Problem Set 5

Design and Analysis of Algorithms

When a dynamic programming algorithm is presented, follow the given procedure.

- Define sub-problem(s) and explain how they help in solving the given problem.
- Design a recurrence relation for the sub-problem and prove its correctness.
- Give pseudocode to show how the sub-problems are computed.

1. Assume you are in charge of the Security management team. You are supposed to pick a project for your team every week. You have the option to pick a low-risk project or a high-risk project or no project at all. For week i, your team will earn h_i points if they do the high-risk project and l_i points if they do the low-risk project. (Your team gets zero points if they are not doing any project). Also, if you select a high-risk project in week i, the team is required not to have done any project in week $i-1$.

   Assume the values for h_i and l_i are known for $1 \leq i \leq n$. Give an algorithm to find the maximum total points you can earn in n weeks.

2. Let $IsWord(S)$ be a function that takes a string S as input and outputs TRUE if and only if S is a valid English word. Assume $IsWord(S)$ takes constant time to run. Given a string S, design an algorithm to decide if S can be broken into one or more valid English words. For example, if S is "ARTISTOIL", the algorithm should return TRUE since S can be broken down to "ARTIST" and "OIL".

3. In the above problem, modify the algorithm so that the algorithm returns the maximum number of ways in which a string can be broken down to valid English words. Therefore, if S is "ARTISTOIL", the algorithm should return 2 since S can be broken down to "ARTIST" and "OIL" and "ART", "IS" and "TOIL".

4. In the above problem, modify the algorithm so that the algorithm returns the minimum k such that string can be broken down to k valid English words. Therefore, if S is "ARTISTOIL", the algorithm should return 2 since S can be broken down to "ARTIST" and "OIL" .

5. Given two strings A, B of length n, decide whether A and B can be partitioned into words at the same indices. For example, the strings HEARTANDBODY and STARTONEWEEK can be partitioned into words at the same indices as follows:
   HEART AND BODY
   START ONE WEEK
   Assume that whether a given string is a valid word can be checked in constant time.

6. There are m houses in a street extending n kilometers along a straight line. It is needed to build transmitters such that for every house in the street, there should exist a transmitter within 500 m of it. Some possible locations for transmitters are identified and for each location, the cost of building a transmitter at that location is given. Give an algorithm to find the optimal locations for transmitters such that the total cost of building transmitters is minimised.

7. Given a string S give an algorithm to find the minimum value of k such that S can be partitioned into k substrings that are palindromes. For example, the string BANANAAXA can be partitioned into B.ANANA.AXA.

8. Give an algorithm to partition an array A into two subsets A_1 and A_2 such that $\sum_{A[i]\in A_1} A[i] - \sum_{A[i]\in A_2} A[i]$ is minimized. Reduce the problem to Knapsack Problem. Is your algorithm polynomial time? .

9. As a cashier, your job also involves giving change to any amount of money using coins. Let $w_1, w_2, \ldots, w_n$ represent the values of different coins available. Assuming you have an unlimited supply of coins of any denomination, design an algorithm to give change to amount of value W using minimum possible number of coins.

10. Describe and analyze an efficient algorithm to find the length of the longest contiguous substring that appears both forward and backward in an input string T[1..n]. The forward and backward substrings must not overlap. For example, "CASH" should return 0, "RECURSION" should return 1 (For the substring "R") and "REDIVIDE" should return 3 (for the substring "EDI").

11. Let T be a tree. You need to find a subset of edges E' in T such that no two edges in E' share an endpoint. Give a dynamic programming algorithm to find such a set of maximum cardinality.

12. Suppose you are given a sequence of integers separated by + and signs; for example:

    $1 + 3 - 2 - 5 + 1 - 6 + 7$
    You can change the value of this expression by adding parentheses in different places. For example:
    $1 + 3 - 2 - 5 + 1 - 6 + 7 = -1$
    $(1 + 3 - (2 - 5) + (1 - 6) + 7 = 9$
    $(1 + (3 - 2)) - (5 + 1) - (6 + 7) = -17$
    Describe and analyze an algorithm to compute, given a list of integers separated by + and - signs, the maximum possible value the expression can take by adding parentheses. Parentheses must be used only to group additions and subtractions; in particular, do not use them to create implicit multiplication as in 1 + 3(-2)(-5) + 1-6 + 7 = 33.