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**Research Outline**


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# Investigating Gaps Between Lucas and Fibonacci Primes Using Regression Analysis and Estimating Subsequent Primes

## Abstract


It has been proven using the Ulam Spiral method and various other sieving methods that some prime numbers satisfy particular equations; that is, some equations can generate a large number of primes. In addition, there are many theorems and conjectures on gaps between primes, such as Bertrand's Postulate and Yitang Zhang's proof that there are infinitely many instances where the gap between 2 successive primes is less than $7 \times 10^7$.

However, there has not been adequate research to prove that Fibonacci Prime numbers or Lucas Prime numbers occur more frequently in certain polynomial equations, or that gaps between them are bounded by certain inequalities. In this paper, we use regression analysis, among other techniques, in order to find such polynomials or inequalities.

The Fibonacci and the Lucas Numbers are both Lucas sequences with the property $N_y = N_{y-1} + N_{y-2}$, where $N_0 = 0$ and $N_1 = 1$ for the Fibonacci Numbers, and $N_0 = 2$ and $N_1 = 1$ for the Lucas Numbers. The prime numbers that appear in these sequences are known as Fibonacci and Lucas primes respectively. To investigate the gaps between Lucas and Fibonacci

primes, we created a program detailed in the paper (also attached along with the outline) which establishes their double exponential nature. Furthermore, the paper will also describe variations to the Ulam Spiral that we created, that can be applied to Fibonacci and Lucas primes.

This information about the prime gaps, along with independent regression analysis on the Fibonacci and Lucas primes and their indices in their respective sequences, will be used to demonstrate the existence of Fibonacci and Lucas prime-rich polynomials.

This paper demonstrates a comprehensive analysis of the Fibonacci and Lucas primes and the polynomials that can yield them, along with a discussion of any inequalities that can limit the gaps between them. Additionally, it discusses how these gaps can aid in estimating further primes in the sequences, expanding our knowledge of primes as a whole, and perhaps even aiding in future research about unsolved problems related to Fibonacci and Lucas Primes.

**Keywords: Prime Numbers, Ulam Spiral, Fibonacci Numbers, Lucas Numbers, Fibonacci Primes, Lucas Primes, Prime Gaps, Regression Analysis, Estimation, Double Exponential Equations**

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