

DAA

Group A

Assignment No: D1

Title of the Assignment: Write a program non-recursive and recursive program to calculate Fibonacci numbers and analyze their time and space complexity.

Objective of the Assignment: Students should be able to perform non-recursive and recursive programs to calculate Fibonacci numbers and analyze their time and space complexity.

Prerequisite:

1. Basic of Python or Java Programming
 2. Concept of Recursive and Nonrecursive functions
 3. Execution flow of calculate Fibonacci numbers
 4. Basic of Time and Space complexity
-

Contents for Theory:

1. Introduction to Fibonacci numbers
 2. Time and Space complexity
-

Introduction to Fibonacci numbers

- The Fibonacci series, named after Italian mathematician Leonardo Pisano Bogollo, later known as Fibonacci, is a series (sum) formed by Fibonacci numbers denoted as F_n . The numbers in Fibonacci sequence are given as: 0, 1, 1, 2, 3, 5, 8, 13, 21, 38, . . .
- In a Fibonacci series, every term is the sum of the preceding two terms, starting from 0 and 1 as first and second terms. In some old references, the term '0' might be omitted.

What is the Fibonacci Series?

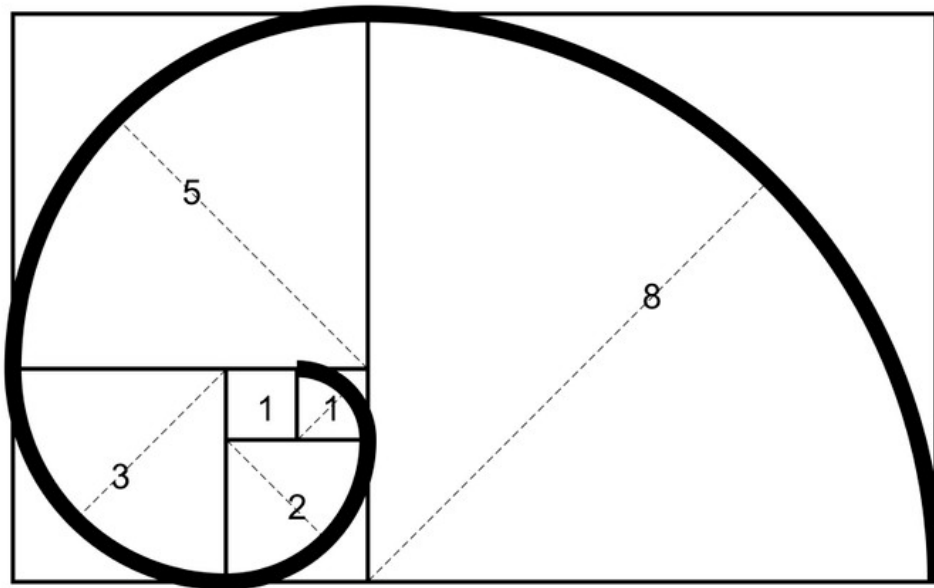
- The Fibonacci series is the sequence of numbers (also called Fibonacci numbers), where every number is the sum of the preceding two numbers, such that the first two terms are '0' and '1'.
- In some older versions of the series, the term '0' might be omitted. A Fibonacci series can thus be given as, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . It can be thus be observed that every term can be calculated by adding the two terms before it.
- Given the first term, F_0 and second term, F_1 as '0' and '1', the third term here can be given as, $F_2 = 0 + 1 = 1$

Similarly,

$$F_3 = 1 + 1 = 2$$

$$F_4 = 2 + 1 = 3$$

Given a number n , print n -th Fibonacci Number.



Fibonacci Sequence Formula

The Fibonacci sequence of numbers “Fn” is defined using the recursive relation with the seed values F0=0 and F1=1:

$$F_n = F_{n-1} + F_{n-2}$$

Here, the sequence is defined using two different parts, such as kick-off and recursive relation.

The kick-off part is F0=0 and F1=1.

The recursive relation part is $F_n = F_{n-1} + F_{n-2}$.

It is noted that the sequence starts with 0 rather than 1. So, F5 should be the 6th term of the sequence.

Examples:

Input : n = 2

Output : 1

Input : n = 9

Output : 34

The list of Fibonacci numbers are calculated as follows:

F_n	Fibonacci Number
0	0
1	1
2	1
3	2
4	3
5	5

6	8
7	13
8	21
9	34
... and so on.	... and so on.

Method 1 (Use Non-recursion)

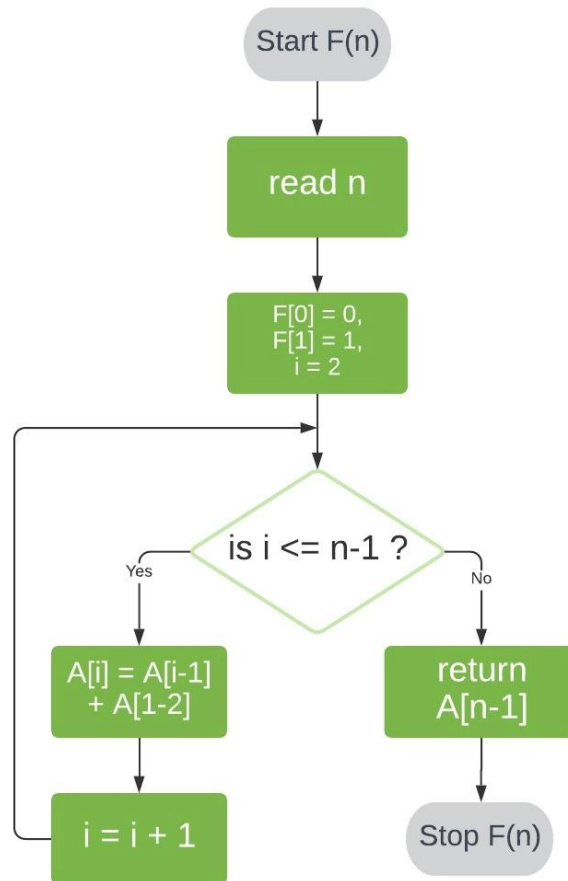
A simple method that is a direct recursive implementation of mathematical recurrence relation is given above.

First, we'll store 0 and 1 in $F[0]$ and $F[1]$, respectively.

Next, we'll iterate through array positions 2 to $n-1$. At each position i , we store the sum of the two preceding array values in $F[i]$.

Finally, we return the value of $F[n-1]$, giving us the number at position n in the sequence.

Here's a visual representation of this process:



Program to display the Fibonacci sequence up to n-th term

```
nterms = int(input("How many terms? "))
```

first two terms

```
n1, n2 = 0, 1
```

```
count = 0
```

check if the number of terms is valid

```
if nterms <= 0:
```

```
    print("Please enter a positive integer")
```

if there is only one term, return n1

```
elif nterms == 1:
```

```
    print("Fibonacci sequence upto",nterms,":")
```

```
    print(n1)
```

generate fibonacci sequence

else:

```
print("Fibonacci sequence:")
```

```
while count < nterms:
```

```
    print(n1)
```

```
    nth = n1 + n2
```

```
    # update values
```

```
    n1 = n2
```

```
    n2 = nth
```

```
    count += 1
```

Output

How many terms? 7

Fibonacci sequence:

0

1

1

2

3

5

8

Time and Space Complexity of Space Optimized Method

- The time complexity of the Fibonacci series is **$T(N)$ i.e, linear**. We have to find the sum of two terms and it is repeated n times depending on the value of n .
- The space complexity of the Fibonacci series using dynamic programming is **$O(1)$** .

Time Complexity and Space Complexity of Dynamic Programming

- The time complexity of the above code is **$T(N)$ i.e, linear**. We have to find the sum of two terms and it is repeated n times depending on the value of n .

- The space complexity of the above code is $O(N)$.

Method 2 (Use Recursion)

Let's start by defining $F(n)$ as the function that returns the value of F_n .

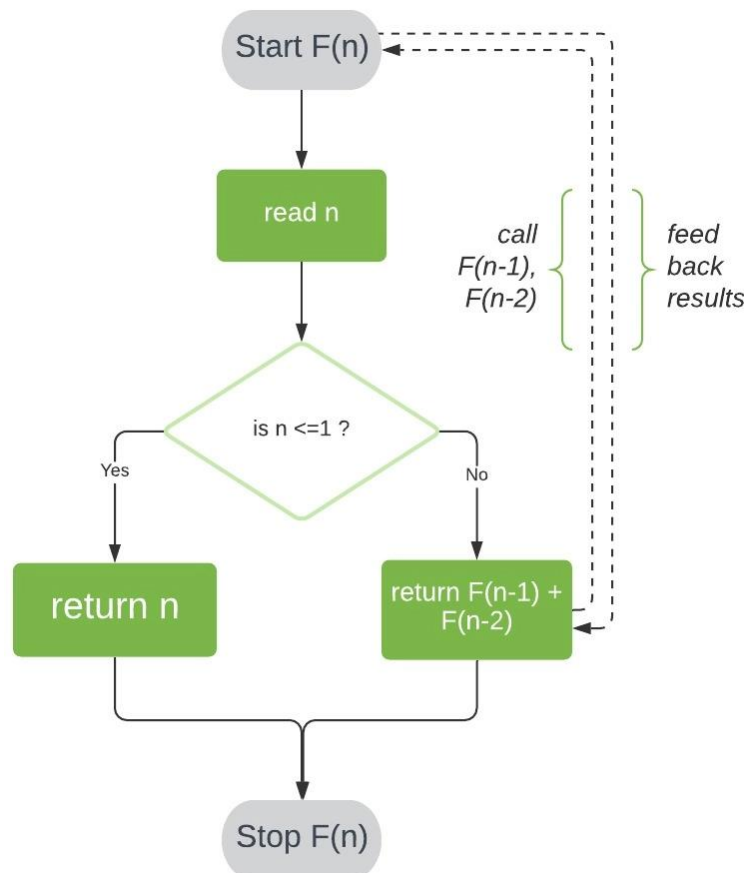
To evaluate $F(n)$ for $n > 1$, we can reduce our problem into two smaller problems of the same kind: $F(n-1)$ and $F(n-2)$. We can further reduce $F(n-1)$ and $F(n-2)$ to $F((n-1)-1)$ and $F((n-1)-2)$; and $F((n-2)-1)$ and $F((n-2)-2)$, respectively.

If we repeat this reduction, we'll eventually reach our known base cases and, thereby, obtain a solution to $F(n)$.

Employing this logic, our algorithm for $F(n)$ will have two steps:

1. Check if $n \leq 1$. If so, return n .
2. Check if $n > 1$. If so, call our function F with inputs $n-1$ and $n-2$, and return the sum of the two results.

Here's a visual representation of this algorithm:



```
# Python program to display the Fibonacci sequence
```

```
def recur_fibo(n):
```

```
    if n <= 1:
```

```
        return n
```

```
    else:
```

```
        return(recur_fibo(n-1) + recur_fibo(n-2))
```

```
nterms = 7
```

```
# check if the number of terms is valid
```

```
if nterms <= 0:
```

```
    print("Plese enter a positive integer")
```

```
else:
```

```
    print("Fibonacci sequence:")
```

```
    for i in range(nterms):
```

```
        print(recur_fibo(i))
```

Output

Fibonacci sequence:

0

1

1

2

3

5

8

Time and Space Complexity

- The time complexity of the above code is $T(2^N)$ i.e, exponential.

- The Space complexity of the above code is **$O(N)$ for a recursive series.**

Method	Time complexity	Space complexity
Using recursion	$T(n) = T(n-1) + T(n-2)$	$O(n)$
Using DP	$O(n)$	$O(1)$
Space optimization of DP	$O(n)$	$O(1)$
Using the power of matrix method	$O(n)$	$O(1)$
Optimized matrix method	$O(\log n)$	$O(\log n)$
Recursive method in $O(\log n)$ time	$O(\log n)$	$O(n)$
Using direct formula	$O(\log n)$	$O(1)$
DP using memoization	$O(n)$	$O(1)$

Applications of Fibonacci Series

The Fibonacci series finds application in different fields in our day-to-day lives. The different patterns found in a varied number of fields from nature, to music, and to the human body follow the Fibonacci series. Some of the applications of the series are given as,

- It is used in the grouping of numbers and used to study different other special mathematical sequences.
- It finds application in Coding (computer algorithms, distributed systems, etc). For example, Fibonacci series are important in the computational run-time analysis of Euclid's algorithm, used for determining the GCF of two integers.
- It is applied in numerous fields of science like quantum mechanics, cryptography, etc.
- In finance market trading, Fibonacci retracement levels are widely used in technical analysis.

Conclusion- In this way we have explored Concept of Fibonacci series using recursive and non recursive method and also learn time and space complexity

Assignment Questions

- 1. What is the Fibonacci Sequence of numbers?**
- 2. How do the Fibonacci work?**
- 3. What is the Golden Ratio?**
- 4. What is the Fibonacci Search technique?**
- 5. What is the real application for Fibonacci series**

Reference link

- <https://www.scaler.com/topics/fibonacci-series-in-c/>
- <https://www.baeldung.com/cs/fibonacci-computational-complexity>

DAA**Group A****Assignment No: D2**

Title of the Assignment: Write a program to implement Huffman Encoding using a greedy strategy.

Objective of the Assignment: Students should be able to understand and solve Huffman Encoding using greedy method

Prerequisite:

1. Basic of Python or Java Programming
 2. Concept of Greedy method
 3. Huffman Encoding concept
-

Contents for Theory:

1. Greedy Method
 2. Huffman Encoding
 3. Example solved using huffman encoding
-

What is a Greedy Method?

- A greedy algorithm is an approach for solving a problem by selecting the best option available at the moment. It doesn't worry whether the current best result will bring the overall optimal result.
- The algorithm never reverses the earlier decision even if the choice is wrong. It works in a top-down approach.
- This algorithm may not produce the best result for all the problems. It's because it always goes for the local best choice to produce the global best result.

Advantages of Greedy Approach

- The algorithm is **easier to describe**.
- This algorithm can **perform better** than other algorithms (but, not in all cases).

Drawback of Greedy Approach

- As mentioned earlier, the greedy algorithm doesn't always produce the optimal solution. This is the major disadvantage of the algorithm
- For example, suppose we want to find the longest path in the graph below from root to leaf.

Greedy Algorithm

1. To begin with, the solution set (containing answers) is empty.
2. At each step, an item is added to the solution set until a solution is reached.
3. If the solution set is feasible, the current item is kept.
4. Else, the item is rejected and never considered again.

Huffman Encoding

- Huffman Coding is a technique of compressing data to reduce its size without losing any of the details. It was first developed by David Huffman.
- Huffman Coding is generally useful to compress the data in which there are frequently occurring

characters.

- Huffman Coding is a famous Greedy Algorithm.
- It is used for the lossless compression of data.
- It uses variable length encoding.
- It assigns variable length code to all the characters.
- The code length of a character depends on how frequently it occurs in the given text.
- The character which occurs most frequently gets the smallest code.
- The character which occurs least frequently gets the largest code.
- It is also known as **Huffman Encoding**.

Prefix Rule-

- Huffman Coding implements a rule known as a prefix rule.
- This is to prevent the ambiguities while decoding.
- It ensures that the code assigned to any character is not a prefix of the code assigned to any other character

Major Steps in Huffman Coding-

There are two major steps in Huffman Coding-

1. Building a Huffman Tree from the input characters.
2. Assigning code to the characters by traversing the Huffman Tree.

How does Huffman Coding work?

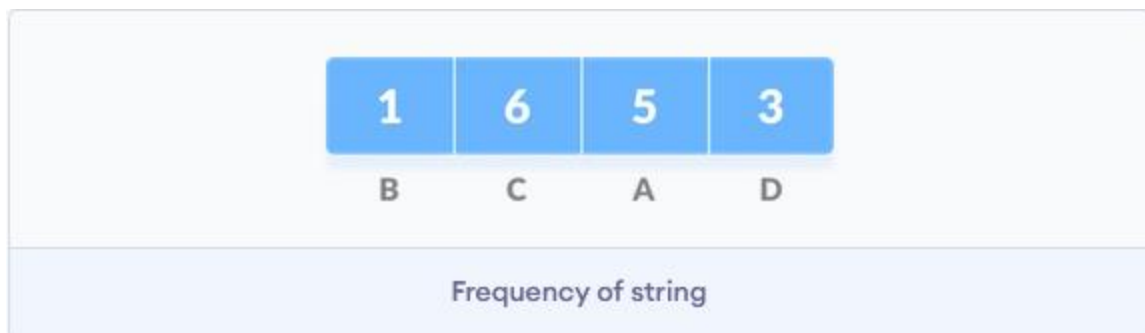
Suppose the string below is to be sent over a network.



- Each character occupies 8 bits. There are a total of 15 characters in the above string. Thus, a total of

$8 * 15 = 120$ bits are required to send this string.

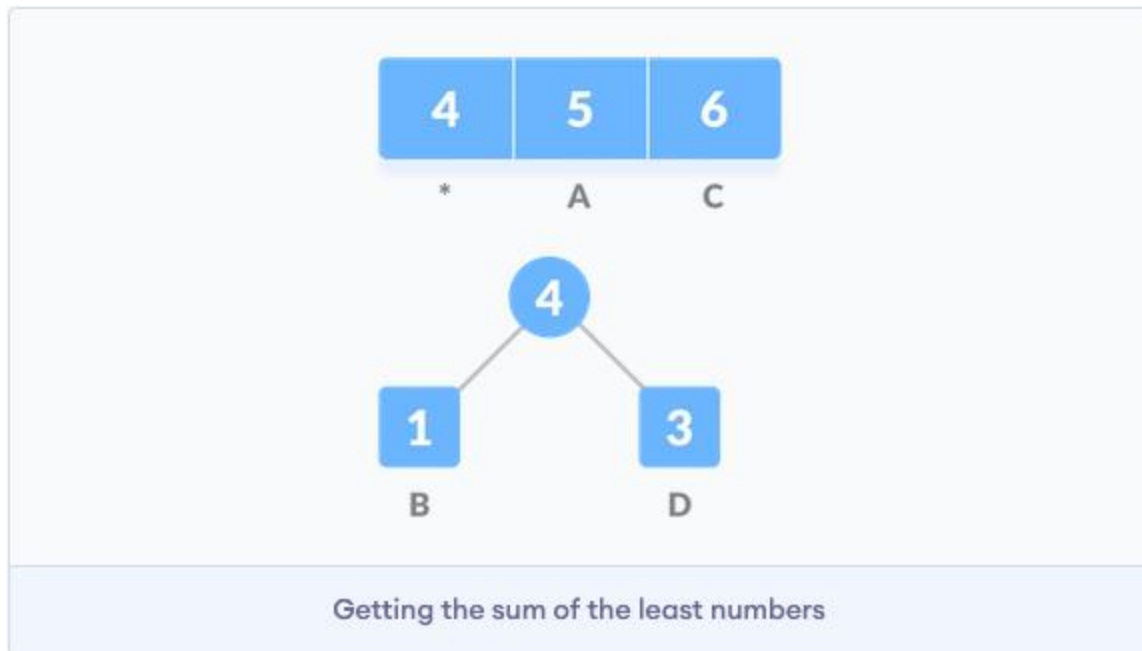
- Using the Huffman Coding technique, we can compress the string to a smaller size.
 - Huffman coding first creates a tree using the frequencies of the character and then generates code for each character.
 - Once the data is encoded, it has to be decoded. Decoding is done using the same tree.
 - Huffman Coding prevents any ambiguity in the decoding process using the concept of **prefix code** ie. a code associated with a character should not be present in the prefix of any other code. The tree created above helps in maintaining the property.
 - Huffman coding is done with the help of the following steps.
1. Calculate the frequency of each character in the string.



2. Sort the characters in increasing order of the frequency. These are stored in a priority queue Q.



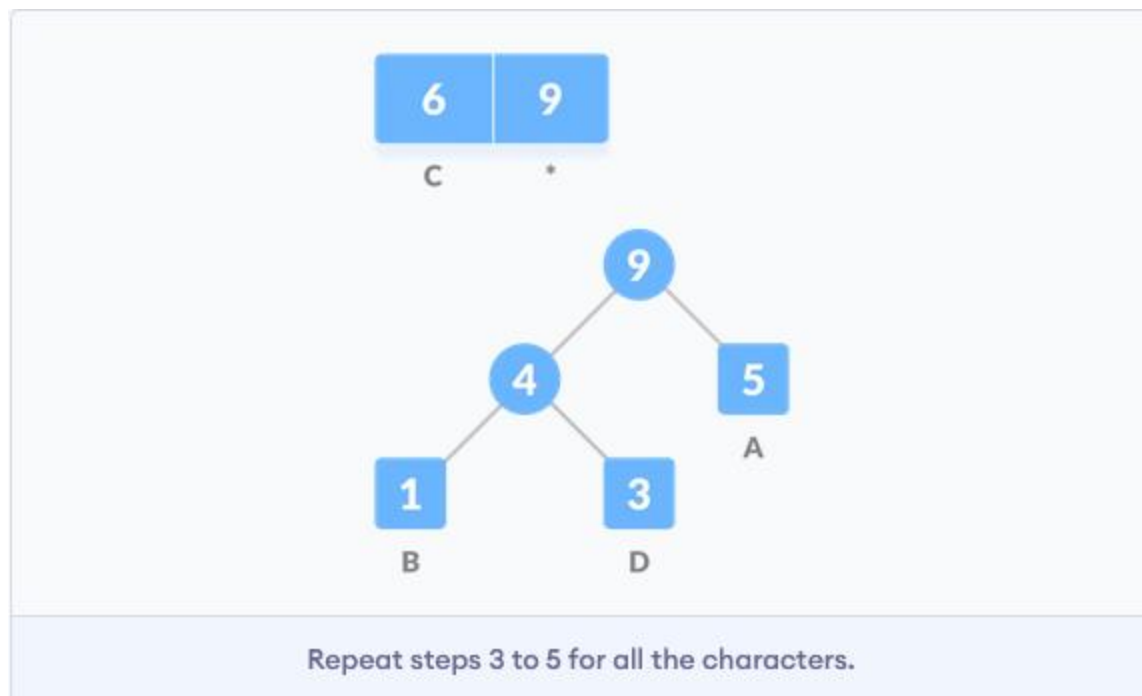
3. Make each unique character as a leaf node.
4. Create an empty node z. Assign the minimum frequency to the left child of z and assign the second minimum frequency to the right child of z. Set the value of the z as the sum of the above two minimum frequencies.

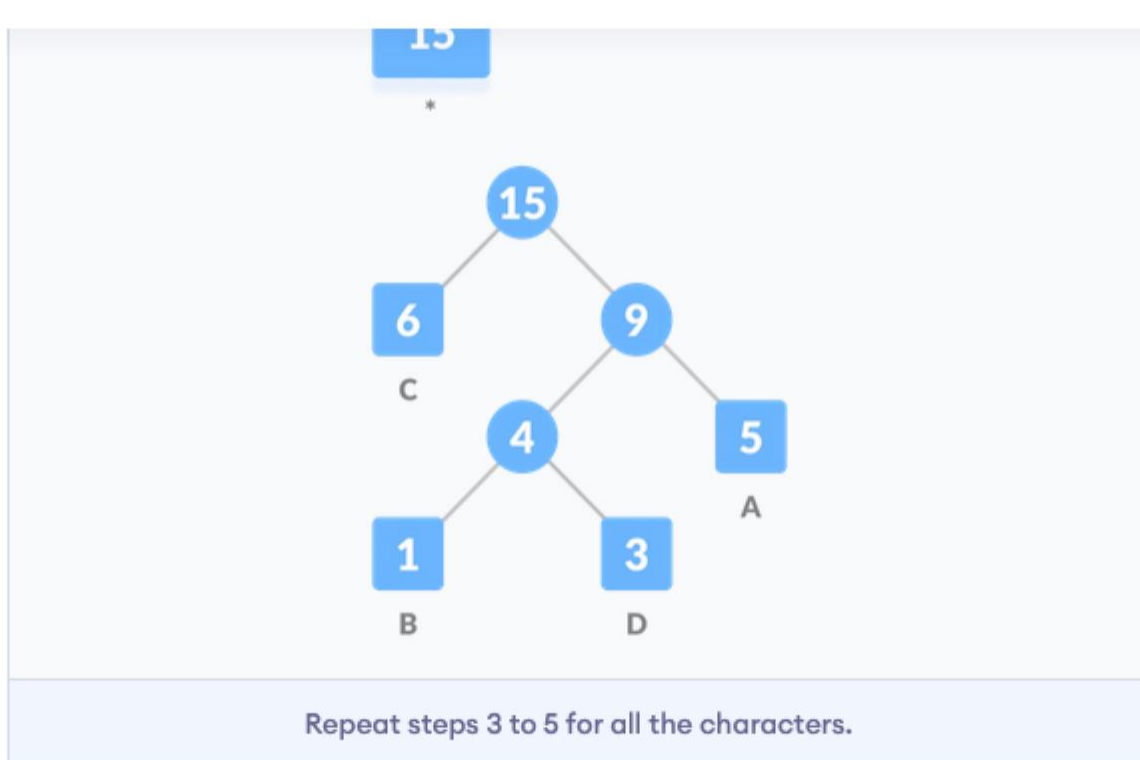


5. Remove these two minimum frequencies from Q and add the sum into the list of frequencies (* denote the internal nodes in the figure above).

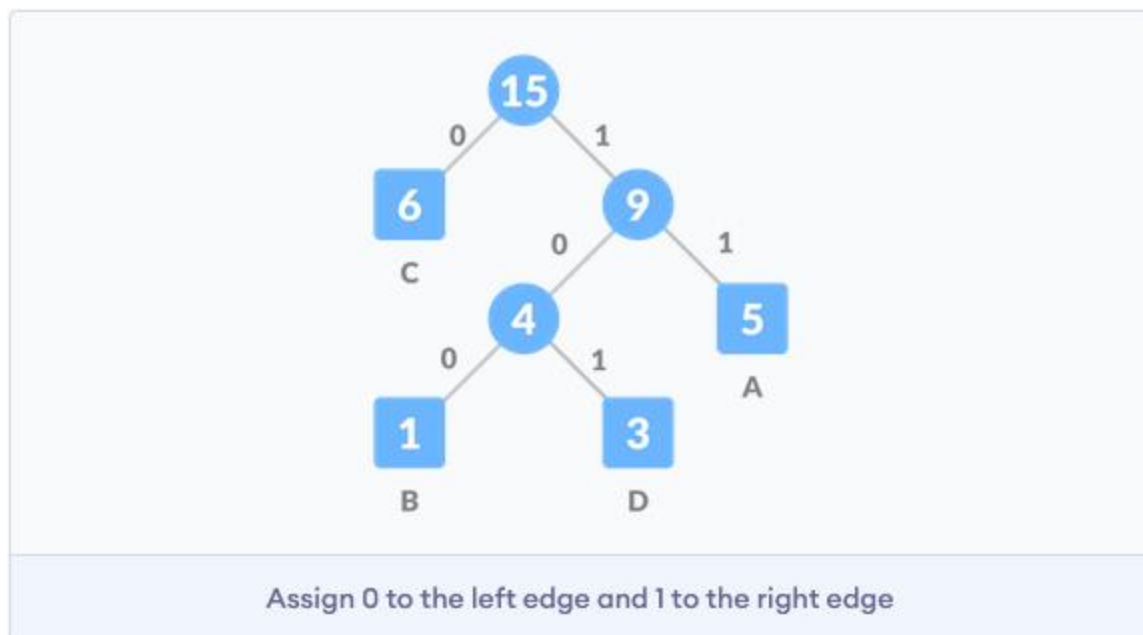
6. Insert node z into the tree.

7. Repeat steps 3 to 5 for all the characters.





8. For each non-leaf node, assign 0 to the left edge and 1 to the right edge



For sending the above string over a network, we have to send the tree as well as the above compressed-code. The total size is given by the table below.

Character	Frequency	Code	Size
A	5	11	$5 \times 2 = 10$
B	1	100	$1 \times 3 = 3$
C	6	0	$6 \times 1 = 6$
D	3	101	$3 \times 3 = 9$
$4 \times 8 = 32 \text{ bits}$		15 bits	28 bits

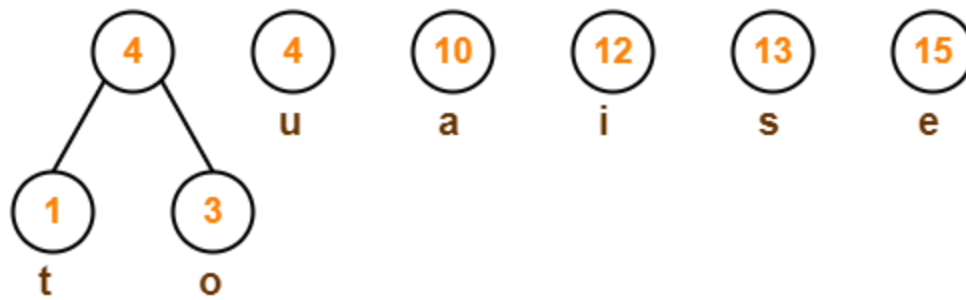
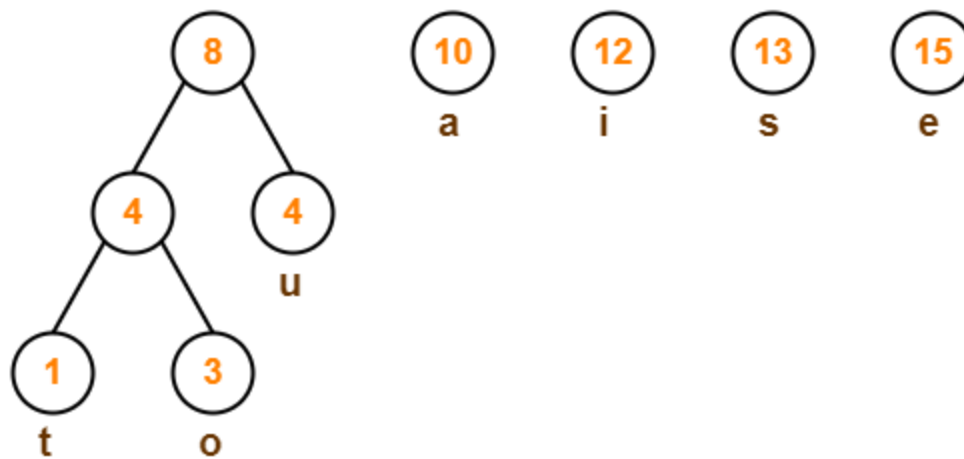
Without encoding, the total size of the string was 120 bits. After encoding the size is reduced to $32 + 15 + 28 = 75$.

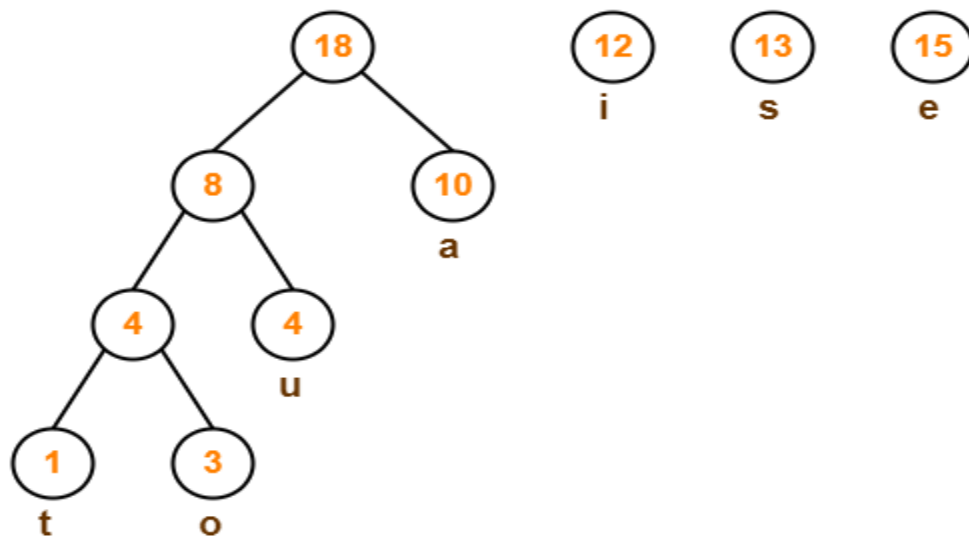
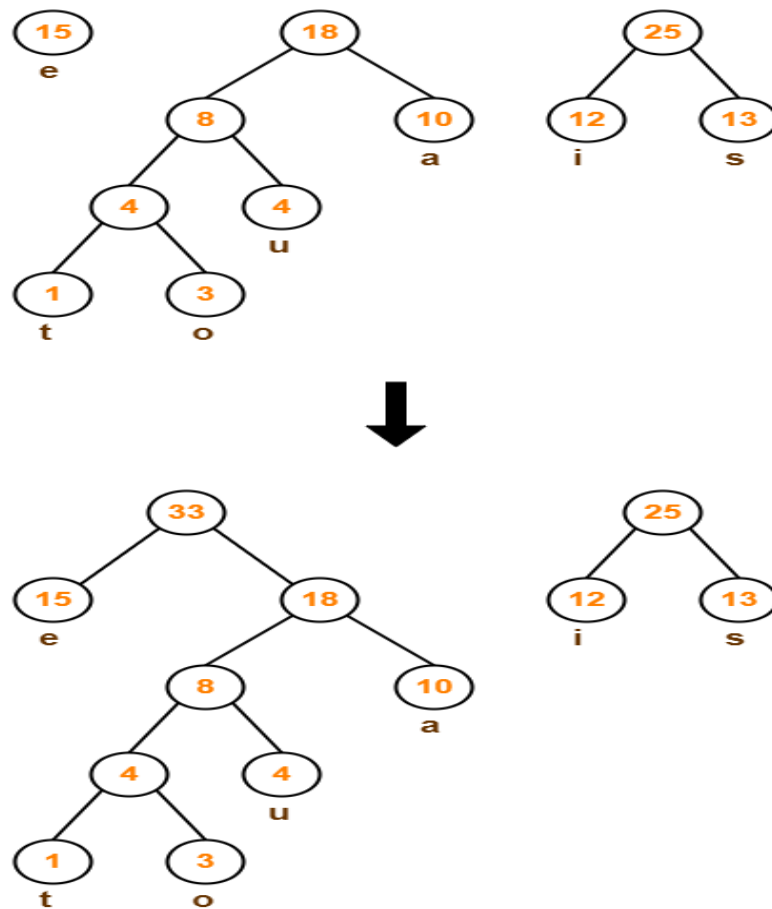
Example:

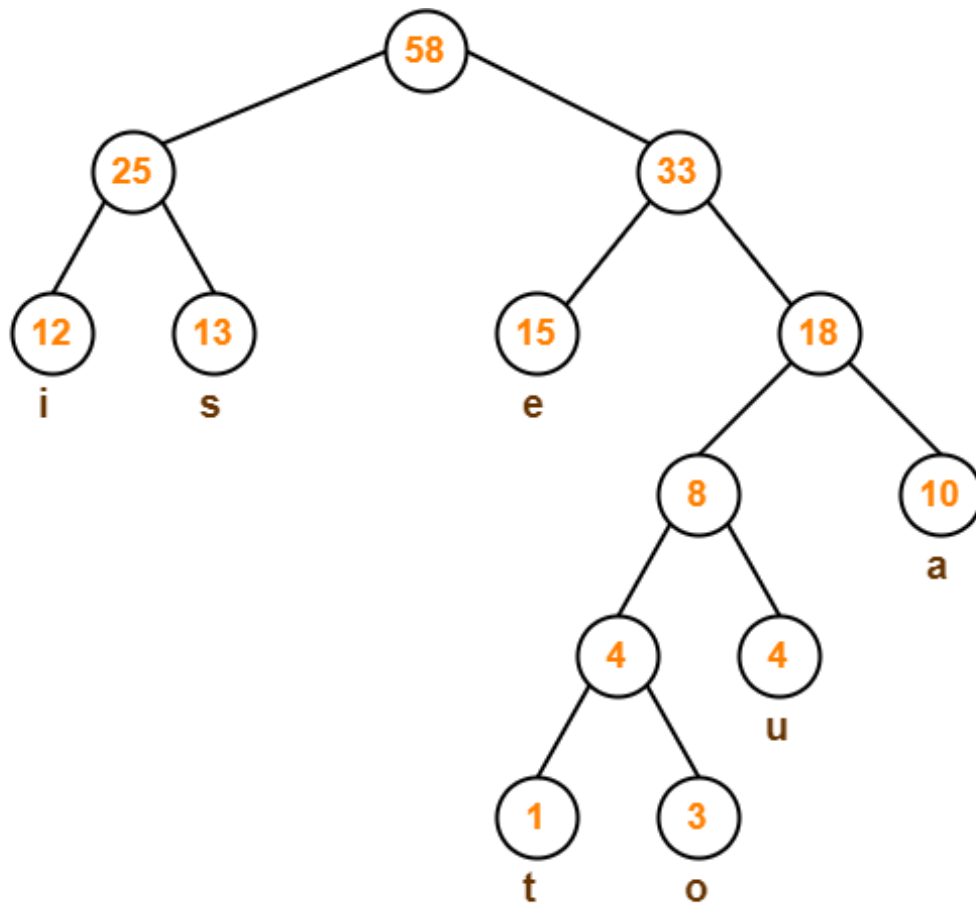
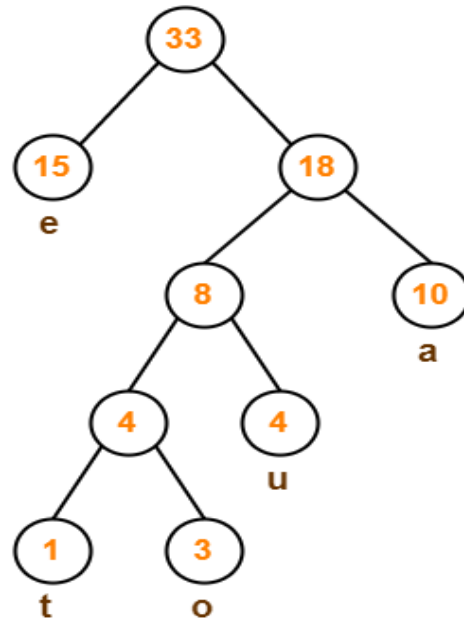
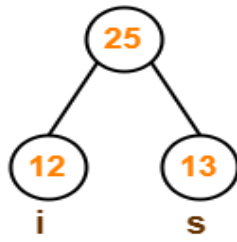
A file contains the following characters with the frequencies as shown. If Huffman Coding is used for data compression, determine-

1. Huffman Code for each character
2. Average code length
3. Length of Huffman encoded message (in bits)

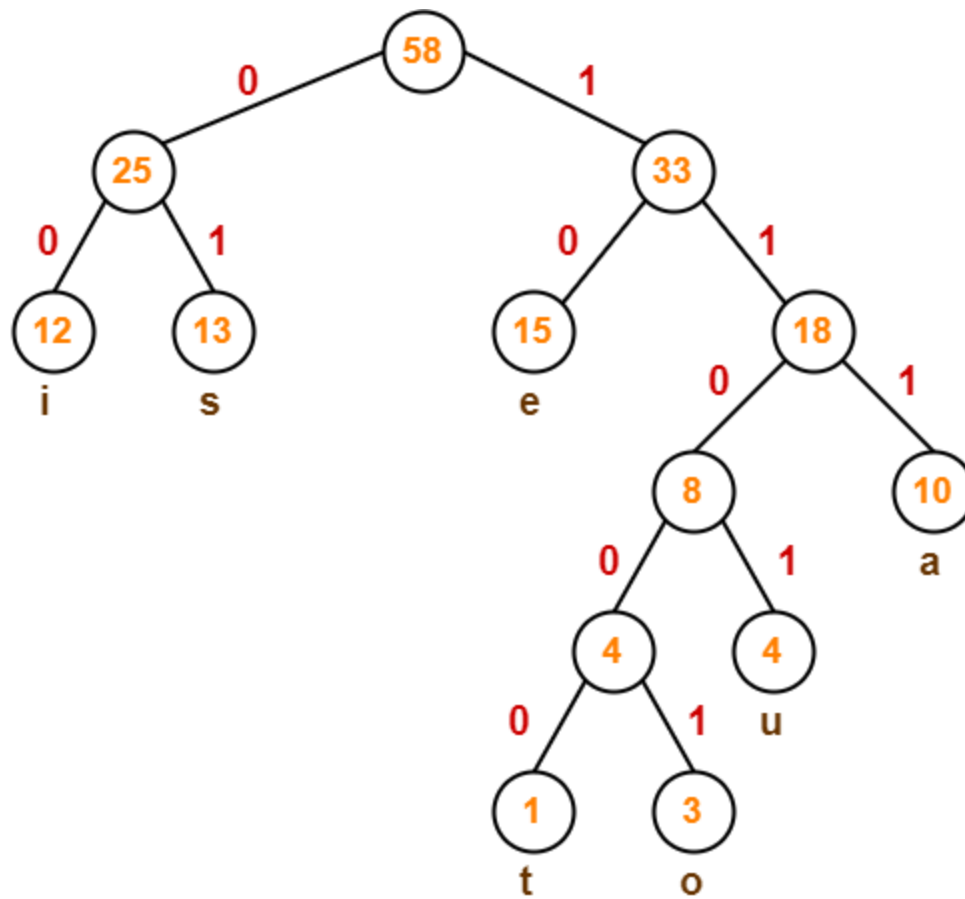
Characters	Frequencies
a	10
e	15
i	12
o	3
u	4
s	13
t	1

Step-01:**Step-02:****Step-03:**

Step-04:**Step-06:**

Step-07:

After assigning weight to all the edges, the modified Huffman Tree is-



Huffman Tree

To write Huffman Code for any character, traverse the Huffman Tree from root node to the leaf node of that character.

Following this rule, the Huffman Code for each character is-

a = 111

e = 10

i = 00

o = 11001

u = 1101

s = 01

t = 11000

Time Complexity-

The time complexity analysis of Huffman Coding is as follows-

- `extractMin()` is called $2 \times (n-1)$ times if there are n nodes.
- As `extractMin()` calls `minHeapify()`, it takes $O(\log n)$ time.

Thus, Overall time complexity of Huffman Coding becomes **$O(n \log n)$** .

Conclusion- In this way we have explored Concept of Huffman Encoding using greedy method

Assignment Questions

1. What is Huffman Encoding?
2. How many bits may be required for encoding the message 'mississippi'?
3. Which tree is used in Huffman encoding? Give one Example
4. Why Huffman coding is lossless compression?

Reference link

- <https://towardsdatascience.com/huffman-encoding-python-implementation-8448c3654328>
- <https://www.programiz.com/dsa/huffman-coding#cpp-code>
- <https://www.gatevidyalay.com/tag/huffman-coding-example-ppt/>

DAA**Group A****Assignment No: D3**

Title of the Assignment: Write a program to solve a fractional Knapsack problem using a greedy method.

Objective of the Assignment: Students should be able to understand and solve fractional Knapsack problems using a greedy method.

Prerequisite:

1. Basic of Python or Java Programming
 2. Concept of Greedy method
 3. fractional Knapsack problem
-

Contents for Theory:

1. Greedy Method
 2. Fractional Knapsack problem
 3. Example solved using fractional Knapsack problem
-

What is a Greedy Method?

- A greedy algorithm is an approach for solving a problem by selecting the best option available at the moment. It doesn't worry whether the current best result will bring the overall optimal result.
- The algorithm never reverses the earlier decision even if the choice is wrong. It works in a top-down approach.
- This algorithm may not produce the best result for all the problems. It's because it always goes for the local best choice to produce the global best result.

Advantages of Greedy Approach

- The algorithm is **easier to describe**.
- This algorithm can **perform better** than other algorithms (but, not in all cases).

Drawback of Greedy Approach

- As mentioned earlier, the greedy algorithm doesn't always produce the optimal solution. This is the major disadvantage of the algorithm
- For example, suppose we want to find the longest path in the graph below from root to leaf.

Greedy Algorithm

1. To begin with, the solution set (containing answers) is empty.
2. At each step, an item is added to the solution set until a solution is reached.
3. If the solution set is feasible, the current item is kept.
4. Else, the item is rejected and never considered again.

Knapsack Problem

You are given the following-

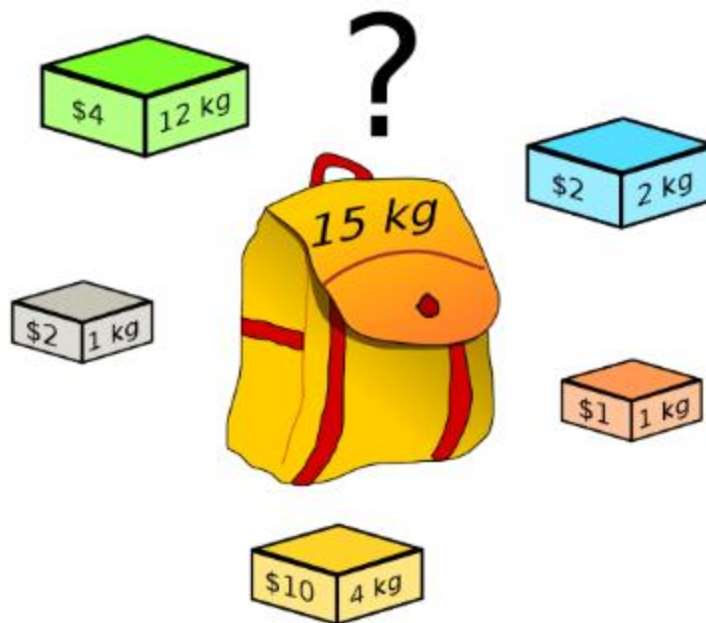
- A knapsack (kind of shoulder bag) with limited weight capacity.

- Few items each having some weight and value.

The problem states-

Which items should be placed into the knapsack such that-

- The value or profit obtained by putting the items into the knapsack is maximum.
- And the weight limit of the knapsack does not exceed.



Knapsack Problem

Knapsack Problem Variants

Knapsack problem has the following two variants-

1. Fractional Knapsack Problem
2. 0/1 Knapsack Problem

Fractional Knapsack Problem-

In Fractional Knapsack Problem,

- As the name suggests, items are divisible here.
- We can even put the fraction of any item into the knapsack if taking the complete item is not

possible.

- It is solved using the Greedy Method.

Fractional Knapsack Problem Using Greedy Method-

Fractional knapsack problem is solved using greedy method in the following steps-

Step-01:

For each item, compute its value / weight ratio.

Step-02:

Arrange all the items in decreasing order of their value / weight ratio.

Step-03:

Start putting the items into the knapsack beginning from the item with the highest ratio.

Put as many items as you can into the knapsack.

Problem-

For the given set of items and knapsack capacity = 60 kg, find the optimal solution for the fractional knapsack problem making use of greedy approach.

Item	Weight	Value
1	5	30
2	10	40
3	15	45
4	22	77
5	25	90

$$n = 5$$

$$w = 60 \text{ kg}$$

$$(w_1, w_2, w_3, w_4, w_5) = (5, 10, 15, 22, 25)$$

$$(b_1, b_2, b_3, b_4, b_5) = (30, 40, 45, 77, 90)$$

Solution-**Step-01:**

Compute the value / weight ratio for each item-

Items	Weight	Value	Ratio
1	5	30	6
2	10	40	4
3	15	45	3
4	22	77	3.5
5	25	90	3.6

Step-02:

Sort all the items in decreasing order of their value / weight ratio-

I1 I2 I5 I4 I3

(6) (4) (3.6) (3.5) (3)

Step-03:

Start filling the knapsack by putting the items into it one by one.

Knapsack Weight	Items in Knapsack	Cost
60	Ø	0
55	I1	30
45	I1, I2	70
20	I1, I2, I5	160

Now,

- Knapsack weight left to be filled is 20 kg but item-4 has a weight of 22 kg.
- Since in fractional knapsack problem, even the fraction of any item can be taken.
- So, knapsack will contain the following items-

< I1 , I2 , I5 , (20/22) I4 >

Total cost of the knapsack

$$= 160 + (20/22) \times 77$$

$$= 160 + 70$$

$$= 230 \text{ units}$$

Time Complexity-

- The main time taking step is the sorting of all items in decreasing order of their value / weight ratio.
- If the items are already arranged in the required order, then while loop takes $O(n)$ time.
- The average time complexity of Quick Sort is $O(n \log n)$.
- Therefore, total time taken including the sort is $O(n \log n)$.

Conclusion-In this way we have explored Concept of Fractional Knapsack using greedy method

Assignment Questions

- 1. What is Greedy Approach?**
- 2. Explain concept of fractional knapsack**
- 3. Difference between Fractional and 0/1 Knapsack**
- 4. Solve one example based on Fractional knapsack(Other than Manual)**

Reference link

- <https://www.gatevidyalay.com/fractional-knapsack-problem-using-greedy-approach/>

DAA

Group A

Assignment No: D4

Title of the Assignment: Write a program to solve a 0-1 Knapsack problem using dynamic programming or branch and bound strategy.

Objective of the Assignment: Students should be able to understand and solve 0-1 Knapsack problem using dynamic programming

Prerequisite:

1. Basic of Python or Java Programming
 2. Concept of Dynamic Programming
 3. 0/1 Knapsack problem
-

Contents for Theory:

1. Greedy Method
 2. 0/1 Knapsack problem
 3. Example solved using 0/1 Knapsack problem
-

What is Dynamic Programming?

- Dynamic Programming is also used in optimization problems. Like divide-and-conquer method, Dynamic Programming solves problems by combining the solutions of subproblems.
- Dynamic Programming algorithm solves each sub-problem just once and then saves its answer in a table, thereby avoiding the work of re-computing the answer every time.
- Two main properties of a problem suggest that the given problem can be solved using Dynamic Programming. These properties are **overlapping sub-problems** and **optimal substructure**.
- Dynamic Programming also combines solutions to sub-problems. It is mainly used where the solution of one sub-problem is needed repeatedly. The computed solutions are stored in a table, so that these don't have to be re-computed. Hence, this technique is needed where overlapping sub-problem exists.
- For example, Binary Search does not have overlapping sub-problem. Whereas recursive program of Fibonacci numbers have many overlapping sub-problems.

Steps of Dynamic Programming Approach

Dynamic Programming algorithm is designed using the following four steps –

- Characterize the structure of an optimal solution.
- Recursively define the value of an optimal solution.
- Compute the value of an optimal solution, typically in a bottom-up fashion.
- Construct an optimal solution from the computed information.

Applications of Dynamic Programming Approach

- Matrix Chain Multiplication
- Longest Common Subsequence
- Travelling Salesman Problem

Knapsack Problem

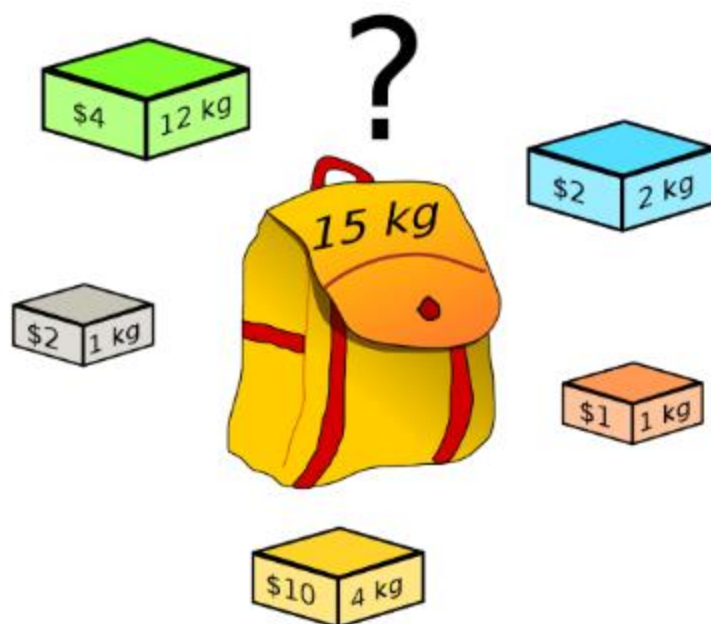
You are given the following-

- A knapsack (kind of shoulder bag) with limited weight capacity.
- Few items each having some weight and value.

The problem states-

Which items should be placed into the knapsack such that-

- The value or profit obtained by putting the items into the knapsack is maximum.
- And the weight limit of the knapsack does not exceed.



Knapsack Problem

Knapsack Problem Variants:

Knapsack problem has the following two variants-

1. Fractional Knapsack Problem
2. 0/1 Knapsack Problem

In 0/1 Knapsack Problem,

- As the name suggests, items are indivisible here.
- We can not take a fraction of any item.
- We have to either take an item completely or leave it completely.
- It is solved using a dynamic programming approach.

0/1 Knapsack Problem Using Greedy Method-

Consider:

- Knapsack weight capacity = w
- Number of items each having some weight and value = n

0/1 knapsack problem is solved using dynamic programming in the following steps-

Step-01:

- Draw a table say 'T' with $(n+1)$ number of rows and $(w+1)$ number of columns.
- Fill all the boxes of 0th row and 0th column with zeroes as shown-

	0	1	2	3	W
0	0	0	0	0	0
1	0					
2	0					
.....						
n	0					

T-Table

Step-02:

Start filling the table row wise top to bottom from left to right.

Use the following formula-

$$T(i, j) = \max \{ T(i-1, j), \text{value}_i + T(i-1, j - \text{weight}_i) \}$$

Here, $T(i, j)$ = maximum value of the selected items if we can take items 1 to i and have weight restrictions of j .

- This step leads to completely filling the table.
- Then, value of the last box represents the maximum possible value that can be put into the knapsack.

Step-03:

- To identify the items that must be put into the knapsack to obtain that maximum profit, consider the last column of the table.
- Start scanning the entries from bottom to top.
- On encountering an entry whose value is not same as the value stored in the entry immediately above it, mark the row label of that entry.
- After all the entries are scanned, the marked labels represent the items that must be put into the knapsack

Problem-. For the given set of items and knapsack capacity = 5 kg, find the optimal solution for the 0/1 knapsack problem making use of a dynamic programming approach.

Item	Weight	Value
1	2	3
2	3	4
3	4	5
4	5	6

$$n = 4$$

$$w = 5 \text{ kg}$$

$$(w_1, w_2, w_3, w_4) = (2, 3, 4, 5)$$

$$(b_1, b_2, b_3, b_4) = (3, 4, 5, 6)$$

Solution-

Given: Knapsack capacity (w) = 5 kg and Number of items (n) = 4

Step-01:

- Draw a table say 'T' with $(n+1) = 4 + 1 = 5$ number of rows and $(w+1) = 5 + 1 = 6$ number of columns.
- Fill all the boxes of 0th row and 0th column with 0.

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0					
2	0					
3	0					
4	0					

T-Table**Step-02:**

Start filling the table row wise top to bottom from left to right using the formula-

$$T(i, j) = \max \{ T(i-1, j), \text{value}_i + T(i-1, j - \text{weight}_i) \}$$

Finding T(1,1):-

We have,

- $i = 1$
- $j = 1$
- $(\text{value})_i = (\text{value})_1 = 3$
- $(\text{weight})_i = (\text{weight})_1 = 2$

Substituting the values, we get-

$$T(1,1) = \max \{ T(1-1, 1), 3 + T(1-1, 1-2) \}$$

$$T(1,1) = \max \{ T(0,1), 3 + T(0,-1) \}$$

$$T(1,1) = T(0,1) \text{ \{ Ignore } T(0,-1) \}}$$

$$T(1,1) = 0$$

Finding T(1,2):-

We have,

- $i = 1$
- $j = 2$
- $(\text{value})_i = (\text{value})_1 = 3$
- $(\text{weight})_i = (\text{weight})_1 = 2$

Substituting the values, we get-

$$T(1,2) = \max \{ T(1-1, 2), 3 + T(1-1, 2-2) \}$$

$$T(1,2) = \max \{ T(0,2), 3 + T(0,0) \}$$

$$T(1,2) = \max \{ 0, 3+0 \}$$

$$T(1,2) = 3$$

Finding T(1,3)-

We have,

- $i = 1$
- $j = 3$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

$$T(1,3) = \max \{ T(1-1, 3), 3 + T(1-1, 3-2) \}$$

$$T(1,3) = \max \{ T(0,3), 3 + T(0,1) \}$$

$$T(1,3) = \max \{ 0, 3+0 \}$$

$$T(1,3) = 3$$

Finding T(1,4)-

We have,

- $i = 1$
- $j = 4$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

$$T(1,4) = \max \{ T(1-1, 4), 3 + T(1-1, 4-2) \}$$

$$T(1,4) = \max \{ T(0,4), 3 + T(0,2) \}$$

$$T(1,4) = \max \{ 0, 3+0 \}$$

$$T(1,4) = 3$$

Finding T(1,5)-

We have,

- $i = 1$
- $j = 5$
- $(value)_i = (value)_1 = 3$
- $(weight)_i = (weight)_1 = 2$

Substituting the values, we get-

$$T(1,5) = \max \{ T(1-1, 5), 3 + T(1-1, 5-2) \}$$

$$T(1,5) = \max \{ T(0,5), 3 + T(0,3) \}$$

$$T(1,5) = \max \{ 0, 3+0 \}$$

$$T(1,5) = 3$$

Finding T(2,1)-

We have,

- $i = 2$
- $j = 1$
- $(value)_i = (value)_2 = 4$
- $(weight)_i = (weight)_2 = 3$

Substituting the values, we get-

$$T(2,1) = \max \{ T(2-1, 1), 4 + T(2-1, 1-3) \}$$

$$T(2,1) = \max \{ T(1,1), 4 + T(1,-2) \}$$

$$T(2,1) = T(1,1) \{ \text{Ignore } T(1,-2) \}$$

$$T(2,1) = 0$$

Finding $T(2,2)$ -

We have,

- $i = 2$
- $j = 2$
- $(\text{value})_1 = (\text{value})_2 = 4$
- $(\text{weight})_1 = (\text{weight})_2 = 3$

Substituting the values, we get-

$$T(2,2) = \max \{ T(2-1, 2), 4 + T(2-1, 2-3) \}$$

$$T(2,2) = \max \{ T(1,2), 4 + T(1,-1) \}$$

$$T(2,2) = T(1,2) \text{ \{ ignore } T(1,-1) \}}$$

$$T(2,2) = 3$$

Finding $T(2,3)$ -

We have,

- $i = 2$
- $j = 3$
- $(\text{value})_1 = (\text{value})_2 = 4$
- $(\text{weight})_1 = (\text{weight})_2 = 3$

Substituting the values, we get-

$$T(2,3) = \max \{ T(2-1, 3), 4 + T(2-1, 3-3) \}$$

$$T(2,3) = \max \{ T(1,3), 4 + T(1,0) \}$$

$$T(2,3) = \max \{ 3, 4+0 \}$$

$$T(2,3) = 4$$

Similarly, compute all the entries.

After all the entries are computed and filled in the table, we get the following table-

	0	1	2	3	4	5
0	0	0	0	0	0	0
✓ 1	0	0	3	3	3	3
✓ 2	0	0	3	4	4	7
3	0	0	3	4	5	7
4	0	0	3	4	5	7

T-Table

- The last entry represents the maximum possible value that can be put into the knapsack.
- So, maximum possible value that can be put into the knapsack = 7.

Identifying Items to Be Put Into Knapsack

Following Step-04,

- We mark the rows labelled "1" and "2".
- Thus, items that must be put into the knapsack to obtain the maximum value 7 are- Item-1 and Item-2

Time Complexity-

- Each entry of the table requires constant time $\theta(1)$ for its computation.
- It takes $\theta(nw)$ time to fill $(n+1)(w+1)$ table entries.
- It takes $\theta(n)$ time for tracing the solution since tracing process traces the n rows.
- Thus, overall $\theta(nw)$ time is taken to solve 0/1 knapsack problem using dynamic programming

Conclusion-In this way we have explored Concept of 0/1 Knapsack using Dynamic approach

Assignment Questions

1. What is Dynamic Approach?
2. Explain concept of 0/1 knapsack
3. Difference between Dynamic and Branch and Bound Approach. Which is best?
4. Solve one example based on 0/1 knapsack (Other than Manual)

Reference link

- <https://www.gatevidyalay.com/o-1-knapsack-problem-using-dynamic-programming-approach/>
- <https://www.youtube.com/watch?v=mMhC9vuA-70>
- https://www.tutorialspoint.com/design_and_analysis_of_algorithms/design_and_analysis_of_algorithms_fractional_knapsack.htm