AC Quantum Transport: Formalisms and Applications

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EE 755 Course Project
May 4, 2023

This talk

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Quite technical, so focus on gathering intuition! The paper is there for the details :)

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- $\rightarrow$ Get observibles! $I_m = \frac{e}{h} \int dE \sum_{m'=1}^{N} (f_m - f_{m'}) Tr[\mathcal{G}_0 \Gamma_{m'} \mathcal{G}_0^\dagger \Gamma_m]$

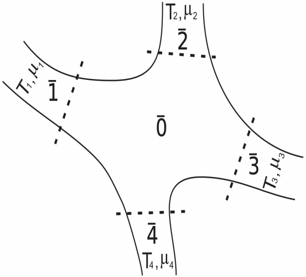

Figure 1: System under study

AC NEGF I [SW13]

- Move into time-domain!

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- Currents! $I_m(t) = e\sum_{i\in\sigma_0,\alpha\in\sigma_m}(\mathcal{V}_{\alpha i}(t)\mathfrak{G}^<_{i\alpha}(t,t) - \mathcal{V}_{i\alpha}(t)\mathfrak{G}^<_{\alpha i}(t,t))$

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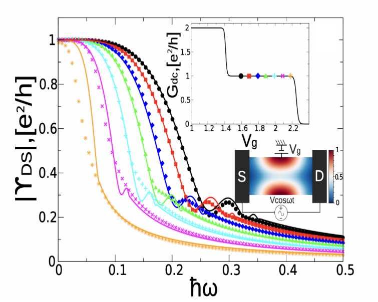

Figure 2: QPC AC conductance

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- $\rightarrow$ Why care? Simplified equations, inefficient, but doable!

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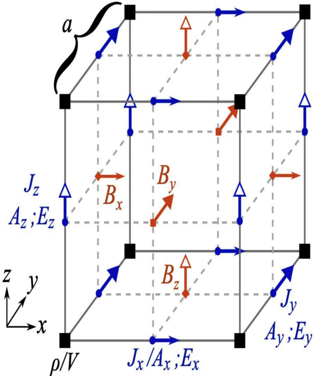
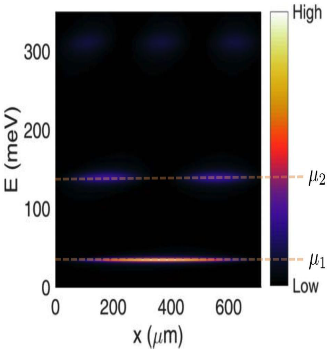


(a)



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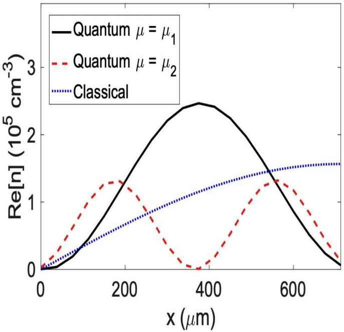

(c)

AC NEGF Others [CS09; PD17; KVL10]

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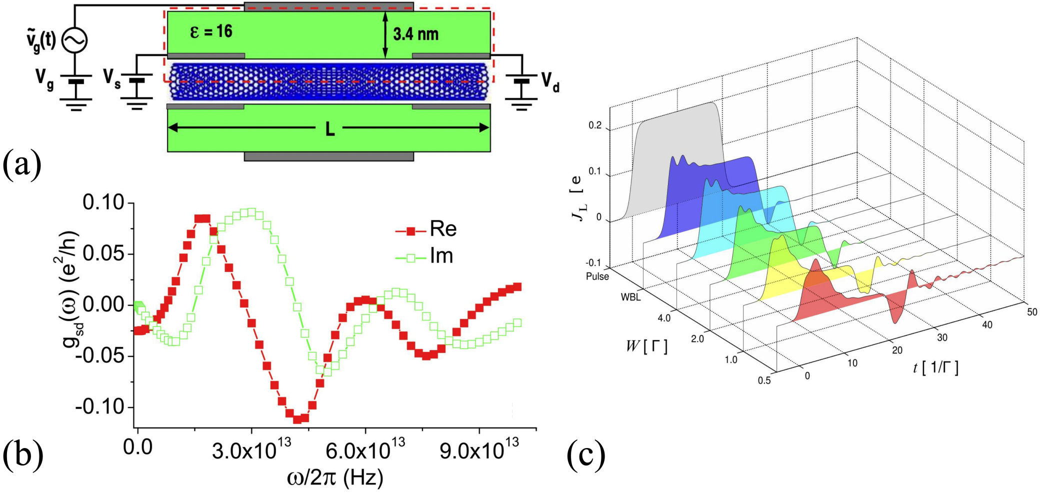

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$$I_{ij}(t) = -2\,\mathrm{Im}\sum_{\alpha}\int \frac{dE}{2\pi} f_\alpha(E)\psi^*_{\alpha E}(t,i)\mathbf{H}_{ij}\psi_{\alpha E}(t,j), \tag{11}$$

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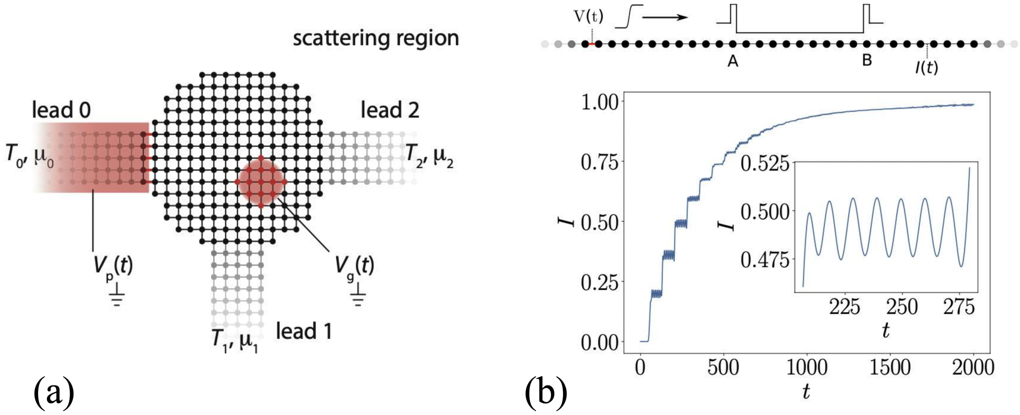

Numerics: Basic I

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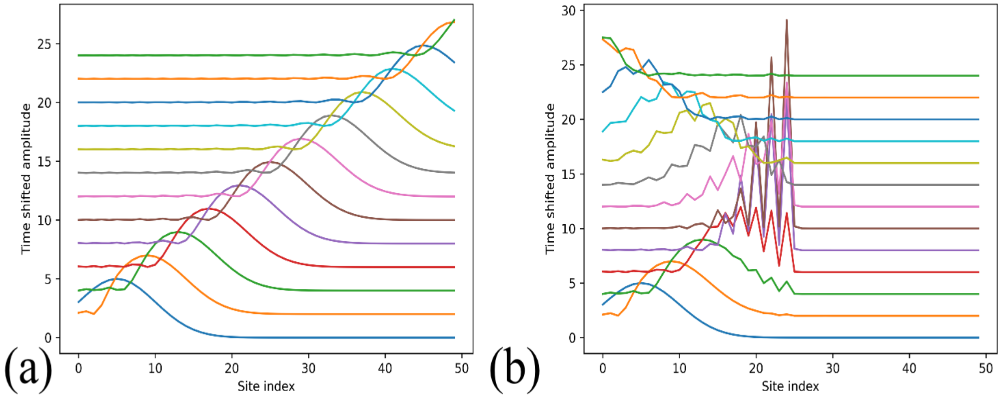

Figure 6: Gaussian wave packet propagation (a) free (b) with barrier

Numerics: Basic II

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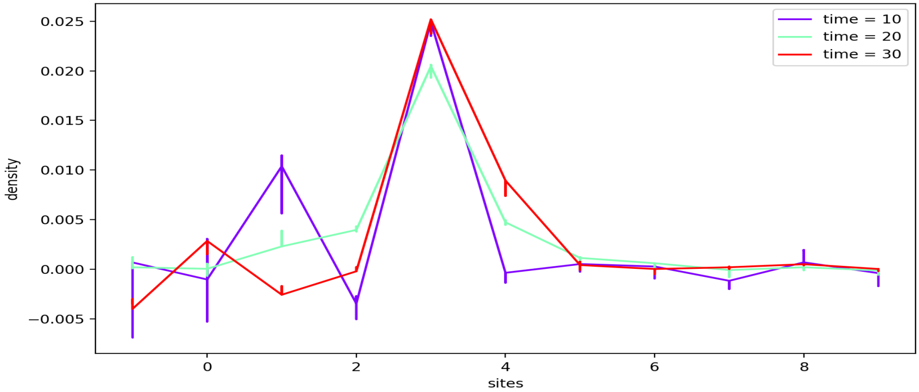

Figure 7: Small square pulse applied across leads of a normal-superconducting junction

Numerics: Basic III

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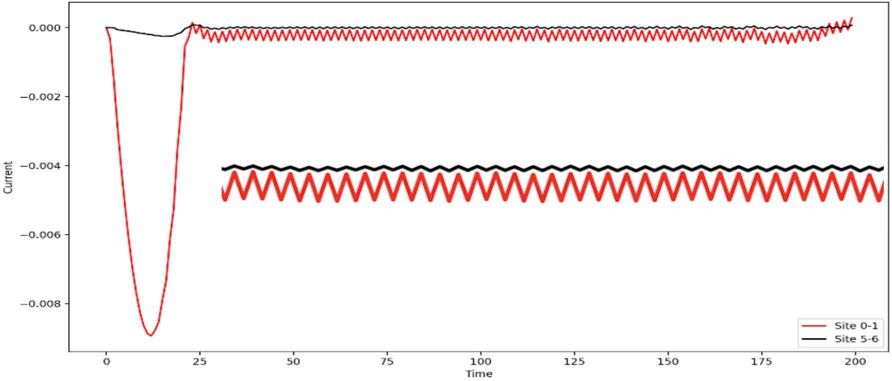

Figure 8: Voltage pulse applied across leads of a topological phase SSH chain

Numerics: Advanced

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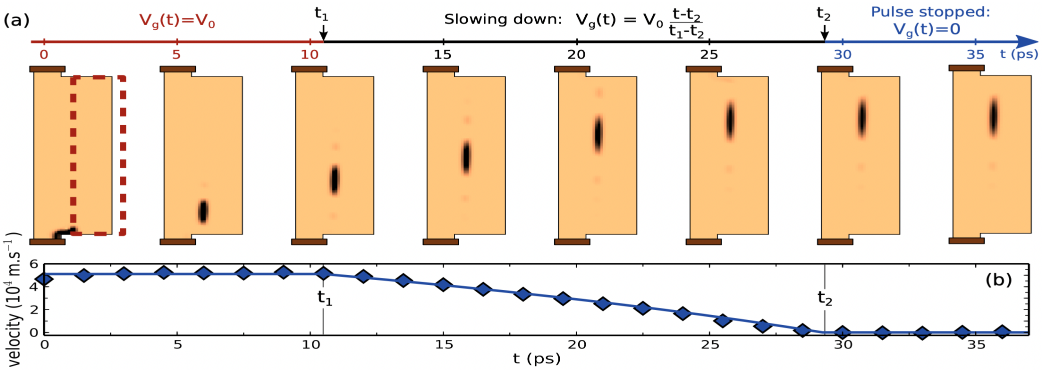

Figure 9: (a) Charge density plotted at various times during the stopping protocol. The gate is polarized for $t < t_1$ (thereby causing the edge mode at the center), and slowly grounded between t_1 and t_2. At $t = t_2$, the pulse is stopped. (b) Velocity $v(t)$ of the pulse as a function of time. Diamonds correspond to numerical data, the full line to the analytical result. Figure from [GWW14]

That's a wrap!

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Woop, heavy stuff for 12 minutes, no?

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References I

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