MATLAB LAB-5

1. In the fundamental interval the signal $x_1(t)$ is defined as

$$x_1(t) = \left(1 - \left|\frac{t}{2}\right|\right)(u(t+1) - u(t-1))$$

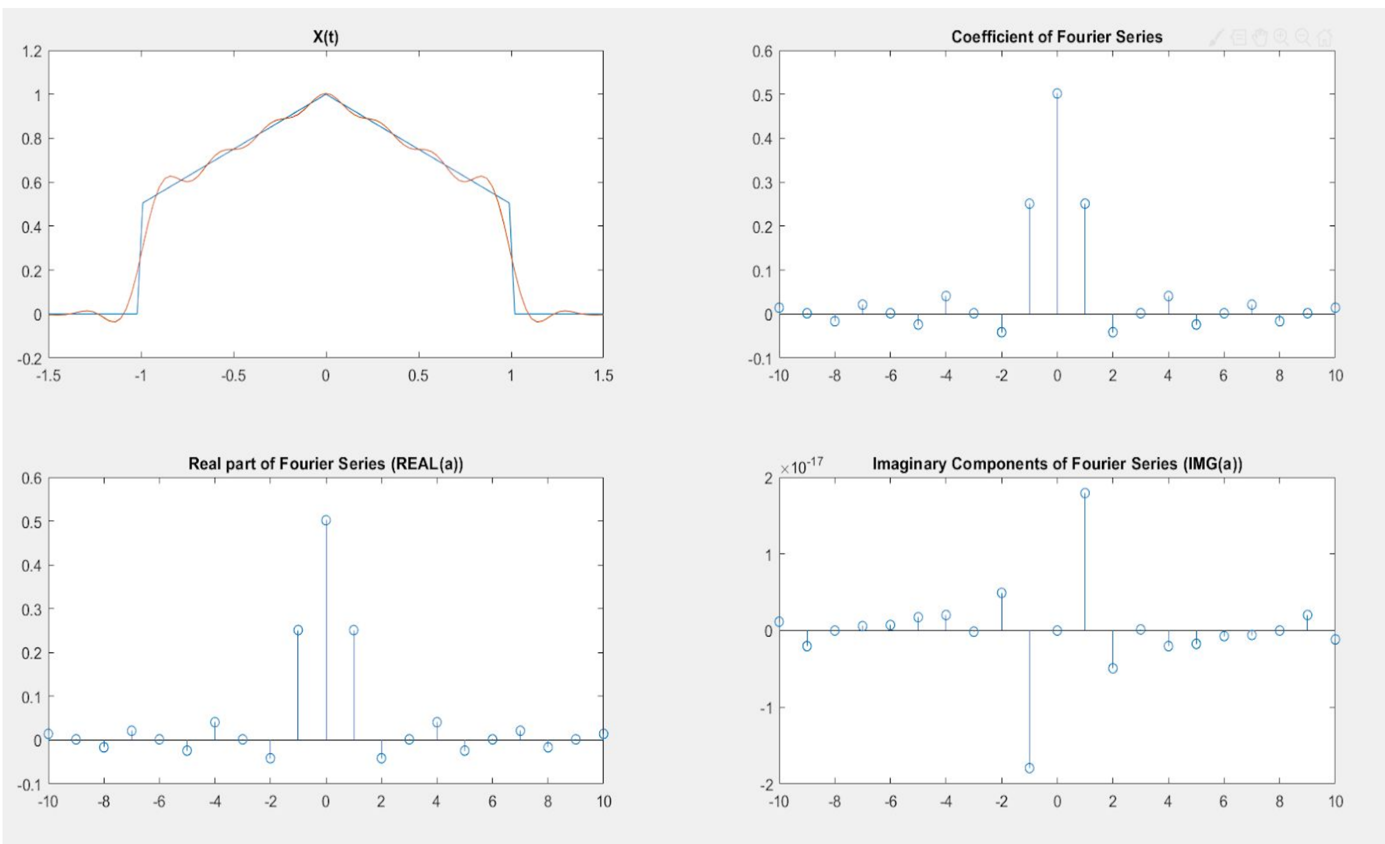
- For each of the periodic signals given above assume a time period T = 3 and compute the Fourier coefficients. Next, plot the following: The Fourier coefficients; both the real and imaginary components vs the theoretical values.
- Next, for each of the periodic signals mentioned above (with the period T = 3), reconstruct the original signal from the Fourier coefficients.
  - Plot the original and reconstructed signal on the same figure.
  - Demonstrate the convergence of the reconstructed signal with respect to the original signal

```
clc;clear all;close all;
T=3;
N=5;
M=10;
x1=zeros(size(N*T));
x1=[];
w=2*pi/3;
t=-5:0.1:5;
tn=0:0.1:N*T;
tv=-1.5:3/100:1.5;
x1t=f(tv).*(u(tv+1)-u(tv-1));


kv=-M:M;
for i=1:length(N*T)
    x1=[x1,x1t];
```

```
end


for i=1:length(kv)
   k=kv(i);
    basis=exp(-1i*w*k*tv);
    a(i)=trapz(tv,x1t.*basis)/T;
end

xn=zeros(1,length(t));
for mx=1:M

    kv=-mx:mx;

    xn=zeros(size(tv));

for j=1:length(kv)
    p=kv(j)
    fun=exp(1i*w*p*tv);
    xn=xn+a(j)*fun;
end
    plot(tv,x1);
    hold on;
     plot(tv,xn,'r','LineWidth',2);
    xlabel('Time');
    ylabel('Signal');
    hold off;
    pause(0.5);
    drawnow;
    error(mx)=mean((abs(xn)).^2);
end

   subplot(221);
    plot(tv,x1t);
    hold on;
    plot(tv,xn);
    title('X(t)');
    subplot(224);
    stem(kv,imag(a));
    title('Imaginary Components of Fourier Series (IMG(a))');
    subplot(223);
    stem(kv,real(a));
    title('Real part of Fourier Series (REAL(a))');
```

```
subplot(222)
stem(kv,a);
title('Coefficient of Fourier Series');
```

```
function x = f(t)
x = zeros(size(t));
x(t<0)=1+(t(t<0)/2);
x(t>=0)=1-(t(t>=0)/2);
end
function x = u(t)
x = zeros(size(t));
x(t>=0)=1;
End
```



① $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t} dt = \frac{1}{3}\int^{1}_{-1} \left(1 - \left|\frac{t}{2}\right|\right) e^{-jk\omega_0 t} dt$

$= \frac{1}{3}\int_{-1}^{0}\left(1+\frac{t}{2}\right) e^{-jk\omega_0 t} dt + \frac{1}{3}\int_{0}^{1}\left(1-\frac{t}{2}\right) e^{-jk\omega_0 t} dt$

$= \frac{1}{3}\left(\int_{-1}^{0} e^{-jk\omega_0 t} + \frac{t}{2} e^{-jk\omega_0 t} dt + \int_0^1 e^{-jk\omega_0 t} - \int_0^1 \frac{t}{2} e^{-jk\omega_0 t} dt\right)$

$= \frac{1}{3}\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{t}{2}\frac{e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{1}{2}\frac{e^{-jk\omega_0 t}}{k^2\omega_0^2}\right]_{-1}^{0}\Bigg)$

$+ \frac{1}{3}\left(\frac{e^{-jk\omega_0 t}}{-jk\omega_0} + \frac{t}{2}\frac{e^{-jk\omega_0 t}}{-jk\omega_0} - \frac{1}{2}\frac{e^{-jk\omega_0 t}}{k^2\omega_0^2}\right)$

$= \frac{1}{3}\left[\left(\frac{-1}{jk\omega_0} - \frac{e^{jk\omega_0}}{-jk\omega_0}\right) + \left(0 + \frac{1}{2}\frac{e^{jk\omega_0}}{-jk\omega_0}\right) + \left(\frac{1-e^{jk\omega_0}}{2k^2\omega_0^2}\right)\right]$

$\frac{1}{3}\left[\left(\frac{e^{-jk\omega_0} - 1}{-jk\omega_0}\right) + \left(\frac{e^{-jk\omega_0}}{2jk\omega_0}\right) - \frac{1}{2}\left(\frac{e^{-jk\omega_0}-1}{k^2\omega_0^2}\right)\right]$

$\frac{1}{3}\left\{\frac{e^{jk\omega_0} + e^{-jk\omega_0}}{-jk\omega_0} + \frac{2 - e^{jk\omega_0} - e^{-jk\omega_0}}{k^2\omega_0^2}\right]$

$\frac{1}{6}\left[2\cos(k\omega_0) + \frac{2 - 2\cos(k\omega_0)}{k^2\omega_0^2}\right]$

$\Rightarrow \frac{-1}{3}\left[\cos(k\omega_0)\right] + \frac{\sin^2(k\omega_0/2)}{3k^2\omega_0^2}$

$a_k = \frac{2\sin^2\left(\frac{k\omega_0}{2}\right)}{3k^2\omega_0^2} + j\left(\frac{-\cos(k\omega_0)}{3}\right)$

$\uparrow$ $\uparrow$

(a_k)

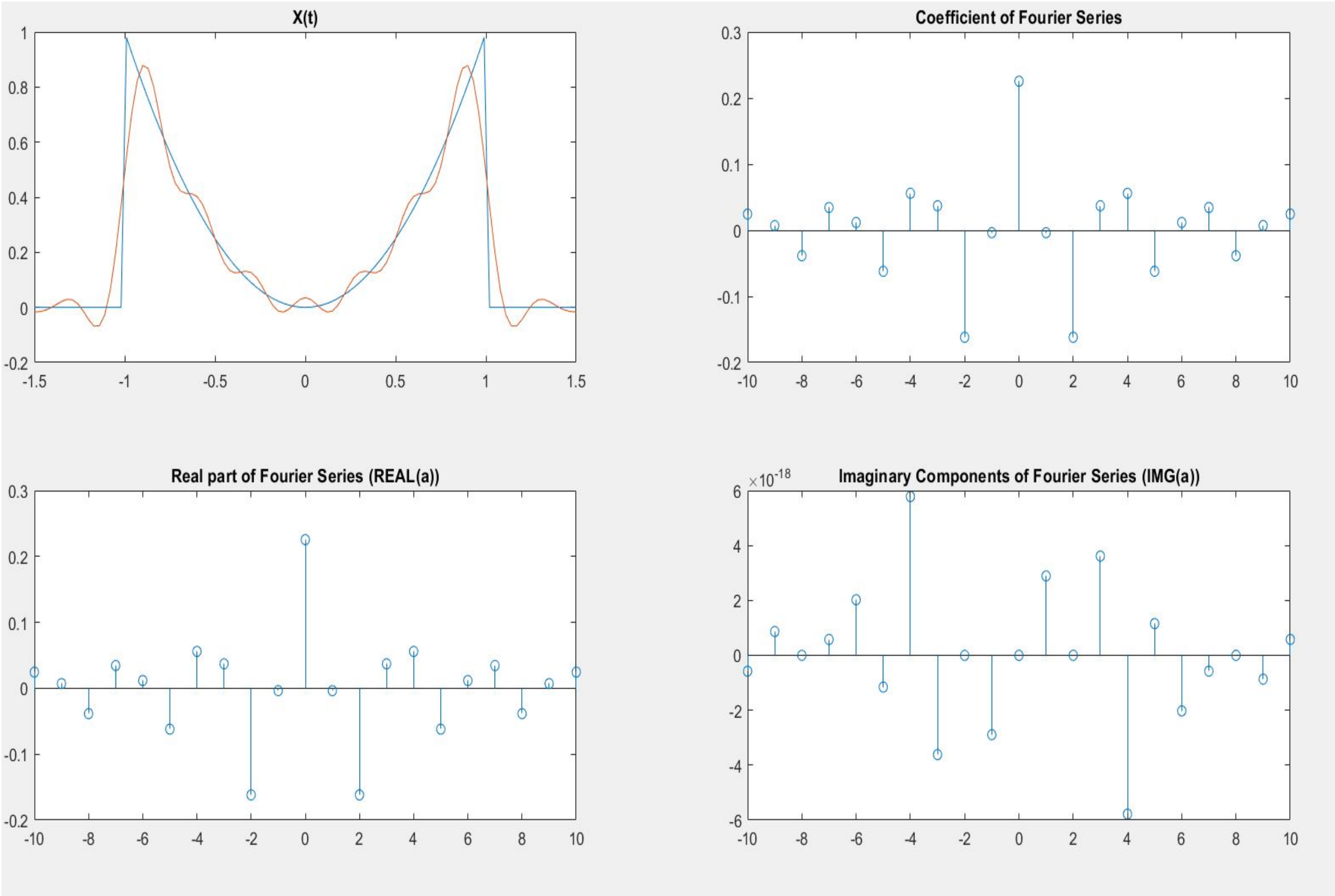
2. In the fundamental interval the signal $x_2(t)$ is defined as

$$x_2(t) = t^2(u(t+1) - u(t-1))$$

- For each of the periodic signals given above assume a time period T = 3 and compute the Fourier coefficients. Next, plot the following: The Fourier coefficients; both the real and imaginary components vs the theoretical values.
- Next, for each of the periodic signals mentioned above (with the period T = 3), reconstruct the original signal from the Fourier coefficients.
  - Plot the original and reconstructed signal on the same figure.
  - Demonstrate the convergence of the reconstructed signal with respect to the original signal

```
clc;clear all;close all;
T=3;
N=5;
M=10;
w=2*pi/3;
x2=zeros(size(N*T));
x2=[];
t=-5:0.1:5;
tv=-1.5:3/100:1.5;
x2t=tv.^2.*(u(tv+1)-u(tv-1));
kv=-M:M;
for i=1:length(N*T)
   x2=[x2,x2t];
end


for i=1:length(kv)
  k=kv(i);
   basis=exp(-1i*w*k*tv);
   a(i)=trapz(tv,x2t.*basis)/T;
end
xn=zeros(1,length(t));
for mx=1:M
```

```
    kv=-mx:mx;

    xn=zeros(size(tv));

for j=1:length(kv)
    p=kv(j)
    fun=exp(1i*w*p*tv);
    xn=xn+a(j)*fun;
end
    plot(tv,x2);
    hold on;
     plot(tv,xn,'r','LineWidth',2);
    xlabel('Time');
    ylabel('Signal');
    hold off;
    pause(0.5);
    drawnow;
     error(mx)=mean((abs(xn)).^2);
end
    subplot(221);
    plot(tv,x2t);
    title('X(t)');
    hold on;
    plot(tv,xn);
    subplot(224);
    stem(kv,imag(a));
    title('Imaginary Components of Fourier Series (IMG(a))');
    subplot(223);
    stem(kv,real(a));
    title('Real part of Fourier Series (REAL(a))');
    subplot(222)
    stem(kv,a);
    title('Coefficient of Fourier Series');
 function x = u(t)
x = zeros(size(t));
x(t>=0)=1;
end
```


X(t)
Coefficient of Fourier Series
Real part of Fourier Series (REAL(a))
×10^-18
Imaginary Components of Fourier Series (IMG(a))

$$a_k = \frac{1}{T}\int_{-1}^{1} t^2 e^{-jk\omega_0 t}\,dt$$

$$\Rightarrow \frac{1}{3}\left(\left[\frac{t^2 e^{-jk\omega_0 t}}{-jk\omega_0}\right]_{-1}^{1} - \int_{-1}^{1}\frac{2t\, e^{-jk\omega_0 t}}{-jk\omega_0}\,dt\right)$$

$$\Rightarrow \frac{1}{3}\left(\frac{e^{-jk\omega_0} - e^{jk\omega_0}}{-jk\omega_0} + \frac{2}{jk\omega_0}\int_{-1}^{1} t\, e^{-jk\omega_0 t}\,dt\right)$$

$$\frac{1}{3}\left(\frac{-2j\sin(k\omega_0)}{-jk\omega_0}\right) + \frac{2}{3jk\omega_0}\left(\left[\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\right] - \int_{-1}^{1}\frac{e^{-jk\omega_0 t}}{-jk\omega_0}\,dt\right)$$

$$\Rightarrow \frac{2}{3}\frac{\sin(k\omega_0)}{k\omega_0} + \frac{2}{3jk\omega_0}\left(\frac{e^{-jk\omega_0} - e^{jk\omega_0}}{-jk\omega_0} + \frac{1}{jk\omega_0}\left(e^{-jk\omega_0} - e^{jk\omega_0}\right)\right)$$

$$\frac{2}{3}\frac{\sin k\omega_0}{k\omega_0} + \frac{2}{3jk\omega_0}\left(\frac{-2j\sin(k\omega_0)}{-jk\omega_0} - \frac{2j\sin(k\omega_0)}{jk\omega_0}\right)$$

$$\frac{2}{3}\frac{\sin(k\omega_0)}{k\omega_0} - \frac{2}{3}\frac{j}{k\omega_0}\left(4\sin(k\omega_0)\right)$$

$$a_k = \underbrace{\frac{2}{3}\frac{\sin(k\omega_0)}{k\omega_0}}_{\text{Real}(a_k)} + j\underbrace{\left(-\frac{8}{3}\frac{\sin k\omega_0}{k\omega_0}\right)}_{\text{Imag}(a_k)}$$