superconducting qubits

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Abstract

Your abstract.

1 Introduction

DiVincenzo criteria:

- *scalability.* current flows in superconducting loops on an integrated circuit
- *initialization.* cooling at cryogenic temperatures (~ 10mK)
- *coherence.* superconducting ground state!
- *gates.* single qubit: microwave photons. two qubit: circuit QED.
- *measurement.* EM fields.

2 Superconductivity

The BCS form of superconductivity manifests as a macroscopic degenerate ground state formed by *pairs* of electrons. A pair looks like this:

$$\sim |k, \uparrow\rangle + |-k, \downarrow\rangle \tag{1}$$

These pairs, known as *Cooper pairs* are bosonic excitations which result in a macroscopic occupation of the ground state. This is nothing but Bose-Einstein condensation. The ground state is described by the BCS order parameter

$$\Psi = |\Psi| e^{\iota\theta} \tag{2}$$

with $|\Psi|^2 = n_s$ denoting the density of the pairs.

3 Quantum Oscillators

Consider an oscillator described by the following differential equation

$$\ddot{Q} + \frac{1}{LC} Q = 0 \tag{3}$$

This dynamics is generated from the following Lagrangian

$$L = \frac{1}{2} L\dot{Q}^2 - \frac{Q^2}{2C} \tag{4}$$

as can be seen by a direct application of the Euler-Lagrange equations

$$\frac{d}{dt}\frac{dL}{d\dot{Q}} = \frac{dL}{dQ}. \tag{5}$$

The natural frequency of this oscillator is $\omega = 1/\sqrt{LC}$. The 'momentum' variable here is

$$\Phi = \frac{dL}{d\dot{Q}} = L\dot{Q} \tag{6}$$

which is nothing but the magnetic flux through the inductor. These canonical variables upon quantizing satisfy

$$[\hat{Q}, \hat{\Phi}] = \iota\hbar \tag{7}$$

and the Hamiltonian can be written as

$$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}. \tag{8}$$

The ladder operators for this oscillator can be defined as

$$\hat{a} = \frac{1}{\sqrt{2C\hbar\omega}}\hat{Q} + \iota\frac{1}{\sqrt{2L\hbar\omega}}\hat{\Phi} \tag{9}$$

$$\hat{a}^\dagger = \frac{1}{\sqrt{2C\hbar\omega}}\hat{Q} - \iota\frac{1}{\sqrt{2L\hbar\omega}}\hat{\Phi} \tag{10}$$

which satisfy $[\hat{a}, \hat{a}^\dagger] = 1$ as usual. What is the Hilbert space? Working in the number/charge basis, we have a set of eigenstates $|n\rangle$ for $n \geq 0$ which are eigenvalues of the number operator $\hat{N}|n\rangle = n|n\rangle$ and also of the charge operator as $\hat{Q} \propto \hat{N}$ where the constant of proportionality does not matter for now. Then, we can write the Fourier transform of basis states as

$$|n\rangle = \int e^{-\iota n\phi} |\phi\rangle \, d\phi \tag{11}$$

where $|\phi\rangle$ is the flux basis.

4 Josephson junctions

$$\Delta V = \frac{\hbar}{2e}\frac{d(\Delta\phi)}{dt} \tag{12}$$

$$I = I_c \sin\Delta\phi \tag{13}$$

5 Quantum Circuits

Energy in the oscillator,

$$U = \int_{-\infty}^{t} I(t')V(t')dt' = \int_{-\infty}^{t} I_c \sin\phi \frac{\hbar}{2e}\dot{\phi}dt = \frac{I_c\hbar}{2e}\int_{\phi(-\infty)}^{\theta(t)} \sin\theta d\theta = -\frac{I_c e}{2\hbar}\cos\theta + c \tag{14}$$

$$H = \frac{Q^2}{2C} - E_J\cos\phi \tag{15}$$

with $E_J = eI_c/(2\hbar)$. Expanding the cosine $\cos\phi \approx 1 - \phi^2/2$ and ignoring the constant offset,

$$H = \frac{1}{2}CV^2 + \frac{1}{2}LI^2 = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} \tag{16}$$

with $\omega = 1/\sqrt{LC}$, $\phi = -2e\Phi/\hbar$ and $V = -LdI/dt = Q/C$.

Since now we are dealing with Cooper pairs, we have that $\hat{Q} = 2e\hat{N}$. Thus, $\hat{Q}^2/2C = (2e^2/C)\hat{N}^2 \equiv E_C\hat{N}^2$ where we define $E_C = 2e^2/C$. This results in

$$\boxed{H = E_C\hat{N}^2 - E_J\cos\hat{\Phi}} \tag{17}$$

Expanding this out,

$$H = \sum_N E_C N^2 |N\rangle\langle N| - \frac{E_J}{2}(|N\rangle\langle N+1| + h.c.) \tag{18}$$

The anharmonicity of this oscillator scales as $\sqrt{E_C/E_J}$.

Let's quantize this oscillator.

6 trivia

1. $k_B T = \hbar\omega$
2. 5GHz $\rightarrow T = 250$mK.
3. Cooper pair box $T_2^* \sim 100$ns
4. the effective dipole moment, which enters in the transition matrix elements while driving between two states, can be made much larger in superconducting qubits than atoms