Transformation of R.V.
and
Multivariate Gaussian

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Contents

1 Introduction

In this article, we will study about the following topics of statistics:

- Transformation of random variables
- Multivariate Gaussian random variable

2 Transformation of Random Variable

Given any continuous r.v. X with PDF $P_X(x)$ and given any function $g(X)$(defined on range of X) we intend to find PDF associated with the r.v. $Y = g(X)$.
For simplicity, let's assume $g(.)$ is monotonic increasing.
Then by probability mass conservation,

$$P(a < X < b) = P(g(a) < Y < g(b) = \int_{g(a)}^{g(b)} Q(y)dy$$

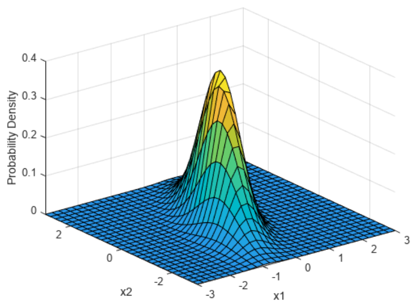
Using $y = g(x)$, we get the below relation upon simplification,

$$Q(y) = P(g^{-1}(y))\frac{dg^{-1}(y)}{dy}$$

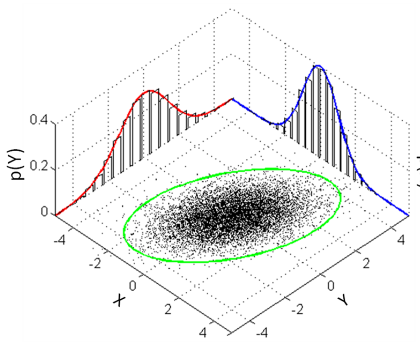
To handle monotonically decreasing $g(.)$ as well[1],

$$Q(y) = \begin{cases} +P(g^{-1}(y))\frac{dg^{-1}(y)}{dy}, & \text{for } g(.) \text{ monotonically increasing.} \\ -P(g^{-1}(y))\frac{dg^{-1}(y)}{dy}, & \text{for } g(.) \text{ monotonically decreasing.} \end{cases} \tag{1}$$

For more information, refer [1]



(a) Example 1



(b) Example 2

3 Multi-variate Gaussian Distribution

3.1 Definition

Let X be a vector of random variables of dimension D.
A r.v. X has a joint PDF as multi-variate Gaussian distribution $\exists$ finite i.i.d. standard Gaussian

[1]we could have used modulus operator but I wanted things to look more complicated

r.v. $W_1, W_2, \ldots W_N$ with $N > D$ such that

$$A = XW + \mu$$

Refer fig[1a] and fig[1b] for visual examples.This has many applications in machine learning, refer [2] and [3].

3.2 A is diagonal

In this case, the X_i are independent. The standard deviation of distribution of X_i is A_{ii}.

3.3 A is non-singular square matrix

Let's take $\mu = 0$ for simplicity.
Similar to univariate case, where scaling was determined by $|\frac{dg^{-1}(y)}{dy}|$ the scaling for multi-variate case is determined by determinant of matrix of derivatives, Jacobian matrix.
Also, $W = A^{-1}X$.
We intend to find the volume of the parallelepiped formed due to this transformation.
ClaimThe volume of parallelepiped described by column vectors of matrix A^{-1} is given by $det(A^{-1})$

$$P(X) = \frac{1}{(2\pi)^{\frac{D}{2}}} \cdot \frac{1}{\sqrt{det(C)}} \cdot exp(0.5 \cdot XT \cdot C^{-1} \cdot X) \tag{2}$$

Sample Values of bivariate normal distribution		
x	y	f(x,y)
0	0	1.6
0	1	0.096
$\sqrt{2}$	$\sqrt{2}$	0.02

References

[1] https://www.example.com.

[2] https://www.example.com.

[3] Donald E. Knuth. *The TEX Book*. Addison-Wesley Professional, 1986.