fact: Dyadic points $\left(\frac{j}{2^i}, \frac{l}{2^k}\right)$ are dense in $\mathbb{R}$

fact #2 given any point $x \in \mathbb{R}$, and any $\varepsilon > 0$, you can find a point of the form $\left(\frac{j}{2^k}, \frac{l}{2^k}\right)$

$j, l \in [0, 2^k]$

s.t.

$\left| x - \left(\frac{j}{2^k}, \frac{l}{2^k}\right) \right| < \varepsilon$

Thus, you can find a sequence of points $x_n \to x$ s.t. $f(x_n) = 1 \ \forall n$

$\therefore$ f is discontinuous at all "non diadic points"

$\therefore$ Set of discont does not have measure zero.

But, for g, use the kind of argument done in Tut 1 Q3, and see that it will be continuous at non dyadic points. (the power of 2 will need to get bigger to approach)

$\therefore$ g is int.

now, take $x \in [0, 1]$

consider $\int_0^1 f(x, y)\, dy = \begin{cases} 0 & \text{if } x \text{ is not dyadic} \\ g_x(y) & \text{if } x \text{ is} \end{cases}$

let $x = \frac{(\ )}{2^k}$

there can be only finite diadics w/

dnom power $= k$.

$\therefore$ set of discont. of $g_x(y)$ is of measure zero.

$$\Rightarrow \int_0^1 f\,dy = 0$$

Sim,

$$\int_0^1 f\,dx = 0$$

now let $x = \frac{i}{2^j}$ be dyadic.

consider $\phi_x(y) = g(x,y)\big|_x = 0$ if y not diadic

$= \frac{1}{2^j}$ if y is diadic

now, given any nondiadic $x \in [0,1]$, I can find $x_n \to x$ s.t. x_n is diadic

But, $f(x_n) = \frac{1}{2^j}\ \forall n$ — this is constant, not dep. on. of g.

$f(x) = 0$

$\therefore$ Set of discont of $\phi_x(y)$ is not of measure zero!