6.0 INTRODUCTION

Heat i.e., the thermal energy is one of the forms of energy. It is transfered from one body or system to another as a result of temperature difference. Heat is positive when energy is transfered to a system's thermal energy from its environment and heat is negative when energy is transfered from a system's thermal energy to its environment.

Heat can produce the effects

(i) Expansion,

(ii) Change of temperature,

(iii) Change of state,

(iv) Change of physical property,

(v) Chemical change and

(vi) Transformation to other forms of energy.

Temperature : Temperature is the property of a body or region of space, which determines the rate at which heat will be transfered to or transfered from it. In other words, temperature is a thermal condition of the substance and measures its relative hotness or coldness.

When the universe began, its temperature was about $10^{39}K$. As the universe expanded, it cooled and it has now reached an average temperature of about 3 K (i.e., $-270^{o}C$). We on the earth are little warmer than that because we are living near Sun. Without our Sun we could not exist at the average temperature of 3K.

6.1 MODES OF TRANSMISSION OF HEAT

Heat energy can be transferred from one body to the other or within the body itself with a temperature difference. Heat is transmitted from high temperature to low temperature by three modes. They are :

(i) Conduction,

(ii) Convection and

(iii) Radiation

(i) **Conduction :**

Definition : The mode of transmission of heat without the actual motion of the heated molecules is called ***conduction.***

Explanation : Generally, conduction takes place in solids. The molecules of the solids are strongly held together in the structure by intermolecular forces of attraction. Molecules vibrate at their positions when heat energy is supplied to them. As they vibrate, they transfer their energy to the surrounding molecules causing them to vibrate. Thus heat transfer takes place as a result of vibratory motion of the molecules, but not by the actual motion of the heated molecules from one place to the other.

Example : When one end of a metal rod is kept in fire by holding the other end with bare hand, we feel warmness after some time because of conduction of heat from one end to the other end.

(ii) **Convection :**

Definition : The mode of transmission of heat by the actual motion of heated molecules from one place to the other is called ***convection.***

Explanation : Generally, convection takes place in liquids and gases. Heat is transmitted in fluids from a region of high temperature to a region of low temperature by the actual motion of the heated molecules.

Example : When water is boiled in a vessel, the water near the fire place gets heated first and becomes lighter. This hot water moves upwards forcing the cold and denser water at the top to come down. We can observe this convection during boiling of water.

(iii) Radiation :

Definition : The mode of transmission of heat without the aid of material medium is called ***radiation***.

Explanation : Radiation does not require any material medium for transmission of heat. By this process, heat can be transmitted through vacuum also. In this method, electromagnetic waves transfers heat from one place to the other.

Example : Heat energy from the Sun, reaches the earth by radiation because most of the space in between is vacuum.

6.2 THERMAL EXPANSION OF SOLIDS AND COEFFICIENTS OF EXPANSION OF SOLIDS

6.2.1 THERMAL EXPANSION OF SOLIDS

The tendency of solids to increase in their dimensions in response to increase in temperature is called ***thermal expansion of solids.***

When a solid is heated, size of the atoms does not increase but the volume occupied by them increases because of the vibrations of heated atoms. In a solid, atoms are closely packed together. On heating, kinetic energy of the atoms increases and vibrates rapidly. Every vibrating atom pushes away the neighboring atoms slightly resulting in slight increase of inter atomic distances. This adds up to a larger size for the whole body. Hence, the dimensions of the body increase on heating.

There are three types of thermal expansion of solids depending on the dimensions that undergo change on heating. They are

(i) Linear expansion,

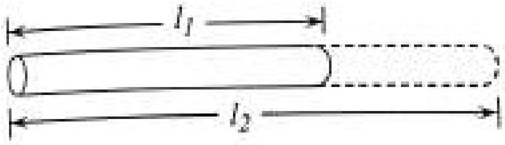
(ii) Superficial expansion, and

(iii) Cubical expansion.

6.2.2 LINEAR EXPANSION

Definition : Whenever there is an increase in length of the body due to heating, then the expansion is called ***linear expansion*** or longitudinal expansion.

Explanation :



When a thin metal rod or wire is heated, its length increases. The increase in length depends on its original length, the rise in temperature and nature of the material of the rod or wire.

Coefficient of Linear Expansion (α) : Consider a thin metal rod of length l_1 at a temperature of t_1 °C. If the rod is heated, its length increases. Let l_2 be the length of the rod at t_2 °C.

$\therefore$ Change in length $= l_2 - l_1$ and

Change in temperature $= t_2 - t_1$

The change in length is directly proportional to its original length and to the change in temperature.

$$\therefore \quad (l_2 - l_1) \propto l_1 \quad \text{and}$$

$$\propto (t_2 - t_1)$$

$$\Rightarrow \quad (l_2 - l_1) \propto l_1 (t_2 - t_1)$$

$$\Rightarrow \quad (l_2 - l_1) = \alpha\, l_1 (t_2 - t_1) \qquad \text{..........} \quad (1)$$

Where α is constant of proportionality and is also known as ***"Coefficient of linear expansion"***.

$$\Rightarrow \quad \boxed{\alpha = \frac{l_2 - l_1}{l_1 (t_2 - t_1)}} \qquad \text{..........} \quad (2)$$

This is the expression for coefficient of linear expansion of a solid.

In the above relation if $l_1 = 1$ and $(t_2 - t_1) = 1$ then $\alpha = (l_2 - l_1)$. Hence, the coefficient of linear expansion can be defined as follows.

Definition : The ***coefficient of linear expansion*** is defined as the change in length per unit original length for every degree rise in temperature.

Its SI unit is kelvin $^{-1}$

The coefficient of linear expansion is different for different materials.

Equation (1) is given by

$$(l_2 - l_1) = \alpha\ l_1 (t_2 - t_1)$$

$$\Rightarrow \quad l_2 = l_1 + \alpha\, l_1 (t_2 - t_1)$$

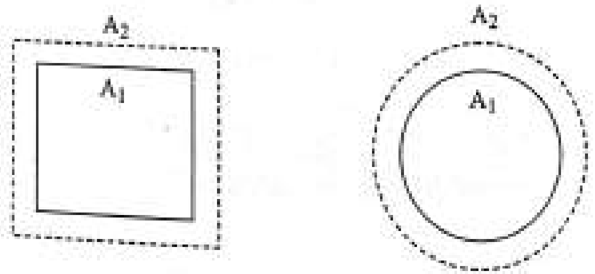
$$\Rightarrow \quad l_2 = l_1 [1 + \alpha\ (t_2 + t_1)]$$

This expression gives the final length of the rod after expansion.

6.2.3 SUPERFICIAL EXPANSION

Definition : Whenever there is an increase in area of a solid body due to heating, then the expansion is called ***superficial expansion*** or areal expansion.

Explanation :



When a solid is heated, it expands in all directions. In case of solids having definite shape like a rectangle, circle etc., there

will be incre... time which causes increase in area.

Coefficient of Superficial Expansion (β) : Consider a thin metal plate of area A_1 at a temperature of t_1 °C. If the metal plate is heated, its area increases. Let A_2 be the area of the plate at t_2 °C.

$$\therefore \quad \text{Change in area} = A_2 - A_1 \text{ and}$$

$$\text{Change in temperature} = t_2 - t_1$$

The change in area is directly proportional to its original area and to the change in temperature.

$$\therefore \quad (A_2 - A_1) \propto A_1 \text{ and}$$

$$\propto (t_2 - t_1)$$

$$\Rightarrow \quad (A_2 - A_1) \propto A_1 (t_2 - t_1)$$

$$\Rightarrow \quad (A_2 - A_1) = \beta\ A_1 (t_2 - t_1) \quad \text{..........} \quad (1)$$

When β is constant of proportionality and is also known as "***coefficient of superficial expansion***".

$$\therefore \quad \boxed{\beta = \frac{A_2 - A_1}{A_1 (t_2 - t_1)}} \quad \text{..........} \quad (2)$$

This is the expression for coefficient of superficial expansion of a solid.

In the above relation, if $A_1 = 1$ and $(t_2 - t_1) = 1$ then $\beta = (A_2 - A_1)$. Hence, the coefficient of superficial expansion can be defined as follows.

Definition : The ***coefficient of superficial expansion*** is defined as the change in area per unit original area for every degree rise in temperature.

Its SI unit is kelvin^{-1}

The change in [illegible] proportional to its original volume and to the change in temperature.

$$(V_2 - V_1) \propto V_1 \text{ and}$$

$$\therefore \qquad \propto (t_2 - t_1)$$

$$\Rightarrow \quad (V_2 - V_1) \propto V_1 (t_2 - t_1)$$

$$\Rightarrow \quad (V_2 - V_1) = \gamma V_1 (t_2 - t_1) \qquad \text{.........} \quad (1)$$

Where γ is the constant of proportionality and is also known as ***"coefficient of cubical expansion"***.

$$\therefore \quad \boxed{\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)}} \qquad \text{.........} \quad (2)$$

This is the expression for coefficient of cubical expansion of a solid.

In the above relation, if $V_1 = 1$ and $(t_2 - t_1) = 1$ then $\gamma = (V_2 - V_1)$. Hence, the coefficient of cubical expansion can be defined as follows.

Definition : The *coefficient of cubical expansion* is defined as the change in volume per unit original volume for every degree rise in temperature.

Its SI unit is kelvin^{-1}.

The coefficient of cubical expansion is different for different materials.

Equation (1) is given by

$$(V_2 - V_1) = \gamma V_1 (t_2 - t_1)$$

$$\Rightarrow \quad V_2 = V_1 + \gamma V_1 (t_2 - t_1)$$

$$\Rightarrow \quad V_2 = V_1 [1 + \gamma (t_2 - t_1)]$$

This expression gives the final volume of the metal cube after expansion.

Note : The three coefficients of expansion of solids are related as follows.

(i) $\beta = 2\alpha$ and

(ii) $\gamma = 3\alpha$

6.3 THERMAL CONDUCTIVITY OF A SOLID

Thermal conductivity is the ability of a material to conduct heat through it. It expresses the rate at which heat passes through the specified material. Higher the thermal conductivity faster the heat transfer between two places in the material. Metals are good conductors of heat and their thermal conductivity is great when compared with that of the poor conductors like rubber. So, heat transfers at lower rate in materials of low thermal conductivity than in materials of high thermal conductivity.

The amount of heat conducted from hotter place to colder place in a material is

(i) directly proportional to the area of cross-section,

(ii) directly proportional to the time for which heat flows,

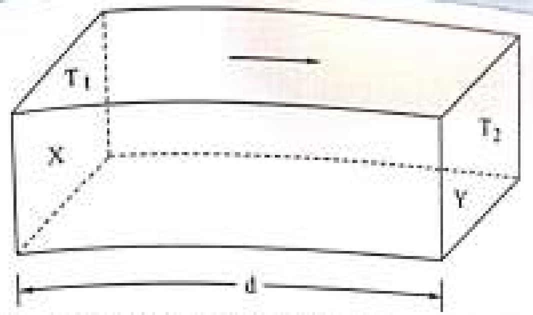
(iii) directly proportional to the temperature difference between the places and

(iv) inversely proportional to the distance between the two places.

Definition : The rate at which heat is transferred by conduction through unit area of cross-section of a material, when a temperature gradient exisits perpendicular to the area, is called ***thermal conductivity.***

EXPRESSION FOR THERMAL CONDUCTIVITY

Consider a metal slab of area of cross-section A and length d, whose end faces X and Y are at temperatures T_1 and T_2 respectively.



If $T_1 > T_2$, according to the laws of thermal conductivity, the amount of heat Q conducted across its end faces in a time t is given by

$$Q \propto A$$
$$\propto t$$
$$\propto (T_1 - T_2)$$
$$\propto \frac{1}{d}$$

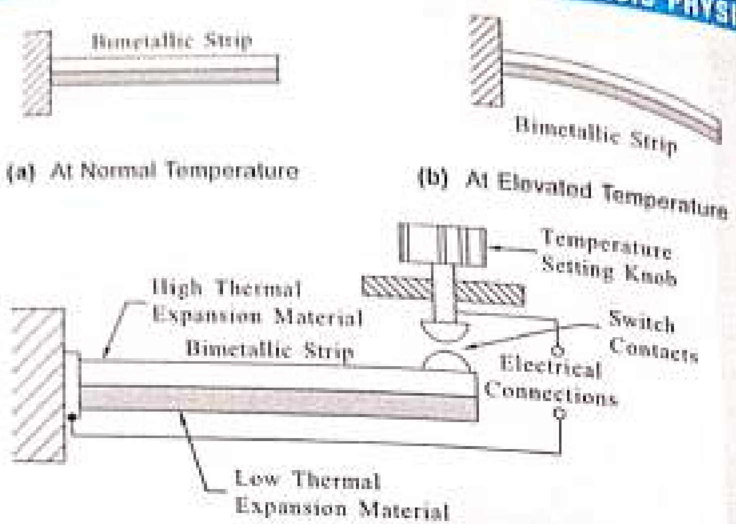
$$\Rightarrow \quad Q \propto \frac{At\,(T_1 - T_2)}{d}$$

$$\text{or} \quad Q = K\,\frac{At\,(T_1 - T_2)}{d}$$

where K is constant of proportionality and is called '***coefficient of thermal conductivity***'.

4 BIMETALLIC STRIP

A bimetallic strip is made by bonding two different thin strips of metals usually steel and brass or steel and copper. The coefficients of thermal expansion of these metals are different at the same temperature. On heating, these two strips expand at different rates and create a bending effect. When temperature increases, the bimetallic strip bends towards the metal having lower coefficient of thermal expansion and when temperature decreases, it bends towards the metal having higher coefficient of thermal expansion.



(c) Bimetallic Strip Thermostat

FIGURE 6.1 :

6.5 APPLICATION OF BIMETALLIC THERMOSTAT AS AUTOMATIC TEMPERATURE CONTROL DEVICE IN ELECTRICAL GADGETS

In thermostat, a bimetallic strip is used to control power supply to the heating appliance. A dial is connected to the electrical circuit of heating element of the appliance through a bimetallic strip which switches the circuit on and off by bending. When the power supply is switched on, the bimetallic strip starts getting heated. As the temperature increases, the two metallic strips expand in different lengths. The metal having higher coefficient of thermal expansion expands more than the metal having lower thermal coefficient. As the two strips are bonded together, there will be a bending effect.

The dial can be set at a desired temperature at which the bimetallic strip switches the circuit on and off. When the bimetallic strip is cool, it is straight and acts as a bridge through which electricity can flow. Then the circuit is on and heating process is carried out in the appliance.

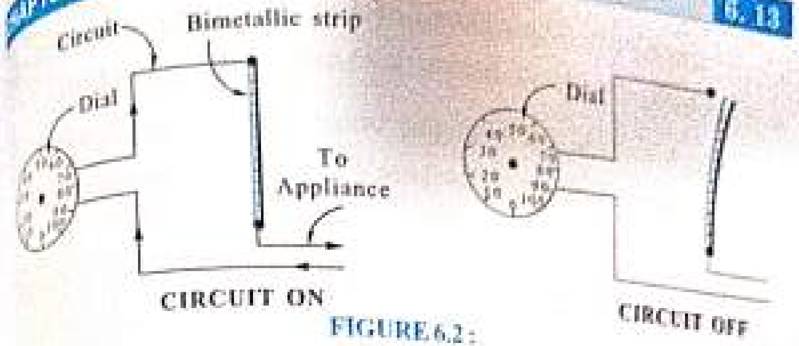


FIGURE 6.2 :

When the set temperature is reached, the bimetallic strip becomes hotter and it bends and breaks the circuit. Then the circuit is off and no electricity flows. As a result, the heating process stops in the appliance.

When heat is drawn from the appliance or due to loss of heat with time, the temperature in the appliance decreases and also the temperature of bimetallic strip. As a result, the bimetallic strip bends back and becomes straight. Then the circuit is on and heating process is carried out again in the appliance till the set temperature.

Thus the bimetallic strip used in thermostat automatically controls the temperature in electrical gadgets.

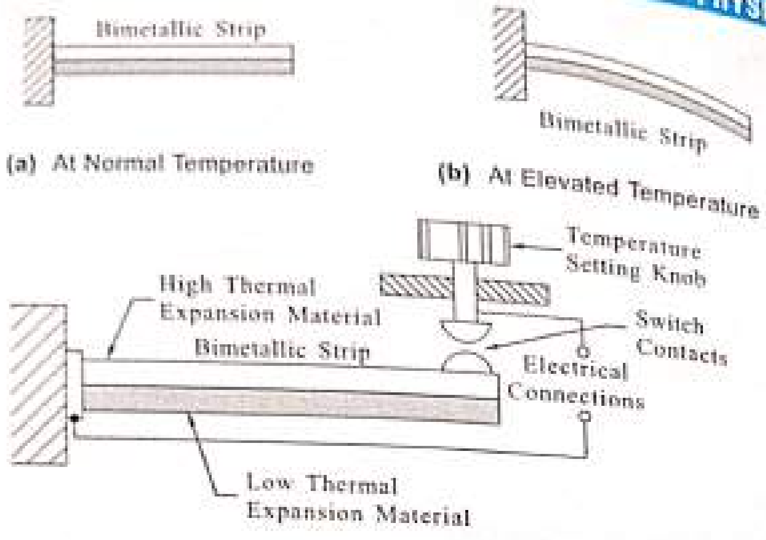
BOYLE'S LAW AND ITS LIMITATIONS

EXPANSION OF GASES

The effect of pressure is very small in solids and liquids. But in the case of gases volume changes due to change in pressure. If the temperature of a gas changes, then both of its volume and pressure changes. Thus in the case of gases there are three variables - volume (V), pressure (P) and temperature (t) which depends upon one another. If one of these factors is kept constant, we get a gas law that connects the other two.

BOYLE'S LAW

The Boyle's law gives the variation of volume with pressure when temperature is kept constant.



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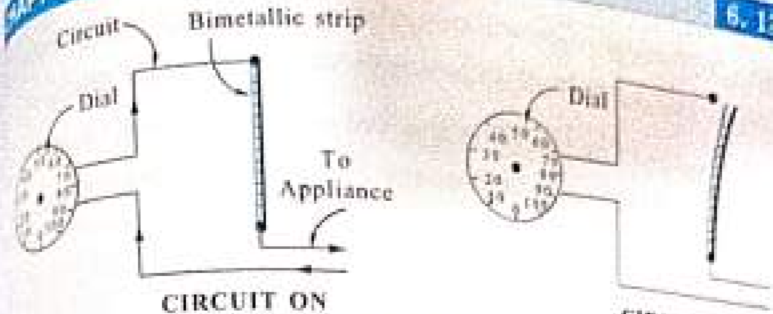

FIGURE 6.2 :

When the set temperature is reached, the bimetallic strip becomes hotter and it bends and breaks the circuit. Then the circuit is off and no electricity flows. As a result, the heating process stops in the appliance.

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6 BOYLE'S LAW AND ITS LIMITATIONS

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BOYLE'S LAW

The Boyle's law gives the variation of volume with pressure when temperature is kept constant.

Statement : The volume of a given mass of gas is inversely proportional to its pressure at constant temperature.

Explanation : If V is the volume of a given mass of gas corresponding to its pressure P at constant temperature, then

$$V \propto \frac{1}{P}$$

$$\Rightarrow \quad V = K \cdot \frac{1}{P}$$ (Where K is the constant of proportionality whose value depends on mass and temperature of the gas)

$$\Rightarrow \quad \boxed{PV = K}$$

If the volume of a given mass of gas increases, its pressure decreases and if the volume decreases, its pressure increases in such a way that the product of pressure and volume is constant at a particular temperature.

Example : Consider a rubber ball containing a certain amount of gas. The gas enclosed in the ball possess certain volume V_1 and certain pressure P_1. Suppose that the ball is kept in hand and compressed a little. Then its volume decreases and pressure increases. The increase in pressure can be felt by our sense. From this we can conclude that the volume of the gas is inversely proportional to its pressure. Let the changed volume and pressure be V_2 and P_2 respectively.

Now suppose that the ball is compressed a little more. Let the changed volume and pressure be V_3 and P_3 respectively. This activity was done at the room temperature which can be considered as constant during the short period of activity.

According to Boyle's law, we can write that

$$P_1V_1 = P_2V_2 = P_3V_3 = \text{Constant}$$

The graph plotted between pressure and the corresponding volume of a given mass of gas at constant temperature is shown in the Fig. 6.3.

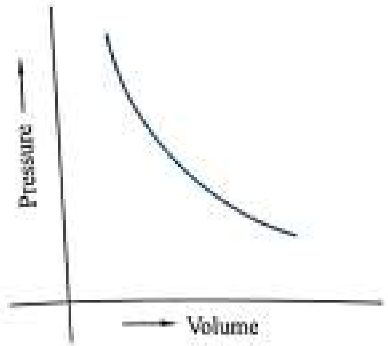

FIGURE 6.3 :

LIMITATIONS OF BOYLE'S LAW

1. Gases obey Boyle's law at high temperatures and low pressures.
2. Gases do not obey Boyle's law at low temperatures and high pressures.

SOLVED PROBLEM

PROBLEM-1

Pressure of a gas is $100\ Nm^{-2}$ and its volume is 10 litres. If pressure is changed to $200\ Nm^{-2}$ at constant temperature, find the volume. [Oct/Nov. 2018 (CME)]

Solution :

Given that :

temperature t = constant

initial pressure $P_1 = 100\ Nm^{-2}$

initial volume V_1 = 10 litres $= 10 \times 10^{-3} = 10^{-2}\ m^3$

final pressure $P_2 = 200\ Nm^{-2}$

final volume $V_2 = ?$

Thus at $-273^{\circ}C$, the pressure and volume of a given mass of gas becomes zero. If the temperature is less than $-273^{\circ}C$, the pressure and volume of the gas becomes negative which is absurd. Hence $-273^{\circ}C$ is the lowest possible temperature theoretically. In reality all the gases liquify and even solidify before reaching this temperature.

ABSOLUTE ZERO

Definition : The temperature ($-273^{\circ}C$) at which the pressure and volume of a given mass of gas becomes zero is called '*Absolute zero*'.

ABSOLUTE SCALE OF TEMPERATURE

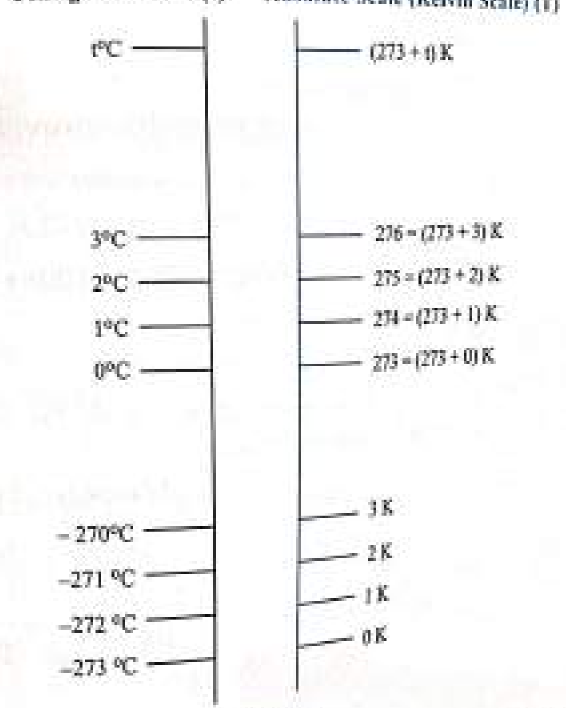

FIGURE 6.4 :

A new scale of temperature was established by taking Absolute zero as the starting value. This scale is called ***Absolute scale*** or ***Kelvin scale***. The magnitude of 1° on the Kelvin scale is equal to that of on centigrade scale. The temperature on absolute scale is represented by *T*.

It is clear from the Fig. 6.4 that

$$\boxed{T^{o}A = 273 + t^{o}C}$$

Examples of Conversion :

1. Melting point of ice is 0°C = (273 + 0) = 273 K.
2. Boiling point of water is 100°C = (273 + 100) = 373 K.

6.7.3 SOLVED PROBLEMS

PROBLEM-2

Express the following in absolute scale of temperature :

(a) Melting temperature of ice.

(b) Boiling temperature of water.

[Oct/Nov. 2017 (EEE)]

Solution :

(a) Melting point of ice is 0°C = (273 + 0) = 273 K.

(b) Boiling point of water is 100°C = (273 + 100) = 373 K.

PROBLEM-3

Normal human body temperature is 37 °C. Convert into absolute scale of temperature.

[Mar/Apr. 2016 (DME)]

(or)

Convert 37 °C into kelvin.

[Mar/Apr. 2016 (ECE)]

Solution :

The relation between centigrade scale and absolute scale (Kelvin scale) is given by,

$$T^{o}A = 273 + t^{o}C$$

$$\Rightarrow \quad T^{o}A = 273 + 37 = 310\ K$$

∴ The normal human body temperature is 310 *K*

CHARLES' LAWS IN TERMS OF ABSOLUTE TEMPERATURE

CHARLES' FIRST LAW

Charles' Law at Constant Pressure : This law gives the variation of volume with absolute temperature at constant pressure.

Statement : The volume of a given mass of gas at constant pressure is directly proportional to its absolute temperature.

Explanation : If the absolute temperature increaes, the volume of the gas increases and if the absolute temperature decreases, the volume of the gas decreases.

Let *V* be the volume of the gas at absolute temperature *T*.

Then $\quad V \propto T$

$$\Rightarrow \quad V = (\text{constant})\ T$$

$$\Rightarrow \quad \boxed{\frac{V}{T} = \text{constant}}$$

∴ The ratio of volume to absolute temperature of a given mass of gas at constant pressure is constant.

If V_1 and V_2 are the volumes of the gas at absolute temperatures T_1 and T_2 respectively, then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{constant}$$

6.8.2 CHARLES' SECOND LAW

Charles' Law at Constant Volume : This law gives the variation of pressure with absolute temperature at constant volume.

Statement : The pressure of a given mass of gas at constant volume is directly proportional to its absolute temperature.

Explanation : If the absolute temperature increases, the pressure of the gas increases and if the absolute temperature decreases, the pressure of the gas decreases.

Let P be the pressure of the gas at absolute temperature T.

Then $P \propto T$

$\Rightarrow$ $P = (\text{constant})\ T$

$\Rightarrow$ $\boxed{\dfrac{P}{T} = \text{constant}}$

$\therefore$ The ratio of pressure to absolute temperature of a given mass of gas at constant volume is constant.

If P_1 and P_2 are the pressures of the gas at absolute temperatures T_1 and T_2 respectively, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{Constant}$$

6.8.3 SOLVED PROBLEMS

(Adopted SI system for consistency of units)

PROBLEM-4

A litre of air is heated from 27°C to 177°C at constant pressure. Find increase in volume.

[Oct/Nov. 2018 (EEE) ; 2017 (DCE, CME, ECE, Common) ; Apr/May. 2015 (ECE)]

Solution :

Given that,

initial temperature $t_1 = 27°C \Rightarrow T_1 = 273 + 27 = 300\ K$

final temperature $t_2 = 177°C \Rightarrow T_2 = 273 + 177 = 450\ K$

initial volume $V_1 = 1\ lt = 10^{-3}\ m^3$

final volume $V_2 = ?$

From Charles' law at constant pressure,

we have $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$

$\Rightarrow$ $V_2 = \dfrac{V_1 T_2}{T_1}$

Substituting the values, we get

$$V_2 = \frac{10^{-3} \times 450}{300}$$

$= 1.5 \times 10^{-3}\ m^3 = 1.5$ litres

Increase in volume $= V_2 - V_1$

$= 1.5 - 1 = 0.5$ litres.

PROBLEM-5

A gas at 30°C has its temperature raised so that volume is doubled. The pressure remaining consant, what is its final temperature ?

[Oct/Nov. 2019 (Common) ; 2018 (ECE) ; Apr/May. 2012 (CME) ; Sept/Oct. 2001 ; Oct. 1994 ; Dec.1991]

Solution :

Given that,

Pressure is consant.

Initial temperature $t_1 = 30°C$

$\Rightarrow$ $T_1 = 273 + 30 = 303\ K$

Initial volume $V_1 = V$ (say)

Final volume $V_2 = 2V$

Final temperature $t_2 = ?$

From Charles' law at constant pressure, we have

From Charles' law at constant volume, we have

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\Rightarrow \quad T_1 = \frac{P_1 T_2}{P_2}$$

Substituting the given values in the above relation,

we get $$T_1 = \frac{0.72\times13.6\times10^3\times9.8\times375}{0.9\times13.6\times10^3\times9.8}$$

$$= 300\ K$$

Then $$t_1 = 300 - 273 = 27^\circ C$$

$\therefore$ The initial temperature is $27^\circ C$

IDEAL GAS EQUATION

IDEAL GAS

Definition : A gas that obeys gas laws exactly and consists of perfectly elastic and infinitesimally small molecules with negligible intermolecular forces of attraction is called an "*ideal gas*".

It is a theoretical concept of a gas. There is no true ideal gas in nature. All real gases approach the ideal state at low enough densities at which molecules are far apart and do not interact with one another.

IDEAL GAS EQUATION

An equation, connecting the pressure P and volume V of a certain amount of gas with the absolute temperature T is called gas equation.

Initial condition of the gas $\longrightarrow$ $P_1 \quad V_1 \quad T_1$

Intermediate condition $\longrightarrow$ $P_2 \quad V_x \quad T_1$

Final condition of the gas $\longrightarrow$ $P_2 \quad V_2 \quad T_2$

Consider a certain amount of perfect gas enclosed in a container. Let V_1 be the volume of the gas at pressure P_1 and absolute temperature T_1.

At first, suppose that, the pressure of the gas is increased from P_1 to P_2 by keeping the absolute temperature constant at T_1. According to Boyle's law its volume changes and let the changed volume be V_x. Now the gas is at pressure P_2, volume V_x and absolute temperature T_1.

Applying Boyle's law,

we get $\quad P_1 V_1 = P_2 V_x$

$$\Rightarrow \quad V_x = \frac{P_1 V_1}{P_2} \qquad \text{..........} \quad (1)$$

Now suppose that the absolute temperature of the gas is raised from T_1 to T_2 by keeping its pressure constant at P_2. According to Charles' law at constant pressure, the volume of the gas changes. Let the changed volume be V_2. Now the gas is at pressure P_2, volume V_2 and absolute temperature T_2.

Applying Charles' law at constant pressure,

we get $\quad \dfrac{V_x}{V_2} = \dfrac{T_1}{T_2}$

$$\Rightarrow \quad V_x = \frac{V_2 T_1}{T_2} \qquad \text{..........} \quad (2)$$

Comparing the equations (1) and (2) we get

$$\frac{P_1 V_1}{P_2} = \frac{V_2 T_1}{T_2}$$

By re-arranging the quantities

we get $\quad \boxed{\dfrac{P_1 V_1}{T_1} = \dfrac{P_2 V_2}{T_2}}$

If we consider the gas at another condition of pressure P_3, volume V_3 and absolute temperature T_3, then

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} = \text{constant}$$

From the above relation, we can conclude that

$$\frac{PV}{T} = \text{constant.}$$

If we consider the volume occupied by 1 gram mole of the gas at NTP, then the constant is represented by R.

$$\therefore \quad \frac{PV}{T} = R$$

where R is known as '*Universal gas constant*' whose value is the same for all gases.

$$\Rightarrow \quad \boxed{PV = RT}$$

This equation is called ***ideal gas equation.***

1.1.3 ALTERNATIVE METHOD OF DERIVATION OF IDEAL GAS EQUATION

Consider a certain amount of perfect gas in an enclosure having pressure P, volume V at abslute temperature T. According to Boyle's law, the volume of the gas is inversely proportional to its pressure at constant temperature.

$$\Rightarrow \quad V \propto \frac{1}{P} \qquad \text{..........} \quad (1)$$

According to Charles' law at constant pressure, the volume of the gas is directly proportional to its absolute temperature at constant pressure. .... (2)

$$\Rightarrow \quad V \propto T$$

Combining the relations (1) and (2), we get

$$V \propto \frac{T}{P}$$

$$\Rightarrow \quad V = (\text{constant}) \frac{T}{P}$$

$$\Rightarrow \quad \frac{PV}{T} = \text{constant}$$

This constant is represented by R if one gram mole of gas is considered at NTP, and is known as ***'Universal gas constant'***.

$$\therefore \quad \frac{PV}{T} = R$$

$$\Rightarrow \quad \boxed{PV = RT}$$

This is the universal gas equation or ideal gas equation.

6.9.4 VALUE OF UNIVERSAL GAS CONSTANT

The universal gas constant $R = \frac{PV}{T}$

The universal gas constant is defined by considering one gram mole of gas. According to ***Avagadro's hypothesis*** one gram mole of any gas occupies a volume of 22.4 *litres* at NTP. If we substitute the value of volume at NTP, in the relation $\frac{PV}{T}$, we get the value of Universal gas constant.

Normal atmospheric pressure $P = 0.76 \times 13.6 \times 10^3 \times 9.8\ Pa$

Volume occupied by 1 *gram* mole of any gas

$$V = 22.4 \text{ litres} = 22.4 \times 10^{-3}\ m^3$$

Normal temperature $T = 273\ K$.

Substituting the above values in the relation

$$R = \frac{PV}{T}$$

we get $R = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273}$

$$= \mathbf{8.31\ \textit{joules mole}^{-1}\ \textit{kelvin}^{-1}}$$

From the relation $\frac{P_1}{d_1T_1} = \frac{P_2}{d_2T_2}$

We have $d_2 = \frac{P_2 d_1 T_1}{P_1 T_2}$

Substituting the values,

$$d_2 = \frac{0.75\times13.6\times10^3\times9.8\times1.293\times273}{0.76\times13.6\times10^3\times9.8\times303}$$

$$= 1.15\ kg\ m^{-3}$$

∴ The final density of the air is $1.15\ kg\ m^{-3}$.

6.11 ISOTHERMAL AND ADIABATIC PROCESSES

6.11.1 ISOTHERMAL PROCESS

Definition : The thermodynamic process in which the temperature of the gas remains constant but pressure and volume changes is called ***isothermal process.***

If a system is perfectly conducting to the surroundings, then exchange of heat takes place and the temperature remains constant throughout the process. Hence consider a certain amount of gas enclosed in a perfectly conducting cylinder provided with a smooth piston.

Isothermal Compression :

FIGURE 6.5 :

When the piston is moved slowly inwards, the gas in the cylinder is compressed and some work is done on the gas. Consequently its volume decreases and pressure increases. Infact the temperature of the gas has to increase since some work is done on the gas which increases its internal energy. But the cylinder and piston are perfectly conducting which

allows exchange of heat between the gas and surroundings. Moreover the piston is moved slowly in order to provide enough time for exchange of heat. So the gas gives out the excess heat to the surroundings and the rise in temperature cannot be observed. Hence the temperature is constant.

Isothermal Expansion : When the gas is allowed to expand, it pushes the piston outwards and some work is done by the gas. In this case volume of the gas increases and pressure decreases. Here the gas does some work by spending its internal energy and the temperature should decrease. But this fall in temperature cannot be observed because the gas absorbs heat from the surroundings continuously to compensate the external work done by it. Hence the temperature is constant..

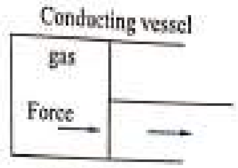

FIGURE 6.6 :

For isothermal process Boyle's law holds good i.e., PV = constant. This is a slow process. The internal energy of the gas is maintained constant during this process.

6.11.2 ADIABATIC PROCESS

Definition : The thermodynamic process in which temperature, pressure and volume of a gas changes is called ***adiabatic process.***

If a system is percfectly non-conducting to the surroundings, exchange of heat cannot take place between the gas and surroundings and the temperature changes during the process. Hence consider a certain amount of gas enclosed in a perfectly non-conducting cylinder provided with a smooth piston.

Adiabatic Compression : When the piston is moved inwards, the gas enclosed in the cylinder is compressed.

6.12.2 EXTERNAL WORK DONE

Definition : The cause of change in internal energy of an isolated system or gas is called ***external work***.

External work is of two types.

(i) Work done on the system and

(ii) Work done by the system.

Consider a certain amount of perfect gas enclosed in a cylinder provided with a smooth and frictionless piston.

1. **Work done on the system :** If the piston is displaced inwards by the application of an external force, then work is said to be done on the gas or system.

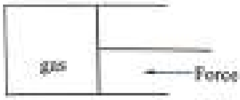

FIGURE 6.9 :

2. **Work done by the system :** If the gas enclosed in the cylinder is allowed to expand, it displaces the piston outwards. Then work is said to be done by the gas or system.

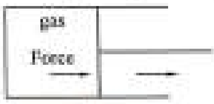

FIGURE 6.10 :

6.13 EXPRESSION FOR WORK DONE BY THE GAS

Consider a certain amount of ideal gas enclosed in a cylinder having a frictionless piston. Let P be the pressure on both sides of the piston.

Now suppose that the gas is heated at constant pressure. To keep the pressure constant, the gas expands and pushes the piston outwards. Let the displacement of the piston be dx.

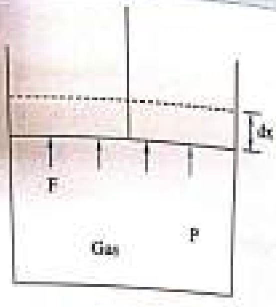

FIGURE 6.11 :

Here some work is done by the gas. If A is the area of the piston, then

Force on the piston = pressure×area = $P \times A$

$\therefore$ Work done by the gas = Force on the Piston × Displacement of the piston

$$\Rightarrow \quad W = P \times A \times dx$$

$= PdV$ ($\because Adx = dV$ = increase in volume of the gas)

If the gas expands from a volume V_1 to volume V_2 at constant pressure, then $dV = V_2 - V_1$

$\therefore$ Work done by the gas $\boxed{W = P(V_2 - V_1)}$

This is the expression for work done by an ideal gas.

6.13.1 SOLVED PROBLEM

PROBLEM-15

At constant pressure of 2×10^5 Nm^{-2}, the volume of the gas is changed from 20 cc to 60 cc. Find the work done by the gas. **[Oct/Nov. 2018 (ECE)]**

Solution :

Given that :

Pressure $P = 2 \times 10^5$ Nm^{-2}

initial volume $V_1 = 20\ cc = 20 \times 10^{-6}\ m^3$

final volume $V_2 = 60\ cc = 60 \times 10^{-6}\ m^3$

Work done by the gas

$$W = P(V_2 - V_1)$$

Substituting the values, we get,

$$W = 2 \times 10^5 (60 \times 10^{-6} - 20 \times 10^{-6})$$

$$= 2 \times 10^5 \times 10^{-6} (60 - 20)$$

$$= 8 \text{ joules}$$

∴ The work done by the gas is 8 joules.

6.14 LAWS OF THERMODYNAMICS

6.14.1 FIRST LAW OF THERMODYNAMICS

Statement : The heat energy supplied to a system is equal to the sum of the increase in internal energy of the system and the external work done by it.

Explanation : When some amount of heat is supplied to a gas enclosed in a cylinder having a piston, the gas absorbs that energy and utilises it in two ways. A part of this energy is utilised to increase its internal energy and the other part is utilised to do work in expanding against constant pressure.

If dQ is the amount of heat supplied to the system, dU is the increase in internal energy and dW is the external work done by the system, then

$$\boxed{dQ = dU + dW}$$

Examples : Working of diesel, petrol and steam engines is based on this law.

6.14.2 PRACTICAL APPLICATIONS

1. **Heat Engines :** Conversion of heat to work requires the use of certain devices called heat engines. Thermal energy is converted into mechanical energy via heat engines. In heat engines like petrol engines, diesel engines, steam engines, jet engines etc., heat is received from combustion of fuel and converts a part of this heat to work. They discharge the remaining heat to the environment.

2. **Refrigerators and Air-conditioners :** In refrigerators and air conditioners, mechanical energy is converted into heat. A mechanical pump transports the working fluid (gas) to the compressor kept outside where it is compressed and hence heated. This heat is radiated to the environment. When the compressed air is allowed to expand into indoors, its temperature falls to a value below the initial temperature. This cool air absorbs heat in the chamber and again transported to the compressor. This is a continuous process and thus heat is removed from the chamber.

3. **Heat Pumps :** In heat pumps, the heat of compressed working fluid (gas) is used to warm the chamber. After discharging the heat in chamber, it is transported to outside where it expands and cools down. It absorbs the heat from the surroundings and transported inside for compression. In heat pumps, the transportation of heat is reverse to air conditioners.

6.14.3 SOLVED PROBLEMS

PROBLEM-16

An amount of 100 joules heat energy is supplied to a gas and simultaneously 50 joules work is done on the gas. Then find the change in internal energy.

[Oct/Nov. 2018 (D…]

Solution :

Given that :

heat energy supplied to the gas $dQ = 100$ joules

work done on the gas $-dW = 50$ joules

increase in internal energy $dU = ?$

From the first law of thermodynamics, we have,

$$dQ = dU + dW$$

$$\Rightarrow \quad dU = dQ - dW$$

Substituting the values, we get,

$$dU = 100 + 50$$
$$= 150 \text{ joules.}$$

$\therefore$ The increase in internal energy of the gas is 150 joules.

PROBLEM-17

When heat energy of 2000 joules is supplied to a gas at constant pressure $2 \times 10^5\ Nm^{-2}$, there was an increase in its volume equal to $0.004\ m^3$. Calculate the increase in internal energy of the gas.

[Mar/Apr. 2017 (DCE, CME); Apr/May. 2015 (EEE)]

Solution :

Given that,

Heat energy supplied, $dQ = 2000$ *joules*

Pressure, $P = 2 \times 10^5\ Nm^{-2}$

Change in volume, $dV = 0.004\ m^3$

Increase in internal energy $dU = ?$

From the first law of thermodynamics, we have

$$dQ = dU + dW$$

$$\Rightarrow \quad dU = dQ - dW$$
$$= dQ - PdV \quad (\therefore dW = PdV)$$

Substituting the given values in the above relation, we get,

$$dU = 2000 - (2 \times 10^5 \times 0.004)$$
$$= 2000 - 800$$
$$= 1200 \text{ joules}$$

$\therefore$ Increase in internal energy is 1200 joules.

6.15 APPLICATION OF FIRST LAW OF THERMODYNAMICS TO ISOTHERMAL AND ADIABATIC PROCESSES

Isothermal Process :

In this process, since the temperature of the gas remains constant, $dU = 0$

We know that, $dQ = dU + dW$

$$\Rightarrow \quad dQ = dW \quad (\because dU = 0)$$

Hence, in this process, the total heat supplied to the gas is used for doing external work.

Adiabatic Process :

In this process, heat is neither given to the system nor taken from it. i.e., $dQ = 0$

We know that, $dQ = dU + dW$

$$0 = dU + dW$$

$$\Rightarrow \quad dU = -dW$$

For adiabatic compression dU is positive and dW is negative.

For adiabatic expansion dU is negative and dW is positive.

6.16 SECOND LAW OF THERMODYNAMICS

Statement : Heat by itself cannot transmit from a body at lower temperature to a body at higher temperature without the aid of an external agency.

Analogy : It is a known fact that water flows from higher level to lower level. It cannot flow from lower level to higher level naturally. But we can make it flow from lower level to higher level by using an external agency like motor and pump.

Explanation : Heat flows from higher temperature to lower temperature naturally. Heat by itself cannot flow from lower temperature to higher temperature. By using an external agency like air conditioner we can transfer heat energy from lower temperature to higher temperature. The air conditioner transports heat energy from the room at lower temperature to the surroundings at higher temperature.

Examples : Working of refrigerators, water coolers and cold storage units is based on this law.

6.16.1 PRACTICAL APPLICATIONS

Natural Process :

1. All types of heat engines and working fluids employed in the engines work as per the second law of thermodynamics. They receive heat from a high-temperature source for their operation and discharge the remaining heat to a low-temperature sink.
2. All types of vehicles like cars and aeroplanes work as per the second law of thermodynamics.

Reverse Process :

3. The refrigerator maintains the refrigerated space at a low temperature by removing heat from it. The removed heat is discharged to a high-temperature medium.
4. Air conditioner removes heat from the room at low temperature and expels the same to the surroundings at high temperature. Here electrical energy is supplied to perform this work by the compressor.

5. A heat pump absorbs heat from the atmosphere and supplies it into the room. The heat pump maintains a heated space at high temperature by absorbing heat from a low-temperature source and supplying this heat to the high-temperature medium. Here also electrical energy is supplied to perform the work.

6.17 SOLVED PROBLEMS

PROBLEM-18

At constant temperature, pressure of a given amount of gas is doubled. Find how the volume of the gas changes.

[Mar/Apr. 2017 (EEE)]

Solution :

Given that,

Temperature	t = constant
Initial pressure	$P_1 = P$ (say)
Final pressure	$P_2 = 2P$
Initial volume	$V_1 = V$ (say)
Final volume	$V_2 = ?$

From Boyle's law, we have,

$$P_1V_1 = P_2V_2$$

$$\Rightarrow \quad V_2 = \frac{P_1V_1}{P_2}$$

Substituting the values,

$$V_2 = \frac{P \times V}{2P} = \frac{V}{2}$$

$\therefore$ The volume of the gas decreases to half of its initial volu