14. Probability and Statistics

Probability & Statistics

1. A fair dice is rolled twice. The probability that an odd number will follow an even number is
[EC: GATE-2005]

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

1. (d) Here the sample space S = 6

Therefore $P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$

and $P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$

since events are independent,

therefore, $P(\text{odd / even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. A probability density function is of the form [EC: GATE-2006]
$p(x) = Ke^{-\alpha|x|}, x \in (-\infty, \infty)$
The value of K is
(a) 0.5 (b) 1 (c) 0.5α (d) α

2. (c)

As (x) is a probability density function

$$\therefore \quad \int_{-\infty}^{\infty} p(x)\,dx = 1$$

$$\Rightarrow \quad \int_{-\infty}^{\infty} ke^{-\alpha|x|}dx = 1$$

$$\Rightarrow \quad \int_{-\infty}^{0} ke^{\alpha x}dx + \int_{0}^{\infty} ke^{-\alpha x}dx = 1 \quad \left[\begin{array}{l} \because |x| = x, \text{for } x > 0 \\ \quad\;\; = -x, \text{for } x < 0 \end{array}\right]$$

$$\Rightarrow \quad k = 0.5\alpha$$

3. Three companies X, Y and Z supply computers to a university. The percentage of computers supplied by them and the probability of those being defective are tabulated below
[EC: GATE-2006]

Company	% of computers supplied	Probability of being defective
X	60%	0.01
Y	30%	0.02
Z	10%	0.03

Given that a computer is defective, the probability that it was supplied by Y is
(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

3. Ans. (d)

4. If E denotes expectation, the variance of a random variable X is given by [EC: GATE-2007]
(a) $E[X^2] - E^2[X]$ (b) $E[X^2] + E^2[X]$
(c) $E[X^2]$ (d) $E^2[X]$

4. ans (a)

$$\text{Varience of } X = E\left\{(X-m)^2\right\}, m = \text{mean of the distribution}$$

$$\therefore \text{Var}(X) = E\left\{\left(X^2 - 2mX + m^2\right)\right\}$$

$$= E\left(X^2\right) - 2mE(X) + m^2$$

$$= E\left(X^2\right) - 2E^2(X) + E^2(X) \quad \left[\because m = E(X), \text{by defination of mean}\right]$$

$$\mathbf{= E(X^2) - E^2(X)}$$

5. An examination consists of two papers, Paper 1 and Paper 2. The probability of failing in Paper 1 is 0.3 and that in Paper 2 is 0.2. Given that a student has failed in Paper 2, the probability of failing in Paper 1 is 0.6. The probability of a student failing in both the papers is [EC: GATE-2007]
(a) 0.5 (b) 0.18
(c) 0.12 (d) 0.06

5.Ans(c).

Let A be the event that 'failed in paper 1'.
B be the event that 'failed in paper 2'.
Given P(A) = 0.3, P(B) = 0.2.

And also given $P\left(\frac{A}{B}\right) = 0.6$

$$\text{we know } P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A \cap B) = 0.6 \times 0.2 = 0.12$$

6. $P_x(x) = M\exp(-2|x|) - N\exp(-3|x|)$ is the probability density function for the real random variable X, over the entire x axis. M and N are both positive real numbers. The equation relating M and N is [EC: GATE-2008]

(a) $M - \frac{2}{3}N = 1$ (b) $2M + \frac{1}{3}N = 1$

(c) M + N = 1 (d) M + N = 3

6. Ans.(a)

Given $P_x(x)$ is the probability density function for the random variable X.

$$\Rightarrow \int_{-\infty}^{\infty}\left(Me^{-2|x|} - Ne^{-3|x|}\right)dx = 1$$

$$\Rightarrow \int_{-\infty}^{0}\left(Me^{2x} - Ne^{3x}\right)dx + \int_{0}^{\infty}\left(Me^{-2x} + Ne^{-3x}\right)dx = 1$$

$$\Rightarrow \left(\frac{M}{2} - \frac{N}{3}\right) + \left(\frac{M}{2} - \frac{N}{3}\right) = 1$$

$$\Rightarrow M - \frac{2}{3}N = 1$$

7. A fair coin is tossed 10 times. What is the probability that ONLY the first two tosses will yield heads? [EC: GATE-2009]

(a) $\left(\frac{1}{2}\right)^2$ (b) ${}^{10}C_2\left(\frac{1}{2}\right)^3$ (c) $\left(\frac{1}{2}\right)^{10}$ (d) ${}^{10}C_2\left(\frac{1}{2}\right)^{10}$

7. (c)

Let A be the event that first toss is head

And B be the event that second toss is head.

$\therefore \quad P(A) = \frac{1}{2}, \qquad P(B) = \frac{1}{2}$

By the given condition rest all 8 tosses should be tail

$\therefore$ The probability of getting head in first two cases

$$= \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^8 = \left(\frac{1}{2}\right)^{10}.$$

8. A fair coin is tossed independently four times. The probability of the event “the number of time heads shown up is more than the number of times tails shown up” is [EC: GATE-2010]

(a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{5}{16}$

8. Ans (d)

Here we have to find

$P(H,H,H,T) + P(H,H,H,H)$

$$= 4c_3\left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right) + 4c_4\left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^0$$

$$= 4.\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4 = \frac{5}{16}$$

ME 20 Years GATE Questions

9 In a manufacturing plant, the probability of making a defective bolt is 0.1. The mean and standard deviation of defective bolts in a total of 900 bolts are respectively

[ME: GATE-2000]

(a) 90 and 9 (b) 9 and 90

9. Ans (a)

It's a poission distribution. Here $n = 900, p = 0.1$

$\therefore$ mean $(m) = np = 900 \times 0.1 = 90$

Standard deviation $(\sigma) = \sqrt{npq} = \sqrt{90 \times .9}$, Here $q = 1 - p$.

$= \sqrt{81} = 9 \quad (\because \sigma > 0).,$

10. Consider the continuous random variable with probability density function

$f(t) = 1 + t$ for $-1 \le t \le 0$

$= 1 - t$ for $0 \le t \le 1$

[ME: GATE-2006]

The standard deviation of the random variables is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{6}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{6}$

10. Ans. (b)

$$\text{Var}(T) = \sigma_t^2 = \int_{-\infty}^{\infty} t^2 f(t)dt,$$ T being the random variable of f(t).

$$= \int_{-1}^{0} t^2(1+t)dt + \int_{0}^{1} t^2(1-t)dt$$

$$= \frac{1}{6}$$

$$\therefore \quad \sigma_t \frac{1}{\sqrt{6}} [\because \sigma_t > 0]$$

11. The standard deviation of a uniformly distributed random variable between 0 and 1 is

[ME: GATE-2009]

(a) $\frac{1}{\sqrt{12}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{5}{\sqrt{12}}$ (d) $\frac{7}{\sqrt{12}}$

11. (a)

Here p.d.f. is $f(x) = \frac{1}{1-0} = 1, \quad 0 < x < 1.$

$$\therefore \quad \text{mean}(m) = E(x) = \int_0^1 xf(x)dx = \int_0^1 xdx = \frac{1}{2}$$

$$\therefore \quad \text{Var}(x) = \sigma^2 = \int_0^1 \left(\frac{x-1}{2}\right)^2 .1.dx = \int_0^1 \left(x^2 - x + \frac{1}{4}\right)dx = \frac{1}{3} - \frac{1}{2} + \frac{1}{4} = \frac{1}{12}$$

$$\therefore \sigma = \frac{1}{\sqrt{12}} [\because \sigma > 0]$$

12 The probability that two friends share the same birth-month is [ME: GATE-1998]

(a) $\frac{1}{6}$ (b) $\frac{1}{12}$ (c) $\frac{1}{144}$ (d) $\frac{1}{24}$

12. (b)

Let A = the event that the birth month of first friend
And B= that of second friend.

$\therefore \quad P(A) = 1,$ as 1st friend can born in any month

and $P(B) = \frac{1}{12}$, by the condition.

$\therefore$ Probability of two friends share same birth-month

is $1 \times \frac{1}{12} = \frac{1}{12}$

13. The probability of a defective piece being produced in a manufacturing process is 0.01. The probability that out of 5 successive pieces, only one is defective, is

(a) $(0.99)^2\ (0.01)$ (b) $(0.99)(0.01)^4$ [ME: GATE-1996]
(c) $5\times(0.99)(0.01)4$ (d) $5\times(0.99)^4(0.01)$

13. (d)

The required probability $= {}^5c_1(.01)^1 \times (.99)^4 = 5 \times (0.99)^4 \times (.01).$

14. A box contains 5 block balls and 3 red balls. A total of three balls are picked from the box one after another, without replacing them back. The probability of getting two black balls and one red ball is [ME: GATE-1997]

(a) 3/8 (b) 2/15 (c) 15/28 (d) ½

14. (c)

Here the possible combination of picking up three balls without replacement is
BBR, BRB, RBB.
(B = Black ball, R = Red balls)

$\therefore \quad P(BBR) = \frac{5}{8} \times \frac{4}{7} \times \frac{3}{6} = \frac{5}{28}$

$P(BRB) = \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} = \frac{5}{28}$

$P(RBB) = \frac{3}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{5}{28}$

$\therefore$ Probability of getting two black balls and one red ball is $\frac{15}{28}$.

15. An unbiased coin is tossed three times. The probability that the head turns up in exactly two cases is [ME: GATE-2001]

(a) 1/9 (b) 1/8 (c) 2/3 (d) 3/8

15. (d)

Required probability $= {}^3c_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right) = \frac{3}{8}$

16. Two dice are thrown. What is the probability that is the sum of the numbers on the two dice is eight? [ME: GATE-2002]

(a) 5/36 (b) 5/18 (c) ¼ (d) 1/3

16. (a)

Here sample space = 6 × 6 = 36

Here, there are five such points whose sum is 8. They are (2,6), (3,5), (4,4), (5,3), (6,2).

$\therefore$ Requireprobability $= \frac{5}{36}$

17. Manish has to travel from A to D changing buses at stops B and C enroute. The maximum waiting time at either stop can be 8 minutes each, but any time of waiting up to 8 minutes is equally likely at both places. He can afford up to 13 minutes of total waiting time if he is to arrive at D on time. What is the probability that Manish will arrive late at D? [ME: GATE-2002]

(a) 8/13 (b) 13/64 (c) 119/128 (d) 9/128

17.Ans(a)

18. Arrivals at a telephone booth are considered to be poison, with an average time of 10 minutes between successive arrivals. The length of a phone call is distributes exponentially with mean 3 minutes. The probability that an arrival does not have to wait before service is [ME: GATE-2002]

(a) 0.3 (b) 0.5 (c) 0.7 (d) 0.9

18.Ans(a)

19. A box contains 5 black and 5 red balls. Two balls are randomly picked one after another from the box, without replacement. The probability for both balls being red is [ME: GATE-2003]

(a) $\frac{1}{90}$ (b) $\frac{1}{2}$ (c) $\frac{19}{90}$ (d) $\frac{2}{9}$

19. (d)

The probability of drawing two red balls without replacement

$= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

20. From a pack of regular from a playing cards, two cards are drawn at random. What is the probability that both cards will be Kings, if first card in NOT replaced

(a) $\frac{1}{26}$ (b) $\frac{1}{52}$ (c) $\frac{1}{169}$ (d) $\frac{1}{221}$ [ME: GATE-2004]

20. (d)

Here sample space S = 52

$\therefore$ The probability of drawing both cards are king without replacement

$$= \frac{{}^4c_1}{52} \times \frac{{}^3c_1}{51} = \frac{1}{221}$$

21. A lot has 10% defective items. Ten items are chosen randomly from this lot. The probability that exactly 2 of the chosen items are defective is [ME: GATE-2005]

(a) 0.0036 (b) 0.1937 (c) 0.2234 (d) 0.3874

21.(b)

Let A be the event that items are defective and B be the event that items are non- defective.

$\therefore P(A) = 0.1$ and $P(B) = 0.9$

$\therefore$ Probability that exactly two of those items are defective

$= {}^{10}c_2(.1)^2(.9)^8 = 0.1937$

22. A single die is thrown twice. What is the probability that the sum is neither 8 nor 9?

[ME: GATE-2005]

(a) 1/9 (b) 5/36 (c) 1/4 (d) 3/4

22. (d)

Here sample space = 36

Total No. of way in which sum is either 8 or 9 are

(2,6), (3,5),(3,6), (4,4), (4,5), (5,3), (5,4), (6,2), (6,3)

So probability of getting sum 8 or 9 = $\frac{9}{36} = \frac{1}{4}$

So the probability of not getting sum 8 or 9 = $1 - \frac{1}{4} = \frac{3}{4}$.

24. A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

[ME: GATE-2006]

(a) 1/5 (b) 1/25 (c) 20/99 (d) 11/495

24(d)

The probability of defective items $= \frac{20}{100}$.

Therefore the probability of first two defective items without replacement

$$= \frac{20}{100} \times \frac{19}{99} = \frac{19}{495}.$$

25. A coin is tossed 4 times. What is the probability of getting heads exactly 3 times?

(a) $\frac{1}{4}$ (b) $\frac{3}{8}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ [ME: GATE-2008

25. (a)

Probability of getting exactly three heads

$$= {}^4c_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right) = 4 \times \frac{1}{2^4} = \frac{1}{4}$$

26. If three coins are tossed simultaneously, the probability of getting at least one head is [ME: GATE-2009]

(a) 1/8 (b) 3/8 (c) 1/2 (d) 7/8

26. (d)

Here the sample space $S = 2^3 = 8.$

No. of ways to get all tails =1.

$\therefore$ probability to get all tails $= \frac{1}{8}$

$\therefore$ Probability to get at least one head is $=1-\frac{1}{8} = \frac{7}{8}$

27. A box contains 2 washers, 3 nuts and 4 bolts. Items are drawn from the box at random one at a time without replacement. The probability **of** drawing 2 washers first followed by 3 nuts and subsequently the 4 bolts is [ME: GATE-2010]

(a) 2/315 (b) 1/630 (c) 1/1260 (d) 1/2520

27. (c)

Here sample space = 9

The required probability of drawing 2 washers, 3 nuts and 4 bolts respectively without replacement

$$= \frac{2}{9} \times \frac{1}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} \times \frac{3}{3} \times \frac{2}{2} \times \frac{1}{1}$$

$$= \frac{1}{1260}$$

28. If 20 per cent managers are technocrats, the probability that a random committee of 5 managers consists of exactly 2 technocrats is [ME: GATE-1993]

(a) 0.2048 (b) 0.4000 (c) 0.4096 (d) 0.9421

28. (a)

The probability of technocrats manager $= \frac{20}{100} = \frac{1}{5}$

$\therefore$ Probability of non technocrats manager $= \frac{4}{5}$

Now the require probability $= {}^5c_2 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right)^3 = 0.2048$

29. Analysis of variance is concerned with: [ME: GATE-1999]
(a) Determining change in a dependent variable per unit change in an independent variable
(b) Determining whether a qualitative factor affects the mean of an output variable
(c) Determining whether significant correlation exists between an output variable and an input variable.
(d) Determining whether variance in two or more populations are significantly different.

29. Ans.(d)
Analysis of variance is used in comparing two or more populations, e.g. Different types of manures for yelding a single crop.

30. Four arbitrary point (x_1,y_1), (x_2,y_2), (x_3,y_3), (x_4,y_4), are given in the x, y – plane Using the method of least squares, if, regressing y upon x gives the fitted line y = ax + b; and regressing y upon x given the fitted line y = ax + b; and regressing x upon y gives the fitted line x = cy + d then [ME: GATE-1999]
(a) The two fitted lines must coincide (b) The two fitted lines need not coincide
(c) It is possible that ac = 0 (d) A must be 1/c

30. (d)

$y = ax + b - (i)$ and $x = cy + d - (ii)$

From (ii) we get $x - d = cy \Rightarrow y = \frac{1}{c}x - \frac{d}{c} - (iii)$

comparing (i) and (ii), $a = \frac{1}{c}$ and $b = \frac{-d}{c}$

31. A regression model is used to express a variable Y as a function of another variable X. This implies that [ME: GATE-2002]
(a) There is a causal relationship between Y and X
(b) A value of X may be used to estimate a value of Y
(c) Values of X exactly determine values of Y
(d) There is no causal relationship between Y and X

31. (b)

32. Let X and Y be two independent random variables. Which one of the relations between expectation (E), variance (Var) and covariance (Cov) given below is FALSE? [ME: GATE-2007]
(a) E (XY) = E (X) E (Y) (b) Cov (X, Y) = 0
(c) Var (X + Y) = Var (X) + Var (Y) (d) $E(X^2 y^2) = (E(X))^2 (E(y))^2$

32. (b).

CE 10 Years GATE Questions

33. A class of first year B. Tech. Students is composed of four batches A, B, C and D, each consisting of 30 students. It is found that the sessional marks of students in Engineering Drawing in batch C have a mean of 6.6 and standard deviation of 2.3. The mean and standard deviation of the marks for the entire class are 5.5 and 4.2, respectively. It is decided by the course instructor to normalize the marks of the students of all batches to have the same mean and standard deviation as that of the entire class. Due to this, the marks of a student in batch C are changed from 8.5 to

(a) 6.0 (b) 7.0 **[CE: GATE – 2006]**

(c) 8.0 (d) 9.0

33.Ans(d). Let mean and stander deviation of batch C be μ_c and σ_c respectively and mean and standard deviation of entire class of 1st year students be μ and σ respectively.

Given $\mu_c = 6.6$ and $\sigma_c = 2.3$

and $\mu = 5.5$ and $\sigma = 4.2$

In order to normalize batch C to entire class, the normalized score must be equated

Since $Z = \dfrac{x-\mu}{\sigma}$

$$Z_c = \frac{x_c - \mu_c}{\sigma_c} = \frac{8.5-6.6}{2.3}$$

Now $Z = \dfrac{x-\mu}{\sigma} = \dfrac{x-5.5}{4.2}$

$$\therefore Z = Z_c \Rightarrow \frac{x-5.5}{4.2} = \frac{8.5-6.6}{2.3}$$

$$\Rightarrow x = 8.969 \cong 9.0$$

34. Three values of x and y are to be fitted in a straight line in the form y = a + bx by the method of least squares. Given $\Sigma x = 6$, $\Sigma y = 21$, $\Sigma x^2 = 14$ and $\Sigma xy = 46$, the values of a and b are respectively. **[CE: GATE – 2008]**

(a) 2 and 3 (b) 1 and 2

(c) 2 and 1 (d) 3 and 2

34.Ans(d)

$y = a + bx$

Given

$n = 3, \Sigma x = 6, \Sigma y = 21, \Sigma x^2 = 14$

And $\Sigma xy = 46$

$$b = \frac{n\Sigma xy - \Sigma x\,\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$a = \bar{y} - b\bar{x}$$

$$= \frac{\Sigma y}{n} - b\frac{\Sigma x}{n}$$

Substituting, we get

$$b = \frac{(3 \times 46) - (6 \times 21)}{(3 \times 14) - (6)^2} = 2$$

$$a = \frac{21}{3} - 2 \times \left(\frac{6}{3}\right) = 3$$

$\therefore$ a = 3 and b = 2

35. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws is defective will be

(a) 100% (b) 50% **[CE: GATE – 2003]**

(c) 49% (d) None of these

35. (d)

Non defective screws =7

$\therefore$ Probability of the two screws are non defective

$$= \frac{{}^3c_0 \times {}^7c_2}{{}^{10}c_2} \times 100\%$$

$$= \frac{7}{15} \times 100\% = 46.6 \simeq 47\%$$

36. A hydraulic structure has four gates which operate independently. The probability of failure off each gate is 0.2. Given that gate 1 has failed, the probability that both gates 2 and 3 will fail is **[CE: GATE – 2004]**

(a) 0.240 (b) 0.200

(c) 0.040 (d) 0.008

36.(c)

P(gate to and gate 3/gate 1 failed)

$$\begin{aligned} &= P(\text{gate } 2 \text{ and gate } 3) \\ &= P(\text{gate } 2) \times P(\text{gate } 3) \\ &= 0.2 \times 0.2 = 0.04 \end{aligned} \quad \left[\begin{array}{l}\therefore \text{ all three gates are} \\ \text{independent corrosponding} \\ \text{to each other}\end{array}\right]$$

37. Which one of the following statements is NOT true?

(a) The measure of skewness is dependent upon the amount of dispersion

(b) In a symmetric distribution, the values of mean, mode and median are the same

(c) In a positively skewed distribution; mean > median > mode

(d) In a negatively skewed distribution; mode > mean > median **[CE: GATE – 2005**

37. (d)

(d) is not true since in a negatively skewed distribution, **mode > median > mean**

38. There are 25 calculators in a box. Two of them are defective. Suppose 5 calculators are randomly picked for inspection (i.e., each has the same chance of being selected), what is the probability that only one of the defective calculators will be included in the inspection?

[CE: GATE – 2006]

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

38. (b)

Probability of only one is defective out of 5 calculators

$$= \frac{{}^{2}c_{1} \times {}^{23}c_{4}}{{}^{25}c_{5}} = \frac{1}{3}$$

39. If the standard deviation of the spot speed of vehicles in a highway is 8.8 kmph and the mean speed of the vehicles is 33 kmph, the coefficient of variation in speed is

[CE: GATE – 2007]

(a) 0.1517 (b) 0.1867 (c) 0.2666 (d)0.3645

39. (c)

$$C\nu = \frac{\sigma}{\mu} = \frac{8.8}{33} = 0.2666$$

40. If probability density functions of a random variable X is

$f(x) = x^2$ for $-1 \le x \le 1$, and **[CE: GATE – 2008]**

$= 0$ for any other value of x

Then, the percentage probability $P\left(-\frac{1}{3} \le x \le \frac{1}{3}\right)$ is

(a) 0.247 (b) 2.47 (c) 24.7 (d) 247

40. (b)

$$P\left(\frac{-1}{3} \le x \le \frac{1}{3}\right) = \int_{-\frac{1}{3}}^{\frac{1}{3}} x^2 dx \qquad = \left[\frac{x^3}{3}\right]_{-\frac{1}{3}}^{\frac{1}{3}} = \frac{2}{81}$$

$$\therefore \quad \text{Percentage probability} = \frac{2}{81} \times 100 \simeq 2.47\%$$

41. A person on a trip has a choice between private car and public transport. The probability of using a private car is 0.45. While using the public transport, further choices available are bus and metro, out of which the probability of commuting by a bus is 0.55. In such a situation, the probability (rounded up to two decimals) of using a car, bus and metro, respectively would be

[CE: GATE – 2008]

(a) 0.45, 0.30 and 0.25 (b) 0.45, 0.25 and 0.30

(c) 0.45, 0.55 and 0.00 (d) 0.45, 0.35 and

41. (a)

Given

p(private car) = 0.45
p(bus / public transport) = 0.55

Since a person has a choice between private car and public transport

$$
\begin{aligned}
p\,(\text{public transport}) &= 1 - p(\text{private car}) \\
&= 1 - 0.45 = 0.55 \\
p\,(\text{bus}) &= p(\text{bus} \cap \text{public transport}) \\
&= p(\text{bus} \mid \text{public transport}) \\
&\quad \times p(\text{public transport}) \\
&= 0.55 \times 0.55 \\
&= 0.3025 \simeq 0.30
\end{aligned}
$$

Now $\quad p\,(\text{metro}) = 1 - [p(\text{private car}) + p(\text{bus})]$

$= 1 - (0.45 + 0.30) = 0.25$

$\therefore \quad p\,(\text{private car}) = 0.45$

$p\,(\text{bus}) = 0.30$

and $\quad p(\text{metro}) = 0.25$

42. The standard normal probability function can be approximated as

$$F(x_N) = \frac{1}{1 + \exp(-1.7255\, x_n\, |x_n|^{0.12})}$$ **[CE: GATE – 2009]**

Where x_N = standard normal deviate. If mean and standard deviation of annual precipitation are 102 cm and 27 cm respectively, the probability that the annual precipitation will be between 90 cm and 102 cm is

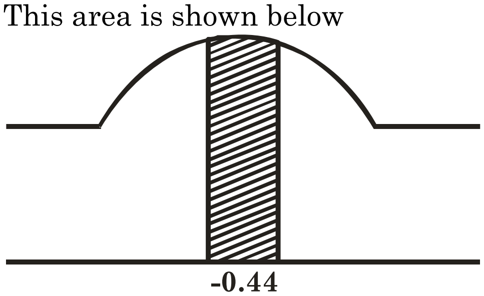
(a) 66.7% (b) 50.0% (c) 33.3% (d) 16.7%

42. (d)

Here $\mu = 102\text{cm}$ and $\sigma = 27\text{cm}$

$$P(90 \le x \le 102) = P\left(\frac{90-102}{27} \le x \le \frac{102-102}{27}\right) = P(-0.44 \le x \le 0)$$

This area is shown below



The shaded area in above figure is given by $F(0) - F(-0.44)$

$$= \frac{1}{1+\exp(0)} - \frac{1}{1+\exp(-1.7255 \times (-0.44) \times (0.44)^{0.12})}$$

$= 0.5 - 0.3345$

$= 0.1655 \simeq 16.55\%$

43. Two coins are simultaneously tossed. The probability of two heads simultaneously appearing is **[CE: GATE – 2010]**

(a) $\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

43.(c)

Probability of two head $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Q3. There are two containers, with one containing 4 Red and 3 Green balls and the other containing Blue and 4 Green balls. One bal is drawn at random form each container. The probability that one of the ball is Red and the other is Blue will be

(a) 1/7 (b) 9/49 (c) 12/49 (d) 3/7 **[CE-2011]**

Ans. (c)

EE All GATE Questions

45. A fair coin is tossed three times in succession. If the first toss produces a head, then the probability of getting exactly two heads in three tosses is **[EE: GATE-2005]**

(a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{3}{4}$

45.(d)

After first head in first toss, probability of tails in 2^{nd} and 3^{rd} toss $= \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$

$\therefore$ Probability of exactly two heads $= 1 - \frac{1}{4} = \frac{3}{4}$

46. Two fair dice are rolled and the sum r of the numbers turned up is considered **[EE: GATE-2006]**

(a) $\Pr(r > 6) = \frac{1}{6}$ (b) Pr(r/3 is an integer)= $\frac{5}{6}$

(c) Pr(r=8 | r/4 is an integer) $= \frac{5}{9}$ (d) Pr(r=6 | r/5 is an integer)= $\frac{1}{18}$

46. (c)

47. A box contains 4 white balls and 3 red balls. In succession, two balls are randomly selected and removed from the box. Given that the first removed ball is white, the probability that the second removed ball is red is **[EE: GATE-2010]**

(a) 1/3 (a) 3/7

(a) 1/2 (a) 4/7

47. (c)

After first ball is drawn white then sample space has 4 + 3 – 1 = 6 balls.

Probability of second ball is red without replacement

$$= \frac{{}^{3}c_0 \times {}^{3}c_1}{6} = \frac{1}{2}$$

14. X is a uniformly distributed random variable that takes values between 0 and 1. The value of $E\{X^3\}$ will be **[EE: GATE-2008]**

(a) 0 (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

14. Ans. (c)

$$f_x(x) = \begin{cases} 1, 0 < x < 1 \\ 0, other\ \text{wise} \end{cases}$$

$$E\left(X^3\right) = \int_{-\infty}^{\infty} x^3 f_x(x)dx = \int_0^1 x^3 dx = \left.\frac{x^4}{4}\right|_0^1$$

$$= \frac{1}{4} - 0 = \frac{1}{4}$$

IE All GATE Questions

48. Consider a Gaussian distributed radom variable with zero mean and standard deviation σ. The value of its cumulative distribution function at the origin will be **[IE: GATE-2008]**

(a) 0 (b) 0.5 (c) 1 (d) 10σ

48 Ans. (b)

49. A random variable is uniformly distributed over the interval 2 to 10. Its variance will be **[IE: GATE-2008]**

(a) $\frac{16}{3}$ (b) 6 (c) $\frac{256}{9}$ (d) 36

49. (a)

The p.d.f $f(x) = \frac{1}{10-2} = \frac{1}{8}$, $x \in (2,10)$

mean of $x = E(x) = \int_2^{10} \frac{1}{8} x\,dx = \frac{1}{8}\left[\frac{x^2}{2}\right]_2^{10} = \frac{1}{16}.96 = 6.$

Varience of x $= (\sigma_x^2) = E\left[(x-6)^2\right]$

$= \int_2^{10} (x-6)^2 \frac{1}{8} dx = \frac{1}{8}\left[\frac{x^3}{3} - \frac{12x^2}{2} + 36x\right]$

$= \frac{16}{3}$

50. The probability that there are 53 Sundays in a randomly chosen leap year is

(a) $\frac{1}{7}$ (b) $\frac{1}{14}$ (c) $\frac{1}{28}$ (d) $\frac{2}{7}$

[IE: GATE-2005]

50. (d)

No. of days in a leap year are 366 days. In which there are 52 complete weeks and 2 days extra.

This 2 days may be of following combination.

1. Sunday & Monday
2. Monday & Tuesday
3. Tuesday & Wednesday
4. Wednesday & Thursday
5. Thursday & Friday
6. Friday & Saturday
7. Saturday & Sunday

There are two combination of Sunday in (1.) and (7).

$\therefore$ Required probability

$= \frac{2}{7}$

51. You have gone to a cyber-café with a friend. You found that the cyber-café has only three terminals. All terminals are unoccupied. You and your friend have to make a random choice of selecting a terminal. What is the probability that both of you will NOT select the same terminal? [IE: GATE-2006]

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1

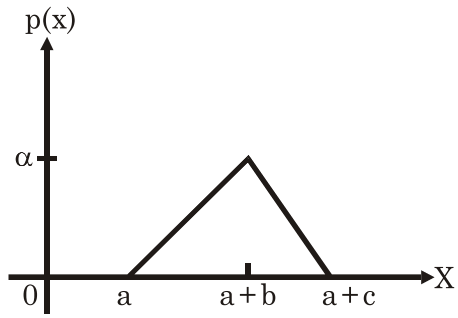
51.(c)

Out of three terminals probability of selecting terminals of two friends is $= \frac{1}{3}$

$\therefore$ Probability of not selecting same terminal $= 1 - \frac{1}{3} = \frac{2}{3}$

52. Probability density function p(x) of a random variable x is as shown below. The value of α is [IE: GATE-2006]

(a) $\frac{2}{c}$ (b) $\frac{1}{c}$ (c) $\frac{2}{(b+c)}$ (d) $\frac{1}{(b+c)}$



52.(a) $p(x)\,\text{is p.d.f.} \Rightarrow \int_{-\infty}^{\infty} p(x)dx = 1$

From figure, area of traiangle $= \frac{1}{2}.c.\alpha = \frac{\alpha c}{2}$

$\therefore \quad \frac{\alpha c}{2} = 1 \quad \Rightarrow \alpha = \frac{2}{c}$

53. Two dices are rolled simultaneously. The probability that the sum of digits on the top surface of the two dices is even, is [IE: GATE-2006]

(a) 0.5 (b) 0.25 (c) 0.167 (d) 0.125

53. (a)

Here sample space S= $6 \times 6 = 36$

Total no. of way in which sum of digits on the top surface of the two dice is is even is 18.

$\therefore$ The require probability $= \frac{18}{36} = 0.5$.

55. Poisson's ratio for a metal is 0.35. Neglecting piezo-resistance effect, the gage factor of a strain gage made of this metal is [IE: GATE-2010]

(a) 0.65 (b) 1 (c) 1.35 (d) 1.70

55. (d)

Poission's ratio $\sigma = 0.36$

Gage factor, $Gr = 1 + 2\sigma = 1 + 2 \times 0.35 = 1.70$

56. Assume that the duration in minutes of a telephone conversation follows the exponential distribution f(x) = $\frac{1}{5}e^{-\frac{x}{5}}, x \geq 0$. The probability that the conversation will exceed five minutes is **[IE: GATE-2007]**

(a) $\frac{1}{e}$ (b) $1-\frac{1}{e}$ (c) $\frac{1}{e^2}$ (d) $1-\frac{1}{e^2}$

56. (a)

Required probability $= \int_5^\infty \frac{1}{5}e^{-\frac{x}{5}}dx = \frac{1}{e}$

22. Using the given data points tabulated below, a straight line passing through the origin is fitted using least squares method. The slope of the line is

x	**1.0**	**2.0**	**3.0**
y	**1.5**	**2.2**	**2.7**

[IE: GATE-2005]

(a) 0.9 (b) 1.0
(c) 1.1 (d) 1.5

22. Ans.(c)

Suppose the line being, $y = mx$

Since, it has been fit by least square method, therefore

$$\sum y = \mu \sum x, \text{ and } \sum xy = \mu \sum x^2$$

$\therefore$ $m = 1.1$

23. The function y = sin ϕ, **(ϕ > 0)** is approximated as y = ϕ, where ϕ is in radian. The maximum value of ϕ for which the error due to the approximation is with in **±2% is**

[IE: GATE-2006]

(a) 0.1 rad (b) 0.2 rad
(c) 0.3 rad (d) 0.4 rad

23. Ans.(c)

CS All GATE Questions

Q3. If two fair coins are flipped and at least one of the outcome is know to be a head, what is the probability that both outcomes are heads?

(a) 1/3 (b) ¼ (c) ½ (d) 2/3 [CS-2011]

Ans. **(c)**

Q18. If the difference between the expectation of the square of a random variable $\left(E[X]\right)^2$ is denoted by R, then

R = 0 (b) R< 0 (c) $\mathbf{R \geq 0}$ (d) R > 0 [CS-2011]

(a)

Ans. **(c)**

Exp. We know,

The second control momnt,

$$\mu_2 = \mathbf{E\{(X-m)\}^2\,[m = \text{mean of the distribution of } X]}$$

$$= \mathbf{E(X^2) - 2m \times E(X) + m^2}$$

$$= \mathbf{E(X^2) - 2[E(X)]^2 + E(X)} \qquad \mathbf{[\because m = E(X)]}$$

$$\mathbf{E(X^2) - [E(X)]^2}$$

$$\mu_2 \geq \mathbf{0}$$

$$\therefore \mathbf{E(X^2) - [E(X)]^2 \geq 0}$$

Q34. A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

(a) 1/5 (b) 4/25 (c) ¼ (d) 2/5 [CS-2011]

Ans. *

57. For each element is a set of size 2n, an unbiased coin is tossed. The 2n coin tossed are independent. An element is chosen if the corresponding coin toss were head. The probability that exactly n elements are chosen is [CS: GATE-2006]

(a) $\dfrac{\binom{2n}{n}}{4^n}$ (b) $\dfrac{\binom{2n}{n}}{2^n}$ (c) $\dfrac{1}{\binom{2n}{n}}$ (d) $\dfrac{1}{2}$

57.(a)

The probability that exactly n elements are chosen

= the probability of getting n heads out of 2n tosses

$$= {}^{2n}c_n \left(\frac{1}{2}\right)^n \times \left(\frac{1}{2}\right)^{2n-n}$$

$$= \frac{{}^{2n}c_n}{2^{2n}}$$

$$= \frac{{}^{2n}c_n}{4^n}$$

59. Suppose we uniformly and randomly select a permutation from the 20! permutations of 1, 2, 3 ….., 20. What is the probability that 2 appears at an earlier position that any other even number in the selected permutation? [CS: GATE-2007]

(a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{9!}{20!}$ (d) None of these

59. (d)

Number of permutations with '2' in the first position = 19!

Number of permutations with '2' in the second position = 10 × 18!

(fill the first space with any of the 10 odd numbers and the 18 spaces after the 2 with 18 of the remaining numbers in 18! ways)

Number of permutations with '2' in 3^{rd} position = 10 × 9 × 17!

(fill the first 2 places with 2 of the 10 odd numbers and then the remaining 17 places with remaining 17 numbers)

and so on until '2' is in 11^{th} place. After that it is not possible to satisfy the given condition, since there are only 10 odd numbers available to fill before the '2'. So the desired number of permutations which satisfies the given condition is

19! + 10 × 18! + 10 × 9 × 17! + 10 × 9 × 8 × 16! + … + 10! × 9!

Now the probability of this happening is given by

$$\frac{19!+10\times18!+10\times9\times17!\ldots+10!\times9!}{20!}$$

Which is clearly not choices (a), (b) or (c)

Thus, Answer is (d) none of these.

60. Aishwarya studies either computer science or mathematics everyday. if the studies computer science on a day, then the probability that she studies mathematics the next day is 0.6. If she studies mathematics on a day, then the probability that she studies computer science the next day is 0.4. Given that Aishwarya studies computer science on Monday, what is the probability that she studies computer science on Wednesday? [CS: GATE-2008]

(a) 0.24 (b) 0.36 (c) 0.4 (d) 0.6

60. (c)

Let C denote computes science study and M denotes maths study.

P(C on monday and C on wednesday)

= p(C on monday, M on tuesday and C on wednesday)

+ p(C on monday, C on tuesday and C on wednesday)

= 1 × 0.6 × 0.4 + 1 × 0.4 × 0.4

= 0.24 + 0.16

= 0.40

61**.** Let X be a randon variable following normal distribution with mean +1 and variance 4. Let Y be another normal variable with mean –1 and variance unknown. If $P(X \leq -1) = P(Y \geq 2)$ the standard deviation of Y is [CS: GATE-2008]

(a) 3 (b) 2 (c) $\sqrt{2}$ (d) 1

61. Ans. (a) Given $\Psi_x = 1,\ \sigma_x^2 = 4 \Rightarrow \sigma_x = 2$ and $\mu_Y = -1,\ \sigma_Y$ is unknown

given, $p(X \le -1) = p(Y \ge 2)$

Converting into standard normal variates,

$$p\left(z \le \frac{-1-\mu_x}{\sigma_x}\right) = p\left(z \ge \frac{2-\mu_y}{\sigma_y}\right)$$

$$p\left(z \le \frac{-1-1}{2}\right) = p\left(z \ge \frac{2-(-1)}{\sigma_y}\right)$$

$$P(z \le -1) = p\left(z \ge \frac{3}{\sigma_y}\right) \quad \ldots \text{(i)}$$

Now since us know that in standard normal distribution,

$$P(z \le -1) = p(z \ge 1) \quad \ldots \text{(ii)}$$

Comparing (i) and (ii) we can say that

$$\frac{3}{\sigma_y} = 1 \Rightarrow \sigma_y = 3$$

62. An unbalanced dice (with 6 faces, numbered from 1 to 6) is thrown. The probability that the face value is odd is 90% of the probability that the face value is even. The probability of getting any even numbered face is the same.

If the probability that the face is even given that it is greater than 3 is 0.75, which one of the following options is closed to the probability that the face value exceeds 3?

[CS: GATE-2009]

(a) 0.453 (b) 0.468 (c) 0.485 (d) 0.492

62. (b)

It is given that

$P(\text{odd}) = 0.9\ p(\text{even})$

Now since $\Sigma p(x) = 1$

$\therefore$ $p(\text{odd}) + p\ (\text{even}) = 1$

$\Rightarrow$ $0.9\ p(\text{even}) + p\ (\text{even}) = 1$

$\Rightarrow$ $p(\text{even}) = \dfrac{1}{1.9} = 0.5263$

Now, it is given that p (any even face) is same

i.e $p(2) = p(4) = p(6)$

Now since,

$p(\text{even}) = p(2)$ or $p(4)$ or $p(6)$

$= p(2) + p(4) + p(6)$

$\therefore$ $p(2) = p(4) = p(6) = \dfrac{1}{3} p(\text{even})$

$= \dfrac{1}{3}(0.5263)$

$= 0.1754$

It is given that

$p(\text{even} \mid \text{face} > 3) = 0.75$

$\therefore \quad \dfrac{p(\text{even} \cap \text{face} > 3)}{p(\text{face} > 3)} = 0.75$

$\Rightarrow \quad \dfrac{p(\text{face} = 4, 6)}{p(\text{face} > 3)} = 0.75$

$\Rightarrow \quad p(\text{face} > 3) = \dfrac{p(\text{face} = 4, 6)}{0.75} = \dfrac{p(4) + p(6)}{0.75}$

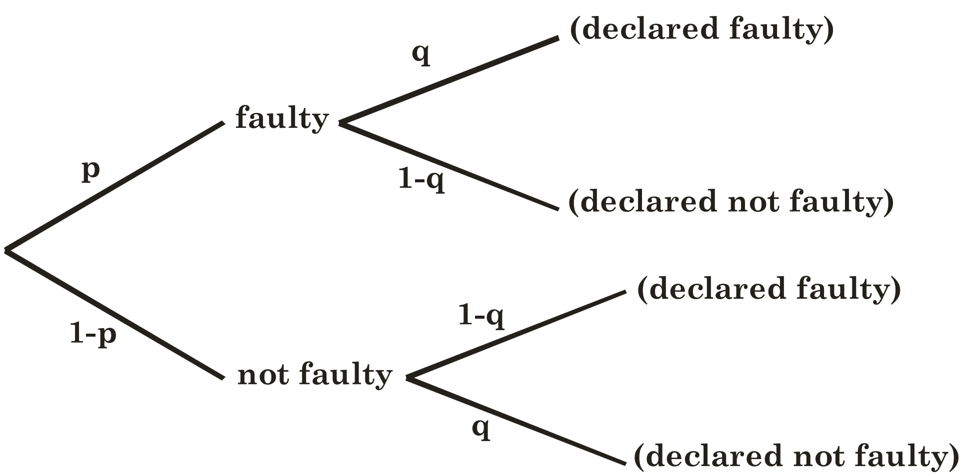
$= \dfrac{0.1754 + 0.1754}{0.75}$

$= 0.4677 \simeq 0.468$

63. Consider a company that assembles computers. The probability of a faulty assembly of any computer is p. The company therefore subjects each computer to a testing process. This testing process gives the correct result for any computer with a probability of q.
What is the probability of a computer being declared faulty? [CS: GATE-2010]

(a) $pq + (1 - p)(1 - q)$ (b) $(1 - q)p$ (c) $(1 - p)q$ (d) pq

63.(a)



From the diagram,

$P(\text{declared faulty}) = pq + (1-p)(1-q)$

64. What is the probability that a divisor of 10^{99} is a multiple of 10^{96}? [CS: GATE-2010]

(a) $\dfrac{1}{625}$ (b) $\dfrac{4}{625}$ (c) $\dfrac{12}{625}$ (d) $\dfrac{16}{625}$

64. Ans. (a)

p(multiple of 10% | divisor of 10^{99})

$$= \frac{n(\text{multiple of } 10^{96} \text{and divisor of } 10^{99)}}{n(\text{divisor of } 10^{99})}$$

Since $10 = 2.5$

$10^{99} = 2^{99} . 5^{99}$

Any divisor of 10^{99} is of the form $2^a . 5^b$ where $0 \le a \le 99$ and $0 \le b \le 99$.

The number of such possibilities is combination of 100 values of a and 100 values of b = 100 × 100 each of which is a divisor of 10^{99}.

So, no. of divisors of $10^{99} = 100 \times 100$.

Any number which is a multiple of 10^{96} as well as divisor of 10^{99} is of the form $2^a . 5^b$ where $96 \le a \le 99$ and $96 \le b \le 99$. The number of such combinations of 4 values of a and 4 values of b is 4 × 4 combinations, each of which will be a multiple of 10^{96} as well as a divisor of 10^{99}.

$\therefore$ p(multiple of 10^{96} | divisor of 10^{99})

$$= \frac{4 \times 4}{100 \times 100} = \frac{1}{625}$$

65. Let P(E) denote the probability of the even E. Given P(A) = 1, P(B) = $\frac{1}{2}$, the values of $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$ respectively are [CS: GATE-2003]

(a) $\frac{1}{4}, \frac{1}{2}$ (b) $\frac{1}{2}, \frac{1}{4}$ (c) $\frac{1}{2}, 1$ (d) $1, \frac{1}{2}$

65.(d)

Here, $P(A) = 1, P(B) = \frac{1}{2}$

Since A, B are independent events,

$\therefore \quad P(AB) = P(A)P(B)$

$$P\left(\frac{A}{B}\right) = \frac{P(AB)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) = 1$$

$$P\left(\frac{B}{A}\right) = \frac{P(A)P(B)}{P(A)} = P(B) = \frac{1}{2}$$

66. A program consists of two modules executed sequentially. Let $f_1(t)$ and $f_2(t)$ respectively denote the probability density functions of time taken to execute the two modules. The probability density function of the overall time taken to execute the program is given by [CS: GATE-2003]

(a) $f_1(t) + f_2(t)$ (b) $\int_0^t f_1(x) f_2(x) dx$

(c) $\int_0^t f_1(x) f_2(t-x) dx$ (d) $\max\{f_1(t), f_2(t)\}$

66.(c)

Let the time taken for first and second modules be represented by x and y and total time = t.

and y and total time = t.

$\therefore t = x + y$ is a random variable

Now the joint density function

$$g(t) = \int_0^t f(x, y)\, dx$$

$$= \int_0^t f(x, t - x)\, dx$$

$$= \int_0^t f_1(x)\, f_2(t - x)\, dx$$

which is also called as convolution of f_1 and f_2, abbreviated as $f_1 * f_2$.

Correct answer is therefore, choice (c).

67. If a fair coin is tossed four times. What is the probability that two heads and two tails will result? [CS: GATE-2004]

(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$

67. (a)

Here $P(H) = P(T) = \frac{1}{2}$

It's a Bernoulli's trials.

$\therefore$ Required probability

$$= {}^4c_2 . \left(\frac{1}{2}\right)^2 . \left(\frac{1}{2}\right)^2$$

$$= \frac{{}^4c_2}{2^4} = \frac{3}{8}$$

68. An examination paper has 150 multiple-choice questions of one mark each, with each question having four choices. Each incorrect answer fetches – 0.25 mark. Suppose 1000 students choose all their answers randomly with uniform probability. The sum total of the expected marks obtained all these students is [CS: GATE-2004]

(a) 0 (b) 2550 (c) 7525 (d) 9375

68. (d)

Let the marks obtained per question be a random variable X. It's probability distribution table is given below:

X	1	-0.25
P (X)	$\frac{1}{4}$	$\frac{3}{4}$

Expected mark per question $= E(x) = \sum x\, p(x)$

$$= 1\times\frac{1}{4} + (-0.25)\times\frac{3}{4} = \frac{1}{16}\text{marks}$$

Total marks expected for 150 questions

$$= \frac{1}{16}\times 150 = \frac{75}{8}\text{ marks per student.}$$

Total expected marks of 1000 students

$$= \frac{75}{8}\times 1000 = 9375\text{ marks}.$$

69. Two n bit binary strings, S_1 and S_2 are chosen randomly with uniform probability. The probability that the Hamming distance between these strings (the number of bit positions where the two strings differ) is equal to d is [CS: GATE-2004]

(a) $\frac{{}^{n}C_{d}}{2^{n}}$ (b) $\frac{{}^{n}C_{d}}{2^{d}}$ (c) $\frac{d}{2^{n}}$ (d) $\frac{1}{2^{d}}$

69.(a)

It's a binomial distribution

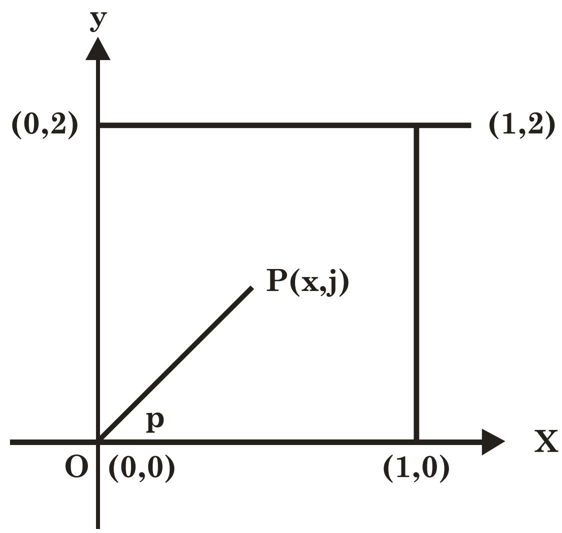
$$P(x = d) = {}^{n}c_{d}\left(\frac{1}{2}\right)^{d}\left(\frac{1}{2}\right)^{n-d}$$

$$= \frac{{}^{n}c_{d}}{2^{n}}$$

70. A point is randomly selected with uniform probability in the X-Y. plane within the rectangle with corners at (0, 0), (1, 0), (1, 2) and (0, 2). If p is the length of the position vector of the point, the expected value of p^2 is [CS: GATE-2004]

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

70. (d)



$\therefore \quad p = \sqrt{x^2 + y^2} \quad \Rightarrow p^2 = x^2 + y^2$

$\therefore E(p^2) = E(x^2 + y^2) = E(x^2) + E(y^2)$

Since x and y are uniformly distributed in the interval $0 \le x \le 1$ and $0 \le y \le 2$ respectively.

$\therefore$ Probability density function of x,

$$p(x) = \frac{1}{1-0} = 1$$

and probability density function of y,

$$p(y) = \frac{1}{2-0} = \frac{1}{2}$$

$$\therefore E(x^2) = \int_0^1 x^2 p(x)dx = \int_0^1 x^2\, dx = \frac{1}{3}$$

$$\text{And } E(y^2) = \int_0^2 y^2 p(y)dy = \int_0^2 \frac{y^2}{2} dy = \frac{4}{3}$$

$$\therefore E(p^2) = E(x^2) + E(y^2) = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

71. Let f(x) be the continuous probability density function of a random variable X. The probability that $a < X \le b$, is [CS: GATE-2005]

(a) f (b –a) (b) f(b) – f(a) (c) $\int_a^b f(x)dx$ (d) $\int_a^b xf(x)dx$

71.(c)

For continuous cases,

$$P(a < \times \le b) = \int_a^b f(x)dx$$