Problem Set 1

Design and Analysis of Algorithms (Jan -May 2023)

January 13, 2023

1. Suppose you have k sorted arrays, each with n elements. Give an algorithm to merge all the arrays to form a sorted array of kn elements.

2. Given an array of n elements, give an algorithm to check whether there exists an element that repeats more than $\frac{n}{2}$ times in the array. The elements in the array are not comparable, therefore a query like $A[i] > A[j]$ is meaningless. However, you can query whether two elements are equal.

3. Given an array A of n elements, $(i, j), i, j \leq n$ is called an inversion if $i < j$ and $A[i] > A[j]$. Design a divide and conquer algorithm that counts the number of inversions in A.

4. Given two sorted arrays A and B of n integers, give an algorithm to find the median element of $A \cup B$.

5. An array A has n cells. Each cell is colored either black or white. It is known that the first and last cells are colored black and white respectively. The colors of the other cells are not known but can be checked in constant time. Give an algorithm to find two consecutive cells that are colored with different colors.

6. An array $A[1 \cdots n]$ is said to be unimodal if there exists an index $p, 1 \leq p \leq n$ such that elements of the array increases from index 1 to p and decreases thereafter. For example, $A = \{5, 6, 7, 9, 8, 4, 3, 2\}$ is a unimodal array with 9 being the peak element. Give an $O(\log n)$ algorithm that finds the peak element in a unimodal array.