

INDIAN INSTITUTE OF TECHNOLOGY KANPUR  
DEPARTMENT OF MECHANICAL ENGINEERING

**ME685A - ASSIGNMENT I**

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The following mathematical model is obtained during the analysis of a tank-and-tube system:

$$\frac{dh}{dt} + \sqrt{h} = q(t) \quad h(t) > 0$$
$$h(t = 0) = H$$

To study the tank emptying process,  $q(t)$  is set to zero while  $H=2$  units. Integrate the differential equation numerically. Thus find the time required for the water level in the tank to fall just below  $H/2$ .

The tank-emptying problem has an analytical solution. Derive this expression and tabulate the results.

For a numerical solution, use the following two approximations of the square-root term:

$$\sqrt{h} = \sqrt{h^n} \quad (\text{i})$$

$$\sqrt{h} = \sqrt{h^{n+1}} \quad (\text{ii})$$

Here,  $n$  is the discrete time level ( $n=1, 2, 3, \dots$ ). Select the time step judiciously and justify your choice (by comparing with the time constant). Tabulate the data obtained for each method and compare with the analytical. Stop integration when  $h(t) < H/2$ .