Assignment I

Note : Each question carries 4 marks, making a total of 20 marks for this assignment. You have to submit within 2 weeks, that is, by September 27.

Q1a) Write your own program to generate an array of random numbers between -1000 and +1000. Pick some random number generation formula. To check how good your random numbers are, divide the interval into 100 bins of width 20 each, and find out how many random numbers fall into each bin. Hence draw a histogram. Also compute the mean and variance of the distribution.

b) Write your own program to sort (arrange in the increasing order) an array (vector) of integers of any arbitrary length. The array elements should be chosen as random numbers between 0 and 1000.

c) Find the largest and smallest element, the mean, median and standard deviation of this array of elements.

Q2a) Write a program to obtain the first three energy eigen values of a particle in a finitely-deep potential well, by finding the first three roots of appropriate functions as discussed in the class. Use Newton-Raphson method.

or

Write a program to calculate the time taken by a body to come back to the ground, when thrown up into the air. Include damping force for air resistance. Choose the damping constant to be such that the maximum damping force varies between 0 t0 20% of the body's weight. See how the time taken varies with the changing value of the damping constant. Choose the initial velocity appropriately.

Q3) Find the point of intersection between i) a torus of inner radius a, outer radius b (mean radius R=(a+b)/2) ii) a sphere of radius r, s.t., a < r < b iii) a plane passing through the origin. The centres of both the torus as well as the sphere are at the origin.

Q4a) Write a function that will do a curve fitting using linear combination of Legendre polynomials up to degree 5, over any arbitrary interval [a,b]. Use Gauss elimination method to solve for the coefficients of the linear sum.

b) As a particular case, generate a random set of data points, distributed within an error bar of 10% of the values of any give function over the interval [a,b] and fit these data points using the function developed in (5a).

Q5) Consider quadratic spline interpolation, where two successive data points are joined by a second-degree polynomial

$$P_i(x) = a_i(x-x_i)^2 + b_i(x-x_i) + c_i \quad i = 1, 2, ..., N-1$$

a) With the condition that the first derivative of one polynomial matches with the first derivative of the subsequent polynomial at the common boundary, derive the following equations

i) $a_i = (y_{i+1} - y_i)/h^2 - b_i/h \qquad i = 1, 2, ,......, N-1$

ii) $b_i + b_{i+1} = 2(y_{i+1} - y_i)/h \qquad i = 1, 2, \ldots\ldots, N-2$

b) As the additional condition, put $P_1''(x_1) = 0.$ Hence, show that

$$b_1 = (y_2 - y_1)/h$$

c) With b_1 known, it is straight forward to determine all the other b_i (i = 2, 3, ..., N-2) from eqs (ii) and from them, all a_i (i = 1, 2, ..., N-1) from eqs. (i). Write a MATLAB function to encode the above procedure and test it on some known functions and compare it with cubic-spline interpolation through graphs.

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