MA 109 D2 T1 Week Two Recap

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November 16, 2022

The Idea



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We defined and spent time understanding convergence of sequences. Now, we will apply those properties to talk about functions over \mathbb{R} .

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Further, we have that, both the definitions are equivalent! Can you give a proof?

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Proposition

Let $x_0 \in \mathbb{R}$, and let $A \subset \mathbb{R}$ such that $N_r(x_0) \subset A$ for some r > 0. The function $f : A \to \mathbb{R}$ is continuous at x_0 iff $\lim_{x \to x_0} f(x)$ exists and is equal to $f(x_0)$. That is,

Continuity at
$$x_0 \Leftrightarrow \lim_{x \to x_0} f(x) = f(x_0)$$

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Also attempt the following: Just like we made precise the handwavy definition of a sequence 'going to infinity', give a definition using the $\epsilon-\delta$ way for a function 'going to infinity' at some $x\in\mathbb{R}$.

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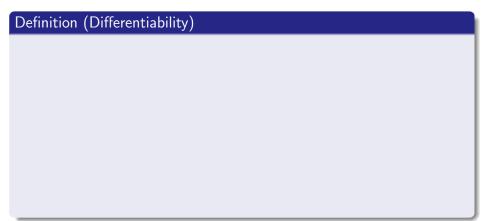
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$$d(h; f, c) := \frac{f(c+h) - f(c)}{h}$$

Then, we say that f is differentiable at c if $\lim_{h\to 0} d(h; f, c)$ exists. If it does, we denote it by f'(c).

- Differentiability

 Continuity? Yes
- Thus, if a function is not continuous at a point, it cannot be differentiable a that point.
 - Continuity \implies Differentiability? No

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