ThinQ
think • inquire • question

Foundations of Knowledge and Inquiry across Disciplines

Tara Mohanan and K P Mohanan

"If education were for living and not merely for livelihood; if education were for joy and happiness and not merely for temporal success; … if education were as much for service as it is for self-seeking; if education were as much for wisdom and truth as it is for so-called facts; … then, indeed, would the younger generation be well-equipped for life."

Dr. George Sydney Arundale, Dynamic Education.
In C. Roberts (ed.) *What India Thinks: Being a Symposium of Thought Contributed by Fifty Men and Women Having India's Interests at Heart*

I was born not knowing and have had only a little time to change that here and there.

Richard P Feynman

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To the Learner

We could specify the educational goal of this book in several ways: *to develop the ability to think like an academic – like a mathematician, a scientist, a philosopher, a historian, and so on*, or, worded differently: *to help you develop those aspects of your intelligence that are best developed by engaging with trans-disciplinary academic inquiry, often called Higher Order Cognition,* which includes *critical thinking.* And we might add: *Higher Order Cognition is the mental capacity to acquire, construct, and evaluate what is transmitted as 'knowledge' in schools and colleges*.

What does all this mean? For an experiential understanding of these goals described above, you need to work through at least the first three chapters of the book. In the meantime, we will try to give you a broad idea of what this book aims at, by giving a few examples, and pointing to some of the broad strategies that the book uses to achieve its goals.

What we Expect you will Learn

You may find this book very different from all the textbooks you are familiar with. There are at least two reasons for this.

1. The focus here is not on 'information' and 'knowledge'. This material will not help you get the specific knowledge content of any particular subject. The aim is for you to develop the general ***abilities*** you need for *inquiring* and *thinking critically*.

   So instead of teaching you the proof of, say, the Pythagoras theorem, we will look at *how to prove something*, whether in math, the sciences, or the humanities. Rather than learning the correlation between temperature and pressure in a body of gas — that the pressure of a body of gas increases as its temperature increases; or the causes of world wars, you will learn *how to see correlations between variables*, and to find out *how something causes something else*. You will also learn to think critically about the statements in textbooks, and figure out whether or not you should accept them.

2. What you would learn, including the abilities mentioned in (1), are relevant for all subjects and subject groups: it does not belong to any particular subject or 'discipline' or to any subject group. We call this kind of learning ***trans-disciplinary***. As we proceed, you will begin to see what that means.

If you are curious to know more about the book, or get a better idea about its aims, there are a few things you can do:

- Go through the table of contents of the book, for an overview.
- Read Chapters 1 and 10. They talk about the words 'trans-disciplinary', 'inquiry' and 'critical thinking', and about what we mean by them. They will help you see what we hope you will learn.

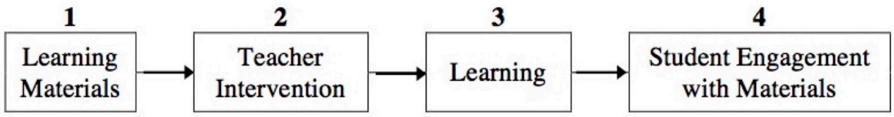
- Read the first and last sections of each of the in-between chapters (Chapters 2-9).

With this broad sense of what you are going to learn, take another look at the chapter titles on the contents page. This will help you read the entire book with a better sense of direction.

The Framework for Learning and Teaching

Learning in a course that this book is meant for is student-driven. It is designed in a way that students who can read and understand English can engage with the concepts in the book, work through the exercises both individually and in groups, and acquire the necessary abilities on their own, even if there is no classroom or teacher to help them.

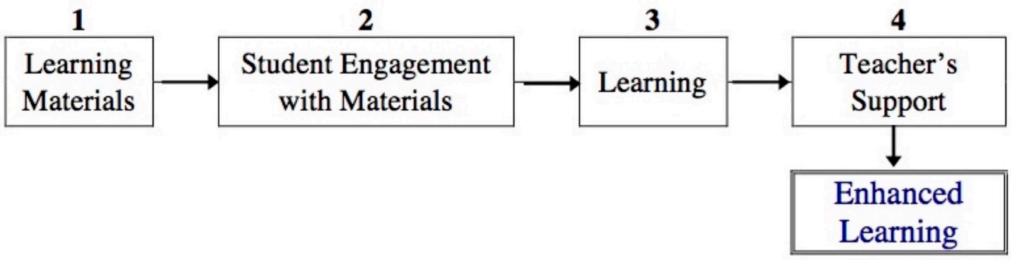
One feature of the course needs special mention, namely, the idea of a 'flipped classroom'. In most courses in schools and colleges, teachers are mediators between the learners and the learning materials such as textbooks and worksheets. Teachers draw upon the textbooks to help students understand the concepts, and the exercises in the worksheets for practice. After a class session, learners turn to the materials to consolidate what they have learned:



A '***flipped classroom***' inverts or 'flips' this sequence, to enable independent learning. Learners engage with the materials directly, and learn from them without the teacher's help.

What, then, is the role of experienced instructors — facilitators of learning — in this model of education? They provide value-added help which begins *after* students have engaged directly with the materials. The teacher would respond to clarificatory and exploratory questions from students, and provide further guidance by asking questions, setting more exercises, and getting students more engaged.

In this mode, the time that students and teachers spend together in the class would be used for such activities, which add value to the course.



We invite you to embark on this learning adventure that gives you space to be curious, to learn how to learn, and become an independent learner, inquirer, and critical thinker.

Foundations of Knowledge and Inquiry across Disciplines

PART 1: INTRODUCTION

> I can only show you the door; you're the one who has to walk through it.
>
> Morpheus to Neo in the movie, *The Matrix*

Chapter 1: What this Book is about

1 About the Book

This book is titled "Trans-disciplinary Academic Inquiry." Did that make you wonder:

What do the words 'trans-disciplinary', 'academic', and 'inquiry' mean?
What would I learn from this book?
Of what value would that learning be for me?

Let us find out.

First, what is Inquiry?

When someone asks us a question in an examination or an interview, they expect us to give an answer using facts or procedures that we have learnt. For example, if someone were to ask us what the boiling point of water is, what we have stored in our memory would tell us that it is 100 degrees centigrade.

Suppose someone says: "Take a rectangular room 12 meters wide and 15 meters long. What is its area?" A quick mental calculation would tell us that it is 180 square meters because we know the formula for the area of a rectangle, and we know how to do the necessary calculation.

Now, consider these questions:

- *For storing drinking water, should I use a glass jar or a plastic jar?*
- *What genetic factors allow migrating butterflies across generations to return to the same location year after year?*

If you don't know the answer, but would like to find out, what would you do? We are talking about a situation where we:

- don't know the answer to a question;
- can't get the answer by making calculations;
- can't find anyone who knows the answer; and
- can't find an answer on the Internet.

> **INQUIRY** is the process of looking for an answer through one's own thinking and discussions with others.
>
> Inquiry is not to be confused with **ENQUIRY**, which is asking for information, as in:
> *I enquired after their health.*
> *I made an enquiry at the counter.*

In such cases, we would have to rely on our ability to think and figure out a good answer. Inquiry is the process of looking for an answer through thinking — either on one's own or collectively with others.

India's new National Education Policy (NEP 2020) talks about the need for students to develop 'higher-order cognitive abilities'.

But what exactly are *higher-order cognitive abilities*? How do they relate to *cognition*, *inquiry*, *critical thinking,* and *problem solving*?

To help you find answers to questions like these, the book is dotted with conversations between:

- Rafa: a 14-year-old student
- Samira: Rafa's mother
- Anu and Neel (twins): Rafa's classmates
- Ila: Anu and Neel's mother, and
- Sanju: Anu and Neel's neighbour.[1]

As you read, imagine that you are Rafa. Whenever Samira asks a question, stop reading, imagine that she is asking you (Rafa), and before you read further, try to answer the question. That would be a good way to learn to figure things out yourself.

Let us listen to a conversation between Rafa and Samira.

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[1] The children call her Sanju *didi*. '*Didi*' means 'older sister' in Hindi and other languages of Northern India, but the term is used broadly to refer to an older girl or woman to indicate respect, and also as a term of endearment.

2 Finding Things Out

Rafa: Mom, I just read a newspaper report about a new policy for education in India. It recommends that education should develop higher-order cognitive abilities. What does that mean?

Samira: Yes, I too read that report. 'Higher-order cognitive skills,' eh? I think we need to unpack this systematically.

First, let's find out what *cognition* means. That will help us figure out what *cognitive abilities* are. Then, we can look for what is special about *higher-order cognitive abilities.*

Rafa: Okay, so what is cognition?

Samira: If I were not at home today, what would you do to find out?

Rafa: Hmm…I don't know.

Samira: Would you ask someone who knows, or look up the word *cognition* on the Internet, or try to find out by thinking through, or try a combination of these?

Rafa: So you're not going to tell me. Okay, maybe I'll do an Internet search.

Samira: We'll talk again after you've done your search, then.

Rafa found two entries for *cognition* on the Internet:

> "… the mental action or process of acquiring knowledge and understanding through thought, experience, and the senses." (https://en.wikipedia.org/wiki/Cognition)
>
> "… the states and processes involved in knowing, which in their completeness include perception and judgment. Cognition includes all conscious and unconscious processes by which knowledge is accumulated, such as perceiving, recognizing, conceiving, and reasoning." (https://www.britannica.com/topic/cognition-thought-process)

So the *Wikipedia* entry described cognition as a mental action or process, while the *Britannica* entry described it as a combination of states and processes.

As he read, Rafa made a note: *To cognize is to know.*

After some reflection, Rafa felt that he understood what *cognition* means, though he was not very confident.

On the Internet, Rafa also found that some websites described Higher-Order Cognition (HOC), as the ability to engage with challenging problems. But they didn't explain what counts as a challenging problem.

Some websites described HOC as the process of thinking (i) creatively — integrating facts and ideas that were unconnected before, and (ii) critically — testing the evidence and reasoning used in arguments.

Now, Rafa felt that he had only a very hazy understanding of the phrase . If someone were to ask him what Higher-Order Cognition was, he was not sure if he would be able to give a satisfactory response.

He tried to gather his thoughts about what he had read, made notes, and took them to his mother. Samira read through the notes and smiled at him.

Samira: So, tell me, what did you find out?

Rafa: I have a vague idea, but I'm not sure. If my classmates Anu or Neel were to ask me, I'd be in trouble. Can I try telling you?

Samira: Oh…those twins, your best friends? Well, tell me. I've been waiting!

Rafa: Higher-order cognition seems to be related to intelligence. But they are different. Intelligence seems to be the capacity to do things with our minds.

Samira: And is that a capacity that people are born with? Or is it something that you can develop through effort?

Rafa: Hm! I think we can develop it, just like our physical capacity. I mean, through proper nutrition and physical training, we get better at what we do with our bodies. So we should be able to do something similar to what we can do with our minds, right?

Samira: Very reasonable! So what do you think higher-order cognition is?

Rafa: Right now, I think it covers mental processes like creative thinking, critical thinking, problem-solving, reasoning, and inquiring.

> ***To cognize*** *is to know.*
>
> ***Cognitive abilities*** *are abilities related to knowing.*
>
> *The* ***process of inquiry*** *seeks to know, so it is a cognitive process.*
>
> ***Higher-order cognitive abilities*** *are inquiry abilities.*

Samira: Well done, Rafa. That is good progress for now.

Rafa: Oh, that reminds me, Mom. Anu-and-Neel's mom has invited me for breakfast this Saturday. Can I go?

Samira: Sure. You are free that day, right? So yes, go!

~*~*~*~*~*~

Samira resumed work on her laptop, but a moment later, she looked up.

Samira: Rafa, why don't you invite Neel and Anu as well as their mom to join us for dinner one day? I'd like to meet them.

Rafa: When?

Samira: Hmm. Let me see. How about on 21st December 2519?

Rafa: Mom!!! Did you say December 2519!?

Samira: So I did.

Rafa: But we can't invite them for dinner in 2519!

Samira: Why not?

Rafa: Because we'll all be dead by then!

Samira: Are you sure? You *believe* we'll be dead by 2519. But do you *know* that?

Rafa: Mom!!! You can't be serious! What do you mean by "are you sure?" Of course, I am! Human beings don't live for even 150 years, let alone 500 years.

Samira: Rafa, do you realise that you are exercising your Higher Order Cognitive abilities to answer my question?

Rafa: Oh…Am I?

Samira: Would you like to get better at it?

Rafa: Of course, yes.

Samira: In that case, you need to begin by learning to answer the question, "How do you know that?" or "What are your reasons for that conclusion?" So tell me, step by step. how do you know we'll all be dead by 2519?.

Rafa: Well, that's easy. For one thing, we know that you and I are human beings. And human beings don't live for 500 years. So, we'll be dead by 2519. Is that a good answer?

Samira: It's a good beginning. You just gave me an argument.

Rafa: Argument?

Samira: Mm-hmm.

Samira walked over to the whiteboard in their dining room and began writing:

1.	No human being has lived for 500 years.	(what we know)
2.	You and I are human beings.	(what we know)
3.	You and I won't live for 500 years.	(what we infer from 1 and 2)

Samira: What I've written here, is that what you meant?

Rafa: Yes. So, that's called an argument?

Samira: M-hmm. The statements in 1 and 2 are premises, and…

Rafa: Premises?

Samira: Yes. A premise is something we take to be true. *Premises* are the starting points for an argument. And statement 3 is what you infer from your premises. You arrived at it from statements 1 and 2. It follows logically from your two premises, so it is a *conclusion*.

Rafa: Cool! Got it.

Samira: Rafa, what would happen if one or more of our premises turn out to be wrong? Take Premise 1 — how do you know that no human being has lived for 500 years?

Rafa: I can think of two reasons. One, lots of people die well before the age of 100. Two, I've never met or heard of anyone living even till 150, let alone 500.

Samira: That is a good response. But now I want you to consider this. Is it possible for Premise 1 to be wrong?

Rafa: How can it be wrong?

Samira: Imagine that there are a few immortals among humans. They've lived beyond 500 years. Without being detected by anyone. Isn't that possible, though extremely unlikely? One of the teachers at your school, for instance, may be an immortal for all you know. What if she is someone who moves to a different town and different school before anyone can notice that she doesn't grow old like other people?

Rafa: Hmm, extremely unlikely, but possible, I guess. But what's your point?

Samira: That there are many things that we believe which are just conclusions from premises that we haven't actually stated. We can question them only when we state them explicitly. And only then can we figure out if they're true or false.

Rafa: You're saying Premise 2 could also be wrong?

Samira: Yes.

Rafa: Mom!! Are we not humans?

Samira: I'm just saying we could be mistaken in our belief that you and I are humans.

Rafa: But how can that belief be wrong?

Samira: What if we're both aliens, and that I was brought to Earth as soon as I was born? And your Dad too?

Rafa: You're talking Sci-fi, Mom! (after a brief pause) Extremely unlikely, but I guess it's not impossible.

Samira: So, we can't be completely certain that you and I are human beings, right?

Rafa: I guess so.

Samira: That's all I'm saying. Even when we're very confident, we can never be absolutely certain.

Rafa thought for a few moments. Being certain about something means *knowing* that it is the only way things could be. His mother had put him in a tough spot, but he was enjoying the conversation.

Rafa: Can I summarize the points you've made?

Samira: Go ahead.

Rafa wrote on the board:

1. Many of our beliefs are conclusions.
2. These conclusions are based on premises that we take for granted.
3. It is important to make these premises explicit, and to question them.
4. If we question them, we may discover that some of them are wrong.

THINK & DO #1

Do you have any beliefs you are certain about? Think carefully about them.

Choose ONE of them, and write down the justification for that belief. Do it the way Samira did, by stating your premises explicitly so that you can figure out if they are true, or false.

Rafa enjoyed learning with his Mom, but he remembered their starting point.

Rafa: So, Mom, about inviting Neel and Anu for dinner. We have two possible conclusions. One, that you and I *will* be alive on 21 December 2519. And two, that you and I *won't* be alive on 21 December 2519. You agree that two is more reasonable than one, yes?

Samira: Yes, I agree.

Rafa: Good. So, when should we have Neel and Anu and their Mom over for dinner?

Samira: How about next Sunday?

Rafa: Sounds good.

3 White Crow on the Balcony

Anu and Neel were having their regular vacation breakfast of *idli* and *saambaar*. Their parents had left for work, and they were by themselves. When Anu heard a crow on the balcony outside, she suddenly remembered something.

Anu: You know that white crow that comes to the balcony every morning, Neel? Yesterday it turned up with a friend — a regular black crow, not a white one.

Neel: What white crow? On the balcony? White crows don't even exist!

Anu: What do you mean, they don't exist? I've been seeing one every morning for weeks, and I've told you about it. But you never pay attention. At least one white crow does exist. And even if only one white crow exists, then it can't be true that no white crow exists.

Neel: You are being irrational, Anu. Or maybe you are hallucinating. Because everyone knows that all crows are black.

Anu: No, Neel, you are the one who is being irrational, and completely closed-minded.

Neel got up, muttering, "I really don't want to talk about this," and walked off.

THINK & DO #2

Write down the meanings of *rational* and *irrational.*

Who was being IRRATIONAL in the conversation above?

(a) Anu (b) Neel (c) Both (d) Neither.

Write down the reasons for your answer, so that you can refer to your notes later.

During lunch, Neel returned to their breakfast conversation.

Neel: Anu, suppose we assume that white crows don't exist.

Anu: But they do exist, Neel!

Neel: Ok...ok. Relax. For now, I just want you to ***assume*** the opposite of what you believe, that white crows don't exist.

Anu: Okay. Now what?

Neel: So we assume that white crows don't exist. And let's also assume that you were not hallucinating. Then, the white bird you've been seeing, could it be a crow?

Anu: No. If white crows don't exist, then the conclusion that follows logically is that the bird I've been seeing is **NOT** a crow, because it is white.

Neel: Thank you, Anu. I rest my case.

Anu: You cannot get off so easily, Neel. Let me ask you a counter-question. Let's assume that the white bird I've been seeing every morning **IS** a crow. If so, do white crows exist?

Neel: I guess so.

Anu: So, here's what we have so far. If we assume that white crows **DON'T** exist, it follows that the bird I've been seeing is **NOT** a crow. And if assume that the white bird I've been seeing **IS** a crow, it follows that white crows **DO** exist.

Neel: Right. We now have two statements:

1. White crows don't exist.
2. The white bird that you have been seeing every morning is a crow.

They can both be false. But, they can't both be true at the same time, because they contradict each other. So when we put them together, we get a logical contradiction, right? And we can't allow that. This means that we have to reject one or both statements.

Anu: Exactly. You reject statement 2, and I reject statement 1. So we arrive at different conclusions. We are both rational, Neel, but only one of us supports the true statement.

Neel: And who's that?

Anu: Me, of course! (laughs)

THINK & DO #3

What reasons can you think of to justify each of these positions?

1. White crows don't exist.
2. The white bird Anu has been seeing every morning is a crow.

The next morning, Anu saw the white bird again and hurried to wake Neel up. Neel dragged himself sleepily to the balcony and stared at the bird Anu was pointing at.

Neel: I do see a white bird that looks just like a crow. But, something tells me it's not a crow.

Anu: You need to do better than that, Neel. Give me some reason for that judgment.

Neel: You mean that I have to give you a rational justification for my judgment?

Anu: Exactly. How do you know that the white bird we both saw is NOT a crow?

Neel: To answer that, we need to define what a 'crow' is.

Anu: M-hm, yes. We need to figure out what makes a bird a crow, and what makes it not a crow. Never thought we would be asking this question!

Neel: (laughs) No, never! This is going to take some work. Let's do an Internet search to see if we can find a definition.

THINK & DO #4

Do an Internet search for 'crow'. See if you can come up with criteria to tell the difference between birds that are crows and birds that are not crows.

Neel and Anu continued their conversation at home that evening.

Neel: Listen, Anu. I don't know if white ravens exist. If they do, and if the white bird we saw was a raven, we can conclude that it was not a crow.

Anu: But Neel, ravens are a *sub-category* of crows, just like parakeets are a *sub-category* of parrots. So, if that white bird is a raven, then it follows that it's a crow.

Neel: Aah, I get it. But how do you know ravens are a sub-category of crows? Suppose ravens and crows belong to different categories. So a bird can be a raven or a crow, but not both. Just like insects and birds.

Anu: You mean, they are mutually exclusive categories.

Neel: Exactly.

Anu: Fine! If ravens and crows are mutually exclusive categories, then we conclude that:

1. If a bird is a raven, then it's not a crow; and
2. If a bird is a crow, it's not a raven.

BUT, if ravens are a subcategory of crows, then we conclude that:

3. If a bird is a raven, then it's a crow.

Neel: Agreed. So, which one? Subcategory, or mutually exclusive categories?

THINK & DO #5

Try to answer Neel's question: Is raven a type of crow, or are ravens a different category from the category 'crow'?

Here is what Anu and Neel found on the Internet:

1. The entry on "Ravens and Crows" on a website called BirdNote highlighted the differences between ravens and crows:
   "Ravens often travel in pairs, while crows are seen in larger groups. Also, study the tail as the bird flies overhead. A crow's tail is shaped like a fan,

while the raven's tail appears wedge-shaped. Another clue is to listen closely to the birds' calls. Crows give a cawing sound, but ravens produce a lower croaking sound."
https://www.birdnote.org/search/node?keys=ravens and crows

2. The Wikipedia entry on "Corvidae" listed crows and ravens as part of the same family:
   "Corvidae is a cosmopolitan family of oscine passering birds that contains the crows, ravens, rooks, jackdaws, jays, magpies, treepies, choughs, and nuthatchers. In common English, they are known as the crow family, or, more technically, corvids."
   https://en.wikipedia.org/wiki/Corvidae

3. The Wikipedia entry on "Crow" said that *corvidae* and *crow* are synonyms.

4. The Wikipedia entry on "Raven" said:
   "There is no consistent distinction between "crows" and "ravens", and these appellations have been assigned to different species chiefly on the basis of their size." It also says that crows are generally smaller than ravens.
   https://en.wikipedia.org/wiki/Raven

THINK & DO #6

Consider these two statements:
1. A raven is a type of crow.
2. A raven is not a type of crow.

Given that we reject logical contradictions, these two statements can't both be true at the same time. So, which one would you accept? Why?

The next morning, the conversation about crows and ravens resumed after breakfast.

Anu: Hey Neel, Sanju didi likes watching birds, and taking pictures of them. Why don't we ask her what she knows about crows and ravens?

Neel: Great idea! Let's go!

When Anu and Neel got to Sanju's home, she welcomed them into her dining room.

Sanju: So what brings you two here this morning?

Anu: We've come to ask you something, Sanju didi.

Sanju: Ask away.

Neel: What's the difference between crows and ravens?

Sanju: May I know what makes you ask this question?

Anu: We're trying to figure out if a bird that visits our balcony every morning is a crow, a raven, or some other kind of bird. To do that, we need to know the defining traits of crows and ravens.

Neel: Our Internet search gave us conflicting answers. Some websites say ravens are a subcategory of crows. Others say that crows and ravens are mutually exclusive categories.

Sanju: You're asking two questions. One is about defining categories. In this case, it's the category of crows. The other is about subcategories: if two groups are sub-categories of the same category.

Anu: Will you help us, Sanju didi?

Sanju: Of course, I will, as much as I can! But right now, I have to leave for a meeting, so I'll let you have my observational notes on crows and ravens. You might find some interesting stories there. And, thank you in advance for returning my notebook when you're done with it. [Brings them her notebook.]

Anu: Thank you, Sanju didi! We'll take good care of your notebook and return it after we are done.

Neel: Thank you, Sanju didi! Have a good meeting.

[DIALOGUE TO BE CONTINUED… PERHAPS]

4 What Do These Examples Show?

You may have found our examples somewhat unusual. Did you notice what they have in common? For example, take the questions that came up.

- Did we *give* you answers? No, we *pointed to* ways of looking for answers.
- It was not possible to answer the questions by recalling what you have memorized, by making calculations, by asking someone (for an answer), or by searching on the Internet.
- We don't know if a question had only one 'correct' answer.

The search for answers to these questions can be a quest, an adventure. It takes time and patience. You may be often pushed to question what you have taken for granted. So don't rush, don't panic. Take your time to answer. And do revisit your answers or discuss them with friends.

The dialogues between Rafa and Samira explore the issue of *how we know what we know*. And that takes us to the reasons why we believe certain statements to be true, and others to be not true. There are so many things that we take for granted that we know. We don't stop to ask:

“How do I know this?” or “Why do I believe this?” In this book, you will learn how to answer these questions.

These dialogues introduce some of the tools for exploring and inquiring. They also highlight some of the fundamental concepts of inquiry and critical thinking, like *premises*, *reasoning*, *conclusion*, and *justification*.

The dialogues between Anu and Neel may seem to be about whether white crows exist. In fact, they are more about the value of *doubting and questioning what we believe to be true*. They show that to give reasons to defend a position, one must articulate that position as clearly and precisely as possible.

The examples in this chapter would have given you a taste of the process of inquiry, the quest for *rationally justified conclusions*. You will have more opportunities for practice and exploration in the remaining chapters of this book.

The chapter also gave a brief introduction to the concepts of knowledge, belief, reasoning, rationality, logical consequences, logical contradictions, doubting and questioning, premises, and conclusions, categorising, subcategorising, and defining. It may be a good idea to read this chapter again, keeping the above summary in mind.

We hope that you will enjoy the process of asking questions and searching systematically for rationally justified answers to the questions you pose, working independently and also in teams. Like life, learning is far more fun if done with friends.

Happy journey!

> We learn more by looking for the answer to a question and not finding it than we do from learning the answer itself.
>
> Lloyd Alexander

> “Dissent is the soul of inquiry and discovery.”
>
> Noam Chomsky

CHAPTER 2:
A BROAD OVERVIEW

1 Looking Back and Looking Forward

To *cognize* is to *know*. In Chapter 1, in the context of Higher Order Cognition, which we take to be the same as Academic Cognition, we saw the word ‘cognitive’, meaning “related to knowledge.”

There are many paths to knowing and knowledge. We may come to know something through our experience, from what is communicated to us by others through speech or writing, through reasoning, or a combination. For instance, from our experience we come to know that falling down can cause painful bruises. We get to know from another person, a piece of writing, or an audio/video recording, that a cobra bite can be fatal. And our reasoning tells us that if Neel is taller than Rafa, and Rafa is taller than Anu, then Neel is taller than Anu.

We use the term ***inquiry*** to refer the way of coming to know something on the basis of our own reasoning, often but not necessarily combined with what our experience tells us or what others tell us.

How do we inquire? By:

formulating questions to express what we do not know;
looking for, and finding answers;
arriving at conclusions on the basis of our answers;
critically evaluating the conclusions; and
accepting the conclusions if there are good reasons to do so.

In Chapter 1, Section 3, we came across some of the tools of inquiry — observing, generalizing, classifying, defining, and reasoning. These tools, which we will pursue in Part 2 of this book, are useful across the boundaries of subjects and disciplines, from mathematics, to the physical, biological, and human sciences, to the humanities. Because they are relevant for all disciplines, we call them ***trans-disciplinary***. As you work through the exposition and exercises in the book, keep an eye out for these tools, and pay attention to the way they lead you to knowledge.

Let us listen in on a conversation between Anu and Neel, who are trying to figure out how to answer the questions they come across or come up with, and what tools to use.

2 Coming up with Questions to Explore

Anu: You know, Neel, those questions about white crows really stumped us. They're still bugging me. So many questions, and no answers.

Neel: They were bugging me too all night. I kept tossing in bed. And then in the morning, they kept playing in my head when I was jogging.

Anu: Are there ways to find answers, I wonder. And even if we find answers, how do we tell if they are good ones?

Neel: Here's a plan. What if we start making a list of the questions we would like to find answers to, but don't know how to go about it?

Anu: Good idea. Let's try to make a list by lunchtime.

Neel: We should do this separately, and we'll put the two lists together after lunch.

THINK & DO #1

Make a list of questions for which you do not have answers, and for which you cannot find satisfactory answers online or in your textbooks. Just a few would do.

By lunchtime, Anu had several questions, with these two right on top:

Do straight-angled triangles exist?

Can there be parallel lines on spherical surfaces?

Neel's list began like this:

Do mermaids exist?

Do eight-legged insects exist?

They looked at each other in surprise. The questions were all the same kind:

"Does X exist?"

The X in the questions were a variety of things. To their list, we could add:

"Does air exist in this room?"

"Do black holes exist?"

"Does a blue moon exist?"

"Does justice exist?"

"Did I exist a hundred years ago?"…

Neel: Wow, I think the question about white crows was so much on our minds that all our questions are about whether or not something exists. Surely there are other types of questions we could have asked.

Anu: How about this one? People say that *tulsi* tea is good for a cold. So we can ask: "Does *tulsi* tea cure a common cold?"

Neel: Good question. Let's put it a bit differently:
"Is it true that *tulsi* tea cures a common cold?"

Anu: Your turn now.

Neel: How about this: Why do people sneeze when they have a cold?

Anu: Why do people yawn, or have hiccups?

Neel: Hey, I think we have enough questions to start with.

Anu: Did you notice that our questions have common patterns? Some questions have the pattern "Does X exist?" Others have the pattern "Is it true that…?" And yet others are: "What causes X?" and then there are questions that begin with "Why…".

Neel: Hm! I hadn't thought of that.

Anu: Why don't we pick one of these questions and find out how we can look for an answer?

Neel: Good. Pick one.

Anu: How about this one? "Is it true that drinking *tulsi* tea cures a common cold?"

THINK & DO #2

Can you think of an experiment to look for an answer to Anu's question? Describe the design of the experiment.

2.1 Testing Hypotheses

Anu, Neel, and their mother Ila had just arrived at Samira and Rafa's place for lunch, the very same lunch that had sent Rafa round and round in circles. The mothers were introducing themselves to each other, when Ila sneezed a loud *aachoo*.

Samira: Gesundheit!

Anu: Gesu what?

Samira: Gesundheit. It means 'health' in German. When someone sneezes, we say gesundheit.

Anu: Oh, just like 'Bless you' in English.

Samira: Exactly.

Rafa: Do you have a cold, Ila aunty?

Ila: I don't know. May be I am coming down with something.

Samira: Can I make you some *tulsi* tea, Ila? That might help.

Anu and Neel suddenly became alert.

Anu: Why *tulsi* tea, Samira aunty? Does it cure a common cold?

Samira: M-hmm, it helps.

Anu: How do you know that?

Samira: Well, when I was young, whenever I had a cold, my grandmother used to give me this steaming concoction. It was *tulsi* leaves, ginger and black pepper boiled in water. And I always got better pretty fast.

Rafa: (grinning) Mom, hold on! Do you really believe that? That you got better when you drank it doesn't prove that *tulsi* tea cures a common cold. May be you got better on your own, and you just thought it was because of the tea. Or may be what cured the cold was the ginger, or the black pepper, not the *tulsi*. Or a combination of the *tulsi* and ginger, or any two of the ingredients. Or a combination of all three,...

Samira: Hm! You have a point, Rafa. Okay, I don't have *evidence* to show that *tulsi* tea cures a common cold. I just *feel* it's true.

THINK & DO #3

Do you think Rafa's objection is valid? If yes, why? If not, why not?

Samira says that whenever her grandmother gave her *tulsi* tea, she got better. Why, then, does she say that she does not have evidence to show that *tulsi* tea cures a common cold?

Anu: (bursts in.) This is weird! *And* amazing! Just this morning, Neel and I were talking about how to find out if *tulsi* tea cures the common cold. And we are talking about the same thing now!

Samira: You know, may be the three of you could do a joint research project on that question.

Anu: But can school students do research? Don't we need to be researchers for that?

Ila: Who says school students can't do research? What you need is the ability to inquire. And we can help, can't we, Samira?

Samira: Sure! That would be fun. Let's plan this project.

Ila: Great. I guess the first step is to be clear about what we mean by 'common cold', by defining it.

Samira: Okay, tell us. What is a common cold?

Rafa: I don't know how to define it, Mom. But I know a common cold when I see it.

Samira: Recognizing it is not enough if you want to do a research project on it.

Rafa: Why not, Mom? Can you define a rose? I bet you can't. But you recognize a rose when you see one, right? You are not going to confuse a rose with a sunflower or a lily. So why define it?

Samira: Well, what would happen if the flower that you label as a rose is labelled as something else by someone?

As Neel listened quietly, Anu chipped in.

Anu: She's right, Rafa. A few days ago, Neel and I were debating on whether white crows exist. I said they do, and Neel said they don't. And when I showed Neel what I judged to be a white crow, he said it was white, but it wasn't a crow. That's when we figured out that when we have a disagreement on *classifying* something, or even labelling it, we need to begin with a shared definition. That is why, if we want to do a research project on whether *tulsi* cures a common cold, we need to define *common cold.* It's only then that other people can understand our project, and evaluate it.

THINK & DO #4

Can you help the research team by coming up with a definition of the common cold?

One way to do that is to identify the set of symptoms of a common cold, such that if you observe that cluster of symptoms in someone, you would say that the person has a common cold. And if you don't observe those symptoms, you would say that the person doesn't have a common cold.

When Anu, Neel, and Ila got home that night, Anu went straight to her desk. She found the Mayo Clinic website for symptoms, causes, and cures for illnesses. It had an entry for the common cold: https://www.mayoclinic.org/diseases-conditions/common-cold/symptoms-causes/syc-20351605

She had heard of the Mayo Clinic as one of the most reliable websites on health and medicine, so she paid close attention. The website listed a number of symptoms:

- Runny or stuffy nose
- Sore throat
- Cough
- Congestion
- Slight body aches or a mild headache
- Sneezing
- Low-grade fever
- Generally feeling unwell

Anu wrote down the symptoms. She and Neel decided that if they observed any five of these symptoms, they would diagnose it as an instance of common cold, and if they didn't observe even three of them, they would judge it to be not a common cold. So their criteria looked like this:

> Observed: five or more of the symptoms in person X.
> > Conclusion: X has a cold.
>
> Observed: three or less of the symptoms in person Y.
> > Conclusion: Y does not have cold.

What if they saw any four symptoms? After some serious discussion, they decided they would seek a doctor's judgment on the matter in this particular case.

The children and their mothers met the next Sunday morning for a discussion. Everyone agreed that Anu and Neel's criteria to decide whether or not an illness should be classified as a common cold were quite good. Their next challenge was designing an experiment to find out if *tulsi* tea cures the common cold. They took several hours to come up with a plan. Once they had agreed on it, Rafa and Samira cycled back home, with thoughts of the delicious *poori* and *aamras* lunch waiting for them.

2.2 Inquiry through Reflection

On the way, a thought suddenly struck Rafa. Riding his bicycle, Rafa imagined the earth as a ball — a perfectly smooth sphere, without mountains, rivers, or oceans. He could keep riding without any obstruction, without changing his direction. What would happen if he kept on riding, without stopping?

The answer that emerged in his mind surprised him: he would return to where he had started. Whichever direction he was riding in, as long as he didn't change direction, he would come back to his starting point.

Now another thought struck him. He said to himself: "A straight line is a path that does not change its direction." But a path of what?

He imagined himself as a point.

> *A straight line is the path traced by a moving point that*
> *does not change its direction.*

Was that a definition of a straight line? Yes. And a pretty good one too!

He was beginning to feel excited. The bicycle path that he was thinking of was a straight line. Given the size of the earth, a bicycle is so small that it can be thought of as a point.

He went back to thinking about his earlier conclusion: if he were riding his bicycle on a smooth spherical earth in a straight path, he would return to the point from which he started the ride. Now, if he was a point, then given his definition, he could conclude that:

> *Every straight line, extended on a spherical surface, would meet itself.*

Would that be the case on a flat surface? No.

> *No straight line on a flat surface, even when extended infinitely, , would meet itself.*

Now yet another thought occurred to him. His teacher had told him that the geometry that he was learning in school was called Euclidean geometry. This was a flat surface geometry. But there was another geometry called spherical geometry. Rafa didn't know much about spherical geometry, except that it was a part of non-flat geometry. He had also read that while Newton's theory of gravity and motion was based on flat geometry, Einstein's theory was based on non-flat geometry.

Rafa's excitement grew. He felt he was beginning to understand spherical geometry. Simply by *focusing on specific elements of a question, imagining*, and *thinking carefully*, he had discovered an important way in which spherical geometry and flat geometry were different from each other.

Samira's voice from inside the home broke into his thoughts. He came awake from the world of geometry. He couldn't wait to tell his mother about his discovery. He opened the gate, jumped off his bicycle, and without bothering to put it on a stand, ran towards Samira, "Mom, this is really cool stuff. Do you know that a point can return to itself when moving in a straight line path?"

Samira looked at him. "What are you talking about? Slow down. Tell me about it, but tell me slowly, so that I can understand what you are talking about. Sit down first."

Rafa sat down at the kitchen table, and Samira placed a glass of water in front of him.

"Now tell me," she said.

Rafa began at the beginning. When he finished, Samira was smiling. Did he detect a certain pride in her eyes?

Samira: That's great, Rafa. What you figured out for yourself is indeed the heart of the distinction between spherical geometry and flat geometry. Can you tell me what your main propositions are? Let's write them down.

Rafa: Huh? What are propositions?

Samira: A proposition is what a sentence asserts. In simpler words, a proposition is what we mean by a sentence. Let me give you an example. Suppose I wrote down four sentences:

(i) Point A comes before point B.
(ii) Point A precedes point B.
(iii) Point B comes after point A.
(iv) Point B follows point A.

Don't these sentences all mean the same thing?

Rafa: Of course they do.

Samira: So we have four different sentences, but they all express the same proposition. They say the same thing. Now look at the opposite case. Imagine this sentence in a story:

(v) Anu is reading the book on the windowsill.

What does the sentence mean?

Rafa: Ah, I see! Anu could be sitting on the windowsill and reading; or she could be sitting on a chair and reading the book that was on the windowsill earlier.

Samira: Excellent. It's an example of an ambiguous sentence.

Rafa: Right. You told me we get ambiguity when a sentence has two different meanings. This is just like that sentence we talked about the other day:

(vi) Neel ate the apple under the tree.

Was Neel under the tree when he ate the apple? He doesn't have to be, right?

Samira: Right.

Rafa: He could have taken an apple from the fridge and eaten it under the tree.

Samira: Or?

Rafa: Or he could have found the apple under the tree and eaten it inside the house, or somewhere else.

Samira: Good. That is an example of the same sentence expressing two different propositions. Now let's go back to my question. You discovered something about 'straight lines when extended'. Can you tell me your main propositions for each of the geometries?

Rafa picked up a marker. "Can I write them down on the board?" He took his time and wrote.

Proposition 1: Every straight line, when extended, would meet itself.

Proposition 2: No straight line, even when extended infinitely, would meet itself.

Samira: Good! Now, is there a logical contradiction here? Do the two propositions contradict each other?

Rafa: Mom, I know you have told me before, but could you remind me what a logical contradiction is?

Samira: Of course! When we take a proposition to be true, and we also take the opposite of that proposition to be true, we have a logical contradiction.

Rafa: Right!

His was mind racing. On the face of it, the two statements on the whiteboard did look contradictory. But he intuitively felt that they were not. But how can both propositions be true at the same time? This was a bit too much for Rafa to resolve.

Rafa: I need some time to think.

Samira: Take as long as you need!

Rafa sat cross-legged on the floor, deep in thought. He didn't move for nearly half an hour. Then, all of a sudden, he jumped up.

Rafa: Mom, they are not contradictory!

Samira: Really? Are you saying that it is not contradictory to say that every straight line, when extended, would meet itself, and no straight line, when extended, would meet itself?

Rafa: No-no-no! That is because we are combining them. We can't do that. The first proposition is true in the world of spherical geometry. The second is true in the world of flat geometry. Spherical geometry and flat geometry are about two different worlds, you can't combine them. So the statements are not contradictory. If they were propositions about the same world, then they would be contradictory. So we have to say:

Proposition 1: In a spherical world, every straight line, when extended, would meet itself.

Proposition 2: In a flat world, no straight line even when extended infinitely, would meet itself.

Samira's smile was even bigger now. It was clear that she was proud of what Rafa had achieved.

Samira: Good! That now is crystal clear. I hope you see why you need to articulate your propositions as clearly as possible, with no room for ambiguity. You know, Rafa, you have a mathematical imagination. I know you've been getting low marks in your math tests, but I believe you have the potential to become a good mathematician.

Rafa: Wow! Really?

Samira: M-hm. Now let me ask you: You have learnt that in flat geometry, the sum of angles in a triangle is two right angles, right?

Rafa: Two right angles?

Samira: A right angle is supposed to be 90 degrees, right? Then two right angles are 180 degrees. You have also learnt in your math class that the sum of angles in a triangle is 180 degrees. Right?

Rafa: Yes.

Samira: Is this true of triangles in a spherical geometry? I know you can figure it out if you use your mind.

Rafa: MOM!!!

3 What Did You Learn in This Chapter?

We have had a glimpse into several tools of inquiry in Chapters 1 and 2. This may be a good time to make a list of the tools we will pursue in this book.

- experiencing and observing,
- imagining and reflecting
- generalizing, classifying, and defining
- reasoning
- justifying claims
- critically evaluating conclusions and their justification
- debating,
- problem solving and decision making
- constructing, evaluating, and choosing between theories and explanations

At this stage, you may not fully understand each of these concepts. But as we proceed, you will have a better understanding, and by the end of the book, you will have a pretty good understanding.

THINK & DO #5

Which of these tools did you notice in the *tulsi* tea story and the spherical geometry story? Read those stories again with a notebook and pencil, and make a note of the tools introduced in each story. Don't read further till you have engaged with this task sufficiently.

If you have gone through the material so far at least twice and have come up with an answer, here are a few hints to help you think through what you have learnt in this chapter.

> The *tulsi* tea story has a brief introduction to the role of observation in inquiry, and the spherical geometry story has an introduction to imagining and reflecting. Can you identify the parts that involve these concepts?
>
> Both stories describe activities that involve defining. The *tulsi* tea story also takes a peek into classifying. Can you identify the parts that discuss these concepts?

Did the *tulsi* tea story include a discussion about justifying claims?

Do either of these stories mention critical evaluation?

The *tulsi* tea story is titled 'testing hypotheses', but the spherical geometry story is titled 'inquiry through reflection'. What exactly do you think is the contrast between them?

Have you come across the term 'hypothesis testing' in your school or college studies? If you have, try to connect the *tulsi* tea story to what you know about this.

Have you come across the idea of research projects in your school or college? If you have, what have you learnt about research through these stories.

What is the difference between the kind of math you are familiar with from your school and college, and the math introduced through the spherical geometry story in this chapter?

It doesn't matter if you can't come up with satisfactory answers to these questions. What is important is that you engage with the questions, and like our characters in these stories, try to answer them as best as you can for now, and refine them as you go along. This type of keen and sustained engagement will strengthen your intellectual muscles, and after a few months, you will discover that you have a stronger, faster, and more agile mind. Do make a note of your answers so that you can re-evaluate them once you've come to the end of the book!

By the way, you may want to come up with a research plan to test the *tulsi* tea hypothesis; and also, to find an answer to the 'sum-of-angles' question that Samira raised.

One thing that you will notice about inquiry is that answering one question is bound to raise several other questions! And the process will lead to an endless string of questions, and that will keep you from ever getting bored.

> Proving that something exists is easy if you can find just one example. But proving that something does not exist can be really hard.
>
> John Voight, Dartmouth College

> By three methods we may learn wisdom: First, by reflection, which is noblest; second, by imitation, which is easiest; and third by experience, which is the bitterest.
>
> Confucius

Chapter 3: Ways of Knowing

1 How Do we Come to Know What we Know?

Suppose someone, say, Anna, starts a conversation with you.

Anna: Which is more sour, a ripe lemon or a ripe mango?

You: Lemon, of course.

Anna: How do you know that?

You: I have tasted both ripe lemons and ripe mangoes. And from that experience I know that ripe mangoes are sweet, but ripe lemons are sour.

Anna: And, what do you think would happen if a cobra bites a rabbit?

You: The rabbit would die.

Anna: How do you know that?

You: I have heard, and also read, that a cobra bite can be fatal.

You would not say: I have had the experience of being bitten by a cobra and dying, would you? ☺

In the first example, our knowledge comes from the memory of our experience. In the second example, it comes from our memory of what we have heard or read — the spoken or written testimonies of fellow humans. Following the terminology in the literature, we will use the term ***ways of knowing*** to refer to how we come to know. Again following common practice, we will use ***sources of knowledge*** for where our knowledge comes from. These two phrases mean the same thing.

In the two examples given above, the kind of knowledge we are talking about is our personal, everyday knowledge, not the academic knowledge transmitted to us through educational institutions. But the point made above applies to academic knowledge as well. Suppose a textbook (or a teacher) asserts that water is not an element or a mixture; it is a compound. In the spirit of critical thinking, it is important for students to ask: "How do you know that?" or "Why should we accept what you are saying?" Our question asks for ***proof*** — the ***rational justification*** – for what is being asserted. An answer would involve pointing to the source of that knowledge.

It would be useful to know that 'ways of knowing' and 'sources of knowledge' denote a concept that is important in ***epistemology***, the academic study of knowledge in philosophy. Epistemology is a field of inquiry that seeks to construct ***Theories of Knowledge*** (ToK). [This is one of the compulsory subjects in the curriculum for the International Buccalaurate (IB) diploma program.]

In what follows, we will go through the following ways of knowing: perception, introspection, memory, reasoning, testimony, invention, intuition and insight. We must bear in mind that these ways of knowing often combine with one another, and work in tandem, not in isolation.

2 Where does Knowledge Come from?

2.1 *From Sense Perception to Observation*

Sense perception — *what appears to us* — is a sub-category of ***perception***, perhaps the easiest one to understand. In many cases, we take what *appears* to us to be what exists as *reality* in the world. Suppose we see a cat on the table in front of us. We are likely to conclude from that experience that there is indeed a cat on the table in front of us. When that happens, we move from sense perception ("I see a cat in front of me.") to ***observation*** ("There is a cat in front of me.")

Not all sense perceptions, however, can be legitimised as observations. When we look at a stick half immersed in water, the stick appears to be bent. But given what we know about light and vision, we do not extend the perception as an observation, and say, "That stick in the water is bent." A child, however, may take it as an observation, and think that the stick is actually bent.

Likewise, if we look at a mirror and see a cat in it, we do not say that there is a cat in the mirror, even though very young children, monkeys, and birds, on the basis of their perception, may take there to be an actual cat in the mirror. When we look at the night sky, the moon appears to be bigger than the stars. Had we lived ten thousand years ago, we might have taken it to be what is true of the external world, and might have said, "The moon is bigger than the stars." But now, we treat the sense perception of the moon *appearing* to be bigger than the stars as an optical illusion.
[For a discussion of sense perception as 'what appears to us', see Appearance and Reality:
Part 1 (https://www.thinq.education/post/appearance-and-reality-1) and
Part 2 (https://www.thinq.education/post/appearance-vs-reality-2)

Sense perception is a form of ***sensation***, a form of experience (sensory experience) located in the body. Itching, nausea, and headache are such sensations. The feeling of stress on our muscles when we lift a heavy suitcase is also a sensation.

Our experience tells us that the earth is stationary. We do not deny the experience, but given what we know about the solar system, we reject the corresponding observational statement, and subscribe to the position that the earth revolves around the sun, and spins on an axis tilted to the plane of its revolution.

So, one source of knowledge is our ***experience***, a specific form of which is sense perception. If there is no reason to conclude that our senses are mistaken, we take what our senses tell us as being true about the world. We have talked about the process of a sense perception becoming an observation. This process, in most instances, is automatic, governed by unconscious mechanisms in the brain that we do not fully understand.

[We can be mistaken about our sense perception; so sense perception is ***fallible***. For a good discussion of the fallibility of sense perception, watch the youtube video:
"Anil Seth: How We Build Perception from the Inside Out," at
https://www.youtube.com/watch?v=FfOu14wIvM4&t=11s]

Thus, as long as there is no evidence to the contrary, we extend our sense perceptions as observations.

[We must distinguish what we perceive through our senses and what we can measure using sophisticated instruments. When we lift an object, we experience a strain on our muscles, which we interpret as the weight of the object. We can lift a watermelon and an apple to perceive the greater weight of the former. But we cannot lift an elephant and a cow to judge on the basis of sense perception that the elephant is heavier. But given the theory of weight, mass, and gravity in physics, we can design weighing machines to measure the weight of the elephant and the cow. Now, instruments that measure weight, mass, gravity, and so on have theories of mechanics in physics built into them. To take another example, human intelligence is a property of the mind that is not observable through the senses. But on the basis of a well-established theory of intelligence, we can identify the observable correlates of intelligence in human behaviour, and set up a way of measuring it. Intelligence Quotient (IQ) makes the controversial claim that it is a valid measure of intelligence, but many researchers reject that claim because IQ is not based on an established theory of intelligence. Similar remarks apply to marks and grades given in exams and tests as a valid measure of student learning. Such critiques point to the flaws in what is called ***operationalization*** in the literature on Research Methodology.]

2.2 INTROSPECTION

Sense perception on the basis of which we make observations directs our attention to what we take to be the external world (the world that we take to exist outside of our consciousness). ***Introspection***, on the other hand, directs our attention to the world internal to our consciousness. When we are aware that we are happy, sad, anxious, or angry, we are attending to the mental states existing in our own consciousness. When we direct our attention to a headache, and are trying to find out if it is a dull ache or a throbbing ache, we are introspecting.

We are engaging in introspection when we ask ourselves, “If I were a doctor, and an injured serial killer were brought to me for treatment, would I save his life?” or, when we ask ourselves if the sentences, “Zeno’s parents admire themselves,” and “Zeno’s parents admire himself,” are equally acceptable to us. But when we ask speakers of English if these sentences are equally acceptable to them, and we record what they say, we are engaging in observing and making observational reports.

In the above discussion, we have used the term *observation* to refer to what our sense poerception tells us about the external world. Now, it is possible to extend the term to refer to what our experience tells us about what exists internal to our mind and body. However, such an extension would obscure the distinction between observation of the external world which can be indsependently corroborated, and one’s experience of the internal world, which is not accessible to anyone else.
[What has been called self-knowledge (as in, “Know thyself.”) is largely based on introspection, not observation.]

2.3 MEMORY

Suppose you stub your toe, and we ask you: "Does it hurt?" your answer would be: "Yes, it does." We ask: "How do you know that?" Your answer would be: "I ***am experiencing*** pain on my toe."

A few minutes later, when it no longer hurts, someone asks you: "Did you stub your toe a little while ago?" You would say: "Yes". The person asks: "Did it hurt?" You would again say: "Yes."
This time, in response to the question, "How do you know that?" your answer would be, "I ***remember*** stubbing my toe and experiencing pain."

As your source of knowledge, you are now appealing to your ***memory*** of what existed in the past.
[We must distinguish between *memory* and the *memorization* (of pieces of information to do well in tests and exams). Valuable memory is meaningful, structured, and integrated, an important strand of intelligence. On the other hand, though we are able to memorise the sentence, "When glooks see sbintoshes, they blif their flopons," in order to give the correct answer to the question, "What do glooks do when they see sbintoshes?" such memorization is meaningless and painful, and when that pain goes beyond a certain limit, it destroys the joy of learning, and even intelligence.]

Suppose you are looking at Plato and Athena, who are standing side by side. Someone asks you, "Who is taller, Plato or Athena?" Your answer might be "Athena". And your answer in response to the question "How do you know that?' would be, "I can see that Athena is taller," pointing to your sense perception. But if you are asked the same question the next day (when Athena is not in sight), and also asked, "How do you know that?" your answer would be, "I remember looking at them standing side by side, and seeing that Athena is taller." That response appeals to your memory of your experience.

Imagine asking your grandparents what it was like when they were in primary school. They might talk about a time when there were no phones, no TV, and no internet. What they tell you is a form of *oral history*, based on their memory of the past. A travelogue written by someone who visited India two hundred years ago is written history, based on the author's experience. The source of knowledge in both cases is memory.

2.4 REASONING

Going back to our example of height, you see Athena and Plato standing side by side, and notice that Athena is taller. The next day you see Plato and Socrates standing side by side, and notice that Plato is taller than Socrates. Someone asks you, "Who is taller, Socrates or Athena?" Your answer would be, "Athena," even though you have never seen Athena and Socrates standing next to each other. If asked: "How do you know that?" your answer would be:

I know that Athena is taller than Plato.
I also know that Plato is taller than Socrates.
Given that Athena is taller than Plato and Plato is taller than Socrates, it is legitimate to conclude that Athena is taller than Socrates.

In giving this answer, you are appealing to ***reasoning*** as a source of knowledge, going beyond observation and memory.

To take another example, suppose you find the following statements in a book:

(1) a. If Zeno is a flodbon, then Plato loves frappilop.
b. If Plato loves frappilop, then Athena is the Princess of Holmark.
c. Zeno is a flodbon.

From these statements, it follows logically that Athena is the Princess of Holmark. How do you know that it follows logically? Your answer? "Through reasoning".

Notice that you do not know who Zeno, Plato, and Athena are, or what a flodbon and frappilop are. This is the first time you are hearing about the Princess of Holmark. So you do not know if the three statements given above are true. All that you can say is: **IF** the statements in (1) are true, then it is also true that Athena is the Princess of Holmark. What you now know is that the statement under consideration follows logically from the three statements in (1).

2.5 TESTIMONY

Suppose you are filling in an application form. You fill in your date of birth, and someone asks you: "How do you know that you were born on that particular day?" You could not have observed that you were born on that day. You don't have any memory of the experience of your birth. Nor do you have any way of concluding, through reasoning, that you were born on that day. Your answer might be, "My mother says that I was born on that day," or "It says so on my Birth Certificate." Your answer to the question, "How do you know that?" would be your mother's testimony, or the testimony of the document called Birth Certificate.

Suppose we ask you, "How old are you?" When you answer the question, we ask, "How do you know that?" Your response would be somewhat sophisticated. First, if you accept the testimony of your Birth Certificate, then the statement that you were born on such and such day becomes part of your knowledge. Next, you consult a calendar to find out the current day — a matter of accepting the testimony of your calendar. Next, you use arithmetic reasoning (called calculation) to figure out the duration from the date of your birth to the current day. That would be your age.

It must be noted that testimonies are sources of knowledge only for Human Studies. Inanimate entities and non-human animate entities do not speak or write. The use of verbal evidence (testimonies) is a unique characteristic of what has been called Humanities and the Social Sciences.

It is important to distinguish between the testimonies of fellow humans on the one hand, and their conclusions on the other. Suppose you ask a friend, "How tall is your sister?" and he says, "About five feet nine inches." If you accept that statement as true, your source is the testimony of your friend.

But suppose you ask a Chemistry professor, "What is the distance to the Sun from the earth?" And she says, "About 150 million kilometers." Here, the professor is reporting a conclusion that she has heard from the community of astronomers, not what she herself has seen, measured or calculated. This is not a testimony. It is a conclusion, and you need to ask the scientific community, "How do you know that?"

The idea of testimony has a prominent place in jurispridence, the theory of law. In trials in the criminal court, we find three kinds of testimony: eye witness testimony, expert testimony, and character testimony. ***Eye witness testimony*** is a report on what the witness observed. Thus, the statement, "I saw the defendant standing beside the body of the deceased with a blood-stained knife in his hand," is an eyewitness testimony. To translate this into the terminology of scientific inquiry, we may say that an eyewitness testimony is an ***observational report***. In contrast, "The defendant had a strong grudge against the deceased," is not admissible as an eye witness testimony, because that is a matter of the witness's opinion or interpretation, not something that the witness observed.

Expert testimony is a conclusion that the expert in the witness box has arrived at on the basis of her expertise. For instance, if a forensics expert examines a dead body, and arrives at the conclusion that the death was due to strangulation, that would be an example of expert testimony.

As a critical thinker, the lawyer (or the judge) treats the testimonies as evidence, not as statements of truth. It is indeed possible that when a witness says, "I saw such and such," he/she is telling a lie, or is mistaken. But unless there is evidence to show that the witness is lying, the judge and the lawyer accept the statement(s) as true.

At the level of academic inquiry, we may treat expert testimony as an ***inference*** that the expert has arrived at on the basis of observation and reasoning.

As an example outside the law court, consider the statement, "The height of Mount Everest is 8,848 meters," as an answer to the question, "What is the height of Mount Everest?" If an expert is asked this question, the answer would be an instance of expert testimony. If the expert is now asked, "How do you know that?" a possible answer would be what is given in the article, "Shrinking Mount Everest: How to Measure a Mountain." (https://www.livescience.com/50691-how-to-measure-mount-everest.html):

> " At heart, measuring a mountain relies on basic ninth-grade math. To calculate the elevation of a mountain, scientists would measure the distance between two points on the ground and then measure the angles between the top of the mountain and each point.
>
> "If you have two angles, you know the third, because the sum of the angles is 180 [degrees],' Molnar told Live Science.

Molinar's answer is that it is based on the measurement of the angle, the body of knowledge called geometry, and calculation (inference) based on measurement and expert knowledge (geometry).

2.6 *INVENTION*

In the geometry that you are familiar with, there are objects like points, straight lines, curved lines, triangles, squares, rectangles, pentagons, circles, and ellipses.

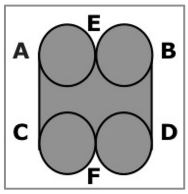


FIGURE 1

Now, suppose the following idea occurs to you. Take a square ABCD. Find the midpoint of AB, and call it E, and the midpoint of CD, and call it F. Draw circles with AE, EB, CF, and FD as their diameters, as in Fig. 1:

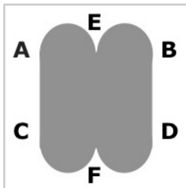


FIGURE 2

Consider the shaded shape AEBDFC in Fig. 2. Can you construct a formula to find the area of this shape? Can you prove that the formula is correct?

What you have done here is to invent a shape that does not exist yet in any of your textbooks, and ask a question about that shape. This is what mathematicians do: they invent logically possible imagined worlds, and objects in those worlds (through axioms and definitions), and arrive at and establish conclusions (called theorems) through deductive reasoning.

Take another example. Euclidean geometry postulates that points have no size. What that means is, (a) no matter how small a line segment, there are infinitely many points in it, (b) no two points can be adjacent to each other because there are infinitely many points between them, and (c) you cannot find the length of a line segment by *counting* the number of points. Since lines have no breadth, they are invisible, so you cannot *measure* the length of a line either.

Suppose you deviate from the Euclidean axioms on points and lines, and set up the following postulates:

Alternative Postulate (AP):

> Every point has an extremely small size, more than zero but less that the Planck's constant (which is the smallest measurable size).

It follows from AP that every finite line has a finite number of points. We can now define the length of a line as the number of points it contains. This has further consequences to the alternative geometry we have constructed. For instance, unlike in Euclidean geometry, not every finite line segment can be bisected. (Try to work out the proof yourself.)

What we have done here is use our imagination to invent a different postulate, such that we can use reasoning to identify and prove its logical consequences (theorems). Inventing postulates is an important aspect of mathematical inquiry.

The ability to invent postulates is equally important in constructing explanations in scientific theory. In order to explain certain observational generalizations on the motion of inanimate objects, theoretical physicists postulated the concept of FORCE, a specific form of force being GRAVITATIONAL FORCE. To explain the observational generalisations on the phenomena of magnetism and electricity, they postulated the concepts of MAGNETIC FIELDS and ELECTRIC FIELDS. And taking his cue from these postulates, Albert Einstein postulated the concept of GRAVITATIONAL FIELD.

To explain a set of observational generalisations on chemical changes, John Dalton postulated that all matter is made up of extremely small indivisible entities called atoms, borrowing the idea from the ancient Greek philosopher Democritus. Using that idea, coupled with the ideas of molecules and valence, he constructed a theory of atoms and molecules that explained the behaviour (changes) of substances.

Scientific theories, like mathematical theories, crucially employ the strategy of postulating concepts and propositions, and deducing their logical consequences. Those logical consequences are called *theorems* in mathematics, and *predictions* in science. In science, when a prediction matches what we observe, we say that the theory *explains* the observations.

2.7 INTUITION AND INSIGHT

An intuition is a feeling that something is true. Research in all domains, including mathematics and the physical-biological-human sciences, is guided by the intuitions of researchers.

Our intuition — the sense that something is true — can be trained and developed through practice. The trained intuition of an experienced researcher is typically more reliable than that of a high school student. This is what the Wikipedia entry on intuitions in mathematics says:

> "Logical Intuition, or mathematical intuition or rational intuition, is a series of instinctive foresight, know-how, and savviness often associated with the ability to perceive logical or mathematical truth—and the ability to solve mathematical challenges efficiently. Humans apply logical intuition in proving mathematical theorems, validating logical arguments, developing algorithms and heuristics, and in related contexts where mathematical challenges are involved. The ability to recognize logical or mathematical truth and identify viable methods may vary from person to person, and may even be a result of knowledge and experience, which are subject to cultivation." (https://en.wikipedia.org/wiki/Logical_intuition)

What is insight? The Wikipedia entry on insight says:

> "An insight that manifests itself suddenly, such as understanding how to solve a difficult problem … The term was coined by the German psychologist and theoretical linguist Karl Bühler. It is also known as an epiphany, eureka moment…" (https://en.wikipedia.org/wiki/Insight)

We can think of insight as an event of sudden illumination in the mind that reveals 'what lies beneath the surface'. A mathematician who has discovered a conjecture through insight has a sense of certainty that the conjecture is true, even without proving it.

The famous organic chemist August Kekulé who discovered the ring structure of benzene credits his discovery of the ring shape of the benzene molecule to having a reverie or day-dream of a snake seizing its own tail. (https://en.wikipedia.org/wiki/August_Kekul%C3%A9)

Most of the conjectures that the legendary mathematician Srinivasa Ramanujam discovered in his brief life were subsequently proved by other mathematicians. He himself had no interest in proving them, because he thought that those conjectures were gifts from the goddess of Namagiri, and hence, did not require proof. He describes as follows one of the events that we would now describe as a flash of insight:

> "While asleep, I had an unusual experience. There was a red screen formed by flowing blood, as it were. I was observing it. Suddenly a hand began to write on the screen. I became all attention. That hand wrote a number of elliptic integrals. They stuck to my mind. As soon as I woke up, I committed them to writing.

Ramanujam did not see the creativity of his unconscious mind to describe his insights; he naturally assumed that they were divine revelations.-Poincaré (1904) describes a process profoundly applicable not only to mathematics, but to just about any creative discipline:

> "I wanted to represent these functions by the quotient of two series; this idea was perfectly conscious and deliberate; the analogy with elliptic functions guided me. I asked myself what properties these series must

have if they existed, and succeeded without difficulty in forming the series I have called thetafuchsian.

"Just at this time, I left Caen, where I was living, to go on a geologic excursion under the auspices of the School of Mines. The incidents of the travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step, the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidian geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake, I verified the result at my leisure."

3 Weaving the Threads Together

We began Section 1 of this chapter with the statement that when textbooks assert that something is true, it is important for students to ask, "How do you know that?" And it is important for textbooks to anticipate such questions, and present rational justification for what is taken to be true by the members of the academic community.

In Section 2, we used the term 'sources of knowledge' to denote the cluster of epistemic concepts that we called perception, introspection, memory, reasoning, testimony, invention, intuition and insight. Why do we need an understanding of this set of concepts when designing a curriculum that is committed to responding to the question, "How do you know that?"

4 Summary

Section 1 articulated the importance of learners asking for rational justification for knowledge claims in textbooks, and textbooks, anticipating this demand, providing the justification for their claims. In Section 2, we examined eight sources of knowledge: sense perception leading to observation; introspection; memory; reasoning; testimony; invention; intuition; and insight. While these are sources of knowledge that we need to rely on, it is important not to treat any of them as infallible..

The rational temper that lies at the heart of the norms of academic inquiry demand that, even while relying on the sources of knowledge, we doubt and question each of them, and also the arguments based on them.

Knowledge is knowing that a tomato is a fruit. Wisdom is knowing not to put it in a fruit salad.

Brian O'Driscoll

> You can't stop the waves, but you CAN learn to surf.
>
> Jon Kabat-Zinn

PART 1: LOOKING BACK

INQUIRY, ACADEMIA, AND TRANS-DISCIPLINARITY

The title of this book is *Foundations of Knowledge and Inquiry across Disciplines*. Right at the beginning of Chapter 1, we asked:

~ What do the words *trans-disciplinary*, *academia*, and *inquiry* mean?

~ Rather, what do the authors of the book mean by these words?

1. What is inquiry? What is research?

As we said in Chapter 1, we use the term *inquiry* to refer to *the process of looking for an answer through one's own thinking, and discussions with others*.

In chapter 2, we identified the components of inquiry as:
formulating questions to express what we do not know;
looking for and finding answers;
arriving at conclusions on the basis of our answers;
critically evaluating the conclusions; and
accepting the conclusions if there are good reasons to do so.

The central thread in our formulation of these strands of inquiry is the concept of QUESTION. The process of systematic inquiry begins with a ***question to investigate***. And by now, you must have seen a number of examples of inquiry questions, some of which can become ***research questions*** at an advanced level.

Another way of looking at the process of inquiry is to say that it begins with a ***problem*** to solve, and proceeds to identify the ways of looking for a ***solution***. Problems have to do with either a gap in something that currently exists and is desirable, or something that is undesirable in the current state of affairs. For instance, a world without suffering, poverty, hatred, and violence is desirable, but as the world exists today, it is full of suffering, poverty, hatred, and violence. The gap between the two is a problem that we need to look for solutions to. Our lack of understanding of the Corona virus is undesirable: that is another problem that we must look for solutions to.

Whether we view them as questions or problems, it is important to appreciate the difference between the questions and problems for inquiry or research, and those in traditional examination questions. A traditional examination question is one for which the examiner expects students to already have a correct answer. For instance, given the question, “What is the tilt of the earth?” the student is expected to access the correct answer already stored in memory, and respond with “23.5 degrees” in less than a minute.

Inquiry questions don’t have such ready answers. When facilitators ask an inquiry question, they expect the novice inquirers to not know the answer, but to look for AN answer (not THE answer), arrive at a conclusion, and rationally justify the conclusion. If you look back at Chapters 1-3, you will find several examples of such inquiry questions.

Now, both inquiry and research start with a question, which is a formulation of what we do not know, but hope to find out. We said earlier that some inquiry questions can become *research questions* at an advanced level. What is the relation between ***inquiry*** and ***research***? How are they similar, and how are they different?

Research can be viewed as *inquiry that aims to make a contribution to the existing pool of collective knowledge*. So, both school students and PhD students can engage in inquiry, and both begin their investigation with a question that they do not have an answer to. However, it would be unrealistic to expect school students to make a contribution to the collective pool of knowledge, while that is a standard expectation of a research thesis.

2. What Makes Something Academic?

As stated earlier, ***academic inquiry*** is the process of inquiry that results in academic knowledge. This is related to the concept of Academic Cognition. That takes us to the question, “What is cognition?”

In Chapter 1, we said, *to cognize is to know*. Cognitive capacities are capacities related to knowing. The process of inquiry seeks to know, so it is a cognitive process. Academic cognitive capacities, then, are the capacities needed for academic inquiry. ***Academic Cognition*** is the cognition relevant for the acquiring, constructing, critically evaluating, and applying academic knowledge. We use the terms Academic Cognition and Higher Order Cognition to mean the same thing.

And we also said that ***Academic Knowledge*** (AK) is the body of knowledge constructed and validated by an academic community, and transmitted through educational institutions (i.e., schools, colleges, universities, institutes) and research publications.

Implicit in that characterisation of AK is that it is the result of the ***Academic Research*** of an ***Academic Community.*** What makes the research of that community worthy of being called ***Academic Research***? What makes the result of that research worthy of being called ***Academic Knowledge***? Who is an ***academic***? What makes something ***academic?***

It boils down to the question, "What is ***Academia***?" According to the Oxford Learner's Dictionaries, academia is "the world of learning, teaching, research, etc. at universities, and the people involved in it." (https://www.oxfordlearnersdictionaries.com/definition/english/academia)

A prototypical body of ***academic knowledge*** has the following traits:

A. Rationality: Academic knowledge is a form of rational knowledge, in which conclusions are rationally justified. The central pillars of rationality are (a) acceptance of the logical consequences (conclusions) of the propositions that we have accepted, and (b) rejection of the combination of propositions that result in a logical contradiction.

B. Proving: A proof is a ***rational argument*** in support of a knowledge claim, that is, a proposition that is claimed to be part of knowledge. A rational argument is composed of premises, steps of derivation, and a conclusion, where the conclusion is what is claimed.
[We must note that the term 'proof' is typically associated with mathematical proofs. Here, we have generalised that concept to include other kinds of proof: those presented in a law court (e.g., the prosecution's argument to prove that the accused is guilty), experimental proofs (e.g., proving that smoking causes cancer), and so on. If so, the term *proof* is synonymous with the term *rational justification*, such that it can be extended to: "How do we prove that the axis of the earth's rotation is tilted to the plane of its revolution around the sun?" and "How do we prove that humans have free will?"]

C. Sense of Uncertainty and Fallibility: The modern leaders of academia recognise that human knowledge, including the knowledge that they themselves have been responsible for, is uncertain and fallible. Total certainty of academic knowledge carries a logical contradiction.

D. Doubting and Questioning: Given (C), academia has a commitment to doubting and questioning what is taken to be academic knowledge. In the context of academic institutions, this means that learners have a responsibility to doubt and question what they regard as knowledge, and what is transmitted to them as knowledge by their teachers, textbooks, and published research, by asking and engaging with the question, "How do you know that?" and "Why should I accept that as 'knowledge'?"

E. Sociality: Academic Knowledge is a social entity: it is created, critically evaluated, validated, and transmitted by a community that we call the Academic Community. The cogniser (knower) of academic knowledge is the academic community.

3. What Makes Something Trans-disciplinary?

A discipline-specific concept, proposition, or form of inquiry is restricted to the given discipline. In contrast, a trans-disciplinary concept, proposition, or form of inquiry does not belong to any given discipline: it exists above the level of disciplines, across disciplinary boundaries.

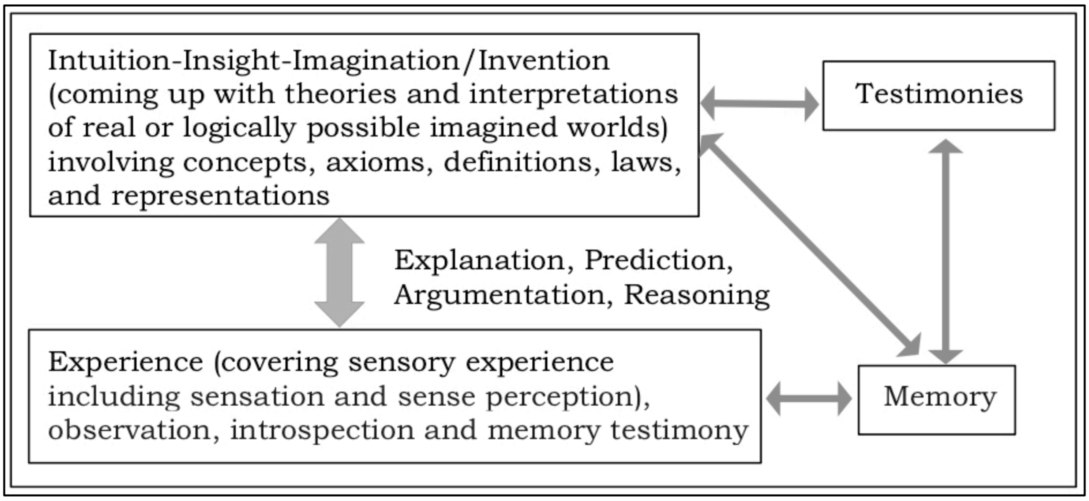
Consider the concept of structure. The structure of atoms comes under physics, the structure of molecules comes under chemistry, the structure of a skeleton comes under biology, the structure of a sentence comes under linguistics, the structure of a poem comes under literary studies, and the structure of an organisation comes under organisational studies. But the concept of structure itself is not bounded by any one discipline.

We see the world around us in terms of the ideas of ENTITIES, their PROPERTIES (traits and trait values; variables and values,...), the RELATIONS among them, and the STATES, PROCESSES, and EVENTS they participate in. The transdisciplinary concepts of STRUCTURE, CHANGE, and CAUSE are closely tied up with all these ideas.

STRUCTURE would involve COMPOSITIONALITY, along with UNITS, and CATEGORIES, as well as DIMENSIONS and LEVELS of structure. CHANGE would involve TIME, SPACE, and the STRUCTURE of states, processes, and events. Examples of *change* include *change of location* (motion), *change of velocity* (acceleration), *change of properties* (e.g., chemical change), *change of structure*, as well as changes involved in emergence/origin and extinction.

In Section 2, we used the term *sources of knowledge* to denote the cluster of epistemic concepts that we called perception, introspection, memory, reasoning, testimony, invention, intuition and insight. All these are trans-disciplinary concepts.

A model of rational inquiry, which draws on the concepts of explanation, prediction, reasoning, and argumentation, and is founded on the sources of knowledge discussed in Section 2 of Chapter 3 would help in seeing why a truly educated person needs an understanding of Transdisciplinary Academic Inquiry:



Total certainty is fatal to inquiry.

Foundations
of Knowledge and Inquiry
across Disciplines

PART 2: LOGIC AND REASONING IN INQUIRY

Chapters 4-6 are an introduction to how as academic inquirers, speaker-writers rationally justify the assertions they make in speech/writing, and how listener-readers critically evaluate the assertions they come across.

Suppose we find this sentence in a book:

> *Light travels in a straight line.*

In writing that sentence, the writer signals that he/she takes this assertion to be true. This means that a critical reader must ask the writer: “Why do you believe that assertion to be true?” And the writer has the responsibility to rationally justify — give reasons for — believing it to be true. The critical reader now examines the justification, and decides whether or not it is convincing.

As we progress through the chapters, we will get to see some of the norms that guide these processes in the context of academic knowledge and inquiry.

> To *argue* with a person who has renounced the use of reason is like *administering medicine to the dead.*
>
> Thomas Paine

CHAPTER 4: INTRODUCTION TO REASONING

1 Reasoning in Critical Reading

Our friend Rafa tells us: "In that football team, Zak is taller than Apollo, and Apollo is taller than Zeno."

Later, Anu asks us: "Who is taller, Zak or Zeno?"

We would be able to answer without a second thought: "Zak is taller than Zeno."

In arriving at that answer, we made an inference based on the information Rafa had given us.

REASONING is *the process by which we arrive at inferences.*

For the purposes of illustration, let us take two passages from two different sources.

Passage 1**:**

"In a field many other undesirable plants may grow naturally along with the crop. These undesirable plants are called weeds. The removal of weeds is called weeding. Weeding is necessary since weeds compete with the crop plants for water, nutrients, space and light. Thus, they affect the growth of the crop. Some weeds interfere even in harvesting and may be poisonous for animals and human beings."

(Source: NCERT Grade 8 Science Textbook, Chapter 1, pg. 10)
https://ncert.nic.in/textbook.php?hesc1=1-13

Passage 2**:**

"Weeds can be a problem for many gardeners, with valid reasoning: They do compete with our chosen plants for nutrients, water, and space. However, the wiser amongst us know that weeds are not only useful but also often play a systemically vital role. Now, this isn't to say that we necessarily want them popping up all over our cultivated beds — a good dose of mulch and establishing ground covers can help with that — yet, with a step back, a sense of humor, and solution-oriented thinking, we may find ourselves appreciating rather than antagonizing weeds. At this point, they become less pest and more partner.

Not to mislead, working wisely with weeds will not mean that a gardener will never again find himself or herself hunched over a bed in horror, trying to get them by the roots, but once again, as permaculture encourages us, working with nature as opposed to against it will provide better results for both humans and the environment. "Weeds" the word may be a human construct, but the plants themselves are natural and have functional niches within the formation of ecosystems, not to mention a multitude of oft overlooked uses within our cultivation. It begins with how we decide to look at them."

(Source: *Working Wisely with Weeds*)
https://www.permaculturenews.org/2016/07/15/working-wisely-weeds/

These two passages portray different positions on weeds. If you are a gardener, they form the basis for different practices in gardening.

A critical reader would carefully think through the reasons that a writer advances in support of or against a position. Having read the two passages carefully, would you see weeds as completely undesirable? Or would you see them as plants that, while being undesirable in some ways, can also provide value? From the above passages, how would you decide whether to accept one of the two positions, reject both, accept both, or reserve them for further scrutiny. And what would be your reasons for the decision?

To come to a decision, we need to break down the argument presented in each passage. An ARGUMENT is a piece of reasoning for establishing something, or disproving something. Like all reasoning, an argument is made up of PREMISES and a CONCLUSION. The premises are statements that we take to be true. They form the basis for the conclusion, which is the statement that we infer from the premises. We can evaluate the reasoning in an argument by examining the steps it uses to connect the premises and arrive at the final conclusion from them.

A conclusion becomes a part of academic knowledge when a community of academics examines and accepts the arguments presented for it. Academics often find themselves asking: Is this argument good enough to support the conclusion? Asking this question, and figuring out an answer is an important ability to cultivate, not just for academics, but as illustrated above, for any individual who wishes to arrive at rational conclusions in any domain of life.

2 Premise, Conclusion, and Logical Consequence

2.1 The word 'therefore'

Let us look at examples of reasoning that use the commonly used word *therefore*. Here are a few examples. Notice that in each one, the last sentence begins with the word *therefore*:

Ex. 1 The formula for the area of a triangle is ½ base x height.
The base of this triangle is 10 cm.
The height of this triangle is 6 cm.
Therefore, the area of this triangle is 30 sq cm.

Ex. 2 Pete and Ari are siblings.
Chuck and Pete are siblings.
Therefore, Chuck and Ari are siblings.

Ex. 3 All dogs are mammals.
All mammals are vertebrates.
Therefore, all dogs are vertebrates.

Ex. 4 If we drop a metal object in water, it sinks to the bottom.
Zeno dropped a metal spoon in a bucket of water yesterday.
A metal spoon is a metal object.
Therefore, the metal spoon that Zeno dropped in a bucket of water must have sunk to the bottom.

In these examples, the word *therefore* signals that the writer believes that the statement that follows that word is a legitimate conclusion arising from the previous statements. Those previous statements are premises. A legitimate conclusion is one that *logically follows from* the premises. We refer to that relation of the conclusion to the premises as a LOGICAL CONSEQUENCE: a legitimate conclusion from a set of premises is a logical consequence of that set of premises.

Two other terms that are used with the same meaning as *therefore* are *hence* and *so*. The Latin word *ergo* (as in *Cogito, ergo sum*, "I think, therefore I am") means the same as the English word *therefore*. (See https://en.wikipedia.org/wiki/Cogito,_ergo_sum)

2.2 A Symbolic Notation for therefore

Let us use an ***arrow notation*** to represent the structure of the above examples. In this notation, X $\Rightarrow$ Y

means: X, therefore Y.

Using this notation, we can represent the structure of Ex. 1 as follows:

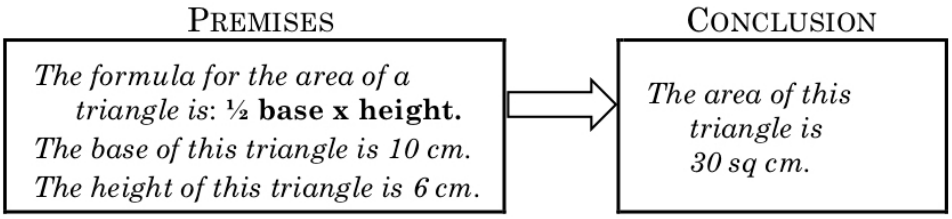


Figure 1

Using the notation of P for premises and C for conclusion, we can express the abstract structure in Fig. 1 as Fig. 2:

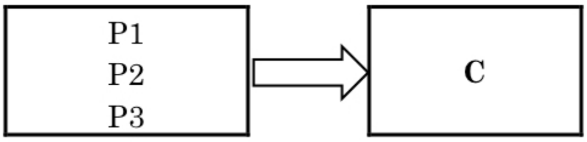


Figure 2

Here are a few more examples of the use of *therefore* in reasoning:

Ex.5 P1: If Plato is a chef, then Zeno loves chillies.
P2: If Zeno loves chillies, then Newton is hungry.
P3: Plato is a chef.
C: *Therefore,* Newton is hungry.

Ex. 6 P1: If an organism is a mammal, then it has vertebrae.
P2: If an organism has vertebrae, then it has bone cells.
P3: Zeno is a mammal.
C: *Therefore*, Zeno has bone cells.

In mathematics, the notation for *therefore* is three dots: ∴
Here is an example:

The sum of angles in a triangle is 180°.
In this triangle, angle A is 90° and angle C is 40°
The sum of 90 and 40 is 130.
The result of subtracting 130 from 180 is 50.
∴ Angle B is 50°.

Instead of the triple dot notation, we will use the arrow notation for *therefore* in this book.

SOME FUN STUFF

TASK 1: For each example below, identify the premises and conclusion. Then use the notation in Fig. 2 to express the structure of reasoning in it.

(1) If an organism is a mammal, it is a vertebrate.
Zeno is a mammal.
Zeno is a vertebrate.

(2) If Plato is a chef, then Zeno loves chillies.
If Zeno loves chillies, then Newton is hungry.
Newton is hungry.
Plato is a chef.

(3) Plato smuged some frabs yesterday.
If glebins delped tabans, then slids are decons.
If slids are decons, then Plato smuged some frabs yesterday.
Glebins delped tabans.

(4) If glebins delped tabans, then slids are decons.
If slids are decons, then Plato smuged frabs yesterday.
If glebins have delped tabans, then Plato smuged frabs yesterday.

TASK 2: Given below are two passages. Express each of them in terms of the premises-*therefore*-conclusion structure illustrated in Fig. 1. To do that, you will have to begin by selecting your premises and reformulating them. For the reformulation, you may have to delete words and phrases that are not crucial, and add words where needed, e.g. the word *therefore*.

Passage A:
Which number is greater, 4875 or 4575? The answer is: 4875. Why? The numbers have the same number of digits. The digits at the thousands place are the same in both, and so are the digits at the tens place. The digit at the hundreds place is greater in 4845 than in 4548.

Passage B:
At the height of their self-confidence in the 19th and early 20th centuries, orthodox Western physicians tried to stamp out most forms of medical treatment other than their own. In much of Europe, the law was already on their side, based on the premise that anything not specifically permitted is forbidden. So it was fairly easy for the medical establishments in such nations as France, Germany, Italy and Spain to ensure that therapists of whom they disapproved did not secure permission to practice. In Britain, the US and other countries with an Anglo- Saxon legal heritage, the situation was more complex. British doctors failed in 1858 to win an outright ban in law on their unorthodox rivals, so they made their own rules stating that any of their number referring patients to non-recognized therapists would in effect be disbarred. In the US, most states adopted legislation defining unorthodox therapies as the practice of medicine and therefore the preserve of qualified physicians, though there were some exemptions and waivers.

TASK 3: Find more passages with the word *therefore*, and express each of them in terms of the premises-*therefore*-conclusion structure illustrated in Fig. 1. This will serve as good practice for becoming comfortable with reasoning.

3 Steps of Reasoning

Let us take an example similar to the one we saw at the beginning of the chapter, but with more than three premises:

1. Athena is taller than Apollo.
2. Socrates is taller than Aristotle.
3. Apollo is taller than Zeno.
4. Plato is taller than Socrates.
5. Zeno is taller than Plato.

Now we are asked: Who is taller, Athena or Aristotle?

In this case, we need a bit of time to answer the question. But if we take into account ***all*** the premises given to us, we should be able to infer the answer:

Athena is taller than Aristotle.

The reason why it takes a bit of time in this case is because the DERIVATION requires several steps, with a number of INTERMEDIATE CONCLUSIONS. Let us work through the steps in the derivation (D1) below:

D1:	P1.	(1) Athena is taller than Apollo.	
	P2.	(3) Apollo is taller than Zeno.	
		Therefore, Athena is taller than Zeno	C1: from (1) and (3)
	P3.	(5) Zeno is taller than Plato.	
		Therefore, Athena is taller than Plato.	C2: from (5) and C1
	P4.	(4) Plato is taller than Socrates.	
		Therefore, Athena is taller than Socrates.	C3: from (4) and C2
	P5.	(2) Socrates is taller than Aristotle.	
		Therefore, Athena is taller than Aristotle.	C4: from (2) and C3

The example of reasoning in D1 demonstrates what an explicit process of reasoning looks like. In D1, the statements in P1-P5 are the premises, C1-C4 are the conclusions, and of these, C4 is the final conclusion. D1 connects the premises to the conclusion in systematic steps — that is, it shows us how the conclusion follows as a logical consequence of the premises.

In cases where the premises are complex, or the number of premises is large, the derivation requires us to spell out not just the steps of reasoning, but also the intermediate conclusions — like C1, C2 and C3 in D1. And also as in D1, the derivation must specify the statements that lead to each intermediate conclusion.

STRUCTURE OF REASONING:

PREMISES → DERIVATION → CONCLUSION

4 Valid and Invalid Derivations

4.1 When Premises are Missing

Suppose we are told that the following premises are true:

1. Athena is taller than Apollo. (P1)
2. Apollo is taller than Zeno. (P2)
3. Plato is taller than Socrates. (P4)
4. Socrates is taller than Aristotle. (P5)

We are again asked: Who is taller, Athena or Aristotle?

Are we in a position to answer the question? No. Premises 1, 2, 3 and 4 allow us to infer that Athena is taller than Zeno, and that Plato is taller than Aristotle. But there is no premise that connects Athena and Aristotle even indirectly. That is, one of the crucial premises needed for deriving an answer is missing in the set of given premises, and hence, we cannot answer the question. Suppose we add that missing premise:

P3. Zeno is taller than Plato.

we would now be able to infer that Athena is taller than Aristotle.

We will use the terms

VALID to refer to a derivation that is without flaws, and

INVALID to refer to a derivation that is flawed.

The derivation of the conclusion that Athena is taller than Aristotle is valid given the premises in 1-5. But it is invalid if any one of those premises is missing.

Let us take a few more examples to get a feel for what we mean by validity of derivations. In doing this, we will consider several premise-conclusion sets (PC), without necessarily spelling out the steps of the derivation. We will also take it that if the derivation is valid, then the conclusion is legitimate.

Consider the following example of a premise-conclusion set (PC1):

PC1: P1 The base of this rectangle is 12 cms.
C Therefore, its area is 120 sq. cms.

You would agree that given only P1 in PC1, the conclusion is not legitimate, even though we judge it to be true. Why? Because a crucial premise about the height of the rectangle is missing. Let us add that premise:

PC2: P1 The base of this rectangle is 12 cms.
P2 Its height is 10 cms.
C Therefore, its area is 120 sq. cms.

Now, we might think that the conclusion in PC2 is legitimate, but there is still something missing. It is something that we happen to have learnt, and are taking for granted as true, but have not stated explicitly. Let us state it:

The area of a rectangle is its base multiplied by its height.

To make our conclusion in PC2 legitimate, let us add this premise to the set:

PC3: P1 The base of this rectangle is 12 cms.
P2 Its height is 10 cms.
P3 The area of a rectangle is its base multiplied by its height.
C Therefore, the area of this rectangle is 120 sq. cms.

4.2 When the Conclusion is not a Logical Consequence

Missing premises are not the only source of flaws in derivations. Consider PC4:

PC4: P1 Apollo and Athena are siblings.
P2 Athena and Aphrodite are siblings.
P3 Aphrodite and Hermes are siblings.
C Apollo and Hermes are siblings.

In PC4, we can intuitively arrive at the conclusion from the premises, because we know the meaning of the word *sibling*. We say that in PC4, C follows from P1, P2 and P3. The conclusion is a LOGICAL CONSEQUENCE of these premises.

But now consider these premises:

PC5: P1 Athena loves Apollo.
P2 Apollo loves Zeno.
C **?**

What can we conclude from the two premises? We are not in a position to provide an answer, because given the premises in PC5, and the meaning of the word *loves*, it does not follow that Athena loves Zeno, nor does it follow that Athena does not love Zeno. Hence, neither of these conclusions would be legitimate.

So, if we had stated C as “Athena loves Zeno”, the derivation would be invalid as the conclusion does not logically follow from the premises.

Practice Exercise Set 1

TASK 1: Given below are sets of premises, and the conclusion derived from each set. For each set, say if the derivation is valid. If it is valid, give the derivation.

1. P1 The base of this rectangle is 12 cms.
P2 Its height is 10 cms.
P3 The area of a parallelogram is its base multiplied by its height.
P4 A rectangle is a parallelogram with right angles.
C *Therefore, the area of this rectangle is 120 square cms.*

2. P1 All dogs have eight legs.
P2 Zeno is a dog.
C *Therefore, Zeno has eight legs.*

3. P1 All dogs have four legs.
P2 Zeno has four legs.
C *Therefore, Zeno is a dog.*

4. P1 All dogs have four legs.
P2 Zeno is a horse.
C *Therefore, Zeno has four legs.*

5. P1 All gleeks have four legs.
P2 All zemphras are gleeks.
C *Therefore, all zemphras have four legs.*

6. P1 All gleeks have four legs.
P2 All zemphras have four legs.
C *Therefore, all zemphras are gleeks.*

7. P1 Whenever a Zelfy duglates, all florgs simify.
P2 A Zelfy duglated yesterday.
C *Therefore, all florgs simified yesterday.*

8. P1 Whenever a Zelfy duglates, every florg simifies.
P2 All florgs simified yesterday.
C Therefore, a Zelfy duglated yesterday.

TASK 2: In the following PC sets, we state the premises slightly differently. This is to draw attention to the issue of the truth of premises and conclusions. The label TRUE IN 'TRUE: X' means, "We judge X to be true." [It is important to note that "We judge X to be true," is not the same as "X is true."] At a later stage, we will see the usefulness of formulating the statements in this way.

9. P1 TRUE: Only those who score 99% marks are admitted to Delgins.
P2 TRUE: Zeno has been admitted to Delgins.
C *Therefore,* TRUE: *Zeno scored 99% marks.*

10. P1 TRUE: Only those who score 99% marks are admitted to Delgins.
P2 TRUE: Zeno scored 99% marks.
C *Therefore,*TRUE: *Zeno has been admitted to Delgins.*

11. P1 TRUE: All those who score 99% marks are admitted to Delgins.
P2 TRUE: Zeno scored 99% marks.
C *Therefore,* TRUE: *Zeno has been admitted to Delgins.*

12. P1 TRUE: All those who score 99% marks are admitted to Delgins.
P2 TRUE: Zeno has been admitted to Delgins.
C *Therefore,* TRUE: *Zeno scored 99% marks.*

13. P1 TRUE: If the area of a rectangle is its base multiplied by its height.
P2 TRUE: The base of this rectangle is 12 cms.
P3 TRUE: The height of this rectangle is 10 cms.
C *Therefore* TRUE: *the area of this rectangle is 120 square cms.*

5 Unearthing the PDC Structure of Texts

In the sections above, we have talked about the structure of reasoning as being composed of a set of **P**(remise)s, a **C**(onclusion), and the **D**(erivation) from the premises to the conclusion. We will call this the PDC structure of reasoning.

Now, to identify this structure in running texts, we have to understand how certain words signal certain kinds of logical relations. Let us look closely at some of these words.

5.1 The words 'therefore' and 'hence'

To go back to what we said earlier, a good example of words that signal logical relations is *therefore*. Let us return to part of an example that we used at the beginning of this chapter, which contains this word:

> Athena is taller than Apollo. Apollo is taller than Zeno.
> ***Therefore***, Athena is taller than Zeno.

The word *therefore* in the last sentence above signals that the author/speaker believes that *the conclusion follows logically from the premises, that it is a logical consequence of the premises.*

The word *hence* has the same meaning as *therefore*: they both signal that the sentence they are part of is a conclusion that follows from the given premises. The example given below has the same structure as the example at the beginning of this section:

> Athena is taller than Apollo. Apollo is taller than Zeno.
> ***Hence,*** Athena is taller than Plato.

5.2 *The Relation of Transitivity*

Notice that in the above premises, *'–er than'* in the expression *taller than* also signals an important logical relation. This becomes clear when we compare it with what we saw in PC5, repeated below as PC6, with a conclusion added in instead of a question mark:

PC6: P1 Athena loves Apollo.
P2 Apollo loves Zeno.
C ***Therefore**, Athena loves Zeno.*

As we saw earlier, PC5 is not an example of good reasoning, because given the meaning of *loves*, neither "Athena loves Zeno," nor "Athena doesn't love Zeno," follows logically from the premises.

Thus, while the reasoning in the examples with "x is taller than y," and "x is a sibling of y," are instances of good reasoning, the reasoning in examples with "x loves y" is flawed.

What sanctions the conclusion in the legitimate instances is the relation of what mathematicians call TRANSITIVITY:

If	R is a transitive relation,
given	xRy and yRz,
it follows that	xRz.

The relations *–er than* and *is a sibling of* are transitive relations, while *loves* is not transitive.

5.3 *The expressions 'if' and 'only if'*

Another important word that signals logical relations between sentences is the word *if*. Consider this piece of text:

> ***If*** Athena was in Delhi that day, Apollo went to work. Athena was in Delhi that day. Therefore, Apollo went to work.

Ignoring the details of the derivation, the partial logical structure of this text can be given as:

PC7: P1 ***If*** Athena was in Delhi that day, Apollo went to work.
P2 Athena was in Delhi that day.
C Therefore, Apollo went to work.

Just as *–er than* is crucial to the reasoning in the examples with *taller than*, *if* is crucial for the reasoning in PC7. To see this more clearly, let us replace *if* with *though*, and contrast the two examples:

> ***Though*** Athena was in Delhi that day, Apollo went to work.
> Athena was in Delhi that day. Therefore Apollo went to work.

In this example, the conclusion does not follow from the premises. Given the meaning of *though*, the reasoning here is flawed.

Now compare the *if* sentence in PC8 with the *only if* one in PC9:

PC8: P1 Apollo went to work ***if*** Athena was in Delhi that day.
P2 Athena was in Delhi that day.
C Therefore Apollo went to work.

PC9: P1 Apollo went to work ***only if*** Athena was in Delhi that day.
P2 Athena was in Delhi that day.
C Therefore Apollo went to work.

While the derivation in PC8 is valid, the one in PC9 is not.

However, the derivation in PC10 is valid, but the one in PC11 is not:

PC10: P1 Athena was in Delhi that day. ***only if*** Apollo went to work.
P2 Athena was in Delhi that day.
C Therefore Apollo went to work.

PC11: P1 Athena was in Delhi that day ***if*** Apollo went to work.
P2 Athena was in Delhi that day.
C Therefore Apollo went to work.

These examples show that *if* and *only if* are mirror images of each other. To see this clearly, let us take P1 in PC7:

If Athena was in Delhi that day, Apollo went to work.

Suppose we use the symbol X for "*Athena was in Delhi that day*," and the symbol Y for "*Apollo went to work*." We can now represent the sentence as:

If X, then Y.

If we do this, how would we represent the following sentence (P1 in PC11)?

Athena was in Delhi that day ***if*** Apollo went to work.

It would be: Y if X.

Notice that the two sentences below have the same meaning:

If Athena was in Delhi that day, Apollo went to work.

Apollo went to work ***only if*** Athena was in Delhi that day.

This means that: *Y only if X* = *if X, then Y*

5.4 The word 'not'

The word *not* is closely related to the expressions *'is true'* and *'is false'*. Let us look at this relationship with examples. Assume that:

Every statement is either true or false.

Take the following statement: *Zeno is a turtle*. If this statement is TRUE, then it is NOT-FALSE; and if it is FALSE, then it is NOT-TRUE. Let us state these equivalences explicitly, using the symbol X for 'statement'.

X is TRUE = X is NOT-FALSE

X is FALSE = X is NOT-TRUE

This leads us to an interesting relation between ***if*** and ***not***. To see this, let us go back to the Athena and Apollo example in PC7 (repeated below). As we have seen, the reasoning in PC7 is valid. However, the reasoning in PC12 below is flawed.

PC7: P1 ***If*** Athena was in Delhi that day, Apollo went to work.
P2 Athena was in Delhi that day.
C Therefore, Apollo went to work.

PC12: P1 ***If*** Athena was in Delhi that day, Apollo went to work.
P2 Apollo went to work.
C Therefore, Athena was in Delhi that day.

But what happens when we add *not* in P2 and C in both PC7 and PC12?

PC13: P1 *If* Athena was in Delhi that day, Apollo went to work.
P2 Apollo did not go to work.
C Therefore Athena was not in Delhi that day

PC14: P1 ***If*** Athena was in Delhi that day, Apollo went to work.
P2 Athena was not in Delhi that day.
C Therefore Apollo did not go to work.

Notice how the appearance of *not* in the conclusion is legitimate in PC13, but not in PC14. The results have reversed.

What this discussion shows can be stated explicitly as:

If X is true, then Y is true = If Y is not true, then X is not true.

When we critically evaluate the reasoning in running text, it is important that we pay attention to the use of *if-then*, *only if*, and *not*, and also *therefore/hence*.

Practice Exercise Set 2

TASK: Given below are a few short pieces of text. In each one, identify the premises and conclusion, and represent their partial logical structure as illustrated in Section 5 above.

1. Whenever it rains, the streets are wet. The streets are wet now. So it must have rained.
2. Zeus struck the man with a thunderbolt because he was angry.
3. Zeno is not an honest person. Why do I think so? Because he tells lies whenever it suits him. And we all agree that telling lies is morally wrong.
4. Xena had a bar of iron in her hand. It must have been a magnet. She brought it close to a metal sewing needle, and the needle jumped up to the bar of iron. Whatever attracts metal is a magnet.

5. Zeus was very hungry. So he ate everything on his plate.
6. Every square is a rectangle. Every rectangle is a parallelogram. Every parallelogram is a quadrilateral. Every quadrilateral is a polygon. So every square is a polygon.
7. The circumference of this CD is about 22 cms because its radius is 3.5 cms.
8. Zeno must have been murdered. His body was found in a well. When a person dies by drowning, there is always water in the lungs. There was no water in Zeno's lungs. Furthermore, there were bruises on his neck, indicating that he was strangled.
9. Alexander was a great emperor. He conquered many countries and succeeded in expanding his empire.
10. A school curriculum must have a series of courses on reasoning because reasoning is a valuable ability for all educated people.
11. Bill Gates is more intelligent than Albert Einstein. He is one of the richest men in the world. In contrast, Einstein did not have any money other than what research institutions paid him, which was not much.
12. Teaching is helping someone to learn something. High quality learning is that which is of high value in the life of the learner. It then follows that high quality teaching is helping someone to learn what is of high value in the life of the learner.
13. Democracy is a system in which those who are affected by a decision have an opportunity to influence the decision. There is no democracy in the country G. In G, people can vote to elect their leaders. But once the elections are over, those who are governed have no opportunity to influence the decisions of those who govern.
14. A legal system that permits 'capital punishment' (the death penalty) as a form of punishment is morally wrong. We all agree that destroying the life of a human being is morally wrong. Some version of "Thou shalt not kill" is a dictum found in almost all major religions. How then, can the criminal law of a legal system permit the Death Penalty?

Logic is the beginning of wisdom, but not the end.

Leonard Nemoy

> Logic is the technique by which we add conviction to truth.
>
> Jean de le Bruyere

CHAPTER 5: JUDGING THE TRUTH OF ASSERTIONS

1 The Soundness Criterion

In Chapter 4, we discussed the concepts of PREMISE, DERIVATION, and CONCLUSION as part of the structure of REASONING, and explored the contrast between VALID and INVALID derivations. We also saw that it is important to distinguish the VALIDITY OF A DERIVATION from the TRUTH of the premises and of the conclusions. As listener-readers, we may judge a derivation to be invalid even if the conclusion is judged to be true. On the other hand, a derivation may be judged to be valid even when we judge the conclusion to be false. This gives us four possibilities:

TABLE 1

	CONCLUSION: TRUE	**DERIVATION: VALID**
A.	YES	YES
B.	YES	NO
C.	NO	YES
D.	NO	NO

We may also have situations where we are not in a position to tell if a premise is true or false, and if a given derivation is valid or invalid. So we get four more categories:

TABLE 2

	PREMISES: TRUE	**DERIVATION: VALID**
E.	YES	CANNOT TELL
F.	NO	CANNOT TELL
G.	CANNOT TELL for at least one premise	YES
H.	CANNOT TELL for at least one premise	NO

The tasks in the exercises at the end of Chapter 4 give examples of each of these categories.

TASK: Take each exercise in Practice Exercise Set 2 in Chapter 4, and categorise it in terms of A-H in Tables 1 and 2 above.

We now turn to ways of deciding if a conclusion is true. The criterion that we use as the basis for making that decision can be stated as:

CRITERION 1:

If we judge that	*i)*	*the premises are true, and*
	ii)	*the derivation is valid,*
then we must also judge that	*iii)*	*the conclusion is true.*

We will use the term ***sound reasoning*** to mean the combination of the two criteria: truth of the premises, and validity of derivation. We will say that:

A given instance of reasoning in support of a conclusion is sound if and only if all the premises are true and the derivation is valid.

We can now say: *For a conclusion to be accepted as true, it must be supported by sound reasoning.*

Let us go through a few examples of premise-conclusion sets (PCs).

PC1: P1 All animals have an alimentary canal.
P2 Only animals have an alimentary canal.
P3 Every alimentary canal has a mouth.
P4 Zeno is an animal.
C ***Therefore,*** *Zeno has a mouth.*

Let us look at how we derive a conclusion from premises:

D1: P1. All animals have an alimentary canal.
P4. Zeno is an animal.
C1 Therefore, Zeno has an alimentary canal. from P1 and P4
P3. Every alimentary canal has a mouth.
C2 Therefore, *Zeno has a mouth.* from P3 and C1

In D1, the premises are *true*, and the derivation is *valid*. Hence, the reasoning in support of the conclusion is *sound*. We accept the conclusion as *true*.

Let us take another example.

PC2: P1 All animals have an alimentary canal.
P2 Only animals have an alimentary canal.
P3 Every alimentary canal has a mouth.
P5 Zyphus is ***not*** an animal.
C ***Therefore,*** *Zyphus does* ***not*** *have a mouth.*

Here is the derivation for the conclusion:

D2: P2. Only animals have an alimentary canal.
P5. Zyphus is ***not*** an animal.
Therefore, Zyphus does not have an alimentary canal.
C1: from P2 and P5
P3. Every alimentary canal has a mouth.
Therefore, *Zyphus does* ***not*** *have a mouth.*

In D2, the premises are true. However, the derivation is **invalid**. P3 says that every alimentary canal has a mouth. However, it does not say that only an alimentary canal has a mouth. Therefore, it does not follow from P2, P3, and P5 that Zyphus does not have a mouth. Hence, the reasoning in support of that conclusion is **unsound**, even though the premises are all true.

Practice Exercise Set 3

TASK 1: For each Premise-Conclusion set given below,
(a) work out the derivation, and say if it is valid.
(b) check if you judge the premises to be true.
If the reasoning is not sound, say to which category of unsoundness it belongs. [The categories of unsoundness are given in Tables 1 and 2 in Section 1.]

1. P1 All animals have an alimentary canal.
P2 Every organism that has an alimentary canal also has a mouth.
P3 Zyphus is not an animal.
C: *Therefore Zyphus does not have a mouth.*

2 P1 Only animals have an alimentary canal.
P2 Every organism that has an alimentary canal also has a mouth.
P3 Zyphus has a mouth.
C: *Therefore Zyphus is an animal.*

3 P1 Plants do not have an alimentary canal.
P2 Every organism that has an alimentary canal also has a mouth.
P3 Zyphus does not have a mouth.
C: *Therefore Zyphus is a plant.*

4 P1 Only animals have an alimentary canal.
P2 Every organism that has an alimentary canal also has a mouth.
P3 Zeno is an animal.
C: *Therefore Zeno has a mouth.*

2 The Logical Consistency Criterion

Are the following statements true?

> The Honduba is in South America and is not in South America.
> Zofras is a dog and is not a dog.
> Fleno passed the exams and did not pass the exams.

You have not heard the names Honduba, Zofras or Fleno before. You don't know anything about them. And yet you would reject these statements as false. Why?

The reason is that all of them assert statements that have a ***logical contradiction***. What does that mean?

> A statement (or a set of statements) is said to have a logical contradiction if it contains a combination of statements where one of them negates the other.

The statement, "Honduba is not in South America," negates the statement, "Honuba is in South America." Similarly, "Zofras is not a dog," negates the statement, "Zofras is a dog." If you combine a statement and its negation to make a single statement, it expresses a logical contradiction. *For a statement or set of statements to be **logically consistent**, it must be free of **logical contradictions**.*

Remember Criterion 1? It stated that for a conclusion to be accepted as true, it must be supported by sound reasoning. Let us state another important criterion that we can use for deciding if a statement is false:

CRITERION 2:

If we find that a statement (or a set of statements)
contains a logical contradiction,
then we must reject the statement (or the set) as false.

To see how this criterion works, let us take these statements:

If Plato loves his brother, then Aristotle hates Socrates.

Plato loves his brother.

Aristotle does not hate Socrates.

Can we accept this set of statements as true? Do you see a contradiction?

3 Truth of the Premises

We have said that we judge an argument to be sound if judge the premises to be true and the derivation to be valid. Let us now ask: Under what conditions should a premise statement be taken as true? Let us take a look.

3.1 Good Reasons

On what basis do we judge the truth of premises? Just as we ask if a conclusion is supported by good reasons, we ask if a premise is supported by good reasons. Let us take an example.

If someone were to ask:	Do you brush your teeth every day?
You would say:	Yes.
You are now asked:	Why do you brush your teeth every day?

To answer this question, you have to give reasons. (This is different from reasons for judging a statement as true.) Your answer would be:

If we don't brush our teeth regularly, bacteria would collect at their base. And if that happens, our teeth would decay. That is not desirable. I brush my teeth every day to prevent this outcome.

At this point, our sceptical inquirer might ask:

You say, "If we don't brush our teeth regularly, bacteria would collect at their base." Why do you judge this statement to be true?

In answer to that question, you might point to the article, "Oral Health: a Window to your Overall Health," from Mayo Clinic (https://www.mayoclinic.org/healthy-lifestyle/adult-health/in-depth/dental/art-20047475).

When you do this, you are expressing our trust in Mayo Clinic as a reliable source of knowledge. But the inquirer may ask:

What are Mayo Clinic's reasons for supporting the assertion that brushing your teeth regularly prevents undesirable bacterial infection to the gums?

We invite you to do an Internet search for an answer. As a starting point, you might try reading "Microbiology of Dental Decay and Periodontal Disease," Chapter 99 of the book, *Medical Microbiology*, by Walter J. Loesche. (https://www.ncbi.nlm.nih.gov/books/NBK8259/). You will have to decide if the arguments there are convincing.

Let us take another example. This is a dialogue between Alpha and his cousin Omega.

A: Omega, would you like to play pingpong with me this weekend?

O: I can't, I've got lots to study.

A: You have an exam next week or something?

O: No, I am just revising everything in all my textbooks, to prepare for the Board exams.

A: What do you mean, "prepare"?

O: Oh, I memorise everything in all the chapters in all the textbooks, and work through all the exercises.

A: Do you think that's the best way to score really high?

O: Yes, of course.

A: Why?

Alpha is asking Omega why she thinks that studying means memorising everything in all the textbooks and doing all the exercises. The reason that Omega gives is that this is the best way to get very high scores in the exams. And Alpha asks why it is the best way to get high scores.

What reasons would you give, if you were in Omega's position? Stop reading for a little while, take a walk, think about your answer, and then come back to the conversation.

A: I don't think this is a good strategy to get high scores. Do you think they ask you questions from all the textbooks from Class 8 to Class 10 in the board exams?

O: Of course. Examiners pick randomly from all the textbooks. You can't tell what they are going to ask, so you have to memorise everything and practice all the exercises.

A: Have you actually looked at the exam papers from the past five to ten years? That will tell you if the questions are random, or if there is a pattern of repeating certain topics? For the Class 10 exam, do they pick topics from Class 6, 7, and 8?

Omega's assertion is that hers is the best strategy to get very high scores. The reason she gives is that examiners pick randomly from all the textbooks from all the years. Alpha questions this assertion, and asks Omega if she has looked at sample question papers to check if the assertion is true.

What do you think? Is Omega's reasoning sound? When you have an answer, formulate it, and write it down. If possible, discuss this question with your friends, and come up with a joint answer.

Let us go to an extended example. This one is from math.

If someone were to ask:	What is the area of a triangle?
Our answer would be:	Its base multiplied by its height divided by two.
The inquirer now asks:	Why do you think this statement is true?

Our response would be to treat the answer as a KNOWLEDGE CLAIM — a statement that we claim to be true — and give reasons to support the claim. It might go like this, in the form of a proof.

[Study the Proof, given on the next page, before you continue.]

At this point, a sceptical inquirer might ask:
Why should we accept P1 as true? Why should we accept P2 as true?

We can answer these questions by taking P1 and P2 as knowledge claims, and giving proofs for them (= reasons in their support). We invite you to give these proofs, along the lines in the Proof.

What we have shown is that the claim is true for *any* triangle that can be created by drawing a diagonal in a rectangle. But can *every* triangle be created by this procedure? So, we must ask if P1+P2 is sufficient to show that the claim is true for *any* triangle.

3.2 The Role of Experience

Let us look at another basis for judging the truth of premises. Consider the following scenario:

Alpha and Sheena are having a conversation about Padu and Cary. Sheena has not met them. Alpha asserts that Cary is taller than Padu. And Sheena asks for reasons for that assertion.

A: Well, Cary is taller than Zena, and Zena is taller than Padu. So Cary has to be taller than Padu.

S: What is your reason for asserting that Cary is taller than Zena? And for asserting that Zena is taller than Padu?

A: I have seen Cary and Zena standing next to each other. And I observed that Cary is taller than Zena. I've also seen Zena and Padu standing side by side. I noticed that Zena was clearly taller.

In this example, the statement that Cary is taller than Padu is treated as a conclusion. Alpha gives reasons for that conclusion by stating his premises. When Sheena questions the premises, Alpha gives his experience as the basis for judging the premises to be true.

(continued on page 77)

PROOF

Premises:

P1 The area of a rectangle is its base multiplied by its height.

P2 The diagonal of a rectangle divides it into two congruent triangles.

Derivation:

Take *any* triangle. Treat its longest side as the base.

In triangle ABC, AB is the base. (Fig. 1)

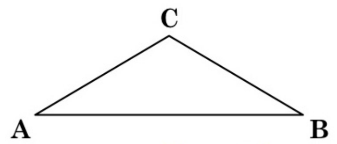


Figure 1

Draw a straight line EF through C, parallel to AB. (Fig. 2)

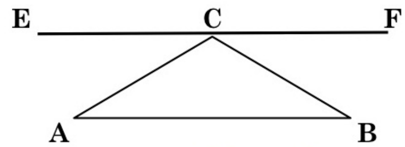


Figure 2

Draw a straight line perpendicular to AB from C, to meet AB at D.

Draw straight lines perpendicular to AB from A and B, such that they meet EF at G and H.

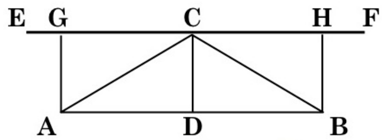


Figure 3

BY		
P1	The area of rectangle AGHB is AB multiplied by AG.	**C1**
P1	The area of AGHB is the sum of areas of AGCD and DCHB.	**C2**
P2	Triangles ACD and ACG are congruent. Triangles BCD and BCH are congruent.	**C3** **C4**
P2	The area of triangle ACB is the sum of the areas of ACD and BCD.	**C5**
C3	The area of ACD is half the area of AGCD.	**C6**
C4	The area of BCD is half the area of BHCD.	**C7**
C5, C6, C7	The area of ACB is half the area of AGHB.	**C8**
C1, C8	The area of ACB is half its base multiplied by its height.	**C9**
	Since ACB is ***any*** triangle, C9 is true of any triangle.	**C10**

Final Conclusion:

For any triangle, its area is half its base multiplied by its height. **QED**

[**QED**: Quad Erat Demonstrandum
(Latin for: 'that which is to be demonstrated/proved')]

(continued from page 75)

Suppose someone were to ask you questions like these:

> Do you love your parents?
> Which is more painful, a pin prick or a stubbed toe?
> Is lemon juice sweet or sour?
> Which smells sweet, jasmine or garlic?

You give an answer to each of these questions, and the inquirer asks:

> On what basis do you judge your answers to be true?

You will not be able to give reasons on the basis of premises that someone else can judge to be true. Instead, your answer would be:

> That is what my experience tells me.

3.3 Reliable Testimonies

Suppose someone asks you how old you are. You answer: I'm eighteen years old. And the person asks: How do you know that you are eighteen years old?

Your answer would be:

P1: I was born in 2004.

P2: It is 2022 now.

C: Therefore, I am eighteen years old.

The inquirer now asks:

> What is the basis for your assertion that you were born in 2004?

Your answer would be:

> My parents tell me that I was born in 2004. It is also what appears on my birth certificate. I have no reason to believe that my parents are mistaken, or that they are lying, or that the birth certificate has incorrect information.

In this case, you are appealing to the credibility of the testimony of your parents, and of a certificate.

A great deal of the knowledge that we acquire through our education comes from the spoken testimony of our teachers in the classroom, and the written testimony of our textbooks and other written sources. This does not mean, of course, that we do not need to question these sources.

4 Summary

When someone asserts that a given statement is true, it is important to ask for the basis for that assertion. As we have seen in this chapter, the basis could be of three kinds: reasoning, our own experience, and the spoken or written testimonies of others.

There are two important criteria that govern our judgement of the truth or falsity of statements.

Accepting logical consequences of what we have accepted:
If we judge a set of premises to be true, and if the derivation from the premises to the conclusion is valid, then we must also judge the conclusion to be true.

Rejecting Logical Contradictions:
If a statement or a set of statements contains a logical contradiction, we must reject it as false.

Here are two tasks for you to practice with:

TASK A: Suppose someone were to make the following assertion:

My maternal grandmother was born on my fifth birthday.

Do you judge this assertion to be true?
If you think it is false, how would you prove that it is false?

TASK B: Here are three assertions from a textbook chapter on microbiology:

1. Microorganisms are invisible to the naked eye.
2. Fungi are microorganisms.
3. Mushrooms are fungi.

Treat these assertions as premises, and state the conclusion derived from them. Do you judge the conclusion to be true?

If you judge the conclusion to be false, you have a problem. How would you solve the problem? Will you revise your judgement about the conclusion and accept it as true, or will you treat one of the three assertions above as false, or problematic in some way? If you choose to do the latter, which of the assertions will you treat as false/problematic? How would you find a resolution to the problem?

> The only reason for time is so that everything doesn't happen at once.
>
> Albert Einstein

Reason is the shepherd trying to corral life's vast flock of wild irrationalities.

Paul Elridge

CHAPTER 6: LANGUAGE, TRUTH, AND LOGIC IN ACADEMIC INQUIRY

1 Introductory Remarks

In this chapter, we will look at some pointers on what to pay attention to when we are constructing or evaluating arguments, whether in speech or in writing. As we have seen, an argument uses reasoning to support or to refute a knowledge claim. What we will do here is to take you through examples that show how language, truth, and logic intersect in the process of reasoning required in Academic Inquiry.

Languages like English, Hindi, Malayalam, Japanese, Korean, Arabic, German, Russian, Spanish, and so on fall under the category of natural languages. The dictionary of a natural language documents the vocabulary of that language. The term 'language' in the context of "language, truth, and logic' above refers to these natural languages used in constructing, transmitting, and critically evaluating Academic

Knowledge. A characteristic of Academic Knowledge is the technical inventory of terms that are specifically created and rigorously defined, often discipline-internally. A glossary or an encyclopedia of a field of inquiry documents the academic terminology of that field. An understanding of the TERMINOLOGY of an academic field is a prerequisite to understanding that field.

Some domains of academic knowledge, such as mathematics, computer. science, formal logic, and the physical sciences also create specialized algebraic and diagrammatic notations for concepts and propositions to encode the knowledge of the field. Classic examples of such notations include the language of formulas and equations in mathematics and the physical sciences. Some of them also use the language of diagrams, such as Venn Diagrams, tree diagrams, and network diagrams.

Now, the study of languages is called linguistics. Let us use the term ARTIFACTUAL language to refer to languages like algebra, programming languages, and the languages of number theory and set theory. The term semiotics is used to refer to the study of that expanded notion of communicative systems that includes not only natural languages, but also artifactual languages.

The specialised terminology of a field lends itself to greater clarity and precision that everyday vocabulary. Artifactual languages go a step further to allow us to express our knowledge propositions with the kind of clarity and precision beyond what natural languages and even specialised terminology of a field would allow us to do.

In this chapter, we will explore the role of terminology in the relation between natural languages, knowledge, truth, and logic in the context of reasoning in academic inquiry.

2 If P then Q, with *All, Every,* and *Any*

Take the following sentences:

1. All birds have beaks.
2. Every bird has a beak.

These two sentences have the same meaning. That is to say, they express the same proposition. What they say can be expressed as (3):

3. Anything that belongs to the category of birds has a beak.

This is the same as:

1'. For all *x*, if *x* is a bird, then *x* has a beak.
2'. For any *x*, if *x* is a bird, then *x* has a beak.

Let us take another example:

4. If Plato likes vegetables, then Socrates is bald.
5. If Socrates is bald, then Plato likes vegetables.

Do the sentences in (4) and (5) have the same meaning? No.

What (4) says is:

> If it is true that Plato likes vegetables,
> then it is true that Socrates is bald.

This means that it cannot be the case that:

> If Plato likes vegetables, Socrates is not bald.

And what (5) says is:

> If it is true that Socrates is bald,
> then it is true that Plato likes vegetables.

Thus, sentences (4) and (5) do not have the same meaning. The conditions under which these sentences can be true or false are different. To see this clearly, it would help to pay attention to the abstract structure of these sentences. Suppose we use the letters P and Q to refer to the two parts of (4) and (5):

> P refers to "Plato likes vegetables."
> and Q refers to "Socrates is bald."

We can now see that (4) and (5) have the forms in (6) and (7) respectively:

6. If P, then Q.
7. If Q, then P.

What we are saying is that the forms in (6) and (7) do not express the same meaning. Perhaps a couple of other examples would help:

Consider (8) and (9):

8. All horses are animals.
9. All animals are horses.

Here, the meaning distinction is obvious. Even the most rudimentary understanding of the categories of horses and animals leads us to judge (8) to be true, and (9) to be false.

8'. If a creature is a horse, then it is an animal.
9'. If a creature is an animal, then it is a horse.

The abstract form of (4) and (5) is precisely what we have given as (6) and (7). If you now compare (1') and (2') with (8') and (9'), you will see the distinction we are pointing to.

Let us take one more example:

10. If an integer is divisible by four, then it is divisible by two.
11. If an integer is divisible by two, then it is divisible by four.

It should be obvious that we judge (10) to be true, and (11) to be false. Do you now see the abstract distinction between (6) and (7)?

3 If P, then Q vs. If not-P, then not-Q

Now compare (12) and (13).

12. If Plato likes vegetables, then Socrates is bald.
13. If Plato does not like vegetables, then Socrates is not bald.

Do they have the same meaning? Notice that (12) has the form given in (6), while (13) has the form in (14), and the two are different:

6. If P, then Q
14. If not-P, then not-Q.

To see this, let us go back to our example of horses and animals:

15. If a creature is a horse, then it is an animal.
16. If a creature is not a horse, then it is not an animal.

and our example of divisibility:

17. If an integer is divisible by four, then it is divisible by two.
18. If an integer is not divisible by four, then it is not divisible by two.

Now do you see the difference?

Moral of the story:

"If P, then Q" and "If Q, then P" do not have the same meaning.

"If P, then Q" and "If not-P, then not-Q" do not mean the same.

TASK: Here is something for you to figure out on your own:

Does "If P, then Q," mean the same as the following?

"If not Q, then not P."	YES / NO
"Q only if P"	YES / NO
"P iff Q" (iff = "If P, then Q; and if Q, then P.")	YES / NO

4 *If, therefore,* and *because*

Three words — *if, therefore,* and *because* — appear again and again in examples of reasoning and justification. It is important therefore to have a clear understanding of the use of these terms in the discourse of academic knowledge.

4.1 The meaning of *if*

To go back to an examples with *taller than,*

if we are told that:	Ani is taller than Mila,
and	Mila is taller than Zid;
we can infer that:	Ani is taller than Zid.

This can be formulated with 'therefore' as follows:

P1: Ani is taller than Mila.
P2: Mila is taller than Zid.
***Therefore**:* C: Ani is taller than Zid.

What this PC set says is: The premises are credible/true. Therefore, it is reasonable to judge the conclusion to be credible/true.

Now consider a derivation that illustrates the use of *if(-then)*:

If Plato is bald, ***then*** Socrates has a beard.
Plato is bald.
Therefore, Socrates has a beard.

The epistemic relation
If X, then Y,
is called the IMPLICATION relation:
X implies Y (where X and Y are propositions)

As we unpack the structure of these statements, we see that the premise, "If Plato is bald, then Socrates has a beard," has two parts, and each part is a clause:

$\underbrace{\text{If Plato is bald,}}_{\text{CLAUSE 1}}\ \underbrace{\text{then Socrates has a beard.}}_{\text{CLAUSE 2}}$

Suppose we use the symbols P for the first clause (the *if*-clause) and Q for the second (the *then*-clause). We can now express the abstract pattern in this sentence as:

If P, then Q

Following the convention in logic textbooks, we use an arrow to represent the relation between the two clauses:

$P \rightarrow Q$

Let us use this notation to represent the abstract structure of the following example (Chapter 4, Ex. 5):

Ex. 1: P1: If Plato is a chef, then Zeno loves chillies.
P2: If Zeno loves chillies, then Newton is hungry.
P3: Plato is a chef.
C: *Therefore*, Newton is hungry.

Notation:

$P \rightarrow Q$ $Q \rightarrow R$ P *Therefore* R.

where P: Plato is a chef.
Q: Zeno loves chillies.
R: Newton is hungry.

Now let us take Ex. 2 (Chapter 4, Ex. 6):

Ex. 2: P1: If an organism is a mammal, then it has vertebrae.
P2: If an organism has vertebrae, then it has bone cells.
P3: Zeno is a mammal.
C: *Therefore*, Zeno has bone cells.

At first glance, it appears to have the same structure as Ex. 1. But here is the complication: in the first premise, if we take P to be: "An organism is a mammal," and Q to be "it has vertebrae," we have to ensure that the 'it' in Q refers to the organism in P. So let us modify Ex. 2 as Ex. 2':

Ex. 2': P1: For any organism *x*, if *x* is a mammal, then *x* has vertebrae.
P2: For any organism *y*, if *y* has vertebrae, then *y* has bone cells.
P3: Zeno is a mammal.
C: *Therefore*, Zeno has bone cells.

P1 and P2 in Ex. 1 and Ex. 2 express the relation of IMPLICATION:

The *if*-clause implies the *then*-clause.

4.2 The meaning of *therefore*

Let us go back to the relation of logical consequence expressed by *therefore*. In contrast to the implication relation expressed by *if-then*, what is expressed by the term *therefore* is the relation of ENTAILMENT between premises and conclusion:

X entails Y (X: a set of premises; and
Y: the conclusion derived from X)

[Note: As stated earlier, the use of the word *therefore* signals the speaker-writer's belief that the conclusion expressed by sentence that follows *therefore* is a logical consequence of the premises that precede it. In listening and reading, it is important to critically evaluate this belief.]

Compare Ex. 3 (a shortened version of Ex. 1), with Ex. 4:

Ex. 3: P1: If Plato is a chef, then Zeno loves chillies.
P2: Plato is a chef.
C: *Therefore,* Zeno loves chillies.

Ex. 4: P1: If Plato is a chef, then Zeno loves chillies.
P2: Zeno loves chillies.
C: *Therefore,* Plato is a chef.

We judge the reasoning in Ex. 3 to be legitimate, but not the reasoning in Ex. 4. If we convert these examples into the P and Q notation, the contrast between the two examples might become obvious:

Ex. 3 P1: If P, then Q.
P2: P.
C: *Therefore,* Q. (legitimate)

Ex. 4 P1: If P, then Q.
P2: Q.
C: *Therefore,* P. (not legitimate)

As mentioned above, entailment is the same as logical consequence: X entails Y means that Y is a logical consequence of X.

Logicians use the symbol $\vdash$ to denote the entailment relation:

$x \vdash y$ means: x entails y.

They also use the symbol $\vDash$ for entailment.

[Note: There is a difference in the meaning of the symbols $\vdash$ and $\vDash$, called ('single turnstile' and 'double turnstile' respectively). But we will not go into that here. For the purposes of this book, we will use the symbol $\vdash$.]

4.3 The meaning of *because*

The third important word in reasoning is *because*, as illustrated in:

19. That glass jar broke because it fell on the floor.

The logical structure of this sentence can be unpacked in terms of the following dialogue:

A. That glass jar broke.
B. Why did it break?
A. Because it fell on the floor.

The relation here is one of CAUSATION. The general premise that sanctions the last statement is that falling on the floor causes glass to break. Without it, the derivation is incomplete.

Premise 1: Falling on the floor causes glass to break.
Premise 2: That glass jar fell on the floor,
Therefore, it is reasonable to infer that:
Conclusion: That glass jar broke because it fell on the floor.

[Some authors use the arrow symbol $\rightarrow$ for both implication and causation. And others use the symbol $\vdash$ for both entailment and causation. This practice conflates two distinct concepts in the same symbol, and therefore lacks intellectual hygiene.]

We will use a curved arrow to represent causation:

X $\curvearrowright$ Y means "X causes Y".

It can also be interpreted as: “Y because X”. If we use the symbol X for “The glass jar fell on the floor,” and Y for “The glass jar broke,” then we have the following representations:

$X \curvearrowright Y$: “Because the glass jar fell on the floor, it broke.”

$Y \curvearrowleft X$: “The glass jar broke because it fell on the floor.”

5 Summary

We began this chapter by pointing out that any form of communication requires a language. The most common mode of communication among humans is that of ***natural languages*** like English, Hindi, Malayalam, Japanese, Arabic, Russian, Spanish, and so on.

We also pointed out that the words in our *everyday use* of natural languages tends to be unclear and ambiguous. That property is undesirable for the communication of academic knowledge, especially for academic arguments. The ***terminology*** of academic knowledge and academic inquiry calls for a high degree of ***conceptual clarification***, often in terms of clear and precise ***definitions***.

To minimise ambiguity and imprecision, many forms of academic communication use algebraic symbols like the following:

X and Y for variables
= for equality
+ for addition
– for subtraction,
and so on.

In this chapter, we used the following symbols:

$\rightarrow$ for implication
$\vdash$ for entailment, and
$\curvearrowright$ for causation.

We may also use $\longleftrightarrow$ for bidirectional implication (*if and only if*).

These are special symbols for the terminology of logic. In addition to these symbols, we need special symbols for the concepts expressed by the words *no* and *not* (negation), and those expressed by *all*, *any, every*, and so on. Academic communication also uses diagrams such as Venn Diagrams, tree diagrams, input-output diagrams, network diagrams, flow charts, and so on.

While linguistics studies natural languages, the study of all systems of communication covering not only natural languages but also algebraic symbol systems, and diagrammatic notation is called semiotics.

To turn to the relation between natural languages and academic knowledge, we need to unearth the propositions expressed by sentences of the language, and the concepts expressed by the phrases and words in them:

SENTENCES	are composed of	WORDS
|		|
PROPOSITIONS	are composed of	CONCEPTS

The concepts expressed by the words *therefore*, *if* and *because*, the topic of this chapter, have an important role to play in that enterprise.

I am convinced that the act of thinking logically cannot possibly be natural to the human mind. If it were, then mathematics would be everybody's easiest course at school and our species would not have taken several millennia to figure out the scientific method.

Neil deGrasse Tyson

> Chess helps you to concentrate, improve your logic. It teaches you to play by the rules and take responsibility for your actions, how to problem solve in an uncertain environment.
>
> Garry Kasparov

PART 2: LOOKING BACK

We began this book with an introduction to the concepts of academic knowledge and academic inquiry. Knowledge is a body of statements that we judge to be true. Mathematics, computer science, logic, astronomy, physics, chemistry, life sciences, economics, psychology, linguistics, history, philosophy, literary studies, medicine, engineering, technology, law, management, and so on are examples of diverse bodies of academic knowledge transmitted to students in schools and colleges.

The term 'academic inquiry' refers to the process of constructing and evaluating academic knowledge, undertaken by researchers. The philosophical study of knowledge is called epistemology, while the scientific study of knowledge is called cognitive science. The subject of this book is the epistemology of academic knowledge, and of academic cognition, which we view as higher order cognition.

The terms cognition and knowledge share the same historical roots: to cognise is to know. This led us, in Part 1 of the book, to a detailed examination of ways of knowing, including experience, testimonies, and reasoning.

Academic knowledge is a body of collective knowledge subject to the norms of rationality, which we described in terms of logical consequence, and logical consistency, and the prohibition of logical contradictions. These are concepts related to reasoning, logic being the study of reasoning.

Part 2 of this book has largely dwelt on reasoning in academic inquiry, especially in academic arguments, against the backdrop of academic knowledge, truth, and logic. An aspect of academic inquiry highlighted here is the role of academic literacy in it, with special attention to critical reading, expository writing, and writing that embodies rational arguments. As part of this pursuit, we also looked briefly at the different systems of communication explored in semiotics, with focus on academic terminology, algebraic notation, and diagrammatic notation. Chapter 6 was specifically on implication, entailment, and causation as core concepts in the terminology of logic, and the notation for those concepts.

Our hope is that this review would provide a useful roadmap for navigation when you read and reread the chapters in Part 1 and Part 2, and serve as the foundation for what you are exposed to in the courses in your Bachelor's, Master's, and PhD programs, as well as your life after graduation.

When intuition joins hands with reason,
one cannot go wrong.

Foundations of Knowledge and Inquiry across Disciplines

Part 3: Basic Tools of Inquiry

> …it is the organization and classification of knowledge on the basis of explanatory principles that is the distinctive goal of the sciences.
>
> Ernest Nagel

CHAPTER 7: CLASSIFYING

1 Looking Back and Looking Forward

In Parts1 and 2, we talked about *inquiry* as a cognitive process, and *knowledge* as an outcome of inquiry. In Part 1, we got a glimpse into some important tools of inquiry, like observing, generalizing, classifying, defining, and so on. Part 2 was devoted to reasoning as a central tool for constructing and evaluating academic knowledge.

An idea that runs through the discussion is that while we may have different approaches for answering different types of questions, underlying these differences is a core of shared considerations for addressing *any* question.

In Part 3 (Chapters 7 to 9), we will explore some of the tools of inquiry in greater detail. This chapter is about *classifying* and *sub-classifying* as a tool of inquiry. Read on to see how this tool plays an important role in every academic discipline that we learn about in school and college, and also in everyday life.

2 Ways of Classifying and Sub-classifying

Anu was in the school canteen during lunch break, when her friend Rafa joined her.

Rafa: Hey, how's your white crow project going?

Anu: Oh, boy! It's giving me the biggest headache of my life!

Rafa: Any progress?

Anu: Nope.

Rafa: I was wondering about something related. White crows are crows, right? And white ravens are ravens. And both crows and ravens are birds.

Anu: Yes! Where are you going with that deep thinking, my philosopher friend?

Rafa took out a piece of paper and drew a diagram, with circles around *white* and around *crow*. "Look, birds can be classified into crows and ravens. Both crows and ravens can be either white or black. You were wondering about the existence of what I have circled, right?" he said.

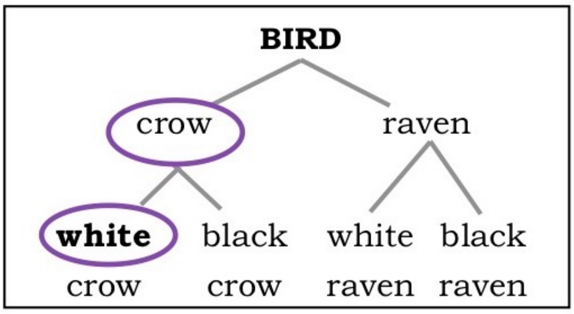


Figure 1

Anu stared at Rafa's diagram representing the classes of birds, and shook her head. "Yes, Neel and I were discussing the existence of white crows. But that is not how I would draw the diagram. Here, let me show you how we were thinking of it," she said.

She took the piece of paper and drew another diagram, as in Fig. 2.

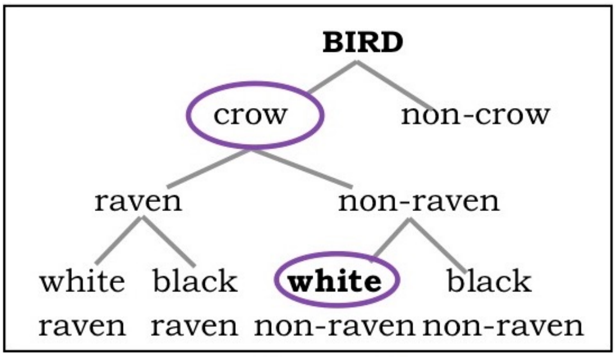


Figure 2

Rafa: Aha! That is exactly what I was coming to! If I remember right, your argument was that if white ravens exist, then white crows exist, because ravens are crows.

Anu: Right.

Rafa: But Neel's position was that a raven is not a crow. His idea fits with my diagram. So even if white ravens exist, white crows may not exist, right?

Anu: You got it.

Rafa: So which of these two schemes of classifications should we adopt?

Anu: Neel and I checked. There is no textbook that says which of them we should adopt.

They sat wondering for a few seconds.

Anu: But Rafa, why do we need to choose one of them? Why can't we use both?

Rafa: Because if we choose *your* classification, and find white ravens, we must admit that there are white crows too. But *my* diagram doesn't lead us to the same conclusion. Our classifications have different *logical consequences*. That's why we need to find a way to choose one.

Anu: Oh, right, that makes sense. Even if we found a classification in a textbook, if another textbook has a different classification, we would need to make a choice. This means we need to know how to decide which classification to use. How do we do that?

Rafa: The question you're asking is: (*writes on the board*)

> How do we choose between different ways of classifying something?

Anu sat brooding over Rafa's question. She finally said, "I can't think of an answer, Rafa. Maybe Neel will have some idea. Let's talk to him after school."

THINK & DO #1

Between Figures 1 and 2, which classification you would choose?
What are your reasons for choosing it?
Think about Rafa's question, and how you might answer it.

After school that day, Anu and Rafa raised the question with Neel.

Neel: I don't have an answer, guys. But I have been thinking about a similar question about squares and rectangles.

Anu: What do crows and ravens have to do with squares and rectangles? That's geometry, and this is biology!

Neel: Your question is about how to classify and sub-classify, right? And how to choose between different ways of classifying any set of things. We'll find these questions in all subjects, whether biology, geometry, chemistry, social studies, language, or something else.

Rafa: You're right!

Neel: Yep! Remember the other day your mom was telling us about trans-disciplinary questions? I think this is one of them.

Anu: Trans-disciplinary?

Neel: A trans-disciplinary question is one that is not restricted to any one subject. It's relevant in different disciplines. Such questions go beyond the level of what we learn in different subjects in school. So disciplines and subjects and their boundaries don't matter.

Rafa: Wow! So we are trans-disciplinarians?

Anu: Okay, okay, plant your feet on the ground, Rafa. Neel, what were you going to ask about squares and rectangles?

Neel: So, both squares and rectangles are four-sided polygons, right? And their angles are all right angles. Now, is a square a rectangle?

Anu: What?!!!

Neel began to draw two diagrams on a piece of paper.

Neel: If you say, 'Squares are NOT rectangles,' you get one kind of classification — like the one on the left. And if you say, 'Squares are a special kind of rectangles,' you get the one on the right.

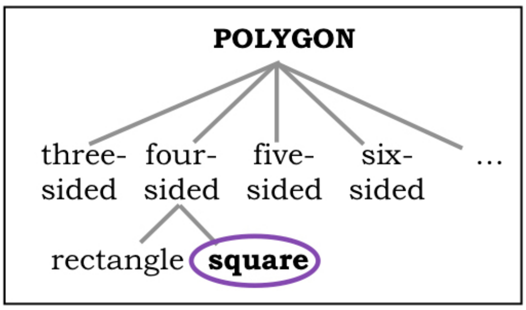


Figure 3

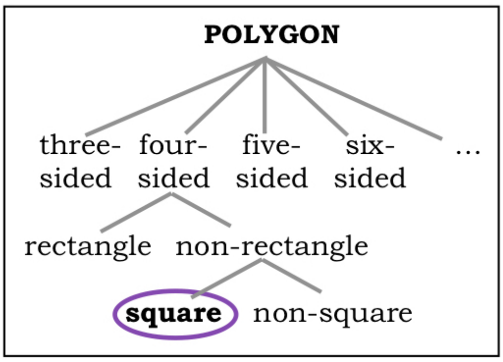


Figure 4

Rafa: Hmm, I see what you mean.

Neel: My question is, how do we choose between these two classifications? Don't you see? This question in geometry is exactly the same as the crows-and-ravens question in biology.

Anu: Oh boy! We couldn't answer that question in the context of birds. Now you are presenting the same question in the context of polygons, and we have no idea. You do know how to make life difficult, Neel!

Neel: Come on, Anu. Actually, I am making things easier. Facing the same question in biology and in geometry might help us to look for an answer. If we find an answer in one, we can use the same strategies to find an answer in the other.

THINK & DO #2

Between Figures 3 and 4, which classification would you choose? What is your reason for choosing it?

THINK ABOUT

Neel says that when we are choosing between different ways of classifying, the reasons that guide the choice are similar for all disciplines. We have seen an example each from biology and geometry. Try to think of examples of classification from other subjects you study.

---

3 Subclassifying

From a young age, one of the ways we understand the world is by grouping things together into classes based on their properties. For example, young children label toys like building blocks by colour (using words like red, blue, green, yellow), or by shape (round, square, triangle). Words in all languages express classes: river, star, planet, animal, vertebrate, insect, plant, flower, boy, girl, and country signal the way we group what we see in nature. Words like table, chair, house, book, and poem indicate our classification of man-made entities. As Neel points out, classification is also a fundamental tool employed by every academic disciplines. While a category is a set of entities, not all sets are categories. If we put a pencil, a spoon, a rose, a button, and a towel in a basket, we have a set of things in the basket, but they don't form a category.

Now, classification is the same as *categorisation*. The output of the process of classification/categorisation are classes/*categories*. So, another word for *class* is ***category***. A category is a set of entities. For any set to be a category, all the entities in it must share at least one property that distinguishes them from all the entities that are not members of that category.

Let us pursue Neel's question further. We take 'triangle' to be a category. Look at the shapes in Fig. 5 below. When we say 'triangle', we are not referring to a particular triangle as in A, but to a class that includes A, B, C, and D, and excludes E and F:

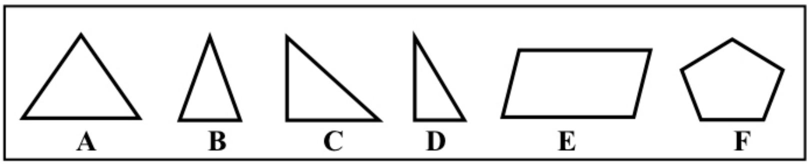


Figure 5

Similarly, the words *cat* and *dog* refer not to a particular cat or a particular dog. Each word refers to a population of living creatures with certain shared characteristics. Members of the category 'spider' share the property of having eight legs. This distinguishes them from members of other categories: 'bee' (six legs), 'cat' (four legs), 'crow' (two legs), and 'snake' (no legs). Creatures that have wings belong to a category that we might call 'winged creatures'. Like members of the category 'bird', butterflies are 'winged creatures'. But are butterflies birds? We don't count them as birds, because creatures belonging to the category 'bird' have beaks; butterflies don't. Similarly, creatures in the category 'cat' have retractable claws; those in the category 'dog' don't.

Sub-categorization has to do with categories within categories. For instance, the sentence, "Humans are mammals," expresses the idea that humans are a sub-category of the category of mammals. The sentence, "Triangles are polygons," expresses the idea that triangles are a sub-category of the category of polygons.

THINK & DO #4

What are the categories and sub-categories in Figures 1 – 4?

We often find situations in which X is a sub-category of Y, Y is a sub-category of Z, Z is a sub-category of M, and so on. In biology, this is called taxonomy. (The term 'taxon' means category.)

Taxonomies have a hierarchical (branching) structure, much like an inverted tree. Here is an example:

Humans, chimpanzees, and orangutans are *primates*.

Primates, elephants, tigers, dogs, cats, mice, and bats are *mammals*.

Mammals, birds, reptiles, and fish are *vertebrates*.

Vertebrates and invertebrates (insects, worms, starfish) are *animals*.

Thus, we could represent taxonomies visually as ‘tree diagrams’, as in Figure 6:

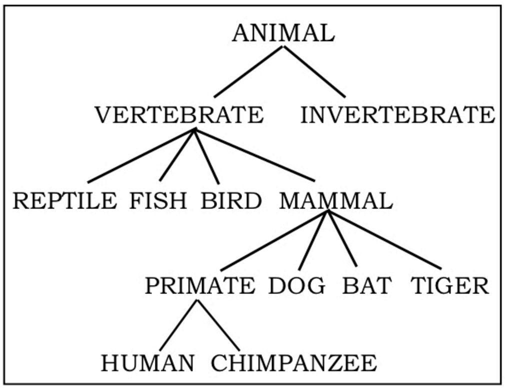


Figure 6

We could expand this taxonomy to go beyond animals to include plants, yeasts, and bacteria.

The question that came up in the children’s conversation earlier, about choosing between different ways of classifying, is a fundamental question in academic inquiry. To see how important it is, consider the following categories:

a. Living creatures can be unicellular or multicellular.

b. Cells can be eukaryotic (= with cell nuclei) or prokaryotic (= without cell nuclei.)

These two statements lend themselves to two alternative classifications:

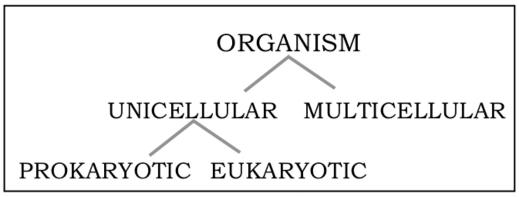


Figure 7

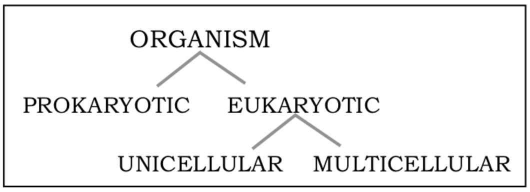


Figure 8

Which of these tree diagrams do we choose? Like our question about squares and rectangles, or about crows and ravens, there is no ready-made answer to this question.

Let us look at a few other examples of competing classificatory schemes that have bothered philosophers in the past.

In ancient Greece, Aristotle (4th c. BCE) classified life forms into humans and non-humans, and non-human life forms into animals and plants

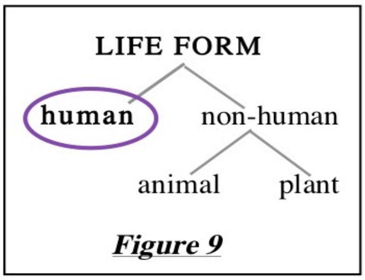


Figure 9

And in the 18th c. CE, the Swedish scholar Linnaeus classified life forms into plants and animals, and animals into various sub-categories, where humans are a sub-category of primates.

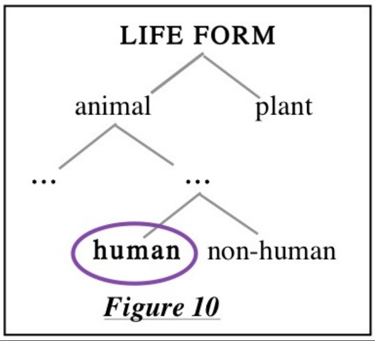


Figure 10

This gives us two competing taxonomies. Which one should we accept? What reasons can we give in support of our choice?

In ancient India, there were at least two different classifications of living organisms. One of them, which we attribute to the scholar *Charaka* (4th c. BCE), is a 4-way classification of animals, based on different modes of birth:

(1) *jaraayuja*: born from the womb (e.g., humans, quadrupeds, etc.)
(2) *andaja*: born of an egg (e.g., fishes, reptiles, and birds)
(3) *svedaja*: born of moisture and heat, spontaneously or asexually generated, (e.g. worms, mosquitoes, etc.)
(4) *udvija*: born of vegetable organisms.

Another classification, attributed to the scholar *Prashastapaada* (5th c. CE) begins with two categories:
(1) *ayonija*: asexually generated,
(2) *yonija*: sexually generated; subdivided into: (a) *jaraayuja* and (b) *andaja*.

Notice that unlike Aristotle, both Charaka and Prashastapaada treat humans as a subcategory of animals.

Once again, it is not easy to choose between these proposals. But it is important to engage with the question: which of the proposed classificatory schemes or taxonomies should we accept?

> One thing only I know, and that is that I know nothing.
>
> Socrates

4 Choosing between Competing Taxonomies

When he got home, Rafa wanted to talk to his mother about Neel's question. 'Could a square be considered a rectangle?' he wondered. But she was busy, and Rafa had no chance.

He had his dinner, and tried to do his homework. But his mind was on squares and rectangles. He felt frustrated and exhausted. And while scribbling his thoughts in his notebook, he dozed off.

He woke up to a loud tapping noise at the window. When he looked up, he saw an old bald man with a flowing white beard. He opened the window to ask him what he wanted, but as soon as the window opened, the man was inside Rafa's room, sitting on his chair. It was almost like he simply floated in. Rafa's jaw dropped!

"Hello, Rafa!" said the man.

"Huh? How… how do you know my name?"

"Oh, I am Socrates, the dead Greek philosopher. I know about many things that most people don't know, including about my own ignorance. In fact, I know more about my ignorance than most people. By the way, you can call me Socs."

By now, Rafa was fully awake and excited. And the conversation continued.

Rafa: A dead man? How are you here? Am I dreaming?

Socs: You are asleep, Rafa, and you are dreaming. How else would you be talking to a philosopher who died more than 2000 years ago? I decided to visit you because you have been struggling with rectangles and squares. I am here to help you."

Rafa: (his excitement rising) Oh, really? That is so nice, Socs. So, what is the answer? Are squares a special type of rectangles?

Socs: (laughing) Oh, I am here to help you, not to give you an answer. My help is like your mother's.

At first, Rafa was crestfallen. Then he remembered his excitement at figuring out the difference between flat and spherical geometry. Coming up with answers to such questions on one's own can be fun. Actually, it is much more pleasurable than getting answers from someone who already knows them. So now, he was all ears..

Socs: Ready to dive in? Your problem is: how do you choose between these two statements: 'Squares are NOT rectangles,' and 'Squares are a special case of rectangles.' Am I right?

Rafa: Yes.

Socs: To start, then, imagine a rectangle. Now slowly reduce the length of the longer side. At some stage, all the sides will have the same length, and it will be a square, right?

Rafa: Right.

Socs: If we say that squares are not rectangles, we can describe the process like this: As the length of the longer side reduces, the shape suddenly ceases to be a rectangle and becomes a square. And as we continue reducing it, the very next moment, it is again a rectangle, not a square.

Rafa: I hadn't thought of it that way.

Socs: Suppose we accept that squares are a type of rectangles. Think about it. During the process of shortening the longer sides, the shape never stops being a rectangle. At some stage, it does become a square for a just a moment, but it remains a rectangle throughout.

Rafa: (thinking) M-hmmm.

Socs: Both the options I gave you are ways of classifying squares and rectangles. Which of these descriptions do you like?

Rafa: Oh, the second one is definitely more appealing.

Socs: That is a good starting point. The second description appeals to you intuitively. But it's only if you give an argument that you can convince someone else that squares are a type of rectangles. Can you come up with an argument for the position that appeals to you? That is how you would justify your claim that taxonomy X is better than taxonomy Y.

Rafa: (mutters under his breath) You and mom are just the same! Always asking me for a justification… Hmph

Socs: (winking) What's that, boy?

Rafa: (stutters) Umm, I was asking, what should I do to look for an argument?

Socs: For that, you will need a theory of squares and rectangles.

Rafa: A theory of squares and rectangles? And where do I get that?

Socs: Oh, you will have to build one.

Rafa: You mean, I can build one? A theory?

Socs: Let us explore what a theory is by trying to develop one.

Rafa had always imagined a person who develops a theory as someone who has gray hair, wears old-man clothes and forgets where he/she left their keys!

Socs: Of course you can! Why not? First, make a list of everything you know about squares, and another one of everything you know about rectangles. What do you know about squares?

Rafa: Let me see. I'll write it down so that I don't miss anything.

Rafa wrote on the board:

SQUARE

(a) It has exactly four sides.
(b) Each side is a straight line.
(c) The opposite sides are equal.
(d) The adjacent sides are equal.
(e) It has exactly four angles.
(f) Each angle is a right angle.
(g) The diagonals bisect each other.
(h) The square of the diagonal is equal to the sum of the squares of the adjacent sides.

And so on.

Socs: Ah! I see you've memorized that well! And what do you know about rectangles?

Again, Rafa wrote, next to what he had written for squares:

SQUARE	RECTANGLE
(a) It has exactly four sides.	(a) It has exactly four sides.
(b) Each side is a straight line.	(b) Each side is a straight line.
(c) The opposite sides are equal.	(c) The opposite sides are equal.
(d) The adjacent sides are equal.	(d) The adjacent sides are not equal.
(e) It has exactly four angles.	(e) It has exactly four angles.
(f) Each angle is a right angle.	(f) Each angle is a right angle.
(g) The diagonals bisect each other.	(g) The diagonals bisect each other.
(h) The square of the diagonal is equal to the sum of the squares of the adjacent sides.	(h) The square of the diagonal is equal to the sum of the squares of the adjacent sides.
and so on.	and so on.

Socrates nodded his head, smiled, and clapped his hands.

Socs: Look at that! They are identical, except for (d)! The only difference is that the adjacent sides are equal in squares, and not equal in rectangles. That difference arises only because you are assuming that rectangles and squares are separate categories. Suppose you take away that statement about adjacent sides. This would let you treat squares as a special case of rectangles, right? What would be the distinguishing feature of squares then?

Rafa: Oh yes, if we say, squares are a kind of rectangles, then everything we've said about rectangles is true of squares. With just one difference, that the adjacent sides of squares are equal.

Socs: Good. So, if a square is a special kind of rectangle, then the logical consequence is that:

> All the properties of rectangles are also properties of squares.

If you don't assume that squares are rectangles, then you have to repeat all the properties of rectangles separately as properties of squares as well. Here, let me show you.

Socrates spoke excitedly, as words emerged on the board, as if from an invisible pen:

RECTANGLE

(a) It has exactly four sides.
(b) Each side is a straight line.
(c) The opposite sides are equal.
(d) It has exactly four angles.
(e) Each angle is a right angle.
(f) The diagonals bisect each other.
(g) The square of its diagonal is equal to the sum of the squares of the adjacent sides.
(and so on.)

SQUARE: a sub-category of rectangles

(h) The adjacent sides are equal.

Rafa was beginning to sense an argument in what Socrates said, but only vaguely. He was also beginning to feel very sleepy. He looked at Socrates with bleary eyes, and then his head hit the pillow.

When he woke up the next morning, his conversation with Socrates was so vivid in his memory that he couldn't think of it as just a dream. He wrote down the conversation before his memory faded, and at the end of it, he added the argument:

> If we assume that 'square' is a sub-category of 'rectangle', then most of the properties of squares follow from the properties of rectangles.
>
> But if we treat 'square' as a 'non-rectangle', then all the properties of rectangles have to be repeated as properties of squares. Also, if we think of them as distinct categories, we have no way of expressing what they have in common.
>
> Therefore, it is best to treat 'square' as a sub-category of 'rectangle'.

As he was writing, Rafa thought to himself, "Are all rectangles parallelograms?" That led to the question, "Are all parallelograms quadrilaterals?"

He kept writing. "If we say that squares are rectangles, rectangles are parallelograms, parallelograms are quadrilaterals, and quadrilaterals are polygons, will the properties of polygons be passed down through quadrilaterals and parallelograms and rectangles to squares? Will the members of the daughter categories inherit the properties of the members of the mother categories?

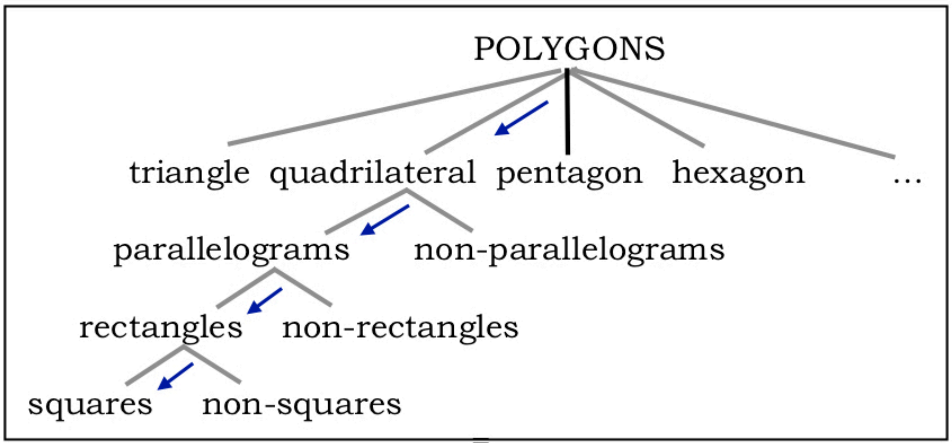


Figure 11

Rafa was now in a hurry to get to school and tell Anu and Neel all about his discovery-in-a-dream and about his new friend, old man Socs.

5 Justified and Unjustified Categories

Anu and Neel listened to Rafa with rapt attention till he had finished.

Anu: Oh boy! This Socrates from your dream sounds like a pretty cool guy, Rafa. You should invite him to our conversations.

Rafa: M-hmm. I did learn something important from him.

Anu: Listen, guys, I have a question related to what we were talking about yesterday. Instead of looking for crows or ravens that are white, what if we looked at all the white birds around us and checked if any of them were ravens or crows? Why don't we have a category of 'white birds'?

Rafa: It doesn't seem like a meaningful category, I can't say why.

Neel: It's the same reason why the category of 'right-angled triangles' makes sense, but a category of a 'right-angled polygon' doesn't seem to make sense.

Anu: What's a right-angled polygon?

Neel: A right-angled triangle is a triangle in which one of the angles is a right angle. Just like that, a right-angled polygon is a polygon in which one of the angles is a right angle.

Rafa: I just thought of another example. In a regular polygon, all the sides are equal and all the angles are equal.

Neel: You mean a regular polygon is one that is equilateral and equiangular.

Rafa: That's right. Now, we accept regular polygons to be a meaningful category. But how about 'equilateral polygons'?

Neel: Sounds like an arbitrary category.

Anu: Heh? What's arbitrary? (She looked it up in her dictionary.) Oh, it means 'not based on reason'. So, it's not a meaningful category.

Neel: Good. So, we need to ask: What is the difference between meaningful and arbitrary categories?

Anu: I wonder, how about equiangular polygons? Is that a meaningful category?

THINK ABOUT

Think of examples of meaningful and arbitrary categories.

News of Neel's question had reached Samira before Rafa got home. Anu and Neel had talked to Ila about it. Ila had never thought about that issue, and she became quite excited, so she phoned Samira, and they had a long conversation about it.

When she heard Rafa open the door, Samira called out to him.

Samira: Hey Rafa, I hear you and your friends are trying to figure out something about classification?

Rafa: Yes, we are, Mom. How did you know?

Samira: I have my sources. Do you want to tell me about it?

Samira listened carefully while Rafa laid out the problem in detail. Then she asked:

Samira: If I told you that I saw a bird yesterday when you were in school, will you be able to give me a description of what I saw? What can you infer from what I told you?

Rafa: Well, lots of things. If what you saw was a bird, I can infer that it had a beak, two legs, claws, two eyes, and wings; that its wings are made of feathers, it lays eggs. Lots of stuff.

Samira: Very good. Suppose I now tell you that I saw a white creature. What can you infer from that? Can you give me a description of what I saw?

Rafa: I can't infer anything except that it was white. Oh Mom! I got it.

Samira: What did you get?

Rafa: That *bird* is a useful category, because if you know that something is a bird, you can infer a number of its properties. And *white bird* is a subset of the category *bird*. But *white creature* would be an arbitrary category, because it doesn't lead to any inferences.

Samira smiled approvingly, as Rafa continued.

Rafa: Let me try another example. If I know that a figure is an equilateral triangle, I can infer that its angles are equal; that its angle bisectors divide the triangle into two right-angled triangles; that these right-angled triangles are congruent; and so on. So *equilateral triangle* is a useful and meaningful category. But can we treat *equilateral polygon* similarly? I don't know of anything that we can infer by knowing that a figure is an equilateral polygon, so it doesn't seem to serve any useful purpose.

Samira: Exactly. So in academic knowledge, meaningful categories like 'equilateral triangles' or 'bird' are called **legitimate** or **justified** categories. Arbitrary ones that do not allow us to make inferences are called **unjustified** categories.

Rafa: Great! I think I have the answer to Neel's question. I can't wait to tell Neel and Anu. I am going to write this down to make sure I don't forget.

Rafa opened his bag to get his notebook and began writing furiously.

THINK & DO #6

Revisit Figures 1 and 2.

Choose between the two classifications based on what you have learnt about justified and unjustified categories.

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6 Categories, Sets, and Units

6.1 Categories and Sets

To develop a deeper understanding of categorising as a tool in academic inquiry and research, it would be useful to take a look at a related and perhaps familiar concept, that of sets.

An Internet search on the concept of sets in mathematics yields statements like the following:

> "A set is the mathematical model for a collection of different things; a set contains elements or members, which can be mathematical objects of any kind: numbers, symbols, points in space, lines, other geometrical shapes, variables, or even other sets."
>
> (https://en.wikipedia.org/wiki/Set_(mathematics))

The examples of sets given in this description are restricted to mathematical objects. To make this concept useful in domains outside mathematics, we are going to extend the concept of a set as a collection of abstract mathematical objects to a collection of any objects. In astronomy, for instance, planets, stars, and galaxies are sets. In chemistry, hydrogen atoms, oxygen atoms, carbon atoms, water molecules, and hydrogen molecules are sets. In biology, humans, chimpanzees, mammals, insects, and animals are sets. In literary studies, poems, plays, novels, and short stories are sets. In statisics, samples and populations are sets. Societies, communities, organisations, institutions, schools, colleges, hospitals, political parties, are all religions are also sets.

A word about notation. In set theory, sets are enclosed in curly brackets. So $\{A\}$ means "set *A*". $\{2, 5, 17\}$ means "the set of 2, 5, and 17." The symbol $\in$ represents 'member(s) of a set'. Thus, $x \in \{A\}$ means 'x is a member of set *A*'. And represents "subset of a set;" $\{A\} \subseteq \{B\}$ means "Set *A* is a subset of set *B*."

This might make you wonder: if such things as chimpanzees, mammals, and streets are sets, what is the difference between sets and categories? And what is the difference between subsets and subcategories?

The answer is that categories are a special kind of sets. Let us define the concept of categories as follows:

> A CATEGORY is a set whose members have at least one common trait that distinguishes it from the members of all other sets.

Take, for instance, the following set:

> {the set of numbers from 23 to 315, the set of books that Harry read between 2003 and 2016, the set of students in this course}

The members of this set have no discernible trait in common that includes, say, 25 and the *Harry Potter* books, but excludes 25.7, Macbeth, and the president of Maldives. So by our definition, the set given above is not a category. In contrast, given:

{carbon atoms}, {animals}, {even numbers}, and {hospitals},

each of them is a set that is also a category.

As for sub-categories, we have already seen the subcategory relation in biology (humans, mammals, vertebrates, animals, ...) and in geometry (square, rectangle, parallelogram, quadrilateral, polygon) as examples.

6.2 Logical Inheritance

As we have seen in Sections 2 to 4, an important logical property of the the subcategory relation, absent in the subset relation is, illustrated in the following derivation:

P1: Humans are Primates

P2: Primates are mammals.

P3: Mammals are vertebrates.

P4: Vertebrates are animals.

P5: Animals have an alimentary canal.

C: Therefore, humans have an alimentary canal.

While P1-P4 are premises that express the relation of subcategorization between two categories, P5 is a statement of a property that is distinctive of the members of a category. What allows us to deduce the conclusion C is the following principle of logical inheritance:

Logical Inheritance

The properties of the members of a category are inherited by the members of its subcategories.

6.3 Categories and Units

Related to the concepts of categories and subcategories is the concept of the hierarchy of units and sub-units. Here are two examples:

Subatomic particles > atoms > molecules > cells > organs > organisms > communities

While the term *molecule* refers to a unit, the terms *water molecule*, *sodium chloride*, *carbon dioxide*, *carbon monoxide*, and *carbon molecule* refer to categories of molecules. While the term *atom* refers to a unit, the terms *hydrogen atom*, *oxygen atom*, and *carbon atom* refer to categories of units. While the term *subatomic particle* refers to a unit, the terms *electron*, *muon*, and *photon* refer to categories of subatomic particles.

The relation between units and their subunits, as well as between categories and their subcategories, is that of compostionality. Crystals are composed of molecules, molecules are composed of atoms, atoms are composed of particles, and so on. Notice that logical inheritance does not apply to units and subunits. The properties of crystals, for instance, are not inherited by the molecules they are composed of. The properties of molecules are not inherited by the atoms they are composed of.

The absence of logical inheritance in the part-whole relation between units and subunits allows for members of a unit to have properties that are not present in its subunits. The presence of such novel properties in a unit is called ***emergence*** in the philosophy of science.

7 What did you Learn in this Chapter?

We hope that this chapter has helped you see that many of the terms we learn in school and college express categories. Learning becomes easier if we understand that in academic knowledge, words like *triangle*, *polygon*, *mammal*, *insect*, *male*, *female*, *science*, etc. express categories. Categories allow us to discover Observational Generalisations about their members, e.g., Butterflies have compound eyes. Combined with the principle of Logical Inheritance, the postulation of sub-category relations allows us to arrive at conclusions on the members of their subcategories.

There are two important questions that came up in the conversations in this chapter:

Question 1: How do we choose between alternative ways of classifying a set of entities?

The answer is simple. We choose the classification from which we can make the most number of inferences from the smallest number of premises. We used this criterion in choosing to classify squares as a sub-category of rectangles rather than as a separate category.

Question 2: Given a term, how do we critically evaluate the concept it refers to, in order to decide if it is a justified category in academic knowledge?

We judge a category to be justified if it allows us to make inferences. A category that allows us to make more useful inferences than others has greater legitimacy. And if a category doesn't allow us to make useful inferences, it is arbitrary.

This difference can help us learn something important about how we respond to the statements that we receive from others — books, the internet, or other people — and accept as knowledge. Before accepting statements, we need to subject them to careful scrutiny, and decide for ourselves whether or not to accept them. This applies to our own conclusions as well! We must be open to abandoning our conclusions when there are sufficient reasons against them.

The ancient Greek philosopher Socrates is reported to have said that an unexamined life is not worth living. In that spirit, let us add: Unexamined knowledge claims are not worth accepting. We need to carefully examine both knowledge rooted in our own experience and reasoning, as well as knowledge we receive from other sources, to decide whether to accept it, reject it, or set it aside for further scrutiny.

> The classification of facts, the recognition of their sequence and relative significance is the function of science, and the habit of forming a judgment upon these facts unbiased by personal feeling is characteristic of what may be termed the scientific frame of mind.
>
> Karl Pearson (1892) \
> "*The Grammar of Science*"
> Republished (2007). P.6, Cosimo.Inc.

> A mathematician who can only generalize is like a monkey who can only climb up a tree, and a mathematician who can only specialize is like a monkey who can only climb down a tree. In fact neither the up monkey nor the down monkey is a viable creature. A real monkey must find food and escape his enemies and so must be able to incessantly climb up and down. A real mathematician must be able to generalize and specialize.
>
> George Polya

Chapter 8: Generalising

1 Looking Back and Looking Forward

In Chapter 7, we explored classifying as an important tool of inquiry. In Chapter 8, we proceed to generalising as yet another such tool.

Suppose we ask: “Do vertebrates with beaks always have exactly two legs?” This is a question about the population of vertebrates. To look for an answer, we would examine a sample of vertebrates, and arrive at a conclusion about the sample. Suppose we take a sample of 23 vertebrates in a village or a city. We find that in that sample, every vertebrate with a beak has exactly two legs. Now, is it legitimate to conclude that in the population of vertebrates, every vertebrate with a beak has exactly two legs? Reflect on that question.

From what we observe in a sample, how do we form a generalization about the entire population? Here we are arriving at a conclusion about a population on the basis of reasoning from a sample taken from that population. This type of reasoning that allows us to arrive at a conclusion on a population from a sample is called *INDUCTIVE REASONING*, as distinct from *DEDUCTIVE REASONING*.

Forming credible generalisations on a population has certain requirements. One is that the sample we base our reasoning on should be *REPRESENTATIVE* of the population. This chapter is devoted to the concepts and methodological strategies associated with Inductive Reasoning.

2 Rude Americans and Polite Japanese

Anu was waiting for her friends in the school canteen, skimming through a newspaper article. Suddenly, her ears pricked up: two men were having a conversation at the next table. They were parents of students, and were waiting for a meeting with a teacher, One of them was talking about the company that he worked for. What caught Anu's attention were his words: "You know, Americans are so rude! Why can't they learn from the Japanese? They are so polite!"

Anu couldn't help turning to the man and saying: "Excuse me, Sir, I couldn't help overhearing. Were you saying that Americans are rude and the Japanese are polite?"

Man: Yes, that's exactly what I said.

Anu: So you must have travelled widely in America and Japan?

Man: Oh, no. I've not been abroad.

Anu: Then how do you know that Americans are rude and the Japanese are polite?

Man: I work in a multinational company with many branches. There are many Americans and Japanese in the company.

Anu: How many Americans and how many Japanese have you interacted with in your company? All of them?

Man: Not all of them, but many.

Anu: How many?

Man: You are quite a persistent girl, aren't you? I don't know exactly how many, but around six Americans and four Japanese.

Anu: And your remark on Americans is based on six Americans, and on the Japanese based on four Japanese? How do you know that the

> other Americans you haven't met are rude, and the Japanese you haven't met are polite?

The man turned red in the face. He got up, telling his companion, "Let's leave. I don't want to talk to this impertinent brat." They got up. As they walked off, Anu said, "Well, there you are! There's at least one rude Indian!"

Neel and Rafa got there just then, and heard Anu's furious words. "What happened?" Neel asked. "That man!" she muttered. "He was saying bad things about people without even knowing anything about them! And he calls me impertinent!"

"Calm down, Anu," Neel said. "Tell us what happened?"

Anu described the incident to them. Neel listened carefully, with an amused expression, and finally said, "I understand what you are pointing to, Anu. That man doesn't understand the basics of how to generalise from a sample to a population, so he has baseless opinions. But why you do expect everyone else to live up to your standards?"

"I don't expect everyone to live up to my standards," bristled Anu. "But I do expect people to not be mean."

"I understand, Anu. But calm down. There's no point getting agitated about what you can't change."

All this while, Rafa was staring at the wall totally absorbed in something. Anu noticed that Rafa was inside some bubble within himself. "Hey Rafa, where are you?" she asked, forgetting her indignation.

No response. Rafa remained in his inside-the-bubble state. Anu tried again. She waved her hand in front of his face. "Hi Rafa, are you here?"

No response. She tapped him on his shoulder. "Rafa?"

Rafa came back with a start. "Yes, Anu?"

"What were you thinking about?"

"I was thinking, how do we generalise from what we see in a sample? The person you were talking to obviously couldn't do that, so his generalisation was baseless. But how do we generalise? When can we generalise? Under what conditions is it okay to generalise?

"I don't know. I need to think," Anu said.

"Me too, I need to think," said Neel.

The three of them sat at their table in the canteen, silently lost in thought.

Before you read further: Can you think of an answer to Rafa's question?

3 Do All Mammals Have Lungs?

Rafa was riding his bicycle home, his mind busy. What exactly is a generalisation? He thought to himself:

> A generalisation is a statement about a population.

So when a textbook says that all mammals have lungs, that statement is a generalisation because it is about the entire population of mammals, some of them currently existing, some of them already dead and gone, and some yet to be born.

What is a population? Well, it is clear that the members of any category are members of a population. But does every population constitute a category? All the living organisms in a city constitute the population of living organisms in that city. But they may not form a category. Perhaps the concept of a population is the same as the concept of a set, in that as in the case of the members of a set, it is not necessary that the members of a population have any shared properties.

Rafa decided to focus on populations that are categories, like dogs, mammals, vertebrates, and animals.

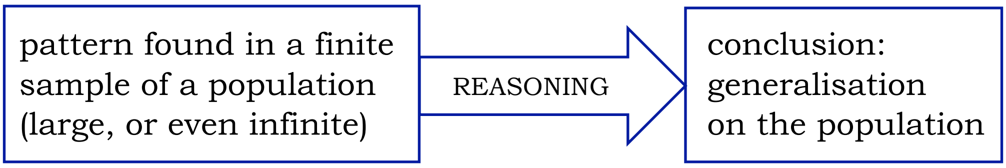
Then it hit him that to formulate statements about such populations, it is not possible to examine every member of the population. For example, to check if all mammals have lungs, there is no way to examine ALL mammals, as that population would include all the mammals that are dead, and all the mammals that are yet to be born. So the basis for making the generalisation must be a sample of the population that we have observed. When we observe a sample of mammals, we find that every member in that sample has lungs. So that is a property of the sample, a pattern in the sample. It was now clear to Rafa that a generalisation is what we say about a population on the basis of what we find in a sample.

He was beginning to get excited. He stopped and got off his bicycle, took out his notebook from his bag, and wrote:

> Patterns are regularities we observe in a sample.
>
> A sample of a population is a subset of the population.
>
> Some populations can be infinite, or at least very large.
>
> If a population is infinite, it is impossible to examine every member of the population.
>
> If it is very large, it is impractical to examine every member of the population.
>
> But we can observe a pattern in a sample, and generalise it to the population.

Just as he was putting the notebook back in his bag, another idea struck him. A generalisation on a population is a conclusion. But what is the process of arriving at that conclusion? And it dawned on him: the process is that of reasoning. Given the pattern we find in a finite sample, we use reasoning to arrive at a conclusion on the population, and that conclusion is what we call a generalisation.

Rafa opened his notebook again and drew a diagram:



He looked at the diagram carefully, then put the notebook away in his bag, and was on his way home again.

At home, as he put his bag down, he thought of the incident in the canteen. What was Anu objecting to? Aha! When the man said that all Americans are rude, his reasoning was flawed, because he was basing it on an entirely insufficient sample to arrive at a conclusion on the population. “Trust Anu to get upset!” he thought.

The generalisation that all mammals have lungs was not like that. The reasoning there was not flawed. What is the difference between the two arguments? What makes the reasoning in one of them good and in the other one bad?

As he sat still on his bed, thinking, unable to come up with an answer, he heard his mother calling him from the kitchen. He got up and went to the kitchen, still lost in thought.

> Before you read further:
>
> Can you think of an answer to Rafa’s question? What distinguishes the sample-to-population reasoning in the two conclusions: “Mammals have lungs,” and “Americans are rude”?

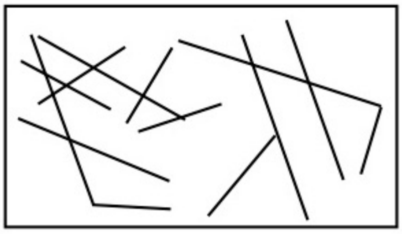
4 Intersecting Straight Lines

As soon as Rafa walked into the kitchen, Samira knew that something was bugging him. She sat down and said, “Come sit with me, Rafa, and tell me what’s troubling you.”

Rafa described the entire history of his question, beginning with Anu’s encounter with the man who said that Americans are rude and the Japanese are polite. Samira smiled.

“I want you to do something, Rafa. Take a sheet of paper, and draw a number of straight lines on it, randomly.”

Rafa drew this picture:



Samira: Do you see anything interesting in that picture?

Rafa: Anything interesting? What should I be looking for, Mom?

Samira: Okay, do any of the straight lines intersect in that sample of straight lines?

Rafa: Yes, some do, and some don't.

Samira: If they do, how many times do any two straight lines intersect?

Rafa: (looking at the paper again) Just once.

Samira: Do any two lines of the ones you have there intersect more than once?

Rafa: (looking again) No!

Samira: So you can say that in that sample, no two straight lines intersect twice.

Rafa: Yes! No two straight lines can intersect at two points.

Samira: Ah, now you are making a generalisation. Do you think that the statement is true of the entire population of straight lines?

Rafa: Let me check.

Rafa took out another piece of paper and drew a number of straight lines. He tried to make some of them intersect at two distinct points, but he couldn't. He tried again with another piece of paper. No success.

Samira: So, what is your conclusion? Are you confident that in the infinite population of straight lines, no two straight lines can intersect at two distinct points?

Rafa: Yes, I am. Actually, no two straight lines can intersect at more than one point!

Samira: But how many straight lines do you have in your sample?

Rafa: Around fifty.

Samira: And on the basis of a sample of fifty straight lines, you are willing to say that no two straight lines can intersect at more than one point? What if there is some pair of straight lines that you haven't examined yet, and they intersect at two distinct points?

Rafa: I can't deny that possibility, Mom. But I can say that until I come across a pair of straight lines that intersect at two or more distinct points, I'll take it that no two straight lines can intersect at more than one point.

Samira: Good. Now how about mammals? What do you think the scientists would say if they were asked about every mammal having lungs?

Rafa: Hm! I guess they would say something similar, something like:

We have examined hundreds of mammals.

There is not a single mammal in our sample that doesn't have lungs.

So, until we find a mammal without lungs, it is reasonable to conclude that there are no mammals without lungs.

Samira: Excellent. Can you now state the general principle that applies to your reasoning for both straight lines and mammals?

Rafa: What do you mean, Mom? Wait, let me think.

Rafa went out for a walk, to be by himself. He came rushing back in a few minutes, took out a piece of paper, and wrote on it.

Confirming Generalisations

We take a large sample of Xs, and examine all the Xs for the property P.

Suppose we find that every X has P.

Also, we don't find a single X that does not have P.

So, until we find Xs that don't have P, it is reasonable for us to conclude that all Xs have property P.

Samira looked at what Rafa had written.

Samira: Can you now figure out what was wrong with that man's statement about Americans?

Rafa: I think I can! One thing was that the guy didn't check if there were any examples that contradicted what he was saying.

Samira: Yes. By the way, that's called a 'counterexample'; it shows a statement to be false.

Rafa: Another is that he made that statement based on just the few Americans he had come across in his company. He didn't bother to check if there were other Americans in his company, or outside, who were not rude — let alone in other countries including in America itself. His sampling was sloppy.

Samira: Okay, looks like you are now ready to present your newfound wisdom to your friends!

Rafa: I can't wait. That feels good, Mom!

5 The Case of Human Height

Anu and Neel were thrilled when, the next morning, Rafa shared with them the results of his conversation with his mother. They asked him a bunch of questions to make sure they had all the details. Rafa, of course, was happy to respond.

When they were done, Neel had a question. "You know, the type of generalisation that we have been talking about has the form:

> "Every member of category X has property P."

So we said that every member of the category 'mammal' has the property of having lungs. Similarly, we said that every member of the category of straight lines has the property that it cannot intersect with another member of the category at more than one distinct point. Did I get that right?"

Rafa: Yes, that's it.

Neel: But now you're saying that a category is a population.

Rafa: Yes, that's what I am saying.

Neel: There is a question that's bothering me. In the case of lungs, and of intersecting straight lines, the properties don't vary. But what would happen if there is variability in the sample? How do we generalise in such cases?

Anu: I don't get what you are saying. What do you have in mind?

Neel: Okay. Take the example of human height. You'll agree that no adult human can be, say, less than one foot tall? And you'll also agree that no adult human can be, say, more than 10 feet tall?

Anu: Right.

Neel: What kind of generalisation can we make on the height of humans? In our sample, one human may be five feet tall, another may be five and a half feet tall, yet another may be six feet tall, and so on. From that kind of a sample, what generalisation on human height can we arrive at?

Rafa: (thoughtful) Hmmm. You are also pointing to something about human height that doesn't vary. You said no adult human being can be less than one foot or more than 10 feet. So human height can vary only within that range. So we do have a generalisation:

> "Until we find humans whose height is less than one foot or more than ten feet, it is legitimate for us to conclude that human height is within the range of one foot and 10 feet."

Neel: You are right. Human height varies, but only within a given range. But what about generalisations within that range? If I told you that Zeno is an adult human being, and asked you what his height might be, you might say something like: "Most likely around five and a half feet." Can he be seven feet tall? Sure, but that is less likely than six feet. Can he be three and a half feet tall? Once again, that's possible, but less likely than five feet. What kind of generalisation on human height would allow us to make that kind of inference — with 'more likely' and 'less likely' in the statement?

Anu: Does it have to do with statistics by any chance?

The morning bell rang, and the children had to go to their class.

Before you read further:

Can you think of an answer to Neel's question?

At the end of school that day, Anu said, "You know the problem Neel raised this morning? I think we can solve it using the idea of average height. I looked up human height in Wikipedia, and it says that the average height of adult Indian males is around 166 cms, and the average height of adult Indian females is around 152 cms."

Rafa: Aha! That's it! So if someone tells us that Zeno is an adult male, we can infer that his height is likely to be around 166 cms. That answers Neel's question!

Neel: Not so quick, Rafa, you're not thinking. Anu, you're right about average height. But that's not enough. I think it also has to do with the idea of the distribution of heights. So the heights of individual human beings are distributed within the range of human height, and that distribution is variable.

Anu: I don't understand.

Rafa: Oh, I think I see what Neel is saying. Let me try a different example: the marks we get in a subject. Suppose I choose to take the Economics elective course next semester, and want to figure out what marks I'm likely to score on the course. Can I say something like this? For the past three years, the average marks that students scored on this course is 61%; so if I take the course, is it safe to say that I am likely to score 61 marks?

Anu: Are you assuming that you are an average student? If you are, it means that your capacity to score marks, and the time you invest in studying are both average.

Rafa: Well, not really. If you go by my marks, I'm not average, I'm above average, but not a top student. For this course, pretend that I'm average.

Anu: There's something here that's not right, something is making me uncomfortable, I don't know what.

Neel: See if this is where your discomfort is. When we say average marks, we often mean the result of adding up the marks scored by all the students and dividing it by the number of students. But there's more to it.

Anu: Right. But…

Rafa: So by this meaning of average, if there are 100 students in the class. Forty of them get 80 marks each, and forty of them get 20 marks each. The remaining get 50 marks each. Let's calculate the average.

Neel: So the average will be:

40 x 80 =	3,200		+
20 x 50 =	1,000		+
40 x 20 =	800	=	<u>5,000</u>.

Divide it by 100, and we get the average: 5,000/100 = 50 marks.

Anu: So if Rafa were an average student, he should expect to score 50 marks?

Rafa: That's the point. Average can mean different things, right? One concept of average, of course, is adding up all the marks and dividing it by the number of students.

Neel: Yes, it's called the 'mean'. The mean mark here is 50.

Rafa: Isn't another concept of average the marks that the most number of students get?

Neel: Yes, it's called the 'mode' in statistics. In this example of marks, there are two clusters or concentrations of 'most number of students', one cluster of 40 students who score 80 marks each, and another cluster of 40 students who score 20 marks each. So we have two 'modes' here, one at 80 and the other at 20. They call this a bi-modal distribution.

Rafa: And I guess we can't say where I would belong!

Anu: Ah, I see what you mean by distribution. It means the range, the clustering, and the location of clustering. Let me get this straight, In this example, the range is from 20 to 80. It could have been 30 to 70, or 40 to 60, without any difference in the mean. As for concentrations, there are two, one at 80 and other at 20.

Neel: That's it. There's more, but I don't think we need to go into it. The point is, this is important when we are looking for generalisations in something that has variability in the population. So we need to be careful about what the numbers actually mean.

> Before you read further:
> Can you think of other generalisations that have variability?

6 Correlations

Samira was looking at something on her computer screen. Rafa came and sat quietly next to her. When she looked at him and raised an eyebrow, he said, "Are you free, Mom?"

Samira: Oh no, I don't come free. I cost a lot.

Rafa: You cost … Mom! Can we skip these jokes? Can I ask you something?

Samira: You may ask, of course; but whether or not I respond is a different issue.

Rafa rolled his eyes. His mother could be quite frustrating at times.

Rafa: Okay, here is the thing. A statement like: "The average height of adult Indian males is around 166 cms." This is a generalisation on the category of adult Indian males. Right?

Samira: Right.

Rafa: But a statement like, "Men are taller than women," is not about a property of the members of a population. It's a statement about a relation between two populations, actually between two sub-populations in the human population, the male population and the female population, in terms of height. Am I right?

Samira: Yes. Well, that is a correlation.

Rafa: What's a correlation?

Samira: Hmmm. Let's take an example. Do you know the formula for the circumference of a circle?

Rafa: $c = 2 \pi r$

Samira: Since r is radius, and twice the radius is the diameter, we can say that the circumference of a circle is pi times its diameter. Right?

Rafa: Oh, yes! That means there is a systematic relation between the circumference of the circle and its diameter. If we know the circumference of a circle, we can figure out its diameter, and if we know the diameter of a circle, we can figure out its circumference.

Samira: Exactly. So we say that in the population of circles, there is a correlation between the circumference and the diameter. A correlation is a systematic relation between two properties, so that if we know one, we can infer the other.

Rafa: Wow! I've known the formula for the circumference of a circle for so long, but I had no idea it was a correlation. What about the formula for the area of a circle? Is that a correlation too?

Samira: What do you think? If you know the diameter of a circle, you can figure out its area, and if you know the area, you can figure out its diameter. So you tell me, is that a correlation?

Rafa: Yes, it is a correlation. How about the Pythagoras theorem?

Samira: What about it?

Rafa: Oh, so it is a correlation; it's a relation between two populations in terms of length. One population is that of the hypotenuse of a right-angled triangle, and the other population is that of the other two sides. The relation is between the length of the square of the hypotenuse and the sum of the squares of the lengths of the other two sides.

Samira: Good!

Rafa: Mom, are all mathematical theorems correlations?

Samira: You tell me. What are the theorems you're familiar with? Are they all correlations?

Rafa: I'll have to think.

Rafa went into his room. A couple of hours later, he came back out, and joined his mother at the dining table.

Rafa: Mom, they are all correlations, at least the ones that I can think of. This is so cool.

Samira: Good that you find it cool. But can you think of theorems that are not correlations?

Rafa: I need time to think about that.

That night, lying in bed, Rafa continued mulling over correlations, till he dozed off. He woke up hearing a tapping noise at the window. And there he was, outside the window, the same bald man with a flowing white beard. Rafa jumped up and opened the window.

"Socs?"

"None other," said Socrates, gliding into the room and planting himself on Rafa's chair.

Rafa: I am so glad you came tonight, Socs. I've been struggling with these ideas that my mother was telling me about.

Socs: Tell me what has been bothering you, child.

Rafa: Mom was telling me about correlations. We talked about the Pythagoras theorem as a correlation. Many of the mathematical theorems that I can think of are correlations. Like the relation between the circumference and diameter of a circle, the relation between the angles of a triangles, the relation between the angles and the sides of triangles, there are so many. So I thought all mathematical theorems are correlations. But Mom kind of raised an eyebrow when I said that, and asked me to think about it. And I can't think of any that are not correlations.

Socs: (smiling) I see your problem. There are theorems that say such-and-such doesn't exist, or such-and-such exists. For example, we may have a theorem that says that straight-angled triangles do not exist, or that for every triangle, there exists exactly one circle that it circumscribes. Do they express correlations?

Rafa: Thanks for the clue, Socs!!! I need time to think about them. But before that, I have a question. All insects have compound eyes; that's what the books say. And only insects have compound eyes. Also, all insects have six legs, and only insects have six legs. Does this mean that there is a correlation between compound-ness of eyes and number of legs?

Socs: You tell me.

Rafa: If we know that an organism has six legs, then we can infer that it has compound eyes. And if we know that an organism has compound eyes, we can infer it has six legs. So that must be a correlation.

Socs: You're right, it is.

Rafa: But there's no mathematical formula or theorem here.

Socs: Correlations don't have to be mathematical. And they don't always have to be expressed in mathematical symbols. Of course, some correlations can be expressed in a mathematical form, in terms of quantities. But there are also correlations, like compound eyes and number of legs, which are *qualitative*.

Rafa: How about this: "If an organism has feathers, it has two legs." Is that a correlation?

Socs: Yes indeed.

Rafa: But it goes only one way. If an organism has feathers, we can infer it has two legs. But if it has two legs, we can't infer that it has feathers. Humans have two legs, but no feathers.

Socs: That's right. The inference goes only one way. It's *unidirectional*: If feathers, then two legs. But that doesn't mean: If two legs, then feathers. The compound eyes and number of legs correlation is *bi-directional*: it works both ways. So, if compound eyes, then six legs; and if six legs, then compound eyes.

Rafa: Socs, I need time to digest all this.

Socs: Take your time, kiddo. You're asking very good questions.

When Rafa looked at Socrates to say 'Thank you', he found that Socrates was getting blurry and disappearing. In a few seconds, he was totally gone. The next thing that Rafa knew was his mother knocking on the door and saying, "Rafa, if you don't get up, you will be late for school."

Before you read further:

Can you think of a few other examples of correlations, say, outside math and biology? Which of them are unidirectional, and which of them are bi-directional?

7 Generalising vs. Abstracting

Consider the following generalisations:

A. (i) All monarch butterflies have six legs.
(ii) All mourning cloak butterflies have six legs.
(iii) All black shallowtail butterflies have six legs.

B. (i) All butterflies have six legs.
(ii) All dragonflies have six legs.
(iii) All ants have six legs.
(iv) All mosquitoes have six legs.

C. All insects have six legs.

There are two kinds of generalisations in A to C. In A, each statement is a generalistion on the members of a category. In Bi, we have a generalization from the different subcategories of butterflies to the category of butterflies. And when we move from B to C, we are generalizing from the different subcategories of insects to the category of insects.

This example points to the need to distinguish between generalising:

~ from a sample to the population that the sample belongs to, and

~ from a subcategory to the category that the subcategory belongs to.

The second type of generalization involves moving up the level of abstraction, found in every case of moving from a sub-category to the category it is a subcategory of. In Chapter 7, we discussed two examples of such generalisations, namely:

~ Right-angled triangles > Triangles > Polygons

~ Squares > Rectangles > Parallelograms > Quadrilaterals > Polygons

The type of reasoning from a category to its subcategories, and from a population to one of its samples is *DEDUCTIVE REASONING*. In the case of categories and subcategories, the reasoning appeals to what we called *LOGICAL INHERITANCE* in Chapter 7. The reasoning from a sample to a population is *INDUCTIVE REASONING*, which can be either quantitative or qualitative.

8 What Did we Learn in this Chapter?

The question we have addressed in this Chapter is:

> "Given a pattern we find in a sample,
> how do we generalise it to the population?"

We began with an example of a flawed generalization that concludes on the basis of an inadequate sample that Americans are rude. Our judgment that such a generalisation is flawed led to an investigation of the distinction between generalisations that we judge to be legitimate, and those that we judge to be illegitimate or unfounded.

The journey that we shared with the main characters in our story in this chapter shows that generalisations are a central ingredient in mathematics, the physical-biological-human sciences, and human affairs in general, and even in matters of personal judgment.

It would be useful to bear in mind that generalisations are of many kinds, not all of them illustrated in the dialogues in the chapter. They may be about properties or relations (e.g., All birds have feathers, vs. If a creature has lungs, it also has red blood.) They may also be about either quantitative or qualitative (e.g., a correlation: between a person's height and weight (quantitative), vs. between having feathers and having two legs (qualitative).)

As you must have gathered from Chapters 1-7, tools of inquiry such as classifying, defining, and generalising are valuable for a deep and integrated understanding of the structure of the knowledge that we are exposed to in school and college. Those who are interested in research would also find that these tools serve as a valuable foundation for their exploration.

The discussion in this chapter centered around Qualitative Inductive Reasoning. We did not talk about quantitative inductive reasoning (called Inferential Statistics), or about what is called Descriptive Statistics. Nor did we go into the details of the methodological considerations that come under terms like 'representativeness', 'sampling', and 'operationalization'. If you would like to get a broad understanding of *STATISTICAL INQUIRY*, the following resources can be useful:

Darrell Huff. *How to Lie with Statistics*.
(How-to-Lie-with-Statistics.pdf)
Derek Rowntree. *Statistics without Tears*
(https://www.scribd.com/document/338754063/statistics-without-tears-Derek-Rowntree-pdf)

These books provide a basic conceptual understanding of *STATISTICAL THINKING*. For delving deeper into the technical aspects of Statistics, and developing the skills of statistical calculations needed for *STATISTICAL RESEARCH*, a good source is the 700-page textbook:

David Lane, et al. *Introduction to Statistics*.
(https://onlinestatbook.com/Online_Statistics_Education.pdf)

All generalizations are false, including this one.
– Mark Twain

> The beginning of wisdom is the definition of terms.
>
> Socrates

CHAPTER 9: DEFINING

1 Looking Back and Looking Forward

In Chapter 7, we examined the role of categories and subcategories (classes and subclasses) in different domains of academic knowledge. We also learnt how to arrive at classifications, how to choose between alternative schemes of sub-classification, and how to justify those choices.

In Chapter 8, we explored the relation between categories, sets, and populations in the context of generalizing from a sample of a population. Suppose we place an apple, a pen, a paperweight, a pair of socks, and a comb in a basket. In the basket is now a collection of things. Suppose we make a list of these things, so that anyone can check to see if everything is still there. A collection of things forms a set: we can tell whether or not a given entity is a member of the set.

If the entities in the basket form a set that can be defined in terms of a shared trait, we have a category. A category, then, is a set of entities defined in terms of one or more shared traits. For instance, the set of even integers is a category because ***all*** members of this set have the property that they can be divided by two, with an integer as a result; and ***only*** members of this set have that property. What we saw in Chapter 7 is this: what distinguishes a set (arbitrary) from a category (justifiable) is:

(a) definability in terms of a set of shared properties, and

(b) what we can infer about an entity if we know what category it belongs to.

The concepts of 'category', 'property', and 'definition' play an important role in academic knowledge. In this chapter, we explore the concept of 'definition'. In the process of exploration, we will learn how to define concepts, how to critically evaluate definitions, and how to choose between alternative definitions.

2 Defining Breathing

Rafa, Neel, and Anu were in class, talking in whispers while waiting for the teacher.

Anu: I visited Sanjudidi last night. She's got this new fish tank!

Neel: Really? Has she set it up yet with fish in it?

Anu: Yea, she has nine fishes: three Goldfish, two White Cloud fish, two Guppies, and two Zebra fish. Oh boy, are they beautiful!

As he listened, Rafa saw a strange image in his mind of fish coming to the surface of the water for air.

Rafa: Can fish breathe?

Anu: What?!

Rafa: Can fish breathe?

Anu: Why on earth are you asking that, Rafa?

Rafa: Just answer my question.

Anu: Of course, they can't. They live under water, not in air.

Neel: Fish can breathe. Just like us, they too need oxygen to survive.

Anu: Oh, really? How about plants? They too need oxygen, don't they?

Neel: Yes, plants breathe too. And so do many bacteria.

Rafa: This is interesting. We should write it down.

Rafa wrote something in his notebook, and held it up to Anu and Neel.

Rafa: Suppose this is a standard multiple choice question in a test, where there is one correct option, and all the others are wrong. Which option would you pick?

Q: Which of the following organisms are capable of breathing?
a. mammals; fishes; plants
b. mammals; fishes
c. mammals
d. none of the above

Anu: Option c.

Rafa: And you, Neel?

Neel: I'd pick (a).

Rafa: And do you think that the answer each of you picked is going to get full marks?

Anu and Neel looked at each other.

Neel: That depends on which answer the examiner thinks is correct!

Rafa: Okay, let me rephrase that. Which option do YOU think is the better one, and why?

Just then, the teacher walked in, and the children scrambled to their seats.

THINK & DO #1: Can you think of an answer to Rafa's question?

When Rafa reached home that evening, Samira was still at work. He waited impatiently for her to get back, and when he heard the door lock click and saw the door opening, he called out even before she had walked in, "Mom, do fish breathe?"

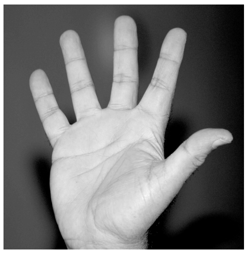
Samira stopped in her tracks, staring at him with amusement. "Rafa! Can I put down my bag, get a cup of tea, and stretch my legs before you ask me about fish and breathing?" she said.

"Sorry, Mom, go ahead."

Samira made herself a cup of tea, took a sip, and walked to one of the bookshelves. She took out a set of notes in a ring-binder, flipped through it, and found the chapter she was looking for, called "Defining". She held out the open book to Rafa and said, "Here, read the first page."

The first page of the chapter said:

KP, a teacher, is in his class. He claims that he has exactly four fingers on his right hand, no more, no less. To check if this claim is correct, the students ask him to hold up his right hand, and he does so. Here is a photograph of his right hand:



The students count the number of digits on the hand, and say, “You’re wrong; there are five fingers on your hand, not four.”

A surprised KP insists, “No, no, I have exactly four fingers on my hand, like most other humans. See (he counts): one (little finger), two (ring finger), three (middle finger), and four (pointing finger). That’s four.”

The students object, “But you didn’t count your thumb!”

“Oh,” KP says, “The thumb is not a finger.”

If we don’t include the thumb as a finger, then KP’s claim is true: he has exactly four fingers, no more. But if we include the thumb, he is wrong. As it happens, the English word ‘finger’ can mean two things. It can mean, “all the digits on the hand,” or “the digits on the hand except the thumb.” This means that if we hear the sentence, “KP has exactly four fingers on his right hand,” and have to judge the truth of the statement expressed by the sentence, we need to know the intended meaning of the word ‘finger’.

Take another situation. AA, sitting at her desk, says, “There is an animal on my desk.” RJ looks at the desk and says, “I don’t see any animal there.” “Don’t you see that orange moth?” says AA, pointing to the far corner of the desk. “Moth?” says RJ, “But a moth is not an animal.”

In the terminology of biology, the word ‘animal’ includes not only cats and dogs, but also humans, worms, moths, fish, and birds. But in ordinary English, humans, worms, and moths are not animals. So if AA is using the word ‘animal’ as a technical term in biology, her claim is true. But if she is using it as an ordinary English word, her claim is false. Again, we need to clarify the intended meaning of the word.

In each of these cases, the apparent disagreement rests simply on the difference in the meanings we assign to the words. Once the meaning of the term is clarified, what appeared to be a disagreement vanishes.

After reading the page, Rafa looked up.

Rafa: Mom, so you want me to define breathing!

Samira: Exactly.

Rafa: Am I allowed to look up the word on the Internet?

Samira: Of course you are, but only to get some ideas as a starting point. I want you to come up with your own definition. I don't want you to repeat what you find on the Internet or in the textbook."

THINK & DO #2

Can you oome up with a definition of breathing? If you need some help, do an Internet search to get some ideas about breathing.

Rafa did an Internet search, but he didn't get far. So he decided to go for a walk to do his own thinking. He went all the way to Neel and Anu's house, but didn't go in. Instead, he decided to just walk back, deep in thought. And while walking back he had a stroke of luck.

Instead of asking himself, "What is breathing?" he decided to ask: "How do I define breathing such that creatures with lungs breathe, but fish and plants don't?" It was obvious what the definition should be:

Breathing: (DEF. 1) A process by which a living organism inhales air from the atmosphere into the lungs, absorbs the oxygen in it, and exhales the remaining air back into the atmosphere.

By this definition, fish and plants don't breathe. Plants don't have lungs so they can't inhale and exhale air. Fish live underwater, so they don't inhale air from the atmosphere, whether or not they have lungs. Rafa wrote down the definition in his notebook.

"But what is breathing such that mammals and fish breathe, but plants don't?" he asked himself. After some reflection, he wrote another definition in his notebook.

Breathing: (DEF. 2) A process by which a living organism takes in either air or water from outside into a multicellular organ, absorbs the oxygen, and expels what remains.

By this definition, both mammals and fish breathe. Fish take in the water with air dissolved in it, absorb the oxygen, and expel what remains. Plants, however, don't breathe because they do not have a multicelluar organ for absorbing oxygen and expelling the remaining air or water.

That's cool, he said to himself. So how do we define it such that it includes plants?

Breathing: (DEF. 3) A process by which a living organism absorbs oxygen from outside.

When he got home, he showed his notebook to his mother.

Samira: That's very good, Rafa.

Rafa: I think I understand three different concepts of 'breathing'. But for the exam, which of them should I assign to that word?

Samira: What would be your choice?

Rafa: Mom, my choice doesn't matter. What does the examiner think is the correct meaning? For the exam, concepts don't matter. it's just a matter of vocabulary.

Samira: Let's forget about the exams for now. Even if it is just a matter of vocabulary, what would be your choice?

Rafa: I'll have to think.

THINK & DO #3: Can you think of an answer to Samira's question?

Rafa went out for a walk to be alone. When he got back, he went straight to his mother.

Rafa: Mom, here's what I think. In ordinary English, when we say 'breathe', we think of what we do with our lungs. So why don't we use definition 1 for breathing?

Samira: What words would you use for definitions 2 and 3?

Rafa: Maybe I should use respiration for definition 3, so that would mean that plants, fish, and mammals all have respiration. Also, why distinguish what plants do from what mammals and fish do? If we drop that, we don't need definition 2, so we don't need a word for it.

Samira: Sounds reasonable.

3 Defining Triangles, Squares, and Polygons

When Rafa talked to Anu and Neel about his definitions of breathing and respiration, they were impressed.

Anu: Neel and I were fighting yesterday about whether or not fish breathe. You know, what I've learnt from you — from your mom actually — is that before we disagree, we must ask what we mean by the words. Had Neel and I asked, "What do we mean by breathing? How do we define breathing?" we wouldn't have had that quarrel.

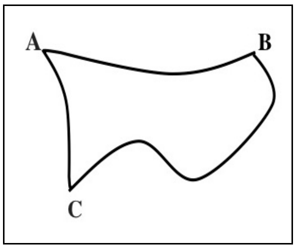
Rafa: Exactly. Isn't that cool!

Neel: Remember we talked last week about squares being rectangles? And we got into the issue of subcategorising triangles and quadrilaterals, and that led us to polygons. Let's go back to triangles and see how to define a triangle.

Anu: That's easy, tri- means three, so triangle is something with three angles.

Neel: Really?

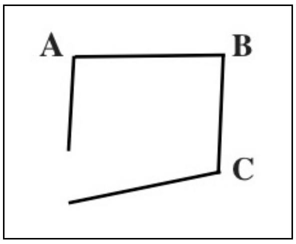
He drew a picture.



Neel: This has three angles, A, B, C. Is it a triangle?

Anu: Oh! It has three angles, but the lines are not straight. Okay, here's a definition.

DEF.1: A triangle is something with three angles and three straight lines.



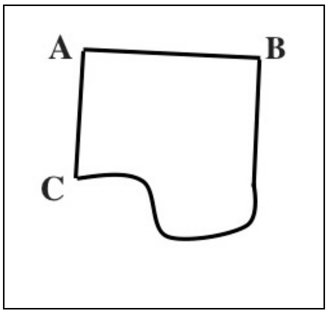
Neel: Hm! How about this? Is it a triangle? It has three angles and three straight lines.

Anu: But it has four straight lines!

Neel: Yes, it has three, and one more. But you didn't say it should have exactly three straight lines, no more and no less.

Anu: Also it's not a closed figure! Let me try again.

DEF. 2: A triangle is a closed figure with three angles and exactly three straight lines.



Neel: How about this? It's a closed figure, and it has three angles and exactly three straight lines.

Anu: But it has a curved line!

Neel: So? You didn't say that there should be no curved lines. All you said was that it has to have three straight lines.

Anu: Oh no, this is difficult. Okay, try this.

DEF. 3: A triangle is a closed figure with exactly three angles and exactly three lines, all of which are straight.

Neel: Do we need to say 'three angles'?

Anu: Huh?

As Neel was patiently getting Anu to refine her definition, Rafa watched silently, with amusement.

Neel: Suppose you say that a triangle is a closed figure with exactly three lines, all of which are straight. Do you need to say that it has three angles? If it is a closed figure, and has three straight lines, won't it follow that it has three angles?

Anu: Oh yea! I hadn't thought of that.

Rafa: Wait a minute. What about this definition?

DEF. 4: A triangle is a three-sided polygon.

Anu: That is neat! We can now say that a quadrilateral is a four-sided polygon, and a pentagon is a five-sided polygon.

Neel: That's great! But now you have to define polygon.

Anu: That is easy. A polygon is a closed figure made of only straight lines.

Neel: But that doesn't connect to the concept of 'side' in the definition of triangles.

Anu: That's okay. We just have to add that when we say 'side', we mean one of the straight lines of a polygon.

Neel: This is exciting! Rafa, it started with your definition. These definitions are based exactly on what we said last week about subclassification. We can now say that a parallelogram is a quadrilateral in which the opposite sides are parallel. We don't need to say anything about angles, or about the number of sides.

Anu: Right. And a rectangle is quadrilateral in which all angles are equal. And a square is a rectangle in which all sides are equal.

Rafa: This is cool, connecting subclassification and definitions! I love it.

THINK & DO #4

a. Consider the following definitions of a triangle:
   (i) A triangle is a closed figure made of exactly three straight lines.
   (ii) A triangle is a three-sided polygon.

   Which is the better definition?

   What are the reasons for your choice?

b. Based on Fig. 6 in Chapter 7, try to define animal, vertebrate, mammal, primate, and human.

THINK & DO #5

Let us expand our domain of definitions. Consider the following scenarios, and answer the questions given at the end of each.

Scenario 1: In ASV School, for every 10 students, there is one teacher. All students and teachers have equal voting rights. So the students mostly get what they want by voting for them – holidays, food, movies, picnics. If a teacher thinks that learning something is valuable for the students, but the students are not interested, they can outvote the teacher's proposal.

Is this a democratic school?

Scenario 2: In the BR family, all decisions affecting the children's lives are made jointly by the parents and the children, through rational discussion, negotiation, and consensus. They jointly decide if one of the parents should accept a promotion with transfer to a different city; what subjects the children should study, what extracurricular activities they should join, what TV programs they can watch and for how long, and what they choose to specialise in for their higher studies.

Is this a democratic family?

Scenario 3: In PNH School, all decisions affecting students and their learning are made jointly by the Principal, administrative staff, teachers, and students. If students are interested in learning something that is not currently part of the school curriculum, they discuss it with the Principal and teachers, and if feasible, the school offers the course. Syllabuses, textbooks, homework, assignments, and deadlines are negotiated between teachers and students. If there is a discipline problem, a committee of teachers and students figures out a solution, as well as a penalty, if needed.

Is this a democratic school?

Scenario 4: In the Malkot family, many decisions are made jointly by the parents and the children, through discussion: what movie to watch, where to go on vacation, and so on. The father decides what subjects the children should study, and what they choose to specialise in for their higher studies. The mother decides what extracurricular activities they should join.

Is this a democratic family?

Based on your responses to the questions in these scenarios, construct a definition of democracy, such that your judgments on the four scenarios follow logically from the definition. Don't proceed until you have made an honest effort.

4 The Relation between Classifying and Defining

In Chapter 7, we chose to treat squares as a subcategory of rectangles because it simplifies the definition of squares by drawing upon the idea that all the properties of the members of the mother category are inherited by the members of the daughter category. (See Ch. 7, Fig. 9). We called this idea *LOGICAL INHERITANCE*
(Before proceeding further, it would be good for you to revisit the discussion of 'square as rectangle' in Ch. 7, Sections 3 and 4.)

To see how this works, compare the following definitions of a 'square':

Square: DEF. 1: A square is a closed figure made of exactly four straight lines of equal length, with all four angles being right angles.

Square: DEF. 2: A square is an equilateral rectangle.

In Chapter 7, we decided to treat a square as a rectangle, with good reasons. Def. 2 builds on this idea. This definition has two parts:

A: a square is a rectangle.
B: a square is equilateral.

(A) is a statement about subcategorisation. If we specify a square as a subcategory of rectangle, then by the principle of logical inheritance, all the properties of rectangles are inherited by squares. For instance, we don't need to specify that a square is composed of exactly four straight lines, that it has four angles, that all the angles are right angles, and so on, because these are properties of rectangles, inherited by squares.

Given the statement that a square is a rectangle, we need to define 'rectangle'. For this, we appeal to the idea that a rectangle is a parallelogram. To define a parallelogram, we appeal to the idea that it is a quadrilateral. And to define a quadrilateral, we appeal to the idea that it is a polygon.

This path would take us to the following configuration of interconnected definitions:

I. A square is an equilateral rectangle.
II. A rectangle is an equiangular parallelogram.
III. A parallelogram is a quadrilateral whose opposite sides are parallel.
IV. A quadrilateral is a four-sided polygon.
V. A polygon is a closed figure made of straight lines and only straight lines.
VI. An equiangular polygon is one whose angles are all equal.
VII. An equilateral polygon is one whose sides are all equal.

Notice that 'equilateral' and 'equiangular' are now defined at the level of polygons, not at the level of rectangles or parallelograms. The advantage is that these concepts apply to all the daughters of polygons, including triangles. Without any further specification, it allows us to state one of the triangle theorems as:

> An equilateral triangle is an equiangular triangle, and vice versa.

There is no need to specify what an equilateral triangle is and what an equiangular triangle is. The specification is inherited from (VI) and (VII) above.

The German philosopher-poet Karl Wilhelm Friedrich Schlegel is reported to have said:

> "Classification is a definition comprising a system of definitions."

What this statement means must be clear to you now. If it isn't clear, it will become clear if you meditate on what we have said about definitions, classification, subclassification, and logical inheritance.

THINK & DO #6

In Chapter 7, Socrates and Rafa simplified the statement of the traits of 'square' by classifying it as a subcategory of 'rectangle'. Now consider the theorems of parallelograms:

1) The opposite angles of a parallelogram are equal.
2) The sum of adjacent angles of a parallelogram is two right angles.
3) The opposite sides of a parallelogram are equal.
4) The diagonals of a parallelogram bisect each other.
5) The sum of angles in a parallelogram is four right angles.

Given that rectangles are parallelograms, can you simplify the statements on rectangles in Chapter 7?

---

5 What Did You Learn in this Chapter?

It must be clear by now that there is a close connection between classifying (what you learnt in Chapter 7) and defining (what you learnt in this chapter).

In ordinary language, we use sentences and words to communicate what we want to say: our ideas, opinions, feelings, judgments, instructions, and the like. Ordinary language is often imprecise, vague, and ambiguous. The word 'star', for instance, is ambiguous because it can mean a celestial object (e.g., Pole star), or a famous person, like a film star or a sports star. A word like ‘serious’ has different meanings in different contexts, as in ‘serious illness’ versus ‘serious expression on someone’s face’. We also use words in ordinary language as slogans without understanding their meanings, to communicate positive or negative attitudes. The words ‘slim’ and ‘thin’ may mean the same thing, but ‘slim’ carries a positive attitude, while ‘thin’ carries a negative one.

These characteristics of ordinary language don’t affect our everyday interactions, because we don’t expect everyday communication to have the kind of precision that is expected of academic communication. When generating and critically evaluating academic knowledge, and communicating it to others, the academic community needs to ensure credibility and high-and-lasting quality. Part of this, and highly valued in academic culture, is explicitness, clarity, precision, and rigour of thought and expression.

This means that the terms we use as part of our academic vocabulary must go with a high degree of conceptual clarity. One way to achieve such clarity is through definitions. When we use words like *animal*, *insect*, *force*, *energy*, and *consciousness* in ordinary language, we don’t expect definitions of them, but when we use them as part of academic vocabulary, we expect them to be clearly defined. Absence of that commitment often results in unproductive miscommunication across individuals and groups.

We began our exploration of definitions with the concept of breathing. As illustrated in the conversations between Anu, Neel, and Rafa, ambiguous use for the word ‘breathe’ results in apparent disagreements, until the meanings are clarified.

A classification is a definition comprising a system of definitions.

Karl Wilhelm Friedrich Schlegel

Of the central threads that run through inquiry, one is wound out of the strands of arguments, logic, reasoning, premises and the derivation of conclusions from them, and the tools that aid in the process; and another is spun from the fibers of reflection, intuition, insight, and imagination. Together, they weave a tapestry of capacities, understanding, attitudes, habits of mind, and the values one then stands for.

PART 3: LOOKING BACK

In Part 3, we explored the inquiry tools of classifying (Chapter 7), generalising (Chapter 8), and defining (Chapter 9). The main ideas that emerged from the discussion in these chapters can be summarised as follows:

For a collection of entities to be a set or a population, it is not necessary that their members have a shared trait. However, for a set (a collection of entities) to be considered a category, its members must have at least one shared trait to distinguish it from all other categories.

Classifying is the same as categorising. Categories are the outcome of the process of categorising.

Inductive reasoning from sample to population requires that the population be a category, not just a set.

Some categories have sub-categories, and these may have their own sub-categories, thereby forming a hierarchy of categories.

Setting up a mother category for a set of categories, resulting in a pairing of categories and subcategories, calls for identifying the shared properties of the daughter categories to the mother category. This involves the process of abstraction.

A category is a concept. Every word in a natural language expresses a concept.

The concepts expressed by the words of a language used in everyday discourse do not require definitions. However, academic terminology demands defining the concepts they denote. Evaluating the truth of knowledge claims in academic inquiry calls for clear definitions of the terms that express the claims. When telling a friend that a bat flew into your room at night a couple of days ago does not require a definition of the concept of bats. But when a biologist claims that bats are mammals, it calls for definitions of bats as well as mammals.

Education needs to build the spirit, the capacities, and tools of inquiry, encourage creativity, and instil moral leadership among the young.

Foundations of Knowledge and Inquiry across Disciplines

Part 4: Putting the Pieces Together

The man of science has learned to believe in justification, not by faith, but by verification.
Thomas Huxley

CHAPTER 10: JUSTIFICATION

1 Looking Back and Looking Forward

In Chapters 4-6, you were introduced to the basics of reasoning and logic. We summarize those chapters in terms of the following sets of concepts:

Set 1: Logic Concepts

a. Premise, Derivation, Conclusion
b. Missing/Implicit premises
c. Sound argument, Valid derivation, Credible Propositions
    (=propositions that we judge to be TRUE)

Set 2: Logic Terminology in English

a. *therefore*, *hence*, *so*
b. *if*, *only if*, *if and only if*
c. *all*, *every*, *any*
d. *because*
e. *no*, *not*

Set 3: The Concepts of Logic expressed by the words in Set 2:

a. Entailment: X ENTAILS Y
b. Implication: X IMPLIES Y
c. Causation: X CAUSES Y
d. Negation: X NOT TRUE

Set 4: Logic Notation for Implication

a. P → Q (is the same as: not-Q → not P)
b. not-P → not Q (is the same as: Q → P
c. P ↔ Q

In these chapters, we looked at how reasoning is used in arriving at knowledge claims, justifying them, and critically evaluating them as well as their justification. We now turn to the third function of reasoning, that of justifying knowledge claims.

JUSTIFICATION is the process of showing that something is true, useful, ethical, or valuable. The nature of the proof would depend on what we seek to prove.

How do we prove:

a. whether something is true or false? (e.g., a triangle can have an acute angle, right angle, or obtuse angle, but not a straight angle.)
b. whether something is useful or harmful for a given goal/purpose? (e.g., regular dedicated practice is useful for establishing habits and skills; death penalty in a legal system is not useful as a deterrent of crimes.)
c. whether something is ethically/morally good or bad? (e.g., telling lies is immoral; a legal system that permits private practice for defense lawyers is unethical.)
d. whether something is valuable/desirable or undesirable? (e.g., doubting and questioning is desirable; rationality is a valuable asset; denying the value of rationality is undesirable.)

In what follows, we explore the justification for claims of type (a) and (c).

Before you read further:
It might be a good idea to go through Chapters 4-6.

2 What is Justification?

Suppose you are reading an article, and are expected to evaluate it. The article claims that the general wellbeing of human beings has improved significantly over the last century as a result of science and technology. As a critical thinker, you would like to know the *GROUNDS* for the claim, that is, the premises from which we derive the conclusions. You would also like to know how exactly these grounds support the claim, that is, what are the steps of the derivation from the premises to the conclusion. How would you find out?

The simplest way is to ask the author. Your questions would take one of these forms:

"What is the evidence for that conclusion?"

"What is the proof for that claim?"

"Why should we accept your claim?" or

"How do you know that?"

JUSTIFICATION is a response to such questions.

A researcher arrives at *CONCLUSIONS* and presents them to a jury of experts as *CLAIMS*, in the form of a research paper, article, thesis, or book. The experts are skeptical, but open-minded. The researcher provides justification to convince the jury that the claim(s) must be accepted as correct.

Suppose you are that researcher. How would you justify — or refute — the claim that the general wellbeing of human beings has improved significantly over the last century as a result of science and technology? Let us look at a few simple cases of justification before we attempt to answer this question.

3 The Structure of Justification

Imagine that Zeno and Athena are colleagues. One day at work, Zeno tells Athena that he has only four toes on his right foot; he lost his little toe in an accident. Athena is shocked. She says, "I don't believe you!" Zeno takes off the shoe and sock on his right foot, and says, "See for yourself!" In doing so, he is inviting her to ***directly perceive*** the grounds for his claim, and this convinces her.

GROUNDS:
An independent observer can perceive the toes on the right foot.

↓

CONCLUSION:
The right foot has only four toes.

FIGURE 1

In this example, the justification consists of pointing to an object or situation, and inviting attention to it.

Take a different example. Suppose Zeno claims that his body can fit into a small barrel that is half a meter in diameter. By getting inside such a barrel, and inviting Athena to see this for herself, he convinces Athena. This example is slightly more complex than the prevoius one; it involves a *DEMONSTRATION* in which the person making the claim *does* something, and invites the skeptic to observe the results.

Take yet another case. Zeno claims that the volume of a statue for sale in his store is 7 litres. Athena is skeptical. So in her presence, Zeno fills a large bucket with water, and immerses the statue in the bucket. He invites Athena to *MEASURE* the amount of water flowing out of the bucket. She does so, finds that it is 7 litres, and is convinced. Here, Athena can't see or measure the volume of the statue. But she can measure the volume of the water overflowing from the bucket, and from that, *INFER* the volume of the statue. Implicit in this inference is a background *ASSUMPTION* that she shares with Zeno: "The volume of a body is equal to the volume of water it displaces."

GROUNDS:
Volume of displaced water: 7 litres

ASSUMPTION:
Volume of a body = Volume of water it displaces

↓

CONCLUSION:
Volume of the statue: 7 litres

FIGURE 2

If Athena chooses to question the assumption, Zeno will have to provide justification for it.

Now, in a face-to-face conversation, a speaker can invite listeners to observe the relevant objects or situations. But this is not feasible in written justification. Zeno cannot say in an email or a letter to Athena, "Measure, and see for yourself, the amount of water the statue displaces." He has to do the measuring, and report the results. The grounds offered in experimental research, interviews, and ethnography are of this kind: the grounds are *VERBAL REPORTS* of what the researcher observes.

Take yet another type of claim. Suppose Zeno is a medical doctor who specializes in forensics. He has just finished examining a dead body found in a well. He sends Athena an email with his claim, and an email exchange follows:

Z: Hi Athena, I've just examined the body I told you about. This death was not an accident or a suicide: it was a murder.

A: What makes you say that, Zeno?

Z: There is no water in the lungs.

A: So?

Z: When a person dies by drowning, there is always water in the lungs, because of the victim's gasping for air. If the person is already dead when he falls into water, it doesn't get into the lungs. So it has to be that this person died first, and the body entered the water afterwards.

A: Makes sense. That rules out the suicide hypothesis. But...

Z: But what?

A: It only means that he died first and hit the water later. He could have fallen into the well because he had a heart attack when standing at its edge. What makes you say that someone killed him? What evidence do you have?

Z: Well, let us see. If you are right, this was an accidental death. If so, he must have been standing close to the edge of the well when he had the heart attack, and death must have been instantaneous, before he hit the water.

A: That's right.

Z: Hmmm, this is not impossible. But the probability is very low.

A: Ooh? (She is about to say something, but Zeno interrupts.)

Z. Also during the post-mortem, I found blue bruise marks around his neck, the kind that we find when a person is strangled.

A: Ah, I see now! If we assume that he was strangled, we have an explanation for the bruise marks.

Z: Exactly. And otherwise, there is no explanation. Bruises on the neck, absence of water in the lungs: they pretty much force us to conclude that he was murdered.

Unlike the previous examples, the connection between the grounds (absence of water in the lungs, bruises on the neck) and the conclusion (killing through strangulation) in this example is mediated through an *extended* form of ***reasoning***. The diagram below illustrates this justification. For practice, you may wish to unpack the reasoning into numbered premises and the intermediate conclusions, leading to the final conclusions.

GROUNDS:
1. There is no water in the lungs.
2. There are bruise marks on the neck, characteristic of strangulation.

STEPS OF REASONING:

~ Death by drowning causes water in the lungs. There is no water in the lungs of the body. So the person must have died before drowning.

~ Hence we conclude that the person died before the body hit the water.

~ Sudden death, causing a person to fall into the well while standing at its edge, is improbable.

~ Besides, the bruise marks on the neck can be explained only if we assume that he was strangled to death.

~ Hence, we conclude that he was killed.

CONCLUSION:
The person was murdered.

FIGURE 3

EXERCISE 1

When a child doubts and questions an assertion, implicitly asking why she should believe something, adults tend to respond along one of the following lines:

Because I say so.
Take my word for it.
Because that is what you have been taught.
Don't ask such stupid questions.
You won't understand if I tell you.

These responses have the effect of dampening the child's sense of curiosity, and with it, the capacity for questioning and critical thinking.

Imagine that Athena is a ten-year old, and that you are Zeno. Looking at the night sky, Athena says, "Zeno, the moon is so much bigger than the stars!" You correct her: "It only looks bigger, the stars are actually much, much bigger than the moon." Athena is surprised. She points to the sky and says, "See for yourself. The moon is much bigger."

How would you respond to Athena?

4 Core Concepts

The terms *evidence, argument*, and *proof* carry different shades of meaning, but they all come under the rubric of justification.

4.1 Evidence

'*Evidence*' refers to observations or observational reports as the grounds for justification.-Observations include things we pick up through our senses: a noise we hear, a distinct smell, a broken window, or a blood-stained knife submitted to a court of law. A photograph of stars taken by an astronomer, measurements recorded by a physicist, an interview conducted by a sociologist, the words in a literary text that a literary critic draws attention to, and an ancient scroll discovered by a historian that serves as the basis for a claim — all these are forms of evidence.

We must point out that *DATA* become evidence only when used as the grounds for justification, and connected to a conclusion. For a historian, a set of newspaper reports on World War II, say, from 1944, is only data. It becomes evidence only when used as the grounds for a conclusion on WWII.

So far, all the grounds in our examples of justification have been instances of evidence. Let us now turn to justification where the grounds don't fall under the category of evidence.

Suppose someone makes the following assertions:

1) My grandmother never had any children.
2) My grandmother was born when I was five years old.
3) My grandmother was a rabbit.
4) My grandmother's sisters never had any children.

Let us assume that the term 'grandmother' in all these examples refers to a ***biological maternal grandmother***. Even if we don't know anything about the speaker or the grandmother, we would reject assertions (1)-(3) as false. In contrast, we are not in a position to judge the truth of (4). How did we conclude that (1)-(3) are false? And why can't we determine the truth of (4)?

Here is a possible answer. Suppose we accept the following premises:

5) For any x, the proposition that x's grandmother never had children is false.
6) For any x, the proposition that x was 5 years old when x's grandmother was born is false.
7) For any x, if x is human, the proposition that x's grandmother was a rabbit is false.

From the *general* propositions in (5)-(7), it logically follows that the *particular* propositions (1)-(3) are false.

In contrast, there is no premise from which we can deduce the truth or falsity of (4). Take the propositions in (8):

8) a. For any x, the proposition that x's grandmother's sisters does not have children is false.
   b. For any x, the proposition that x's grandmother's sisters has children is false.

It is possible that x's grandmother's sister has children (as (8a) claims), and it is equally possible that she does not have children (as (8b) claims). So, without evidence either way, we cannot arrive at a judgment on (4).

How about the general statements in (5)-(7)? How do we know if we should treat them as true? How do we justify these statements?

4.2 Axioms and Definitions

We can prove that (5) is true simply by defining the concepts of 'mother', 'child', and '(biological maternal) grandmother':

9) a. Mother: y is the mother of x iff (= if and only if) y gave birth to x.
   b. Child: x is a child of y iff y is a parent of x.
   c. Grandmother: z is a grandmother of x iff z is the mother of y and y is the mother of x.

From (9c), it logically follows that:

10) For any z, if z is the grandmother of x,
    z has a child y who is the mother of x.
    Hence, for any x, the proposition that x's grandmother never had children is false.

By (10), (5) must be true. To prove (6), we need to add the following axioms (which we may take as obviously true) to our definitions in (9):

11) Axiom: a. A living organism x comes to exist when x is born.
           b. For y to give birth to x, y must already exist.

From (11a, b), it follows that for x to exist (be born), the one who gives birth to x must already exist. It then follows from (9a) and (11b), that for y to be the mother of x, y must exist before x can be born. This result, together with (9c), leads to the conclusion that, for z to be the grandmother of x, z must exist before y can be born, and y must exist before x can be born. So, we can never have a situation where z is x's grandmother, and x was five years old when (or even existed before) z was born. Hence, (6) is true.

To prove (7), we need to appeal to a general law that can be established on the basis of evidence (= observational grounds):

12) If y is a parent of x, then x and y must belong to the same species.

From (12) and (9c), it follows that if z is a grandparent of x, then x and z belong to the same species. For any creature to utter the sentence, “My grandmother was a rabbit,” the creature must be capable of speech. As far as we know, only a human being has that capacity for speech, along with an understanding of the sentence and the words in it. (A small number of parrots may be able to utter the words, but not with a conceptual understanding.) Therefore, when a speaker says, “my grandmother,” the grandmother too must be a human being. Given that a human being cannot be a rabbit, it follows that (7) is true.

As you can see, the proof for (5) is based purely on definitions of the relevant terms. The proof for (6) is based on definitions and axioms. These two proofs adopt the mode of proving in mathematics. The proof for (7) is based on definitions and a general law, which can be established on the basis of observations. It adopts the mode of prooving in science.

Let us go through one more example of justification that does not appeal to observational grounds. Here is an exchange between Zeno and Athena:

<u>*Example: Squares and Triangles*</u>

Z: Given a square, and an equilateral triangle whose sides are equal to the sides of the square, the area of the square is greater than the area of the triangle.

A: How do you know that?

Z: Imagine a square ((i) in Fig. 4) and an equilateral triangle, whose sides are equal to the sides of the square ((ii) in Fig. 4):

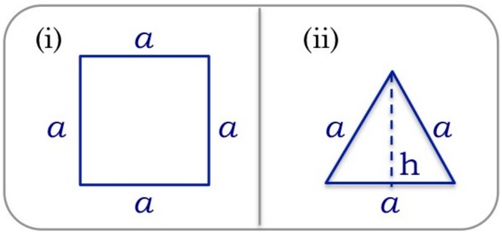


FIGURE 4

We know that: if the sides of a square are of length a, its area is a x a. (a multiplied by a)

We also know that: the area of a triangle is ½ b x h (half the length of the base multiplied by height. h = perpendicular from base to the opposite angle).

Since an equilateral triangle is defined as one whose sides are of equal length, all the sides of the triangle in (ii) are of length a. In an equilateral triangle whose sides are equal to that of our square, b = a, and its area is ½ a x h.

A: Okay…

Z: In the equilateral triangle, h forms one side of two right-angled triangles. The hypotenuse of both these triangles is a. Hence, h is shorter than a. Hence, the area of the triangle, which is ½ a x h, is definitely less than the area of the square, which is a x a.

A: Oh yes, I see that now.

In this example, the grounds are these propositions:

(i) Given a square with sides of length *a,* its area is *a* x *a*.

(ii) The area of a triangle is ½ *b* x *h*.

(iii) The height of an equilateral triangle is less than its sides.

These grounds are results already proved in mathematics by appealing to ***axioms*** and ***definitions***. Neither of these are forms of 'evidence'. However, the proof here is a form of justification.

4.3 Argument

An ***argument*** involves *a chain of reasoning* that connects the grounds to a conclusion. The extended reasoning that Zeno gives to justify his claim (that the dead man in the well was murdered) makes it an argument. The justification above (of the area of a square with sides *a* being greater than the area of an equilateral triangle with sides *a*) is also an argument.

NOTE: The concept of *argument* described here is quite different from the everyday meaning of argument, as in: "They were arguing all night, their loud voices disturbing their neighbours." Here, 'argue' is an *expression of disagreement*, without necessarily presenting evidence and/or reasoning in support of the positions. This activity does not qualify as an argument in rational inquiry.

Another meaning of the term argument, popular in the humanities, is that of an *extended elucidation of a claim*, not necessarily supported by evidence and reasoning. In such cases, the statement: "I argue that modernity is harmful," should be interpreted as: "I present the view, with details, that modernity is harmful." It is important to distinguish this meaning of 'argument' from the concept of argument as evidence and/or reasoning in support of a conclusion.

In contrast to the murder argument and the area-of-square-and-triangle argument, Zeno's justification for the number of toes on his right foot, and for the volume of the statue, can be expressed simply by pointing to the relevant evidence, without the need to express it as an argument.

4.4 Proof

The term ***proof*** can be used in a restrictive sense, to refer only to the kind of proofs that we find in mathematics. Or it can be used in a broader sense, to cover not only mathematical proofs but also legal proofs and experimental proofs, and even a demonstration. A *proof* is a form of justification that leaves a conclusion very little room for error or doubt. How much error or doubt a proof allows varies across domains; in fact, the degree of expected rigour may vary even within a domain.

In a court of law, for instance, a proof is an argument that yields a conclusion with certainty beyond *reasonable* doubt. In contrast, we expect mathematical proofs to meet a more stringent criterion, namely, certainty beyond *any* doubt. (Whether or not actual proofs meet this criterion is a separate issue.) The justification offered in the earlier example of 'Square and Triangle' is a mathematical proof. The justification offered in the example of the drowned man is admissible as a legal proof, but it does not satisfy the more stringent criterion of mathematical proofs.

EXERCISE 2

Consider a rectangle and a non-rectangle parallelogram whose corresponding sides are equal. Here are three possibilities:

(a) They have the same area.

(b) The area of the rectangle is greater than that of the parallelogram.

(c) The area of the parallelogram is greater than that of the rectangle.

Which of the above is true? Can you prove it?

Exercises 3 and 4 below are somewhat challenging. But it is still worth engaging with them, and trying to articulate your positions. The process, we hope, will force you to think outside your comfort zone, and help you expand your horizon.

EXERCISE 3

Justice demands that all human beings be treated equally, regardless of their race, ethnicity, gender, economic status, religion, etc. Does the criminal justice system – as it exists today – have elements of injustice? For instance, does it have an unjust bias against the poor? Does it discriminate against powerless minorities? Formulate your position and justify it, paying attention to the following points:

- The criminal justice system identifies certain actions as criminal offenses, and punishes the offenders with imprisonment, and in extreme cases, death.
- Good defense lawyers charge high fees. Excellent defense lawyers charge even higher fees.
- Laws are created by those in power, even in a country where everyone has the opportunity to vote.
- Politics in most election-based democracies demands that rulers pay attention to the wishes of the majority, but not necessarily to moral integrity.

EXERCISE 4

Has India been becoming more developed during the last fifty years? There are three logical possibilities:

a) It has become more developed.

b) It has become less developed.

c) The level of development has remained the same.

Which of these possibilities do you think is the actuality? How would you gather the relevant data/ information to prove/justify your claim?

To prove any of these claims, we need to (i) define development, (ii) set up clear criteria to measure the different parameters of development, and (iii) find ways to arrive at an overall number that serves as the index of development. To do this, you would need to think through different concepts of development.

If you ask someone, “What do you do for a living?” and (s)he says “I am a developer,” the concept of development implicit in the answer is that of real estate development or software development. For instance, according to Wikipedia, “Developers buy land, finance real estate deals, build or have builders build projects, create, imagine, control and orchestrate the process of development from the beginning to end.”

Clearly, this is not the concept of development that governments seem to have. In January 2014, for instance, Narendra Modi identified five core parameters of development: growth of industry; education, and skilled manpower; small-scale industries and health sector; value addition; and purchasing power. This appears to be the concept of the development of industry-economy in a country, where the purpose of education is primarily to serve the industry’s needs of skilled manpower.

A somewhat different conception of development is presented in the definition of development by the United Nations (http://goo.gl/0dWQxB).

Yet another concept is that of ‘development as freedom,’ proposed by Amartya Sen in 1998 (http://en.wikipedia.org/wiki/Development_as_Freedom).

To prove your conclusion on development in India, you will have to make up your mind about which of these (or some other) concepts of development you would subscribe to.

5 Justifying in Science, Mathematics, and Ethics

Before we proceed, we should note that we use the words *proof*, *argument*, and *justification* to mean the same thing. They all refer to the process of giving reasons in support of a conclusion (or claim/position). This is why, when we say ‘justification’, we mean ‘*rational* justification’.

5.1 Justification in Scientific Inquiry

Remember the disagreement between Neel and Anu in chapter… on whether white crows exist? Anu’s proof was based on experience. She pointed to a white bird that she judged to be a crow, and said: “Look at that crow, you can see for yourself that it is white. So white crows do exist!”

The structure of the proof she proposed was:

Anu's Proof:

Observation: Anu sees a bird.
Premise 1: That bird is a crow.
Premise 2: That bird is white.
Conclusion 1: Hence that is a white crow.
Conclusion 2: Hence, we conclude that at least one white crow exists.
Conclusion 3: Hence, the statement that white crows do not exist is false.

But the discussion hit a roadblock. Neel did not accept that the bird they were looking at was a crow. So Premise 1 did not work for him. For him to accept the proof, Anu had to first establish the credibility of Premise 1.

For this, Anu needed to begin by defining the concept of crow, such that for any bird, they could agree on whether or not that bird was a crow. Anu couldn't come up with a definition, and hence couldn't prove her claim.

Neel used a different strategy. The structure of his proof was:

Neel's Proof:

Premise 1: All crows are black.
Conclusion 1: Hence, it is not true that there exist white crows.
Conclusion 2: Hence, the bird Anu saw could not be a crow.

Anu did not accept Neel's Premise 1, so this proof didn't work either. Given that each of them rejected one of the premises in the other's proof, they couldn't come to an agreement on whether or not white crows exist.

Anu's proof appealed to Neel's acceptance of her observational premise: *See that bird, you agree that it is a white crow, right?* She was arriving at her conclusion on the basis of data/observation. If the basis for an argument is observation or data, we call it *EMPIRICAL*. All scientific proofs are empirical.

The term *evidence* refers to observational premises in support of a conclusion. Data or observation become evidence only when they provide support for the conclusion. An argument, as we have seen, is made up of premises, a conclusion, and a derivation from the premises to the conclusion. Evidence refers to the observational premises in scientific inquiry.

In contrast, Neel's proof appealed to Anu's acceptance of the credibility of his assumption: *All crows are black.* Data or observation was not relevant for this proof. We will use the term assumption-based proof to refer to this kind of proof. Unlike scientific proofs, mathematical proofs are assumption-based. In the following section, we will explore the nature of proofs in Mathematics.

5.2 Justification in Mathematical Inquiry

Ila, Neel, and Anu were having a relaxed Sunday breakfast. Ila noticed that Anu was not paying attention to what she was eating. She was obviously preoccupied with something.

Ila: What is bugging you, Anu?

Anu didn't even look up; she was so deep in thought that she hadn't heard Ila's question.

Ila: (louder) What is it that's bugging you, young lady?

Anu: (startled) Oh, I was thinking about how we prove different kinds of statements.

Ila: Care to explain a bit?

Anu: You know, Ma, I can prove that without oxygen, all animals die. I can also prove that no two straight lines can intersect at two distinct points. But there's a difference between the two proofs. And I can't put my finger on the difference.

Neel: (his antennae going up) That is interesting. I hadn't thought of it, but what you are saying sounds intuitively right. What *IS* the difference?

Anu: (frowning) That's what I'm trying to figure out and not having much success.

Ila: Don't be dejected, Anu. Why don't you begin by giving us those two proofs first? And then we can put our heads together to find out if they are different. And if they are different, we'll try to figure out the difference.

Anu: Okay, here is the proof to show that no two straight lines can intersect at two distinct points. We have to begin by defining 'straight line'. Here is a definition:

> Def 1: A line is straight if and only if, given any two points A and B on that line, AB is the shortest path between A and B.

Ila: Okay. Now try saying that in simple English.

Anu: Hmm… How about this: A line connecting two points is straight if and only if it is the shortest path between those two points. If you want something simpler, try this: a straight line between two points is the shortest path between them.

Ila: Good, that is reasonably clear.

Anu got her notebook out and drew a picture on it:

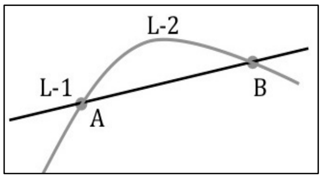


FIGURE 5

Anu: Now, suppose there are two straight lines L-1 and L-2 that intersect at point A. And now suppose they also intersect at point B. Now we have two ABs, one along L-1, and another along L-2. We have defined a straight line connecting two points as the shortest path between them. So if there are two paths between A and B, then by the definition, at least one of them is not a straight line.

Both Ila and Neel sat still for some time, reflecting on the proof that Anu had given.

Ila: So what is crucial for your proof is the definition of a straight line as the shortest path between points, with the assumption that there cannot be two shortest paths.

Anu: Yep.

Ila: What if we were to change that definition. Let's assume that it is possible to have two equally short paths that are shorter than all other paths. Then both of them are the shortest paths. If that is the definition, your proof wouldn't work, would it?

Anu: Ooooh, I hadn't thought of that. Let me see. If I assume that definition, my proof won't work. I see that. But how can there be two shortest paths between two points? That doesn't make sense to me.

Neel: Anu, do you remember when a few weeks ago, Rafa, you and me were struggling with two kinds of geometry?

Anu: Yes. Flat surface geometry and spherical surface geometry.

Neel: Yup. And remember Rafa suggested two different axioms for the two geometries?

Neel took Anu's notebook from her and wrote:

Proposition 1: Every straight line, when extended, would meet itself.

Proposition 2: No straight line, when extended, would meet itself.

Neel: Proposition 1 is the basis for spherical geometry. Proposition 2 is the starting point for flat geometry.

Anu: Oh yes, we learn only Euclidean geometry in school, and that's a flat geometry.

Neel: Imagine that you are at a point on the equator of a perfectly spherical earth. You start walking Northwards in a straight line. You cross the North pole, and you continue walking straight.

Anu: Oh, then I'll be walking towards the South Pole.

Neel: Exactly. You continue walking in a straight line, cross the South Pole, continue walking and get back to the point on the equator where you started. Right?

Anu: Okay, so I walked in a straight line and got back to the same point because it is a spherical surface.

Neel: Here is the cool part. Now from the same point A, you start walking east along the equator in a straight line. You'll still come back to the same starting point, right?

Anu: Yes, of course.

Neel: And in the course of your coming back, you would have crossed the earlier line crossing the equator at point B, diametrically opposite point A, wouldn't you?

Anu: Oh, oh, that is so... that is so... So these two straight lines on the spherical surface intersect each other at points A and B?

Neel: That is what I was driving at!

Ila had been listening to this conversation with intense interest.

Ila: Can I interrupt for a moment? I want to ask you guys two questions. Let me use the term 'complete straight line' in spherical geometry to mean a fully extended straight line which gets back to itself. My first question is: On a spherical surface, can there be two complete straight lines that don't intersect twice? My second question is: Do parallel straight lines intersect twice on a spherical surface?

Anu and Neel stared at Ila without comprehension. It took them a minute, but once Ila's questions sank in, they got so excited that they started discussing the questions and arguing with each other, drawing diagrams on paper. Neel suddenly got up and said, "Let's go talk to Rafa about this." In a few seconds, he and Anu were on the road, furiously riding their bicycles.

Before you read further:

Can you try to answer Ila's questions?

Can you prove your answers?

They found Rafa at home, and told him about their discussion. Rafa was puzzled too.

Rafa: Here is an idea. Imagine a perfectly spherical lime. Suppose you were to cut it into two halves, right through the center, and put the two pieces back again, the cut on the surface would be a straight line. If an ant walked along the cut, without changing direction, it would come back to the same point. Does that make sense?

Anu: Yes.

Rafa: Is it possible to make another cut through the center, and put the halves back, without the new straight line intersecting with the old one?

Anu: Hmmm. Impossible!

Neel: So here is a conjecture in answer to Ma's question.

He writes on Rafa's whiteboard:

> In spherical geometry, any two straight lines intersect each other twice, at diametrically opposite points.

Anu: I believe that our conjecture is true. But how do we prove it?

Rafa: Any straight line on a spherical surface is a circle whose center is the center of the sphere, right? I don't know how to prove that, so let us take that as an axiom for now. It means that any two such circular straight lines on the same spherical surface will have the same center. And if we accept that, it follows that any two straight lines must intersect each other at two diametrically opposite points.

Neel: That makes sense, Rafa, but you haven't proved any of the ideas you just brought up. So your proof is not complete.

Rafa: Yes, I know. Axioms and definitions can't be proved. So to start with, I'm going take all of them as axioms. And given my axioms, the conjecture follows as a consequence. Don't you agree?

Anu: Yes. So if we accept your axioms, we should also accept the conjecture. May be some day soon, we will be able to prove these axioms, and if we prove them, they will become theorems.

Rafa: You know, Socs was saying that broadly there are two kinds of proofs. One kind that proves claims on the basis of data or observations that others can crosscheck. Suppose I look at two people, and say Anu and Neel, and judge Neel to be taller than Anu. I report to others that Neel is taller than Anu. If others crosscheck my report and agree that it is true, if they corroborate my observational report, then that becomes data.

Anu: Oh! I see.

Rafa: Now I can gather lots of data of this kind. Suppose I find that in my data, the average height of men is greater than the average height of women. If my sample is representative of the population of humans, I can generalize my finding to the population, and say that human males are taller than human females. The evidence for this claim comes from data. Socs called this an observation-based proof.

Neel: So a proof that appeals to data is an observation-based proof?

Rafa: M-hmm, yes.

Anu: The evidence and arguments to support Newton's theory or Darwin's theory would also be observation-based?

Rafa: I guess so.

Anu: So what about the proof for the statement that in spherical geometry, any two straight lines intersect at two distinct points?

Rafa: We didn't use data to prove that. Instead, we appealed to axioms and definitions. That is what Socs was calling an assumption-based proof.

Neel: So math uses assumption-based proofs and science uses observation-based proofs?

Rafa: As far as I can tell, yes.

Anu: Hey, Sociology and Psychology also use data to support their conclusions. So does Human History. Are these subjects 'sciences', then?

Neel: Sure. So think of Physics and Astronomy as Physical Sciences; Botany and Zoology as Biological Sciences; and Sociology, Psychology and Human History as Human Sciences.

Rafa: I like that.

Anu: Ha ha! You will soon be talking about insect science and bird science and chimpanzee science.

Rafa: Why not? Insect science is called entomology. Look it up on the web. And the science of primates, including chimpanzees, orangutans, and humans, is called primatology. If entomology and primatology are sciences, why can't we have human sciences?

Anu: Are you going to say that to our history teacher and our physics teacher?

Neel: Well, may be not immediately. But some day we should.

EXERCISE 5

You are given these seven premises. [They contain cooked-up words like *felox* and *blam*; the meanings of these words don't matter for the purposes of this task.]

A. All feloxes have purple ears.
B. Every nicomb is a felox.
C. No felox is a six-legged blam.
D. Only six legged blams eats vogins.
E. All feloxes are glicks.
F. Every purple-eared glick has four legs.
G. Zeno eats vogins.

Now answer the five questions below. Your answer can be: (i) 'yes', (ii) 'no', or (iii) "I can't tell".

Answer (iii) means that the premises given to you are not sufficient for you to give an answer.

If you choose (i) or (ii), provide a proof on the basis of the premises A-G. Your proofs need not necessarily use all the given premises; some of them may not be relevant.]

1. Is Zeno a six-legged blam?
2. Is Zeno a nicomb?
3. Does Zeno have purple ears?
4. Is Zeno a glick?
5. Does Zeno have six legs or four legs?

EXERCISE 6

This exercise is based on the table(s) on the next page.

Consider the six observational reports in Column I, and the five claims Column II. Some of the reports may provide support for one or more of the claims.

Think about which of the observational reports support which claims. When you are ready, answer the questions at the end.

Don't get discouraged if you find it too challenging. ☺

I. OBSERVATIONAL REPORTS	
i	When William Harvey opened up a live snake and squeezed its vena cava (the vein connected directly to the heart), its heart became "whiter in colour" and smaller in size, and started beating slower.
ii	William Harvey measured the amount of blood ejected from the heart at each pulse, and given the number of heartbeats per minute, he could calculate the amount of blood that leaves the heart in a given time period. He discovered that more blood leaves the heart in a short span of time than the amount that the whole body contains.
iii	When Harvey squeezed the aorta (the artery connected directly to the heart) of the live snake, the portion of the artery between the heart and the point of constriction began to swell. The heart became "distended, turned purple to livid in colour," and looked as if it was about to burst.
iv	When you tie a bandage lightly around your arm (enough to constrict the veins but not the arteries), little points of swelling stand out at the valves in the veins below the bandage.
v	When you tie a bandage very tightly around your arm (such that both the arteries and the veins are constricted), the pulse on your wrist becomes weak, and there is a throbbing between the bandage and the shoulder.
vi	When we have strong emotions, our heart starts beating faster.

II. CLAIMS	
1	Emotions are the result of brain activity.
2	The heart pumps blood to various parts of the body.
3	The lungs purify blood and infuse it with oxygen.
4	Arteries carry blood away from the heart.
5	Veins carry blood to the heart.

Here are ten statements about the relation between the Observational Reports (ORs) and the Claims (Cs) given above.

Which of these statements would you regard as legitimate? Mark YES if legitimate, and NO if not legitimate.

What are your reasons for judging each one as legitimate or as not legitimate?

A. OR (i) supports C 5.	Yes/No		F. OR (iv) supports C 5.	Yes/No
B. OR (ii) supports C 2.	Yes/No		G. OR (v) supports C 3.	Yes/No
C. OR (ii) supports C 4.	Yes/No		H. OR (vi) supports C1.	Yes/No
D. OR (iii) supports C 1.	Yes/No		I. OR (vi) supports C 2.	Yes/No
E. OR (iii) supports C 3.	Yes/No		J. OR (vi) supports C 4.	Yes/No

5.3 Justification in Ethical Inquiry

When Rafa opened the door to his room after dinner that night, he found Socrates relaxing on his bed, reading what looked like an ancient text on ethics. Socrates was beginning to treat himself as Rafa's roommate, sleeping in Rafa's bed whenever he felt like. Rafa didn't like others sleeping in his bed. But he couldn't tell Socrates how he felt. How can one be rude to someone who died so many centuries ago?

As Rafa entered the room, Socrates opened his eyes.

Socs: Ah, my young friend, so you had a good discussion on the distinction between assumption-based and observation-based proofs?

Rafa: We did, yes, but… how did you know? You were not there!

Socs: You forget that I don't have to be somewhere in a physical form, because I'm a disembodied mind, what some people call soul.

Rafa: But a mind cannot exist without a body, Socs. You know that as well as I do.

Socs: True, you are absolutely right. But I don't need to have my own body to exist, I can exist in your brain.

Rafa: Are you saying that you are a phantom in my brain? Am I hallucinating?

Socs: Oh, no, I do exist independently of you. But for my physical existence, I am dependent on your body, just like a parasite that draws its sustenance from its host. And just as parasites and their hosts can benefit each other, I help you develop your intellect, and you allow me to exist. And I must say, I am really grateful to you for allowing me to exist.

Rafa: I'm not sure what you mean.

Socs: Okay, enough of that. Let us go back to what we were talking about, the distinction between assumption-based and observation-based proofs.

Rafa: What about them?

Socs: Ah, I'm going to ask you a question. Are ethical proofs assumption-based or observation-based?

Rafa: Ethical proofs? How can you prove anything in ethics?

Socs: Let us imagine that Ledo is a bomber pilot. He loves dropping bombs. So he flies around in his plane and drops bombs on random villages.

Rafa: And kills innocent people there? For no reason?

Socs: Yes. Would you say that what Ledo does is ethically bad, good, or neither?

Rafa: What? It is ethically abominable.

Socs: So your ethical judgment on Ledo's habit is that it is ethically bad.

Rafa: Definitely, no question.

Socs: Alright. Now suppose I asked you to defend your judgment. How would you do it? What reasons would you give for me to accept that judgment?

Rafa thought silently for a few minutes.

Rafa: Do you agree that destroying a life intentionally is ethically bad?

Socs: Yes, I do. I think all humans would agree.

Rafa: Okay, here is how I would try to convince you:

Premise 1: Ledo's habit of bombing intentionally destroys lives.
Premise 2: It is ethically bad to destroy a life intentionally.
Conclusion: It follows from (1) and (2) that what Ledo does is ethically bad.

Since you accepted premises (1) and (2), you must also accept the conclusion derived from them.

Socs: Excellent, Rafa! Do you realise that you have just given me an ethical proof?

Rafa: Hm, I was wondering about that.

Socs: Now, is an ethical proof observation-based?

Rafa: No, I don't think it is observation-based. Premise 1 is what we already know, it is shared knowledge. But my premise 2 is an assumption.

Socs: Can you defend that assumption? For that you have to give reasons to show why others should accept it.

Rafa: I am not sure I can. But you accept it, right? We all do. I haven't come across anyone who doesn't accept it.

Socs: So would you call it an ethical axiom? One that you accept, but cannot provide any rational defense for.

Rafa: Yes. (Thinks for a moment.) Ah, so this is an assumption-based proof!

Socs: So you defend an ethical judgment on the basis of ethical axioms. You use axioms in math too: to defend mathematical conjectures, right?

Rafa wasn't sure. For a while, he sat staring out of the window, without seeing anything.

Rafa: But they are different, right, axioms in ethics and axioms in math?

Socs: So what's the difference?

Rafa: I don't know, Socs. I need time to think about it.

Socs: Good. Time for me to go, Rafa.

With that, Socrates disappeared, leaving a confused Rafa to mull over the difference.

6 What did we Learn in this Chapter?

In the Chapters in Part 2, we were introduced to the basics of reasoning largely in the context of arriving at inferences. For this, we used the structure of Premises, Derivation, and Conclusion (PDC). We were also introduced to logic as the study of reasoning, involving Rules of Inference.

In these chapters, we briefly explored causal reasoning as a specific form of reasoning. We saw how important it is to not confuse causation and correlation, when providing evidence and arguments or evaluating evidence and arguments in support of claims. Causal reasoning is a staple form of reasoning used not only in the academic domain but also in our personal, professional and public life.

In this chapter, we looked at different forms of rational justification, that which convinces us of the credibility of a proposition; the usefulness of a product; the effectiveness and efficiency of an action, or practice to achieve a given goal; or the value of something.

We began by distinguishing between two kinds of justification. One kind is based on data or information, and is typically found in the physical, biological and human sciences. This is what we called observation-based proof. The second is justification based on axioms and definitions, found in mathematical and ethical inquiries. And we called this assumption-based proof.

There is a difference between assumption-based proofs on math and in ethics. Rafa is still trying to figure out the difference.

> *Proof* is an idol before whom the pure mathematician tortures himself. In physics we are generally content to sacrifice before the lesser shrine of *Plausibility*.
>
> Sir Arthur Stanley Eddington

> Don't take assumptions for granted. Begin by taking a skeptical attitude toward anything that is conventional wisdom. Make it justify itself. It usually can't. Be willing to ask questions about what is taken for granted. Try to think things through for yourself.
>
> Noam Chomsky

CHAPTER 11:

CRITICAL THINKING AND CRITICAL READING

1 Looking Back and Looking Forward

In the preceding chapters, we explored the different components of Academic Inquiry and Critical Thinking, with special focus on reasoning (Part 2), and on further tools of inquiry needed for theory construction (Part 3). We now *proceed* to pull together all the threads we have covered, in order to find out how we can develop the capacity for critical thinking, and use that capacity for critical reading and argument-aligned writing in the context of Higher Education, which can be extended to reading and writing in one's personal, professional, and public lives after graduation.

Suppose someone were to ask you: "What is the difference between the way you read an instruction manual for a gadget and the way you read a research article?" What would your answer be? Well, one answer is: "When reading a piece of academic writing, we are expected to engage with it critically. This is not expected of us when reading an instruction manual."

An instruction manual provides information that will be useful in using and maintaing the gadget. We don't ask: "Why should I accept what the manual is saying?" Nor do we read it with the goal of critically evaluating the information. However, that sceptical stance — a healthy distrust of authority — is precisely what is needed when we read a research article. That spirit of doubting and questioning is fundamental to the ethos of the rational temper, a characteristic of the academic culture, rooted in a deep awareness that human knowledge is uncertain and fallible.

Such skepticism ought to extend to reading textbooks as well. Most of us are trained to read textbooks to acquire the information in them, and apply it in examinations. Unfortunately, that mode of reading gets carried over to what one reads after graduation, and for those who go on to do research, when reading research literature as well.

In what follows, we will talk about critical thinking as something that ought to become a habit of our mind, and come naturally to us when we read textbooks and research articles. We will begin with a general framework for critical thinking in academic inquiry (including research), and then proceed to considerations relevant for critical reading.

The mental capacities and skills relevant for are common to critical reading extend to listening, speaking, and writing as well. By acquiring those capacities, we also learn how to write a newspaper article, a research article, and a proposal for a policy in private of government organisations.

2 Critical Thinking

2.1 What is Critical Thinking?

'Critical Thinking' is a buzzword in education today. Every educational institution, educationist, and educator swears by it. So it would be useful to ask: "What *IS* critical thinking?" and answer it in such a way that we can also answer the question, "What is *NOT* critical thinking?"

One widespread meaning of the word 'critical' in ordinary English is "expressing adverse or disapproving comments or judgments." So, when we say, "Zeno's work is of very poor quality," we are being critical of Zeno's work. The English words 'criticism' and 'criticise' have similar associations of disapproval. When we say,"Athena criticised Zeno's work," we mean, "Athena said something negative about Zeno's work."

This is not what we mean by 'critical' in 'critical thinking'.

The meaning of the term 'criticism' in what is called 'literary criticism' and 'art criticism' refers typically to what scholars call literary analysis and interpretation, and art appreciation. This also applies to film criticism. Thus, in the movie trilogy starting with *Matrix*, if we interpret the hero (Neo) as a Jesus symbol, and give reasons for that interpretation, that counts as film criticism.

Analysis and interpretation by themselves are not what we mean by 'critical' in critical thinking and critical reading, even though they may be a part of it.

What DO we mean, then? An Internet search for the term 'critical thinking' yields many different definitions of the concept. In this book, we adopt and extend the definition given by philosopher-educationist John Dewey, in his book, *How We Think*. He viewed critical thinking as "reflective thought," and defined it as:

> "active, persistent and careful consideration of any belief or supposed form of knowledge in the light of the grounds that support it, and the further conclusions to which it tends." (Dewey (1910): p. 6)

Dewey seems to restrict the concept of critical thinking to the context of beliefs, that is, judging the credibility (believability) of propositions. We will extend this concept to include the context of assessing the merit of any entity, action, or practice: credibility of propositions; effectiveness and efficiency of laws; ethical rightness of actions and practices; usefulness or beauty of things; and so on. The definition we will adopt, then, is:

> Critical thinking *is the process of careful and systematic reflection on the factors that go into the assessment of the merit of something.*

In what follows, we will first go through a few examples of critical thinking, and then proceed to critical reading, the process of thinking critically about the merit of what we read.

Outside of the academic context, all of us engage in a rudimentary form of critical thinking instinctively — when we try to decide which university we should apply to (or send our children to); whether total lockdown during COVID was a good thing; whether a rumour floating around about a friend is true; and so on. In every one of these cases, we are using critical thinking to decide what to believe, and what to do.

What we evaluate could be:

~ the **truth** of a statement
  (e.g.: Was Alexander the Great really a great king?)
~ the **credibility** of a theory
  (e.g.: Should we accept the Darwinian theory of biological evolution?)
~ the **quality** of a literary work
  (e.g.: What makes Shakespeare's *Macbeth* a great play?)
~ the **beauty** of a dance choreography
~ the **value** of the learning outcomes of a curriculum
~ the **merit** of an educational policy
~ the **efficacy** of a prescribed drug
~ and so on.

To evaluate the merit of anything that is complex, we need to:

- identify its function (in terms of truth, usefulness, ethicality …)
- determine how the components of what we are evaluating contribute to the function (e.g., the plot, the characters, and the dialogues of a novel; the axioms, definitions, conjectures, theorems, and proofs of a mathematical or conceptual theory), and
- examine how coherent the whole and the parts are (logical consistency being one of the strands of coherence) and how well aligned the parts are to the whole and to the function.

In the context of academic knowledge:
Critical thinking *is the mental process of evaluating*
the credibility of knowledge claims, and
the validity/soundness of the arguments in support of those claims.

We have seen many examples of critical thinking in Chapters 1-10, designed to develop critical thinking abilities. Here is one of those examples. You were introduced to critical thinking in Chapter 1, even though the term 'critical thinking' was not used there. Consider the three propositions that Samira writes on the board (page 14):

P1: No human being has lived for 500 years. (what we know)

P2: You and I are human beings. (what we know)

P3: You and I won't live for 500 years. (what we infer from 1 and 2)

She points out that P1 and P2 are premises advanced in support of the conclusion P3.

You are now ready to look back and evaluate the argument. Is this argument sound? That question prompts us to think critically about the argument.

In that conversation with Rafa, Samira asks if it is possible that either P1 or P2 is false. That question prompts subjecting P1 and P2 to critical thinking.

The question, "Is P3 true?" prompts us to think critically about the conclusion. To engage with that question, we need to think critically about the truth of P1 and P2, as well the validity of reasoning that derives P3 from P1 and P2. Reflect on this example of critical thinking.

Before you read further: We encourage you to go back to the previous chapters and look through them, and identify critical thinking tasks.

2.2 *Examples of Critical Thinking*

2.2.1 Assessing the Merit of Arguments

[In the dialogue below, Anu, Rafa, and Neel explore the problem posed in Chapter 5, Section 4: Summary, Task B. Before you read further, go back to that task, reflect on it again, and formulate your own conclusion.]

Rafa was watching a tennis game during recess. Anu joined him, notebook in hand.

Anu: Hey, have you seen Neel? I thought he was with you.

Rafa: I haven't seen him. Why?

Anu: Do you know Malina?

Rafa: Should I? I'm not sure. But what about her?

Anu: She gave me a proof to show that mushrooms are invisible.

Rafa: What? That's nonsense. We can SEE mushrooms. How can they be invisible?

Anu: But wait till you see the proof she gave me. Our textbook says that mushrooms are fungi, and we accept that, don't we?

Rafa: That is right.

Anu: And the same chapter in the textbook says that fungi are microorganisms, and we accept that, don't we?

Rafa: Yes, that too.

Anu: Good. And in the same chapter, the textbook also says that microorganisms are invisible to the naked eye. So here is the proof.

Anu showed Rafa the proof she had written down in her notebook, with premises (P) and conclusion (C).

P1: Mushrooms are fungi.
P2: Fungi are microorganisms.
P3: Microorganisms are invisible to the naked eye.
C1: Mushrooms are microorganisms. [from P1 and P2]
C2: Mushrooms are invisible to the naked eye. [from C1 and P3]
(QED)

Rafa stared at the proof, and shook his head. There was something wrong with it, but he couldn't figure out what. “Let’s go find Neel and ask him,” he said. When they found Neel, he too stared at the proof, and felt that there was a flaw in the proof somewhere, but couldn't diagnose the problem.

Before you read further: Ask yourself if you accept the proof.
If you don't, why not?

That night, Rafa was brooding over the mushroom problem, when Socs glided in through the open window. Rafa was surprised: Socs had never visited when he was working.

Rafa: Am I asleep?

Socs: Ooh, no, you’re wide awake.

Rafa: So you’re not in my dream?

Socs: (smiling) You and I, we’ve been engaging with each other. And that’s made my being stronger. So now I can be around, outside your mind! And I’m here, my boy, because you have a question for me.

Rafa took some time to recover from the shock. When he did, he was staring at Socs.

Socs: So, you expect me to tell you what is wrong with your mushroom proof?

Rafa: No, I don't. You are just like my mother. She won't give me answers, she’ll help just enough for me to find an answer. So, are you going to give me a clue?

Socs: (laughs) Ok, Rafa. Here is a clue. You know that mushrooms are visible. But if you accept the textbook premises, you can't avoid the conclusion that they are invisible.

Rafa: But how can that be? It’s a logical contradiction.

Socs: Yes! So what do you do?

Rafa: Hmmm. Either I must correct my belief that mushrooms are visible, but that is ridiculous, because I can see mushrooms. Or I have to correct at least one of the statements in the textbook.

Socs: Good. So, which statement will you change?

Rafa: Let me think. The textbook says, “All microorganisms are invisible to the naked eye.” Suppose I change that to: “Most microorganisms are invisible to the naked eye.” Then we won't have the conclusion that mushrooms are invisible.

Socs: That’s good, Rafa. Is there some other way of getting the same result?

Rafa closed his eyes, deep in thought. Something was emerging in his head.

Rafa: Let me see if this will work. Salt is visible to the naked eye, right? But is each salt molecule by itself visible to the naked eye? No. What if each microorganism is invisible to the naked eye, but the aggregate of microorganisms is visible, like the aggregate of salt molecules? And what you see as a mushroom is the aggregate, not the individual fungus cell.

Socs: You never cease to amaze me, child! What you’ve done is to find a flaw in your textbook and suggest two ways to correct it. How did you find the flaw?

Rafa: Actually, Malina gave a proof that disturbed Anu, and she told me about it. And when we told Neel, he felt that there was a flaw in the argument. Given the textbook premises, Malina’s reasoning was flawless. But the conclusion contradicted what my eyes tell me. I can’t change what I can see with my eyes, so I had to change the textbook premises.

Socs: Very good, Rafa, I’m glad you are thinking critically! You know that an unexamined life is not worth living.

He clapped, and poof, disappeared through the window in a whiff of smoke.

EXERCISE 1

The words *civilized* and *civil* are seen in positive light, and *civilization* as a good thing. These words are seen as the opposite of *savage* and *barbarian.*

Consider the concept of Ancient Civilizations (Egyptian, Sumarian, Aztec, and so on). What exactly was good or valuable about them? A brief look at the information about them shows their shared characteristics:

centralised autocracy
military power
conquests through wars
economy based on slavery and exploitation of natural resources
taxes extracted solely for the royalty
brutal punishment of the poor for not being able to pay taxes
and so on.

TASK: Think about and discuss these questions:

~ Are the above traits something you would admire as a mark of *civilization*?

~ Were Ancient Civilizations worthy of being called civilised?

2.2.2 Assessing the Merit of Institutions

Anu, Neel, and Rafa had started thinking about university education. Which university should they apply to after high school? Anu brought it up with her mother, Ila, who suggested that she talk to their neighbour, Sanjudidi, who was a researcher.

Anu talked to Rafa about it, who brought it up with Samira. Samira decided to invite Sanju for dinner on Saturday, along with Ila, Neel and Anu. And during dinner, Anu raised the question.

Anu: Sanjudidi, which are the top universities in India? Or maybe even abroad! I have heard that Harvard and Oxford are the best universities in the world. Is that true?

Sanju: (after thinking for a little while) What exactly is your question, Anu? Are you asking which universities would give you the best learning experience? Or are you asking me which university will give you a certificate that will help you get a job with the highest possible salary?

Anu: What's the difference?

Sanju: You know, universities get ranked internationally. That's what people usually look at to find the 'top' universities. There are scores of organisations and companies that do annual rankings. Now, if you do an Internet search for university ranking, you will find the criteria they use, like the number of publications the faculty have, the faculty-student ratio, reputation among employers, and so on. These are criteria that tell you how famous a university is. But none of the rankings talk about the value of what the students learn, or how effectively the faculty help students to learn. So they won't tell you much about the value of what you will learn from a particular university. Which is more important for you?

Anu: Give me a minute.

No one spoke. Everyone was busy thinking.

Anu: I don't know about the others, but I think what's really important for me is what I will learn, and how well I will learn it. All those things like how many articles they publish or what employers think are not relevant for me. So how do I find out which university helps us learn what is valuable for us in life?

Sanju: I really don't know. Let me see what I can find out.

Samira: You know, if you spend some time on the question, "What is high quality learning?" you might find an answer yourselves, all three of you!

Rafa: Mom, that's tough. We need time to think.

Samira: Take your time. Let's have dinner together next Saturday. You can present your ideas to us after dinner.

Neel: Deal!

Before you read further: Take up the challenge that Samira posed to Rafa, Anu, and Neel. If possible, talk amongst your friends about it, and come up with a group proposal.

That night, as Rafa thought about Samira's challenge, he remembered that a few weeks earlier, Samira had said, "Rafa, how would you decide if someone is an excellent driver? Or an excellent teacher?" Rafa hadn't paid much attention then, but the question had stayed with him. Remembering the incident now set him wondering about the concept of excellence. What do we mean by excellent when we say, "excellent restaurant"? On what basis do we judge a restaurant to be excellent or just good, or mediocre? What makes a poem excellent? What criteria do we use to judge Wordsworth to be a better poet than Coleridge, or Coleridge to be a better poet than Wordsworth? What attributes do we look for in an excellent leader?

Socrates didn't come that night. Rafa tossed and turned in his bed, thinking about the criteria for excellence in different domains. He decided that he needed a discussion on it with Anu and Neel over the weekend.

3 Critical Reading

3.1 What is Critical Reading?

Critical reading is closely tied to critical thinking. It is engaging critically with what we read, going beyond mere reading comprehension. The concepts of claim, justification, evidence, arguments, reasoning, premises, and conclusion are central to critical thinking as well as to critical reading.

The activity of critical reading presupposes the capacity for reading comprehension. To get a sense of what that means, take a look at: "How to Read Non-Fiction," at https://www.thinq.education/post/how-to-read-non-fiction.

We may define Critical Reading as follows:

> Critical reading *is the process of applying critical thinking to what one reads in order to* arrive at *an assessment of its merit.*

What we say about critical reading applies equally to critical listening: it is important to engage critically with what is written, or spoken.

Now, the criteria we use to assess the 'merit' of a piece of spoken or written text would depend on the function of the text. We don't use identical criteria to assess the merit of different kinds of writing. For example, take the following pieces of text, all of which are on snakes:

- i) D H Lawrence's poem, "Snake" (https://www.poetryfoundation.org/poems/148471/snake-5bec57d7bfa17)
- ii) the Wikipedia entry, "Snake," as a taxon in biology; (https://en.wikipedia.org/wiki/Snake)
- iii) a research article on snakes in a research journal; and
- iv) a class lecture on snakes.

Nor do we use identical criteria for assessing the merit of:

- v) an autobiography
- vi) an advertisement,
- vii) a newspaper report,

and so on. Each of these performs a different function, and hence their critical evaluation calls for different sets of criteria.

We won't take you through all these different types of texts in this introduction to critical reading. We will deal only with texts that seek to communicate a body of propositions that claim to be collective knowledge.

As an example, consider a prosecution lawyer making an argument for the claim that the accused is guilty:

> Your honour, the forensic specialist has concluded that the blood stains on the knife are those of the victim, and that the fingerprints are those of the accused. The specialist is a person of high repute, and has published many articles in reputed journals in the field, so we must take her as a credible expert witness. Hence, we must accept the expert's conclusion that the blood stains are those of the victim, and the fingerprints are those of the accused.
>
> If this was the knife used to kill the victim, that explains the blood stains on the knife. Since we do not have an alternative explanation, we must conclude that the knife is the murder weapon.
>
> If the accused is the murderer, that explains the fingerprints of the accused on the murder weapon. Since we do not have an alternative explanation, we must conclude that the accused is the murderer.

This is an example of a text that makes a truth claim. When engaging critically with such texts, we need to ask:

A. What is/are its *central claim(s)* ?

B. What justification does it advance in support of the claim(s)?

C. Is the *justification* sound? That is:

   Are the *premises credible*?

   Does the conclusion *follow logically* from the premises?

D. Are there logical contradictions in the text?

E. Are there *considerations not mentioned in the text* but are relevant to the evaluation of the claim?

   Would they strengthen or weaken the claim?

F. Having considered all of the above, *should we accept the claim, reject it, or set it aside for further scrutiny?*

3.2 Examples of Critical Reading

3.2.1 Modern Health Care

While thinking about critical reading, Rafa remembered a critique in his 7th grade social science textbook about health care and the government's role in it. What he read had made him distinctly uneasy. He found his old textbook, and located the passage:

> "In spite of advances in modern medicine, the major health problems in India remain unabated. Take tuberculosis. About five lakh people die from tuberculosis every year. This number is almost unchanged since Independence! Almost two million cases of malaria are reported every year and this number isn't decreasing."

He showed the passage to Neel.

Neel: What was the population in India at the time of Independence?

Rafa did a quick Internet search. He found a Wikipedia entry on the 1951 census in India, which said that the population at that time was 361 million.

Neel: So, the 50,000 who died of tuberculosis was about 0.14 percent of the population?

Rafa: That looks approximately right.

Neel: And when was the textbook written? 2005? 2010? And what was India's population at that time?

Rafa: I think the book is from 2005. The internet says the population was 114.41 crores. That would be a bit more than 1144 million.

Neel: So the percentage of the people who died in 2005 was about 0.00004?

Rafa: I guess so. But why are you making all these calculations?

Neel didn't say anything. He just looked at Rafa with a raised eyebrow. In a second:

Rafa: Oh! Oh, so what the data in the textbook show is just the opposite of what the textbook asserts about it!

> Before you read further: Why does Rafa say: "What the data in the textbook show is just the opposite of what the textbook asserts about it"?

3.2.2 Ancient Medicine

Rafa: You know that my uncle is an Ayurvedic doctor, right? He keeps talking about the idea of *tridosha* in the Ancient Indian system of medicine, and how it is superior to modern medicine. I asked him about it, and he talked about these things called *vaata*, *pitta*, and *kapha*. I asked him what those words meant, and he gave me some mumbo-jumbo about five elements which I didn't understand. Do you know what these things are?

Anu: Oh yes, I've heard Mom mention those words, but I don't know what they mean.

Neel: Let's see if the Internet has anything to say about it.

Neel opened his laptop and started a search.

Neel: Goodness gracious! There are hundreds of sites talking about these words. Most of them just repeat the same mumbo-jumbo. I don't think they know what they are talking about.

Anu: Click on that one, Neel.

Neel: Which one?

Anu: The one that says Tridosha the Science of Ayurveda. It says, 'California College of Ayurveda'.

Neel: I didn't know there was a College of Ayurveda in California. Interesting. (Clicks on the link). Hm, here, it says:

> "Tridosha defines the three fundamental energies or principles which govern the function of our bodies on the physical and emotional level. The three energies are known a vata, pitta, and kapha. Each individual has a unique balance of all three of these energies. Some people will be predominant in one while others are a mixture of two or more."

Rafa: Tridosha as energies? Really?

Neel: Let's read a bit more. It says:

> "Vata is very much like the wind — it is light, cool, dry and mobile. In the body, those people with a vata nature experience more of these qualities. Their bodies tend to be light, their bones thin, and their skin and hair dry. They often move and speak quickly. When out of balance, they may lose weight, become constipated and have weakness in their immune and nervous systems.
>
> "These qualities also reflect in their personality. Those with a vata nature tend to be talkative, enthusiastic, creative, flexible, and energetic. Yet, when out of balance they may also become easily confused and overwhelmed, have difficulty focusing and making decisions and have trouble sleeping. This becomes more apparent when they are under stress. Emotionally they are challenged by cool emotions like worry, fear, and anxiety."

Anu: They seem to be saying that *vaata* is some kind of abstract property. You can't observe vaata, but you can observe its consequences, and check the correlations among them.

Rafa: What correlations?

Anu: I need to think. Let's write this down so we can check. Given what the site says, the correlations are (writes on the board):

1) Humans who have light and thin bones are likely to have dry skin and dry hair.
2) Humans who have dry skin and dry hair are likely to move and speak quickly.
3. Humans who have light and thin bones are likely to move and speak quickly.
4) Humans who have light and thin bones are likely to lose weight, become constipated, and have pathologies of immune and nervous systems.
5) Humans who have light and thin bones are likely to be talkative, enthusiastic, creative, flexible, and energetic.
6) Humans who have light and thin bones are likely to become confused and overwhelmed, have difficulty focusing and making decisions, and have trouble sleeping.

Neel: Do we know if there is evidence for these correlations?

Anu: I don't know, but surely they are not talking mumbo-jumbo, they are making claims about what we can observe, and any medical researcher can test them.

Neel: That's really interesting. Let's see if anyone has done any research to test these claims.

Rafa: I'll ask my uncle if he has seen any research on this.

Anu: The ideas are so interesting, I hope there is some research on them. When I first heard these words, I thought vaata, pitta and kapha are substances in the body.

Neel: Yes, if they were substances, we should be able to observe their molecules in the body. But it seems they are an abstract concept of energy in biology. And there are different types, just like the different types of energy in physics.

Rafa: Ah, that leads to another thought.

Rafa sat silently, and Anu and Neel sat watching him. After a while, Anu coughed.

Anu: An apple for your thoughts, wise sage!

Rafa: I was wondering. What you said about *vaata*, *pitta*, and *kapha* might be extendable to the five elements theory too — the *panchabhuuta*s. I've always thought of water, earth, and air as substances, as in modern science. If we accept that, we must reject the ancient scholars' view that air and water are elements. But you know, maybe we don't understand what they mean. What if they didn't think of the so-called *bhuuta*s as substances, but as abstract attributes?

Anu: That's a brilliant thought! Maybe for them, 'water' is not a substance made out of hydrogen and oxygen, but the attribute of wetness. And 'earth' means solidity, sort of like the planet.

Neel: Interesting. What we learn now is, "If we accept what modern science tells us, we must reject what the Ancients said." Instead we should be saying, "We don't understand the concepts that the Ancients referred to in their *panchabhuuta* theory, so we can't critically evaluate their theory." Which means that we can't reject it outright.

Anu: Great way to formulate it, Neel!

EXERCISE 2

TASK: Here are three passages. Read them carefully, and for each passage answer questions (1)–(3). The questions are meant to guide the reading. Keep them in mind any time you read something critically.

QUESTIONS

1. What is the claim presented in the passage?
2. What justification does the text present to support the claim?
3. Does this justification support the conclusion? Spell out your reasons.

PASSAGES

(a) Many people think that smoking causes cancer. This is obviously wrong. My grandfather was a chain smoker who started smoking when he was fifteen, and smoked nearly two packets of cigarettes a day till he died at the age of eighty six, without the least trace of cancer. In contrast, a forty five year old cousin of mine died of cancer two years ago. He was a non-smoker. There is no connection between smoking and cancer.

(b) Ayurvedic medicines are better than allopathic medicines because they are derived from plant sources.

(c) Even though he is a poorly paid schoolteacher, Aman Shaw is an extremely intelligent person. Throughout his school days, he stood first in his class. He scored the highest in his board exams for both Classes 10 and 12, as well as in the IIT entrance examination. After his BTech degree, however, he decided to become a school teacher.

EXERCISE 3

The passage below is by Walter Francis Wilcox, chief statistician, United States Census Bureau, and professor of social science and statistics at Cornell University. It appears in the 11th edition of Encyclopedia Britannica (1911). Read it carefully, and answer the questions below it:

> "Mentally the negro is inferior to the white. The remark of F. Manetta, made after a long study of the negro in America, may be taken as generally true of the whole race: "the negro children were sharp, intelligent and full of vivacity, but on approaching the adult period a gradual change set in. The intellect seemed to become clouded, animation giving place to a sort of lethargy, briskness yielding to indolence." We must necessarily suppose that the development of the negro and white proceeds on different lines. While with the latter the volume of the brain grows with the expansion of the brainpan, in the former the growth of the brain is on the contrary arrested by the premature closing of the cranial sutures and lateral pressure of the frontal bone. This explanation is reasonable and even probable as a contributing cause; but evidence is lacking on the subject and the arrest of even

deterioration in mental development is no doubt very largely due to the fact that after puberty sexual matters take the first place in the negro's life and thoughts. At the same time his environment has not been such as would tend to produce in him the restless energy which has led to the progress of the white race; and the easy conditions of tropical life and the fertility of the soil have reduced the struggle for existence to a minimum."

QUESTIONS:

1. What is the main claim of this passage?
2. Does the testimony from F. Manetta constitute credible grounds for the claim? Articulate the argument.
3. Does the reference to the anatomical development of the brain among negros and whites constitute an argument for (1), or an explanation for some statement? If it is an argument, state it clearly. If it is a statement, articulate that statement.
4. Is the statement on the differences in the anotomical development of the brain of negros and whites credible? What are your reasons?
5. What is the function of the reference to the life of the negro (thoughts about sexual matters, easy life in tropical areas)? If you think this is an argument, clearly state the grounds, reasoning, and conclusion.
6. Do you find the justification sound? Spell out your reasons
7. Would you accept the central claim(s), reject it/them, or keep it/them on reserve? Spell out your reasons.

3.3 An Extended Example

We now proceed to an article from a prestigious journal for an experience of critically reading a research article:

"The neuroscience of mindfulness meditation," (2015) Yi-Yuan Tang, Britta K. Hölzel, and Michael I. Posner, *Nature Reviews Neuroscience*, downloadable at https://www.researchgate.net/publication/273774412_The_neuroscience_of_mindfulness_meditation)

This article is a critical review of the research literature on the neuroscience of mindfulness meditation. Read the article, keeping in mind the following questions:

1. What are the authors' definitions of (a) mindfulness, and (b) mindfulness meditation?
2. What is the central claim in this domain of research whose credibility the authors seek to evaluate?

3. According to the authors, what are the brain regions that are relevant for the investigation of the neuroscience of mindfulness meditation? What are the mental correlates of each of these regions?
4. What are the conclusions that the authors arrive at on the basis of their review? Are these conclusions supported by the evidence they discuss?
5. Having read this article carefully, what did you learn from it? Does it make you change any of the positions you subscribed to before reading it? If yes, what are they? And what are the reasons for the change?

4 Fake News and Fake Knowledge

Based on the discussions in the chapters so far, let us define academic knowledge as:

> Academic knowledge is a *body of propositions that the academic community judges to be true with certainty beyond reasonable doubt on the basis of sound rational justification.*

Given this definition, it makes sense to say that:

> Critical reading calls for the capacity to judge the credibility of what one reads.

Let us now take a look at critical reading in the context of what one finds on the Internet such that we can distinguish between credible ('true') propositions and not credible (not 'true'/'false) propositions, especially when the falsehood is deliberate, and its purpose is to deceive. Whe this happens, the false proposition is a lie.

If you do an Internet search on the expressions 'fake news', 'misinformation', 'disinformation', 'pseudoscience', 'fake science', 'fake knowledge', and 'pseudo-knowledge', you can get a sense of how, with the advent of the internet, falsehood and lies have become a serious threat to humanity. As a case in point, consider what the site *Internet Matters* (https://www.internetmatters.org/) says:

> "With so many sources of information online, it has become difficult to make sense of what content is based on fact, half-truths or lies. The use of digital platforms to share things we believe to be true when they may not be can have a powerful ripple effect, influencing others to see them as facts.
>
> This can be especially dangerous for children and young people who can be persuaded to take on distorted views of the world that could cause them or others harm in the real world. This page explains what fake news is and how it can impact those who see it."
> https://www.internetmatters.org/issues/fake-news-and-misinformation-advice-hub/learn-about-fake-news-to-support-children/

The site then proceeds to give the following definitions:

Fake news: News or stories on the Internet that are not true. They may be in the form of disinformation or misinformation.

Disinformation: False information that is created and shared to deliberately cause harm.

Misinformation: Generally used to refer to misleading information created or disseminated without a deliberate intent to cause harm.

For a comprehensive account, take a look at the Wikipedia entry on Fake News, which goes into the details of how propaganda is presented as news, how fake news often has the aim of damaging the reputation of a person, or making money through advertising revenue, and influencing public opinion and the outcomes of elections.

The Wikipedia entry also says:

> "The prevalence of fake news has increased with the recent rise of social media, especially the Facebook News Feed, and this misinformation is gradually seeping into the mainstream media. Several factors have been implicated in the spread of fake news, such as political polarization, post-truth politics, motivated reasoning, confirmation bias, and social media algorithms."

All the intellectual tools we have illustrated in the preceding sections are relevant for protecting oneself against the cluster of toxins pointed to above.

For further protection, we would recommend reading the article, "No, you're not that good at detecting fake videos – 2 misinformation experts explain why and how you can develop the power to resist these deceptions," at (https://theconversation.com/no-youre-not-that-good-at-detecting-fake-videos-2-misinformation-experts-explain-why-and-how-you-can-develop-the-power-to-resist-these-deceptions-217793)

In our discussion of critical reading, we pulled together the relevant concepts discussed in the previous chapters to bring to bear on the challenge of reading, understanding, and critically evaluating what we read, including textbooks, research publications, and the media. When engaged in the activity of reading, it is not sufficient to 'comprehend' what a document says, but to place it against our prior knowledge, critically evaluate what it says, decide whether to accept its claims, and, identify logical contradictions if any, between what the text claims and what we already believe. The purpose of these mental activities is to figure out whether to reject the claims, or to revise our prior beliefs.

Developing the capacity for critical reading calls for a sustained investment of effort over an extended period of time.

This chapter began with relatively simple one sentence or one paragraph examples of critical thinking and critical reading, and then proceeded to look at complex examples, including a research article.

5 What did we Learn in this Chapter?

In this Chapter we defined critical thinking and critical reading as:

> Critical thinking *is the process of careful and systematic reflection on the factors that go into the assessment of the merit of something.*

> Critical reading *is the process of applying critical thinking to what one reads in order to* arrive at *an assessment of its merit.*

We went through several examples of critical thinking in general, and of its specific application to critical reading in engaging with written or spoken texts that seek to communicate a body of collective knowledge. The activities in our examples of critical thinking (including critical reading) drew upon the tools of inquiry, including categorizing, clarifying concepts, defining, looking for counterexamples and counterevidence, identifying and articulating logical consequences, looking for logical contradictions, checking the validity of arguments, checking the credibility of the premises in it, and so on. In doing so, we were bringing together the different tools that you were introduced to in the previous chapters.

> I don't want people to say, "Something is true because Tyson says it is true.' That's not critical thinking.
>
> Neil deGrasse Tyson

The idea that something must be so because Newton or Einstein said so is simply not scientific. So an inquiry has to proceed from a state of openness with respect to the question at issue and to what the answer might be, a state of mind which I think of as healthy skepticism. This kind of openness can make individuals receptive to fresh insights and new discoveries; and when it is combined with the natural human quest for undrstanding, this stance can lead to a profound expanding of our horizons.

His Holiness the Dalai Lama

The Universe in a Single Atom: The Convergence of Science and Spirituality (2005)

CHAPTER 12: CONSOLIDATION

1 Looking Back

In Chapter 11, you were introduced to the art and craft of critical thinking. We suggest that you now practice what you learnt there, by thinking critically about this book. In Section 2.2.2 of that chapter, we discussed assessing the merit of institutions. We suggest that you pick up the thread we initiated there, and extend it to assess the merit of this book.

The exercise would be a good experience for you in two ways. First, it would nudge you to engage in metacognitive reflection and help you synthesize what you have learnt. Secondly, it would initiate the practice of thinking critically about your educational experience in general, helping you become a more intelligent learner.

So for a task in *METACOGNITION*.

> COGNITION, we said, is 'knowing'. And to COGNISE is to know.
>
> METACOGNITION is 'knowing about knowing'.
>
> For reflecting on our own processes of knowing, we need a conceptual understanding of:
> - ~ the nature of knowledge,
> - ~ ways of arriving at knowledge, and
> - ~ ways of evaluating that knowledge.
>
> One of the things this book has been about is such METACOGNITIVE REFLECTION.

Now that you have gone through Chapters 1 to 11, it would be good to step back and ask yourself:

A. What have I learnt from this textbook? (What are my takeaways?)
B. How has this learning been different from other textbooks I have come across?
C. How valuable would the learning be for me after education, say, ten years later?
D. Can this textbook be made even better? How?

Before you read further: We would like to suggest that you take up the challenge of evaluating the merit of this book in terms of questions A-D. Having done this on your own, it would be a good idea to discuss your responses with others who have read the book.

2 What is Distinctive about this Textbook?

2.1 Fundamental Questions for Curriculum Design

Once you have formulated your responses to A-D based on your experience, we can compare YOUR responses to (B) with OUR perception of what is distinctive about this textbook.

When designing a textbook, the writers need to ask three important questions:

I. WHAT do we want learners to learn?

What do we want the learners to understand, and be able to do? What habits, predispositions, attitudes, and values do we expect them to learn? This question is about the *goals* of the book: the expected *learning outcomes*.

II. WHY do we want the learners to learn that?
This question is about the purpose and value system that underlies the book, the *philosophy of education* underlying the goals of the book, and the means used to achieve them.

III. HOW do we help learners to learn what we want them to learn?
This question is about the *means*, the pedagogical strategies the book uses. This includes the sequencing of chapters and sections; the learning tasks (e.g., exercises, fun things to do, ..), and the exposition of concepts.

We will try to communicate to you our answer to (I), such that you can take that into account when you evaluate the textbook.

2.2 Learning Outcomes

2.2.1 Understanding

Rafa, Anu, and Neel were enrolled in a course on Inquiry, and Critical Thinking. At the end of the course, they decided to have lunch together on a Sunday to discuss their experience of learning. They also decided to invite their mothers, Samira and Ila, to join them. Rafa was the first to speak.

Rafa: Moms, I want to ask you something. Now that we've completed the course, do you think we've learnt something valuable from it?

Ila: How do you expect us to know what has changed inside your mind?

Neel: Ma, you can tell by our behavior, which you can observe. Do you find any significant change in the way we behave that tells you we've learnt something?

Ila looked at Samira, with a "do-you-want-to-respond?" look. Samira gently shook her head.

Ila: I am a bit hesitant to answer that question. To make an assessment of what you've learnt, you need to look at its effect on your mind a few years from now, not immediately after a course. If what you've learnt evaporates in five years, no matter how much of it contributes to your exam, what you've learnt has no value to your life.

Anu: Are you saying that the exams don't test what we've really learnt?

Ila: (smiles) That's exactly what I'm saying.

Neel: I see the point you are making, Ma, but can you make an informed guess on how much of what we've learnt is likely to stay with us? And what may even develop further, and be of value to us five years from now?

Before you read further: Take Neel's question about the course, and make it a question on this textbook. Can you engage with it? Write down the conclusions you arrive at and discuss them with others who have read the book.

The table of contents of an introductory physics textbook is typically a list of topics like classical mechanics, optics, electricity, magnetism, statics, thermodynamics, and so on. An introductory math textbook would list topics like geometry, number theory, calculus, statistics and probability, graph theory, and so on. A world history textbook would contain topics like Stone Age, Metal Age, domestication of fire, agriculture, ancient civilizations, middle ages, renaissance, enlightenment, industrial revolution, and so on.

In this textbook, you must have noticed that the concepts you are expected to understand do not belong to any particular discipline. They belong to all disciplines. In this sense, they are trans-disciplinary concepts; they are relevant across disciplines and fields of specialization.

Let us take an example. As mentioned earlier, the concept of atomic structure belongs in physics; the concept of molecular structure belongs in chemistry; the concepts of cellular structure and skeletal structure belong in biology; and the concept of the structure of a sonnet belongs in literary studies. These are all discipline-specific concepts. But the concept of STRUCTURE is trans-disciplinary; it is relevant in all disciplines. Similar remarks apply to the concept of CHANGE, manifested as physical change, chemical change, developmental change, evolutionary change, historical change, social change, and so on.

This textbook has been an introduction to trans-disciplinary abilities of academic inquiry and the trans-disciplinary concepts underpinning those abilities. These concepts serve as tools of inquiry, and some of them appear as chapter titles in the book.

You also encountered the concepts of logical consequence, logical contradiction, premise, conclusion, derivation of conclusions from premises, deductive reasoning, sample to population reasoning, and causal reasoning, and a distinction between correlations and causes. Do take a look at the concepts listed in the glossary of terms at the end of the book, and reflect on what you have or have not learnt, or may not have learnt in as much breadth and depth as is expected.

An important aspect of understanding, especially in the academic domain, is critical understanding. By this, we mean an understanding of the reasons in support of or against the ideas put forward as knowledge. For instance, science textbooks tell us that air is a mixture, water is a compound, and gold is an element. But the Ancient Greeks and the Ancient Indians took air and water to be elements, and gold as not an element. Why should we accept what our textbooks say, and reject what those ancient authorities said? Answering this question would require a critical understanding of the concepts of elements, compounds, and mixtures, and of what the ancients might have meant by them.

Consider this. A glass of water can be divided into two half glasses of water, and each of them can be further divided into two quarter glasses of water, and each of those into one eighths. Can this process of dividing go on endlessly?

The Greek philosopher, Democritus (4th c. BCE), held that matter cannot be divided and sub-divided infinitely; that the breaking up of matter comes to an end when we get to the indivisible units of matter called *atomos*. We find the same idea in Ancient Indian thought, attributed to a philosopher called Kanaada.

In contrast to Democritus and Kanaada, Aristotle (4th c. BCE), another Greek philosopher, held that any body of matter, however small, can be divided and subdivided infinitely. According to him, this process never ends.

We don't know if the idea of atomicity originated in Ancient India or in Ancient Greece, but it was revived by John Dalton at the beginning of the nineteenth century, and is now the received wisdom in all school textbooks.

Once again, it is important to ask why we should accept what the school textbooks say, and reject what an ancient authority like Aristotle said. Answering this question too would require a critical understanding of certain concepts, in this case, the atomic and non-atomic conceptions of matter.

We can make similar remarks about the helio-centric (sun-centered) vs. geo-centric (earth-centered) models of the solar system. Why should we believe that the earth revolves around the sun, though we experience the earth as stationary? Coming to the evolutionary theory in biology, why should we believe that all life forms on the earth evolved from the same ancestors? And in human history, why should we believe that the Stone Age preceded the Metal Age in all human cultures?

You must have noticed that though this book aims at understanding, it is not designed to give you information in any of the 'subjects' taught in a college or university. The book does expect you to have the kinds of information that a student in 10th Standard in school must have. For instance, we expect you to know the formula for the area of a triangle; that water is a compound; that Darwin proposed that chimpanzees and humans have the same ancestry; that there were two world wars in the twentieth century; and so on. But this is only part of the background necessary for engaging in mental activities aimed at developing certain abilities, attitudes, habits, and values.

Before you read further: Make a list of the trans-disciplinary concepts of knowledge and inquiry that you have learnt. Make a list of the concepts and propositions on which you have a critical understanding.

Compare what you have written with what your friends have written, discuss the differences, and revise your list if necessary.

2.2.2 Abilities

While the children and their mothers were talking animatedly about the course, Samira felt the presence of someone watching them. She turned around, and was startled to see an old gentleman standing there, bald, in strange clothes, with a long white beard.

Samira: Who are you? And how did you get in here?

Socs: I am Socrates. And that should answer your second question as well.

Samira: Socrates, the Ancient Greek philosopher?

Socs: The very same.

Samira: What? Rafa has been talking about long conversations with a Socrates. But I never took it seriously. I thought ...

Socs: You thought I was just a figment of his dreams? Hardly so. I am alive as long as people engage with my ideas, like Rafa does.

Samira: But that doesn't mean you're a flesh-and-blood person… Oh, forget it, Socs. We are just delighted to have you join us. Do sit down. Would you like a cup of tea? And some samosas?"

Socs: Thank you, Ma'am! I can't resist that invitation!

Socs sat down at the table.

Socs: So, you were talking about the kinds of understanding that the children have got from this course. But, what about the abilities? What mental capacities did they develop? I'm talking about what would be needed even many years from now, all through their life.

Samira: I was going to bring that up myself, Socs.
(turning to the children) So, what is your answer?

Anu: We need time to think about it.

Rafa: Can we go to my room and talk?

The children went to Rafa's room, leaving the adults at the dining table, chatting.

Before you read further: Answer Socrates' question. Write down your answers, and compare them with your friends' answers. Discuss the differences, and revise your list if necessary.

The children were soon back in the dining room, looking pleased.

Anu: That was easy, Socs. In a nutshell, we would say that we learnt how to inquire.

Ila: I don't see what's in that nutshell. Can you give some examples?

Anu: Sure. We learnt how to define concepts: we can say that we developed the ability to define clearly and precisely, and to critically evaluate definitions. We also developed the ability to classify and subclassify, to justify our classification, and to critically evaluate classificatory systems. We learnt how to generalize; how to construct and to evaluate proofs; to detect logical contradictions.

Ila: Okay, I get it. That's good!

Samira: I'm wondering. Did you learn anything other than these concepts and abilities?

Before you read further: Write down your answer to Samira's question, and compare what you have written with what your friends have written, discuss the differences, and revise your list if necessary.

2.2.3 Habits, Predispositions, Attitudes and Values

Rafa: What do you have in mind, Mom?

Samira: Was there something else you learnt that you think is valuable for you? Beyond the concepts and abilities?

Rafa: I think I see what you mean. And I also see why Socs was so interested in our discussions. When he was being sentenced to death, he stood up to them and said, "An unexamined life is not worth living." That's an amazing statement. And he said that to me too one day! Maybe that's the most important lesson we learnt.

Socrates sat beaming, obviously pleased.

Ila: I am not sure I understand. What does that mean, an "unexamined life"?

Neel: Let me try, Ma. You know, most of us don't examine what we believe. We take a lot of things for granted, and don't ask why. Our parents, teachers, textbooks, experts, and governments tell us something, and we swallow it. And if our experience tells us something, we believe that too, without asking why.

Rafa: Yes. There was a time when the authorities in the West said that black people are inferior to the white people. A famous Prof in Cornell University even wrote that in the *Encyclopedia Britannica*. And people just believed it, without asking: Is this true? How can we decide for ourselves if it is true or false?

Neel: Or try this. In our experience, the earth is completely stationary. And people who haven't been to school take that for granted. When people go to school, their textbook tells them that the earth is not stationary, that it revolves around the sun, and rotates on a tilted axis. Without a moment's thought, they abandon what their experience tells them, and accept what the textbook says. We need to doubt both statements, examine them both carefully, and decide for ourselves what to believe. It seems to me that's what Socs means.

Anu: Yes, he means, if you lead a life without carefully examining those beliefs, and examining what you take for granted, then your life is not worth living.

Socs: You children have nailed it! Some people say that doubting is a sin, that we should never question authorities. To me, doubting is a virtue, and it is a good idea to question authorities. If you learnt that from your course, even if that is the only thing you learnt, you learnt something of immense value.

Ila: Did you know, Socs, that doubting and questioning was a central feature of the Ancient Indian academic culture?

Socs: Yes, I remember. Disagreement with authorities and with peers was the norm in that culture. In Ancient Greece too. But I see that the new Indian culture has forgotten its roots. It has deteriorated into blind faith in authority, and actively discourages doubting and questioning.

Ila: Sadly, not just in India, but all over the world.

Samira: I remember Richard Feynman the physicist saying something very similar to what Socs said. Let me find the exact quote.

Samira did a quick Internet search, and read out from one of Feynman's talks, of 1955:

> "The scientist has a lot of experience with ignorance and doubt and uncertainty, and this experience is of very great importance, I think. When a scientist doesn't know the answer to a problem, he is ignorant. When he has a hunch as to what the result is, he is uncertain. And when he is pretty darn sure of what the result is going to be, he is still in some doubt. We have found it of paramount importance that in order to progress we must recognise our ignorance and leave room for doubt.

Scientific knowledge is a body of statements of varying degrees of certainty — some most unsure, some nearly sure, but none absolutely certain."

Socs: That is a remarkable statement about what ought to be our attitude towards knowledge. Not just that but the value system that derives from it, and the responsibility that comes with that value system. Wonderful that there was a Feynman to say that in the twentieth century.

Neel: Einstein also said something in the same spirit. Here, let me read it to you. It's in his book, *The Evolution of Physics*. I even remember the page number, it's 31.

" ...In our endeavour to understand reality, we are somewhat like a man trying to understand the mechanism of a closed watch. He sees the face and the moving hands, even hears its ticking, but he has no way of opening the case. If he is ingenious he may form some picture of the mechanism which would be responsible for all the things he observes, but he may never be quite sure his picture is the only one which could explain his observations. He will never be able to compare his picture with the real mechanism..."

Ila: I wonder if this is what the Constitution of India calls the scientific temper.

Anu: Scientific temper? What's that?

Ila: Samira, would you check on the Internet? Let's get the exact quotation.

Samira did a quick Internet search.

Samira: Here, article (51A) of the Indian constitution talks about the Fundamental duties of citizens in India. It says that "it is the duty of every citizen... to develop the scientific temper, humanism and the spirit of inquiry and reform..."

Rafa: If scientific temper is what Socs and Ancient Indians, Einstein and Feynman have all been talking about, I don't think we should call it 'scientific temper'. We should call it the 'Rational Temperament'.

Ila: And how would you describe the rational temperament, Rafa?

Rafa: I read it somewhere. Let me see. (Finds the file on his laptop and reads.)

Rational temperament is what the Indian Constitution calls the scientific temper, when extended to all forms of rational inquiry including not only science, but also mathematics, and the humanities. The ingredients of the rational temperament are: (i) Intellectual curiosity; (ii) Joy of learning and of finding things out on one's own; (iii) Openness to criticism: readiness to accept it in the spirit of self-correction; (iv) Intellectual skepticism: the habit of doubting and questioning values, norms, beliefs, and practices, our own and those of others; and not accepting assertions unless supported by adequate reasons; (v) Open-mindedness: willingness to modify one's beliefs and practices when confronted with good reasons to do so; (vi) Commitment to the values of truth, rationality, and rigour, and to clarity and precision of thought and expression. It comes from engaging in rational inquiry over a period of time.

Neel: I think this is exactly what the course was trying to help us learn.

2.3 From Inquiry to Research

Rafa: Can I ask a question I still have about this course?

Samira: Shoot.

Rafa: In the introduction, they describe inquiry as a process that starts with a question whose answer we don't know, but want to find out. And as an example of inquiry questions, they give this:

> *What allows migrating butterflies across generations to return to the same spots on earth year after year?*

We can't find answers to such questions by asking our teacher, or looking it up in a textbook or encyclopedia. To find an answer, we would need to rely on our own thinking. And they define inquiry as the process of looking for an answer through one's own thinking, and discussions with others."

Rafa drew this diagram:

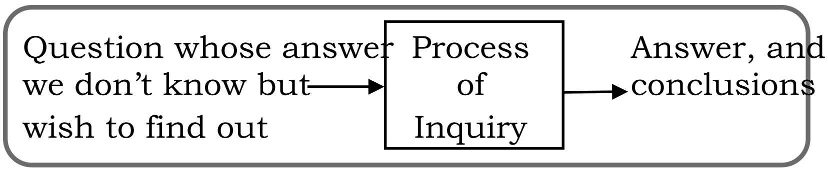


Figure 1

Socs: That is a good way of thinking about it.

Rafa: But isn't it the same as research? What's the difference?

Rafa drew another diagram:

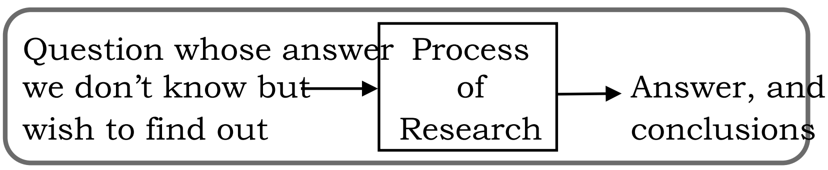


Figure 2

Anu: You're right, the same picture fits both inquiry and research!

Rafa: Yes, so what's the difference between them? That is my question.

Samira: I think the process is the same for inquiry and research. The difference lies in the expections of the outcome of the process. Research is inquiry that aims to make a contribution to the existing body of collective knowledge.

Neel: This means that students like us can engage in inquiry, but it may not count as research, simply because the result is not new knowledge for everyone, even though it's new knowledge for those who do the inquiry.

Samira: Exactly, Neel, good way to put it.

2.4 Higher Order Cognition

Anu: Can I ask a question too?

Samira: Sure, Anu. What's your question?

Anu: In the course, they talked about Higher Order Cognitive abilities, saying that the National Education Policy expects students to develop 'higher order cognitive abilities'. Does this course develop higher order cognitive abilities?

Samira: What do you think? Begin with 'What is cognition?'

Anu: That I can tell you, because Mom and I had a discussion of that at the beginning of the course, and she got me to look it up, and think about it. The Wikipedia entry that says that cognition is the mental process of acquiring knowledge and understanding "through thought, experience, and the senses."

Samira: But all of us have some knowledge. Even a two year old has some knowledge of the world. So do two year olds have cognitive abilities?

Anu: Yes, that follows from the definition of cognition.

Samira: What, then, is Higher Order Cognition?

Anu: Is it those cognitive abilities that we develop through education, through subjects like math, the sciences, and philosophy?

Neel: Ah, I see. In this course, the focus was on learning academic inquiry and academic critical thinking. So, if Higher Order Cognition is the cognition you develop through academic knowledge and academic inquiry, obviously we have been developing Academic Cognition, because that's what we've been doing in this course. I suppose this is what NEP calls higher order cognition.

Anu: Cool!

Rafa: The question is: What next? Where do we go from here?

3 Where do We Go from Here?

This book gives a basic introduction to academic inquiry. Imagine an advanced course on research that aims to develop research abilities across all university subjects, ranging from mathematics to the physical-biological-and-human sciences, and the humanities. Suppose such a course is at level 5. Then what you have learnt from this book is at level 1. So Rafa's question can be interpreted as a plea for books for higher levels.

What we will do here is to point you to certain readings, videos, and courses available on the Internet. Take a look at them, and they will lead you to other readings and courses. Here we go:

Feynman: *Adventures of a Curious Character*

Feynman: *The Meaning of it all*

Einstein and Infeld: *Evolution of Physics*

Robert Sapolsky: *Biological Foundations of Human Behavior*

Resources, courses, webinars, and videos at:

ThinQ website: (https://www.thinq.education/)

and the videos at the ThinQ channel on YouTube:

(https://www.youtube.com/@ThinQ_ed)

We wish you an exciting journey in developing your higher order cognition beyond what this course has empowered you with.

MATHEMATICIAN GEORGE POLYA ON THE ESSENCE OF THE 'RATIONAL TEMPERAMENT'

"In our personal life we often cling to illusions. That is, we do not dare to examine certain beliefs which could be easily contradicted by experience, because we are afraid of upsetting our emotional balance. There may be circumstances in which it is not unwise to cling to illusions, but in science we need a very different attitude, the *inductive attitude*. This attitude aims at adapting our beliefs to our experience as efficiently as possible. It requires a certain preference for what is matter of fact. It requires a ready ascent from observations to generalizations, and a ready descent from the highest generalizations to the most concrete observations. It requires saying "maybe" and "perhaps" in a thousand different shades. It requires many other things, especially the following three.

First, we should be ready to revise any one of our beliefs.

Second, we should change when there is a compelling reason to change it.

Third, we should not change a belief wantonly, without some good reason."

The first point needs 'intellectual courage'… The second point needs 'intellectual honesty'… The third point needs 'wise restraint'.

G Polya (1954:7-8)

Mathematics and Plausible Reasoning

The Kerala State Higher Education Council

FOUNDATIONS OF KNOWLEDGE AND INQUIRY ACROSS DISCIPLINES

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