{ij}], B = [b_{ij}]$
- Sum

$$A + B := [a_{ij} + b_{ij}]$$

- Scalar Multiplication

$$\alpha A := [\alpha a_{ij}]$$

- Multiplication (when defined!)

$$c_{ij} := \sum_k a_{ik} b_{kj}$$

Exercises

- Prove that

$$A + B = B + A$$

- Define the transpose in terms of this notation, and show that

$$(A^T)^T = A$$

- Define symmetric and skew symmetric using this notation.
- (Assume correct dimensions and) Show that

$$(AB)C = A(BC)$$

$$(AB)^T = B^T A^T$$

Exercises

Definition

Given a matrix

$$A = [a_{ij}]$$

the *trace* of the matrix is defined as

$$Tr(A) := \sum_i a_{ii}$$

Show that

- $Tr(A + B) = Tr(B + A)$
- $Tr(AB) = Tr(BA)$

Invertibility

Definition

Let $A \in \mathbb{R}^{n\times n}$. It is said to be invertible if there exists $B \in \mathbb{R}^{n\times n}$ such that

$$AB = BA = I$$

where I is the $n \times n$ identity matrix. Such a B is unique and is denoted as A^{-1}

Invertibility

Definition

Let $A \in \mathbb{R}^{n\times n}$. It is said to be invertible if there exists $B \in \mathbb{R}^{n\times n}$ such that

$$AB = BA = I$$

where I is the $n \times n$ identity matrix. Such a B is unique and is denoted as A^{-1}

Exercises:

- If A and B are invertible, show that
  - $(A^{-1})^{-1} = A$
  - $(AB)^{-1} = B^{-1}A^{-1}$
- If A has a row of zeroes, it is not invertible.

Invertibility

- If there exist L and R such that
$$LA = AR = I$$
show that $L = R$.
- (Stronger form of the above) If there exists a B such that $AB = I$ **or** $BA = I$, then A is invertible and B is its inverse.

Table of Contents

Basics

Column Vectors

- Belong in $\mathbb{R}^{n\times 1}$.
- Denoted as

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}^T$$

Basics

Column Vectors

- Belong in $\mathbb{R}^{n\times 1}$.
- Denoted as

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}^T$$

Row Vectors

- Belong in $\mathbb{R}^{1\times n}$.
- Denoted as

$$\begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

Products of Vectors

Inner Product

Let u, v be column vectors. The inner product is the real number

$$u^T v := \sum_i u_i v_i$$

Products of Vectors

Inner Product

Let u, v be column vectors. The inner product is the real number

$$u^T v := \sum_i u_i v_i$$

Outer Product

Let u, v be column vectors. The outer product is the matrix

$$uv^T = [c_{ij}]$$

$$c_{ij} := u_i v_j$$

Table of Contents

Matrix Vector Product

Let

Matrix Vector Product

Let

$$A = [a_{ij}]$$

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$$

Consider $y = Ax$.

Matrix Vector Product

Let

$$A = [a_{ij}]$$

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$$

Consider $y = Ax$. Further define

$$A_i := \begin{bmatrix} a_{1i} & a_{2i} & \dots a_{ni} \end{bmatrix}^T$$

Can I express y in terms of A_i's?

Matrix Vector Product

Let

$$A = [a_{ij}]$$

$$x = \begin{bmatrix} x_1 & x_2 & \dots & x_n \end{bmatrix}^T$$

Consider $y = Ax$. Further define

$$A_i := \begin{bmatrix} a_{1i} & a_{2i} & \dots a_{ni} \end{bmatrix}^T$$

Can I express y in terms of A_i's? Now let x be a row vector and consider $y = xA$. What now?

Question

Consider the system of equations,

Question

Consider the system of equations,

$$\begin{aligned} x + 9y &= -14 \\ 2x + 6y &= 25 \\ 3x + 2y &= -12 \end{aligned}$$

Question

Consider the system of equations,

$$\begin{aligned} x + 9y &= -14 \\ 2x + 6y &= 25 \\ 3x + 2y &= -12 \end{aligned}$$

Given what we discussed before, how do we proceed here?