# MA 109 D2 T1
# Week Seven Recap

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# Tangent Lines and Planes

- Let $F : D \subset \mathbb{R}^2 \to \mathbb{R}$ have partial derivatives at an interior point $P = (x_0, y_0)$ and let $\nabla F(x_0, y_0) \neq (0, 0)^T$. Consider the curve, $C := \{(x, y) \mid F(x, y) = 0\}$ and assume that $P$ lies on $C$.

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- Consider a $C$ lying on the surface defined by $F(x, y, z) = 0$. Then, for any $t_0 \in [a, b]$, we have,

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Let $D \subset \mathbb{R}^2$, and let $(x_0, y_0)$ be an interior point of $D$. Suppose $f : D \to \mathbb{R}$ has a local extremum at $(x_0, y_0)$. If $u$ is a unit vector and $(\nabla_u f)(x_0, y_0)$ exists, then $(\nabla_u f)(x_0, y_0) = 0$.

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## Definition (Saddle Points)

For a 'nice' function $f : D \subset \mathbb{R}^2 \to \mathbb{R}$, and some interior point $P \in D$, we call $P$ to be a saddle point of $f$ if $\nabla F(P) = (0, 0)^T$ but $P$ is **not** a local extrema.

# Hessian Test

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$$(\Delta f)(x_0, y_0) := f_{xx}(x_0, y_0) f_{yy}(x_0, y_0) - f_{xy}(x_0, y_0)^2$$

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The discrimimant test is inconclusive if $(\Delta f)(x_0, y_0) = 0$.

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Setting: Let $f, g : D \subset \mathbb{R}^2 \to \mathbb{R}$ be nice functions. We aim to find extrema of $f$ on $D$ constrained to the fact that $g = 0$.

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Then, there is $\lambda_0 \in \mathbb{R}$ such that

$$\left(\nabla f\right)\left(x_0, y_0\right) = \lambda_0 (\nabla g)\left(x_0, y_0\right).$$

The real number $\lambda_0$ is called a Lagrange multiplier.

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5. Conclude that $f$ attains extrema on those points.
6. Note that this procedure only tells us that these points are the extrema – does not comment on whether they are minima or maxima.