Assignment II

Answer Any Five Questions

Q1a) Consider the elliptic integral of the first kind:

$$F(k,\phi) = \int_0^{\phi} d\phi' / \sqrt{1-k^2 \sin^2 \phi'} \qquad (k \ \& \ \phi \text{ are constants})$$

Write a function to numerically evaluate the elliptic integral for given values of $k \,\&\, \phi$, by using any of the numerical integration function.

b) Consider a simple pendulum, whose exact equation of motion is

$$\ddot{\theta} = -\,\omega_0^2 \sin\theta \qquad \omega_0^2 = g/\ell$$

Solve this equation numerically (it cannot be solved exactly) when the pendulum is released from rest at angle α with the vertical. Take $\alpha = 10^0, 20^0, ...170^0$, that is at intervals of 10 degrees and plot a graph of time period vs. Angle of release. You may take a meters pendulum $(\ell = 1\text{m})$.

c) The time period of a pendulum can be expressed in terms of the elliptic integral as follows.

$$T = 4\sqrt{\frac{\ell}{g}}\ F(k,\pi/2) \qquad \text{where, } k = \sin(\alpha/2)$$

Plot the time period vs. α using your function of part a) and compare with the similar plot of part b.

Q2) Consider the motion of a charged particle of charge q and mass m in crossed electric and magnetic fields

$$\vec{E} = E_0 \hat{x} \qquad \vec{B} = -\,B_0\,\hat{z}$$

a) Write down the equations of motion in three dimensional space and reduce them to a system of first order equation.

b) Write a program to solve the equations of motion with initial conditions.

c) If the particle is released at rest at the point x = y = z = 0, then calculate and plot the trajectory on the x-y plane. Study the motion by changing the values of $E_0 \,\&\, B_0$.

d) By putting $E_0 = 0$, obtain the helical path of the particle around the magnetic field.

Q3) Consider a hanging chain. The differential equation for the shape of the chain is

$$\frac{d^2y}{dy^2} = a\sqrt{1+y'^2}$$

Here, x axis is the horizontal axis, y axis is the vertically up axis, and 'a' is a constant equal to μ/T, μ being mass per unit length of the chain and T being the tension at the bottom of the chain. Obviously, 'a' is not known.

The chain is hung from the two points (0, 0) and (10, 0). Take the length of the chain to be 10n (n=2,3,4,5)

Solve the differential equation as a boundary value problem using the shooting method. In order to solve it, assign some estimated value to a. From the solution you obtained, determine the length of the chain. Keep on changing the value of 'a', until you get the given length of the chain. Plot the contour of the chain.

Q4) Consider a double pendulum, in which two identical pendulums are hung one below the other. Each pendulum consists of a mass less rod and a bob of mass m. The generalised coordinates are θ_1 & θ_2, the angles made by the two pendulums with the vertical.

Write down the equations of motion for θ_1 & θ_2 and solve them using RK4 method for different initial conditions. Graphically display the configurations of the pendulums at several time instants for the initial conditions : $\theta_1(0) = 30^0$, $\theta_2(0) = 60^0, \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0.$

Q5) A potato (same parameters as discussed in the class) at room temperature (25 degrees Celsius) is put inside boiling water until the temperature at its centre reaches 70 degrees Celsius. It is then removed from inside the boiling water and allowed to cool until its temperature at the centre reaches 5 degrees above room temperature. Assume that cooling takes place only by heat diffusion outward from the surface, that is, there is no radiation cooling.

a) Find the time required for each process.

b) Plot temperature as a function of r both while heating as well as cooling, at five time instants during each process.

c) Plot the temperature at the centre as a function of time during the entire process.

Q6) Consider a string (same dimensions, mass/length) as discussed in the class. Both ends of the string are completely free to move vertically. Generate a pulse at rest at the centre and plot it at several time instants, after repeated reflections at the two ends.

Q7) Consider a string fixed at both ends. There is a heavy mass at position $3L/4$ (the problem discussed in the class). Generate a right moving pulse at the centre and study its reflections and transmissions at the heavy mass.