Problem Set 4

Design and Analysis of Algorithms

1. You are given a bag of capacity W and n items such that for $1 \leq i \leq n$, c_i is the maximum available quantity of item i and p_i is the total value of item i. Choose an amount $x_i \leq c_i$ of each item i to be packed in the bag subject to $\sum_i x_i \leq W$ and maximizing the total value of the packed items.

2. Given a graph $G(V, E)$, a *dominating set* of G is a subset of vertices $D \subseteq V$ such that for every vertex $v \in V$, either v is in D or/and v has a neighbor in D. The *Minimum Dominating Set* problem is to find a dominating set of minimum cardinality. Prove that the following greedy strategies do not always return a minimum dominating set.

   (a) Select the vertex v with the maximum degree. Add v to the dominating set and mark the neighbors of v as dominated. Delete v from G and repeat if there are vertices that are not marked dominated.

   (b) Initially all vertices are unmarked. Select the vertex v with the maximum number of unmarked neighbors. Add v to the dominating set and mark the neighbors of v as dominated. Delete v from G and repeat if there are vertices that are not marked dominated.

3. There are m houses in a street extending n kilometers along a straight line. It is needed to build transmitters such that for every house in the street, there should exist a transmitter within 500 m of it. Give an algorithm to find the optimal locations for transmitters such that minimum number of transmitters are built?

4. Assume that the schedule of n lectures for a day is given and lecture halls need to be allotted for each of them. Clearly the same lecture hall cannot be allotted for two lectures if their timings overlap. Give a greedy algorithm that finds an allotment of lecture halls that minimizes the number of lecture halls used.

5. Given a graph $G(V, E)$ the maximum independent set problem is to find a maximum cardinality subset of pairwise independent (non-adjacent) vertices. Show that the following greedy algorithm to solve the maximum independent set problem in a graph $G(V, E)$ is not optimal.
   Repeat till G is empty
   Select the vertex v with the minimum degree, delete the neighbors of v.

6. You have to schedule n computing jobs. Every job needs to be "preprocessed" on a supercomputer first and then run on a desktop computer. For every job i, the time needed on supercomputer is f_i and the desktop is s_i. You have limited access to supercomputer so only one job can run on the supercomputer at a time. However any number of jobs can run in parallel on desktop computers. Given the values of f_i and s_i for $1 \leq i \leq n$, find a schedule of the jobs so that the end time of the last job is minimized.

7. There are n kids of different ages. Give a greedy algorithm that group the kids into minimum possible number of groups such that the age difference between the eldest and youngest kid in any group is 2 years.

8. Let T be a tree. You need to find a subset of edges E' in T such that no two edges in E' share an endpoint. Give a greedy algorithm to find such a set of maximum cardinality.