

Deep Surrogate Models for Fast Prediction and Parameter Inference of Li-ion Batteries



Siddhant Singh, Haonan Zhao, Atila Haimiti

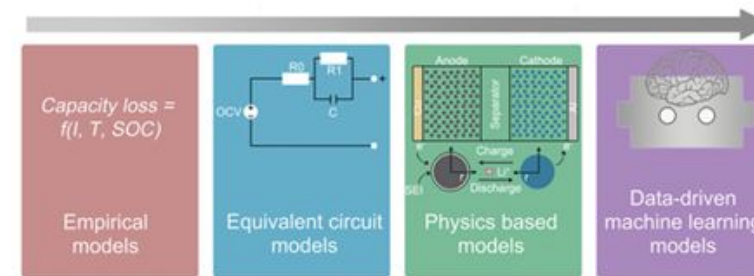
Motivation

Motivation for Creating Battery Models

- Real-World experiments are expensive



Battery cycling can be expensive and time-consuming



Battery modeling can reduce the need for testing as extensively

$$\varepsilon_k C_{\{k,max\}} \sigma_k D_{\{s,k\}} R_k a_k m_k L_k \\ U_{\{k,ref\}} C_{\{e,typ\}} D_{\{e,typ\}} \kappa_{\{e,typ\}} b t^+$$

Requires accurate parameter identification to use in the models

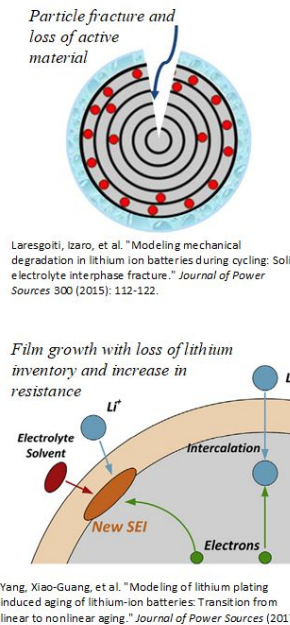
Motivation-continued

Motivation for Creating Surrogate Model for Battery Models

- Even reduced-ordered systems involves system of several highly interdependent ODEs → Computationally Costly

SPM model combined with degradation

Solution: Use AI for Science methods to build a data-driven, and to what extent possible, a physics-informed model.



Mechanical Degradation

$$\sigma_h(r, t) = \frac{2E\Omega}{3(1-\nu)} \left(\frac{1}{R_p^3} \int_0^{R_p} \rho^2 c_s(\rho, t) d\rho - \frac{c_s(r, t)}{3} \right)$$

$$\frac{d\epsilon_s}{dt} = \beta_{LAM} \left(\frac{\sigma_{h,surf}}{\sigma_{critical}} \right)^{m_{LAM}}$$

SEI

$$j_{SEI} = -k_{SEI} c_{EC}^s \exp \left(\frac{-\alpha_{SEI} F}{RT} \eta_{SEI} \right)$$

$$-D_{SEI} \frac{c_{EC}^s - c_{EC}^0}{\delta_{film}} = -a_s j_{SEI}$$

$$\eta_{seI} = \phi_s - V_R - U_{SEI}$$

$$\frac{\partial c_{SEI}}{\partial t} = \frac{-a_s j_{SEI}}{2}$$

$$\delta_{film} = \frac{1}{a_s} \left(\frac{c_{SEI} M_{SEI}}{\rho_{SEI}} \right)$$

$$R_{film} = \omega_{SEI} \frac{\delta_{film}}{\kappa_{SEI}}$$

$$n_{Li,loss} = \int_0^{t_{end}} \left(\int_0^{l^-} A a_s \left(\frac{j_{SEI}}{2} \right) dx \right) dt$$

Single Particle Model

$$\frac{\partial c_s}{\partial x}(r, t) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[D_s r^2 \frac{\partial c_s}{\partial r}(r, t) \right]$$

$$\frac{\partial c_s}{\partial x}(0, t) = 0$$

$$D_s \frac{\partial c_s}{\partial r}(R_p, t) = -j_{tot}$$

$$c_s(r, 0) = c_{init}$$

$$a_s = \frac{3\epsilon_s}{R_p}$$

$$j_{tot} = \frac{I}{a_s l A}$$

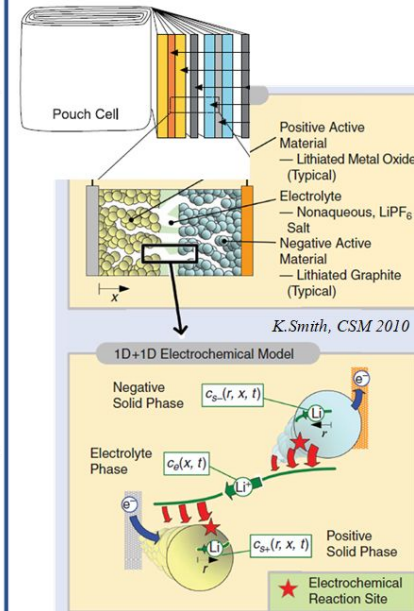
$$j = \frac{i_0}{F} \left(e^{\frac{(1-\alpha)F}{RT} \eta} - e^{\frac{-\alpha F}{RT} \eta} \right)$$

$$i_0 = k_0 (\bar{c}_e)^\alpha (c_{s,max} - c_{ss})^\alpha (c_{ss})^\alpha$$

$$\phi_s = \eta + V_R + U(c_{ss})$$

$$V_R = R_{film} F j$$

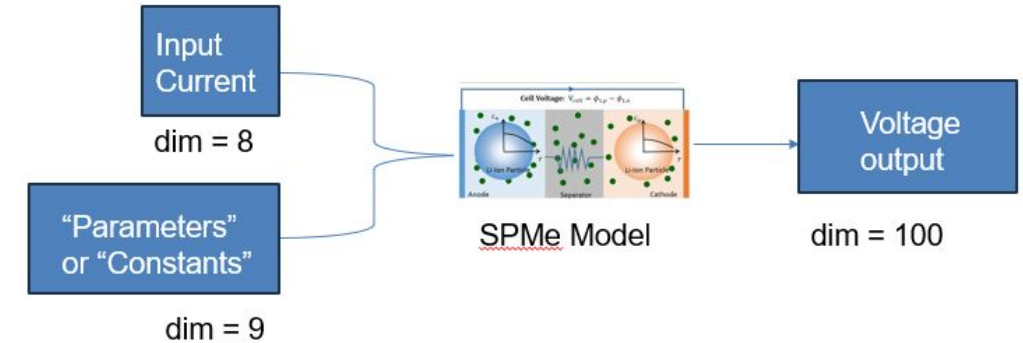
$$V(t) = \phi_s^+ - \phi_s^-$$



Datasets Descriptions

Physical System Characteristics/Assumptions

- ❑ Single Particle Model with Electrolyte Dynamics
- ❑ Solver: Python library - Pybamm



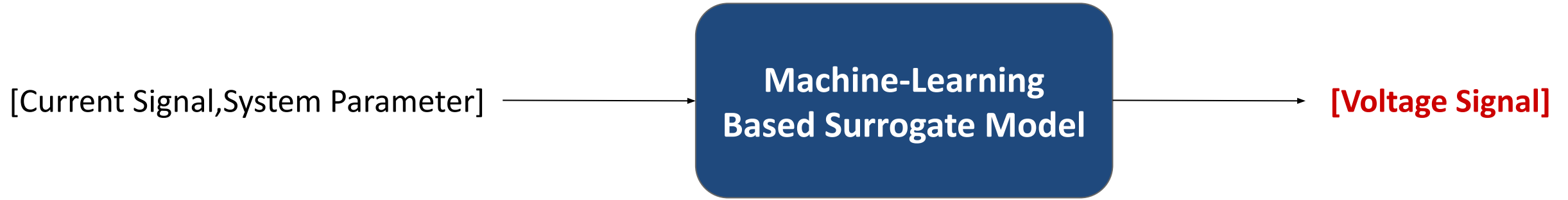
System Parameters

- ❑ Inputs: Nine parameters define the behavior of the model
- ❑ Inputs: Eight parameters define the current signal (time-series)
- ❑ Outputs: Voltage response (time-series)

Number of Sample and Size

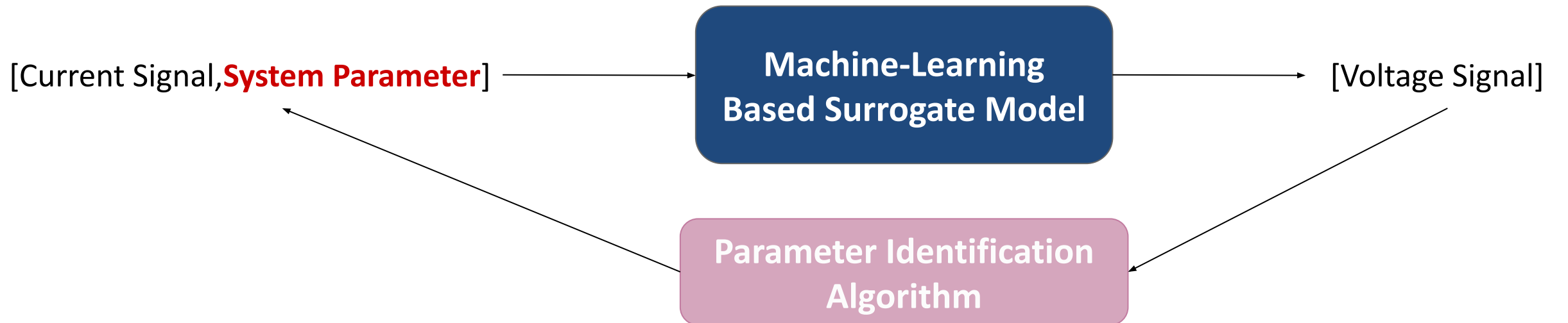
- ❑ 4000 samples for training and testing
- ❑ Each samples has 105 time-steps (used adaptive time-stepping for data generation, so timesteps are non-uniformly spaced)
- ❑ Occupies around 20 MB

Overview Proposed Approach



Step 1: Build Surrogate Model

Step 2: Parameter Identification



Overview Proposed Approach - For Surrogate

Baseline Approach

❑ DeepOnet → Standard neural operator

Proposed Approaches

❑ LSTM → Good at sequence-to-sequence learning

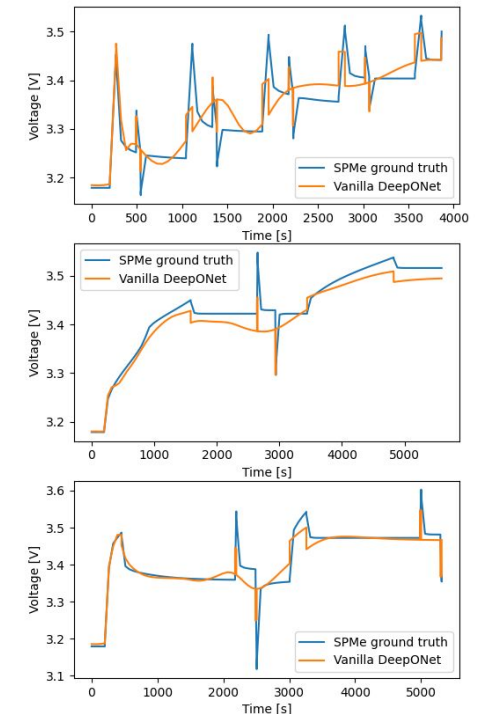
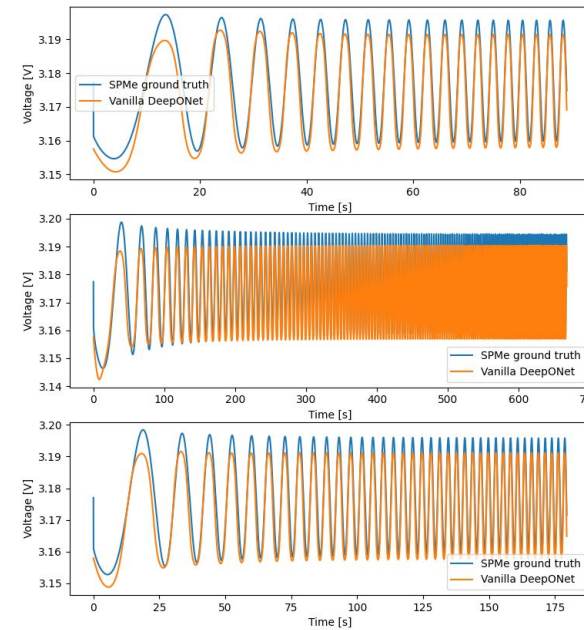
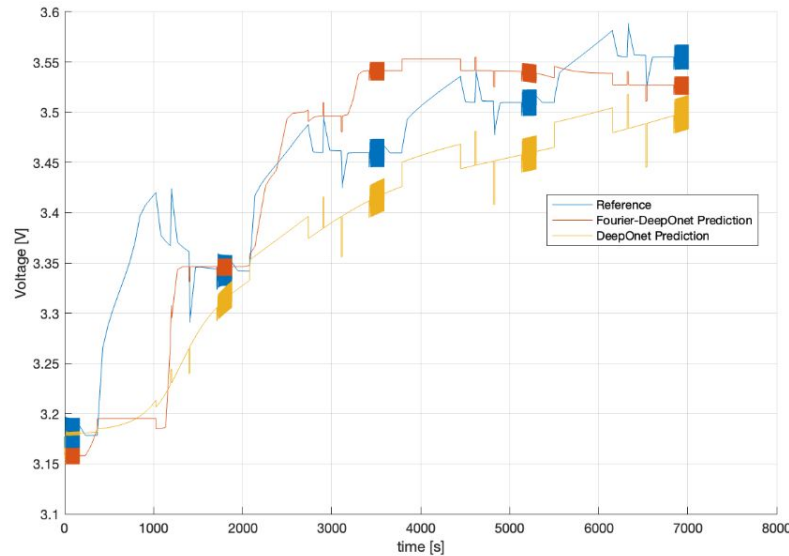
❑ Mamba-DeepOnet → Neural operators natural for solving the dynamic systems.

Baseline Approach - DeepONet

The full battery dynamics:

- ❑ Hybrid pulse power characterization (low frequency) + chirp signal (broad-band excitation).
- ❑ DeepONet fails to capture both of them at the same time.

Baseline: DeepONet



Proposed Approach - LSTM

Inputs-Outputs to the LSTM

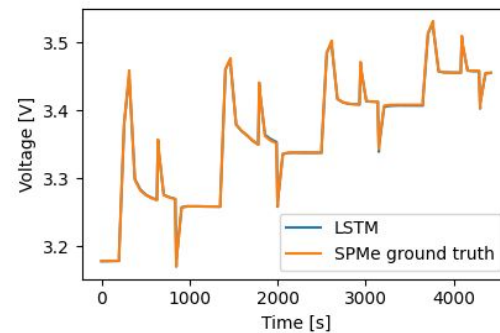
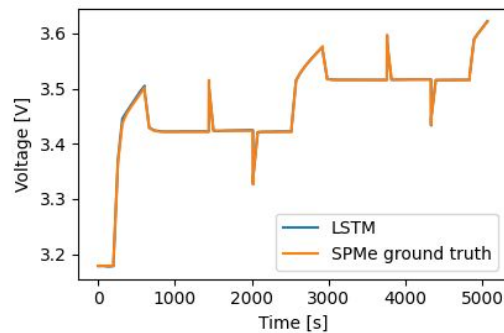
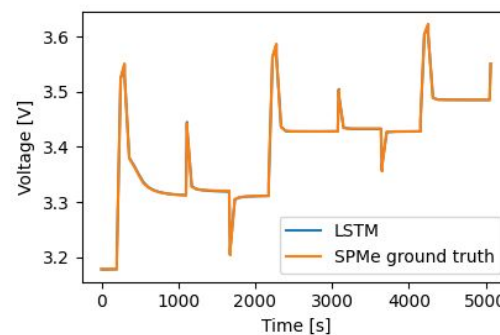
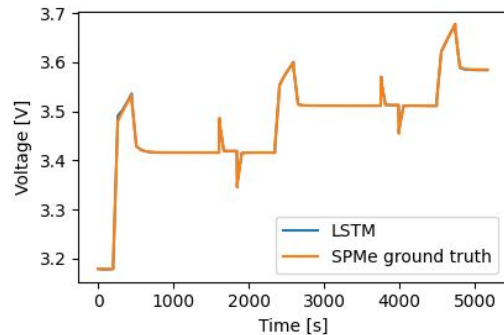
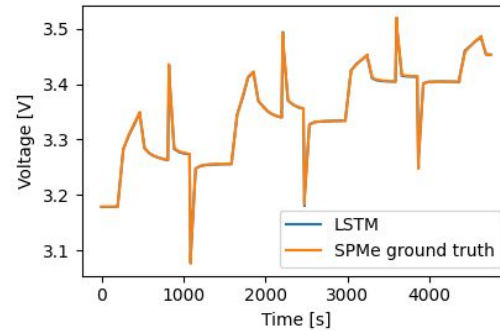
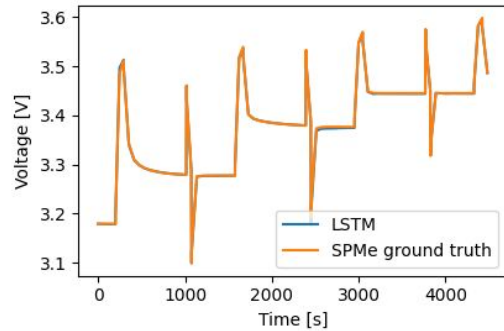
- ❑ Inputs: Current sequence along with the system parameters
- ❑ Output: Voltage sequence
- ❑ Sequence-Length: 105

Why Use LSTM?

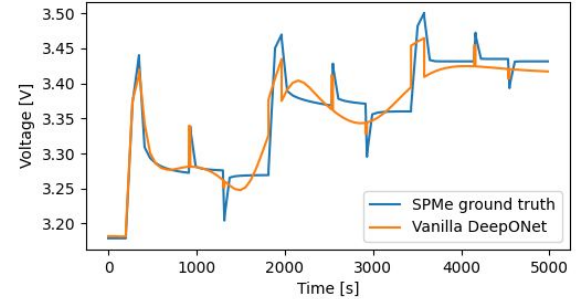
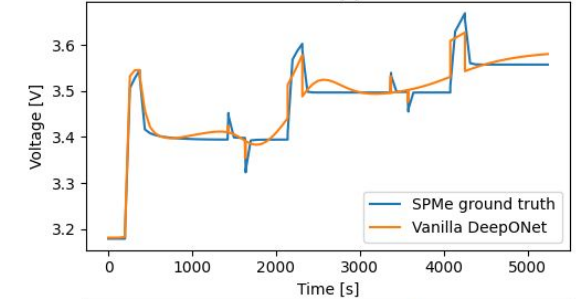
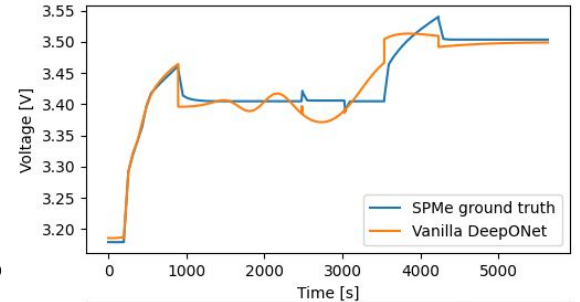
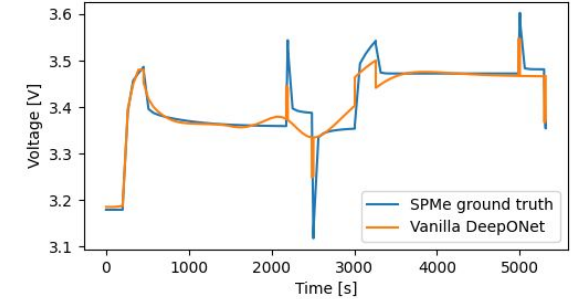
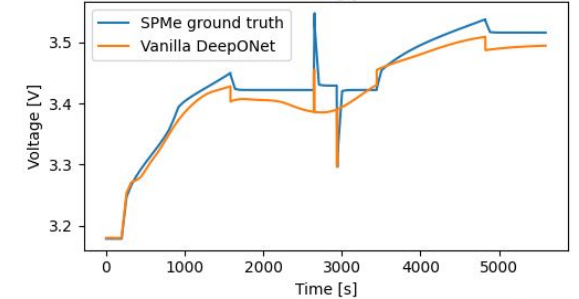
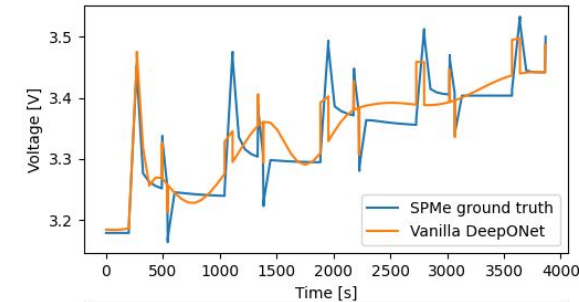
- ❑ The process of the physical battery computational model resembles recurrent networks
- ❑ LSTM is suitable for mitigating gradient explosion and vanishing

Case Study Results For Surrogate - LSTM

LSTM



Vanilla DeepONet



Proposed Approach - Mamba-DeepOnet

Why combine Mamba and DeepONet

- ❑ If one model learns a dynamic system fast and accurately:

$$\dot{\mathbf{x}} = \mathbf{v}(t, \mathbf{x})$$

$$\mathbf{x}|_{t=0} = \mathbf{x}_0$$

- ❑ Then the neural operator for this family of dynamic systems should learn the following fast and accurately:

$$\dot{\mathbf{x}} = \mathbf{v}[\xi(t), \mathbf{x}]$$

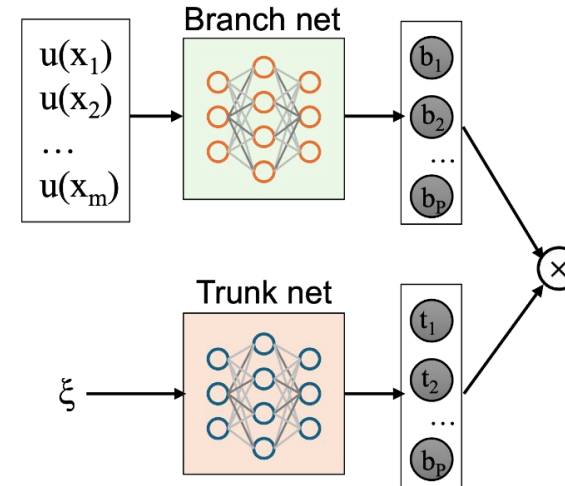
$$\xi = f(t)$$

$$\mathbf{x}|_{\partial\xi} = \mathbf{x}_0$$

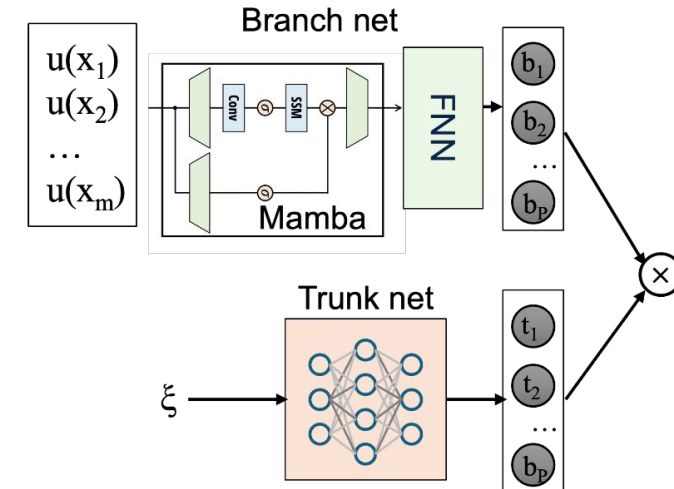
$$\xi|_{t=0} = \xi_0$$

- ❑ Mamba branch net to encode the function space, trunk to encode the output domain.

Vanilla DeepONet



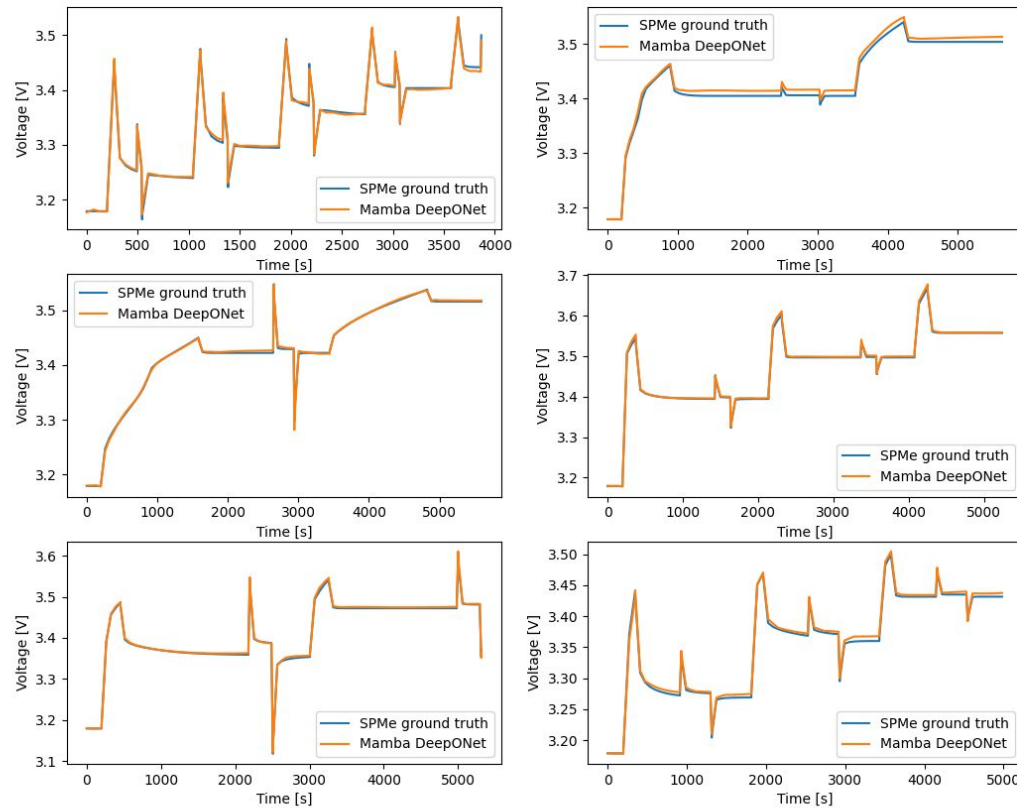
Mamba-DeepONet



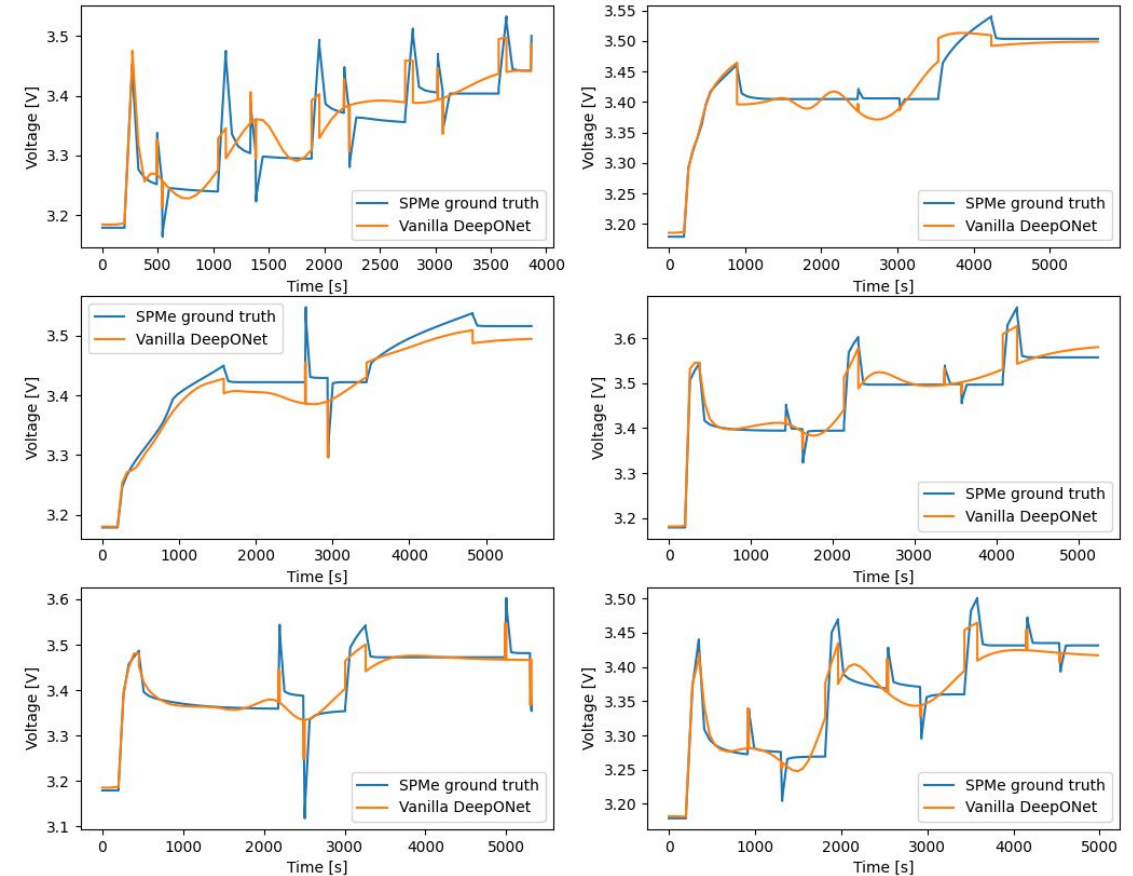
$$\left| G(u)(y) - \underbrace{\sum_{k=1}^p \sum_{i=1}^n c_i^k \sigma \left(\sum_{j=1}^m \xi_{ij}^k u(x_j) + \theta_i^k \right)}_{\text{branch}} \underbrace{\sigma(w_k \cdot y + \zeta_k)}_{\text{trunk}} \right| < \epsilon$$

Results For Surrogate - Mamba-DeepONet

Mamba-DeepONet



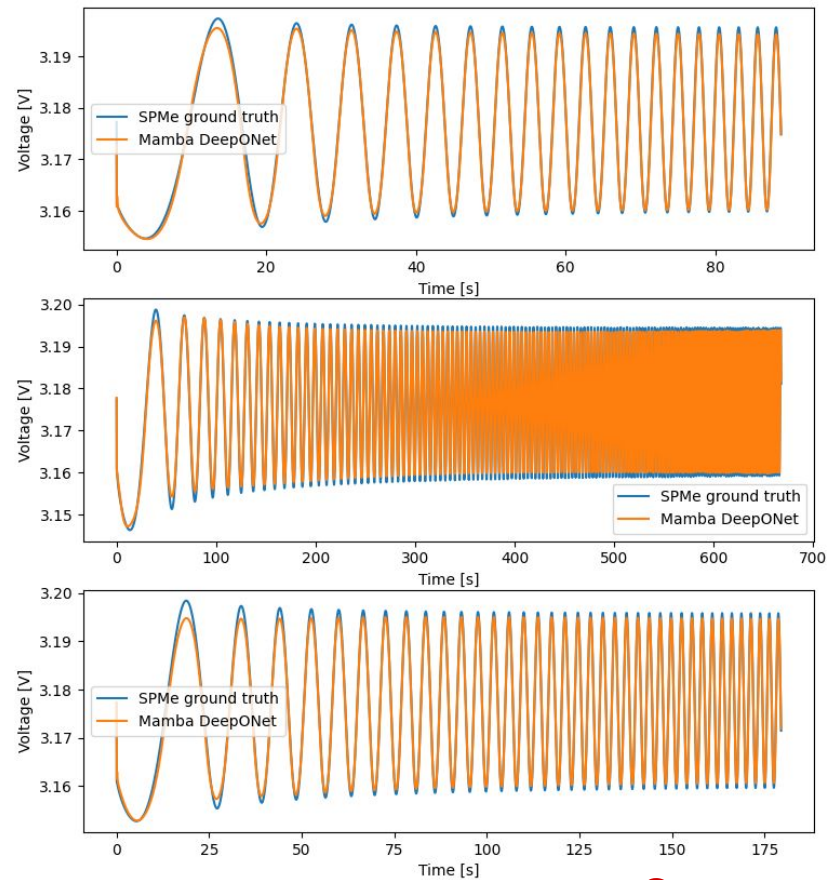
Vanilla DeepONet



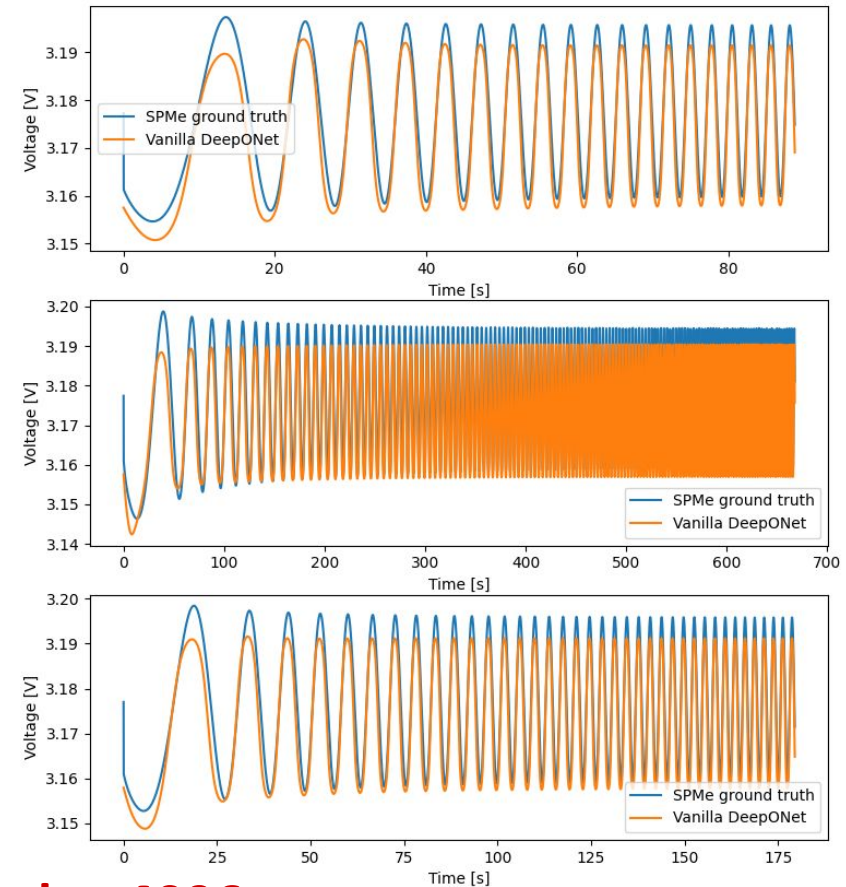
Sequence length = 105

Results For Surrogate - Mamba-DeepONet

Mamba-DeepONet



Vanilla DeepONet



Sequence length = 4096

Results For Surrogate - Summary

	MSE Loss of HPPC fitting	MSE Loss of chirp
DeepOnet	1.41e-03	7.33e-06
Mamba-DeepOnet	1.43e-05	7.57e-07
LSTM	3.71e-06	N/A



Proposed Approach - For Parameter Identification

Tree-structured Parzen Estimator Algorithm (TPE)

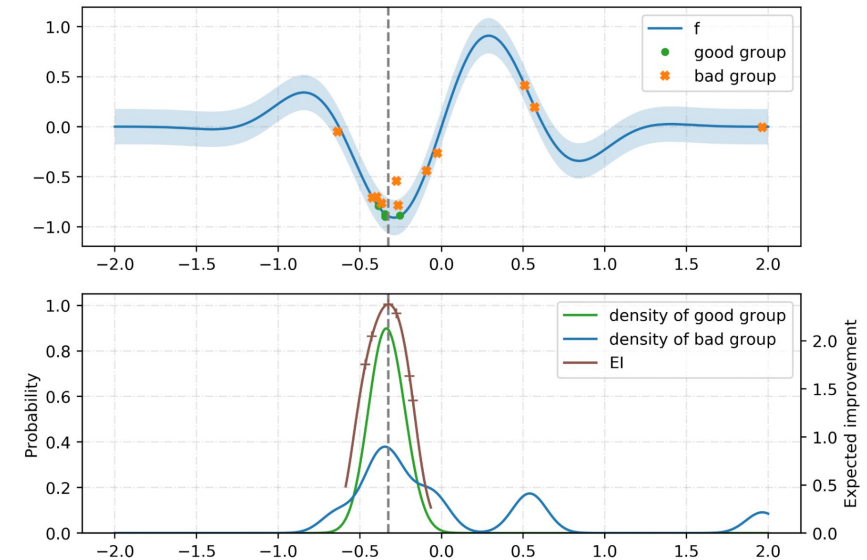
- ❑ Given past search history, suggests the next trial for a single parameter search (ignore correlation)
- ❑ Model $P(\text{single parameter value} \mid \text{loss})$ instead of $P(\text{loss} \mid \text{single parameter value})$
- ❑ Chose next trial single parameter value base on **“Promisingness”** → Proportional to Expected Improvement (EI)

$$\text{Promisingness} = \frac{P(\text{single parameter value} \mid \text{loss} < \text{loss}^*)}{P(\text{single parameter value} \mid \text{loss} > \text{loss}^*)}$$

density of good parameter configurations

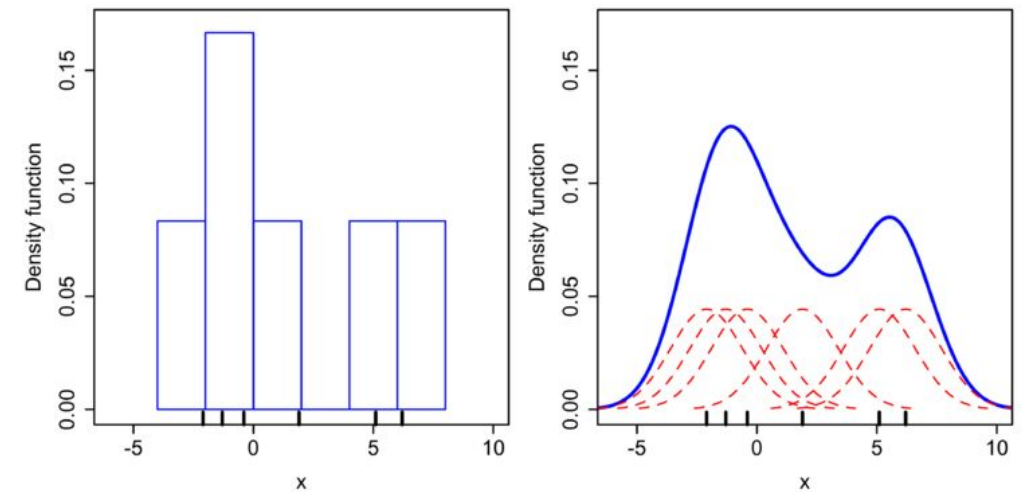
argmax to find next parameter value

density of bad parameter configurations



More on Tree-Structured Parzen Estimator (TPE)

- ❑ Non-parametric method (does not assume any pre-defined distribution)
- ❑ Uses Kernel Density Estimation (KDE) to estimate the densities of good parameter values (low loss, numerator in promisingness) and bad parameter values (high loss)
- ❑ Uses a Gaussian Kernel with an adaptively-determined bandwidth
- ❑ Balances exploration and exploitation by varying the loss threshold. A lower threshold prioritizes exploitation. A higher loss threshold prioritizes exploration.



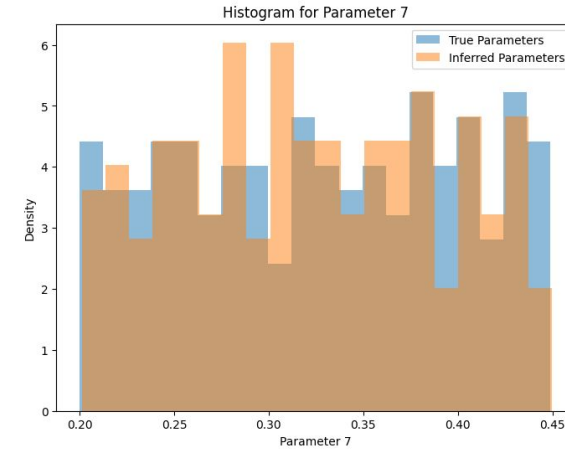
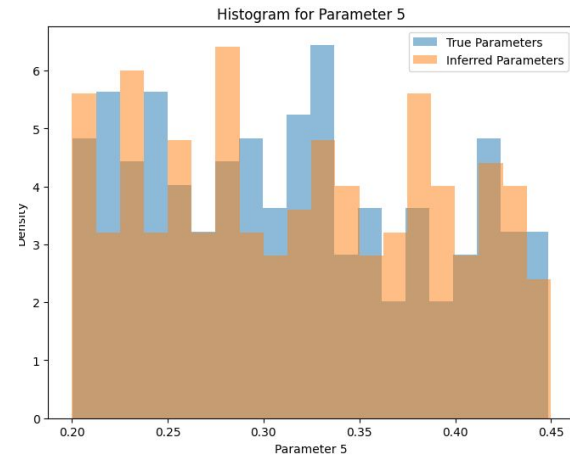
Parameter Identification Results

Parameter Index	LSTM Error (Normalized)	Mamba-DeepONet Error (Normalized)
0	1.285372	1.91261
1	13.12026	18.138
2	30.7648	35.69853
3	2.425946	3.031413
4	0.011579	0.012378
5	0.245476	0.245792
6	0.202349	0.251153
7	0.250003	0.232978
8	0.567224	0.607218

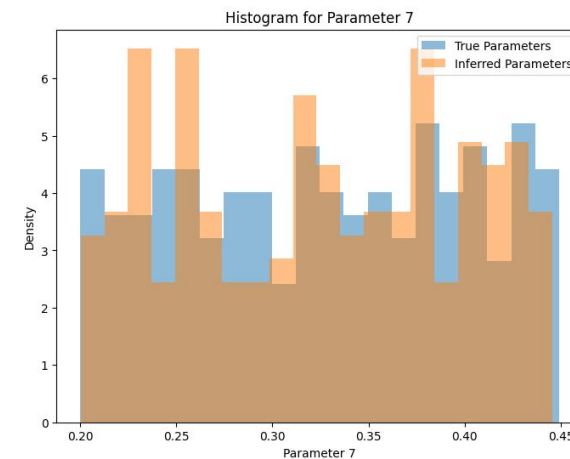
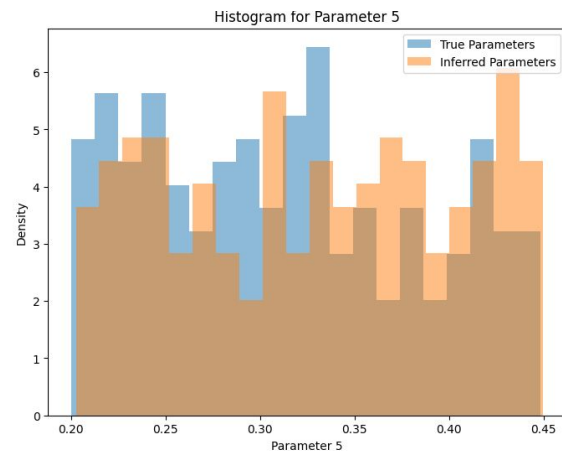


Case Study Results - Example Histograms

LSTM



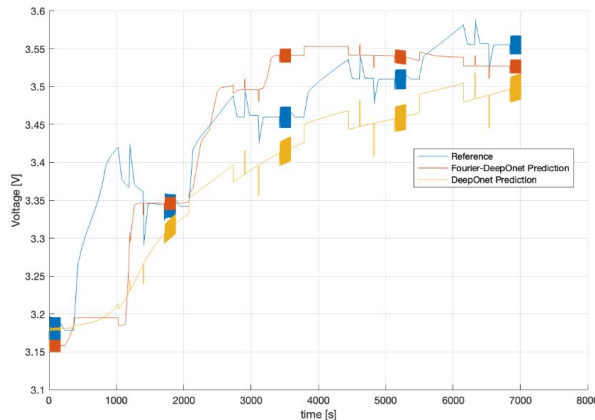
Mamba-DeepONet



Future Work: Incorporate Chirp Signal

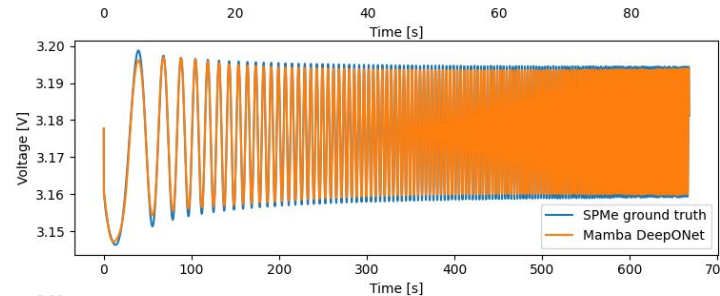
With chirp, fast and increasing frequency

- ❑ There is interest in using chirp signals (a high frequency to low frequency sweep) for greater excitation of battery dynamics enabling faster parameter identification
- ❑ Mamba-DeepONet can handle long sequences better than LSTM
- ❑ Mamba-DeepONet can handle multidimensional domain (trunk net)



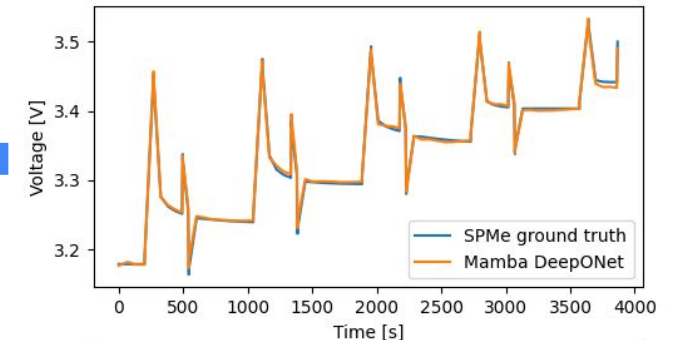
Broadband frequency data

=



Chirp signal
long sequence length
short time step

+



HPPC
short sequence length
long time step

Conclusion

1. Effective Surrogate Modeling:

- Mamba-DeepONet and LSTM outperform DeepONet as surrogate models for the lithium-ion battery single particle model with electrolyte dynamics.

2. Machine Learning for Parameter Inference using Probabilistic Methods:

- Accurate machine-learning-based surrogate models enable advanced parameter inference techniques.
- Bayesian optimization methods, such as Tree-structured Parzen Estimators (TPE), are well-suited for this purpose.

3. TPE Effectiveness:

- TPE demonstrates strong performance in estimating key model parameters, and is computationally efficient and parallelizable.

Appendix

Proposed Approach - For Parameter Identification

Algorithm 1 Tree-structured Parzen estimator (TPE)

N_{init} (The number of initial configurations, `n_startup_trials` in Optuna), N_s (The number of candidates to consider in the optimization of the acquisition function, `n_ei_candidates` in Optuna), Γ (A function to compute the top quantile γ , `gamma` in Optuna), W (A function to compute weights $\{w_n\}_{n=0}^{N+1}$, `weights` in Optuna), k (A kernel function), B (A function to compute a bandwidth b for k).

- 1: $\mathcal{D} \leftarrow \emptyset$
- 2: **for** $n = 1, 2, \dots, N_{\text{init}}$ **do** ▷ Initialization
- 3: Randomly pick \mathbf{x}_n
- 4: $y_n := f(\mathbf{x}_n) + \epsilon_n$ ▷ Evaluate the (expensive) objective function
- 5: $\mathcal{D} \leftarrow \mathcal{D} \cup \{(\mathbf{x}_n, y_n)\}$
- 6: **while** Budget is left **do**
- 7: Compute $\gamma \leftarrow \Gamma(N)$ with $N := |\mathcal{D}|$ ▷ Section 3.1 (Splitting algorithm)
- 8: Split \mathcal{D} into $\mathcal{D}^{(l)}$ and $\mathcal{D}^{(g)}$
- 9: Compute $\{w_n\}_{n=0}^{N+1} \leftarrow W(\mathcal{D})$ ▷ See Section 3.2 (Weighting algorithm)
- 10: Compute $b^{(l)} \leftarrow B(\mathcal{D}^{(l)}), b^{(g)} \leftarrow B(\mathcal{D}^{(g)})$ ▷ Section 3.3.4 (Bandwidth selection)
- 11: Build $p(\mathbf{x}|\mathcal{D}^{(l)}), p(\mathbf{x}|\mathcal{D}^{(g)})$ based on Eq. (5) ▷ Use $\{w_n\}_{n=0}^{N+1}$ and $b^{(l)}, b^{(g)}$
- 12: Sample $\mathcal{S} := \{\mathbf{x}_s\}_{s=1}^{N_s} \sim p(\mathbf{x}|\mathcal{D}^{(l)})$
- 13: Pick $\mathbf{x}_{N+1} := \mathbf{x}^* \in \arg\max_{\mathbf{x} \in \mathcal{S}} r(\mathbf{x}|\mathcal{D})$ ▷ The evaluations by the acquisition function
- 14: $y_{N+1} := f(\mathbf{x}_{N+1}) + \epsilon_{N+1}$ ▷ Evaluate the (expensive) objective function
- 15: $\mathcal{D} \leftarrow \mathcal{D} \cup \{\mathbf{x}_{N+1}, y_{N+1}\}$
