Advanced Data Structure and Algorithm

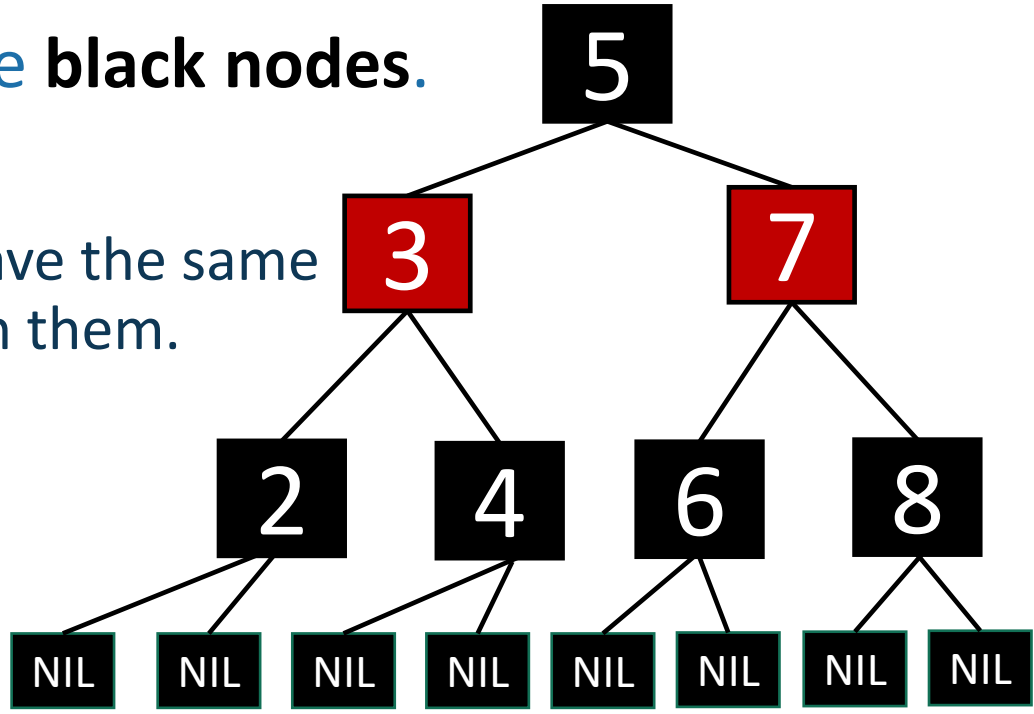
Red-Black Trees

Deletion

Red-Black Trees

obey the following rules (which are a proxy for balance)

- Every node is colored **red** or **black.**
- The root node is a **black node**.
- NIL children count as **black nodes**.
- Children of a **red node** are **black nodes**.
- For all nodes x:
  - all paths from x to NIL's have the same number of **black nodes** on them.



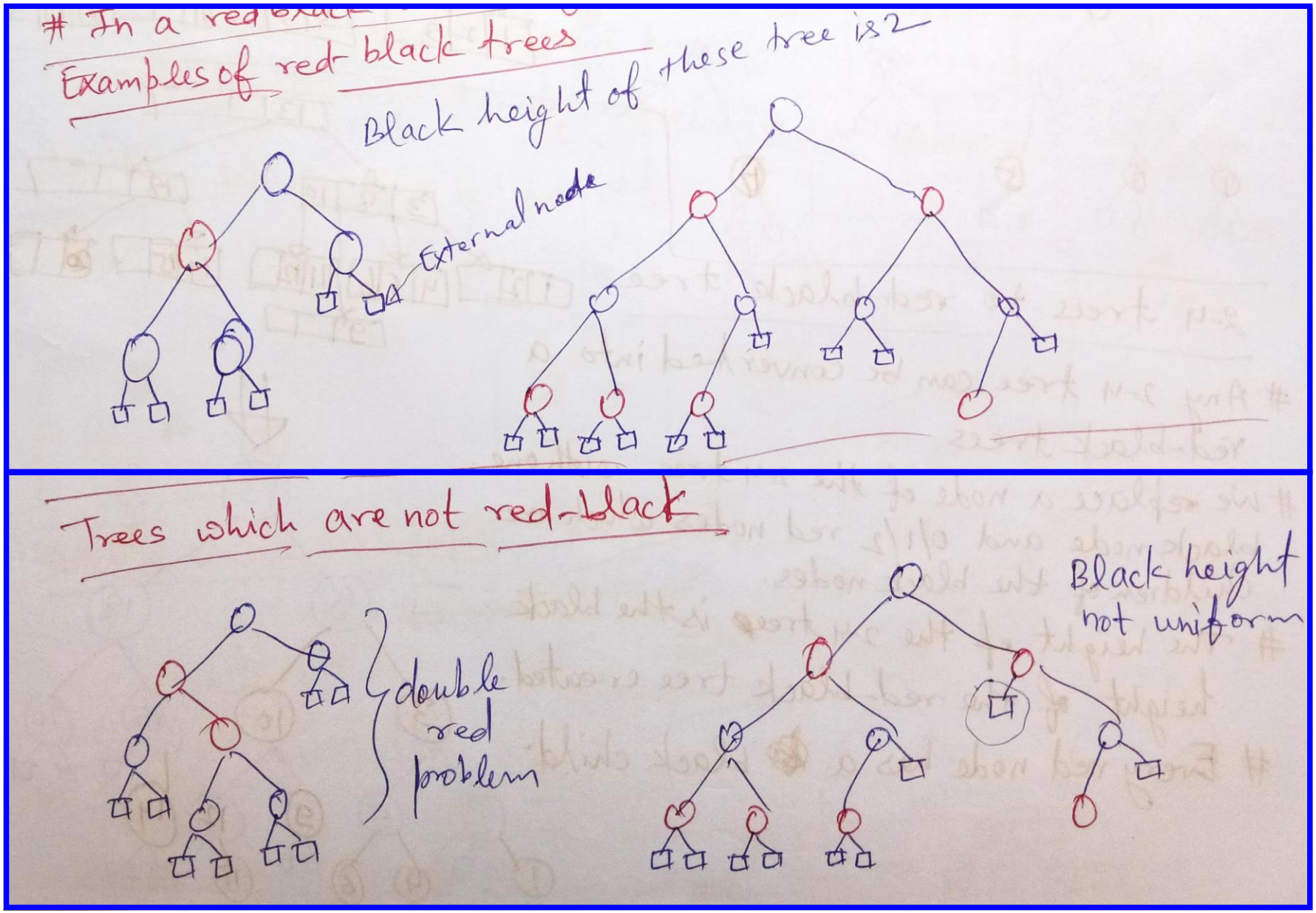
I'm not going to draw the NIL children in the future, but they are treated as black nodes.

Red-Black Trees

Red Black Trees

A red-black tree is a binary search tree in which each node is colored red or black.
The root is colored black.
A red node can have only black children.
If a node of the BST does not have a left and/or right child then we add an external node.
External nodes are not colored.
The black depth of an external node is defined as the number of black ancestors it has.
In a red black tree every external node has the same black depth.

Red-Black Trees



Red-Black Trees

④ Height of a red-black tree

Let h be the black height of a red-black tree on n nodes

n is smallest when all nodes are black. In this case tree is a complete binary tree of height h and $n = 2^h - 1$.

n is largest when alternate levels of tree are red. Then height is 2h and $n = 2^{2h} - 1$.

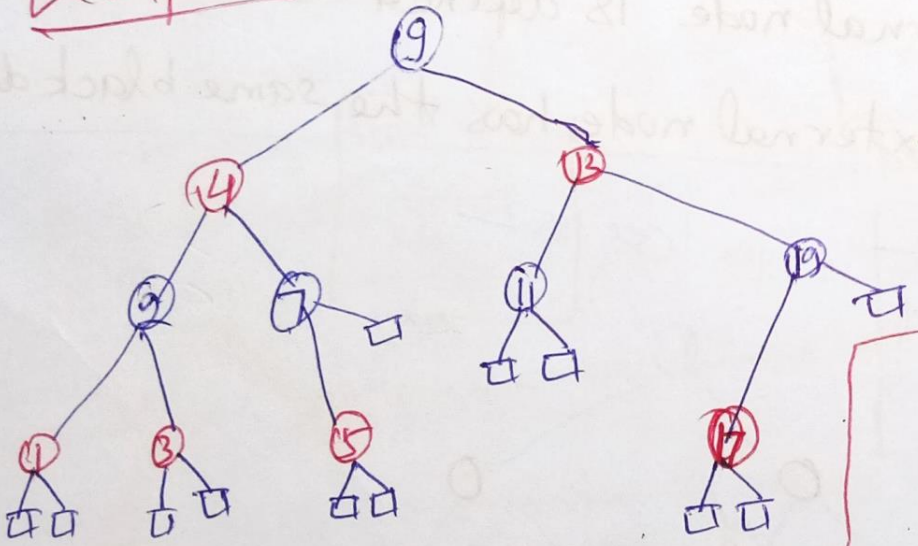
Hence, $\log_4 n < h < 1 + \log_2 n$.

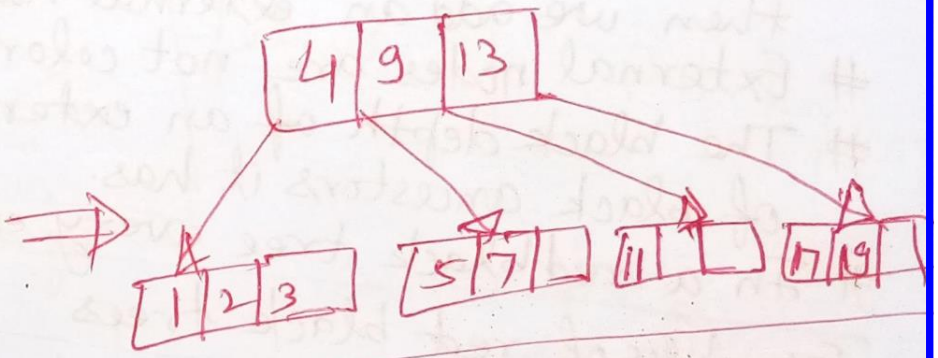
Red-Black Trees

Red-Black tree to 2-4 trees

Any red-black tree can be converted into a 2-4 tree.
Take a black node and its red children (at most 2) and combine them into one node of a 2-4 tree.
Each node so formed has at least 1 and at most 3 keys.
Since black height of all external nodes is same, in the resulting 2-4 tree all leaves are at same level.

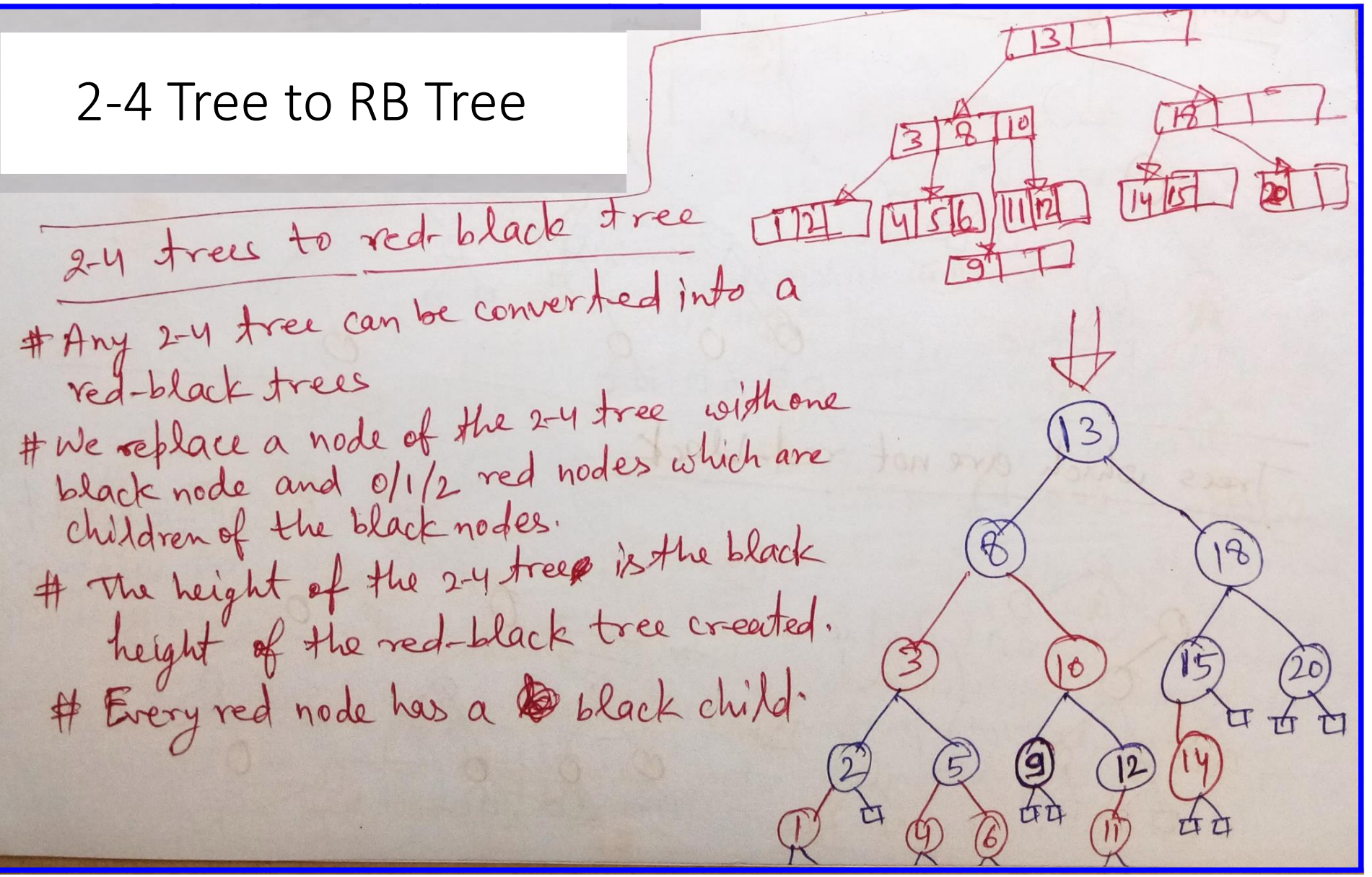
Example.





Red-Black Trees

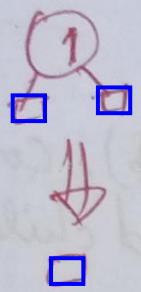
2-4 Tree to RB Tree

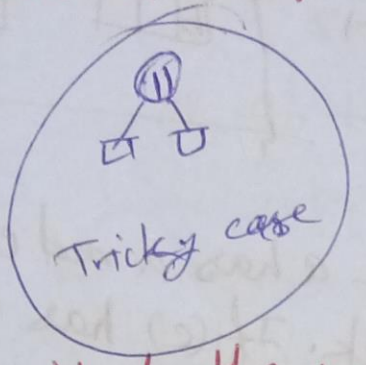


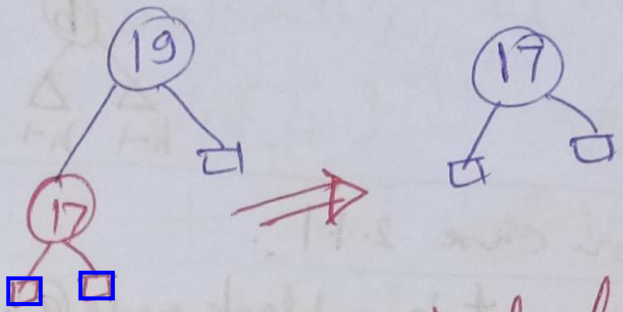
Red-Black Trees

Deletion: preliminaries

To delete a node we proceed as in a BST.
Thus the node which is deleted is the parent of an external node.
Hence it is either a leaf or the parent of a leaf.



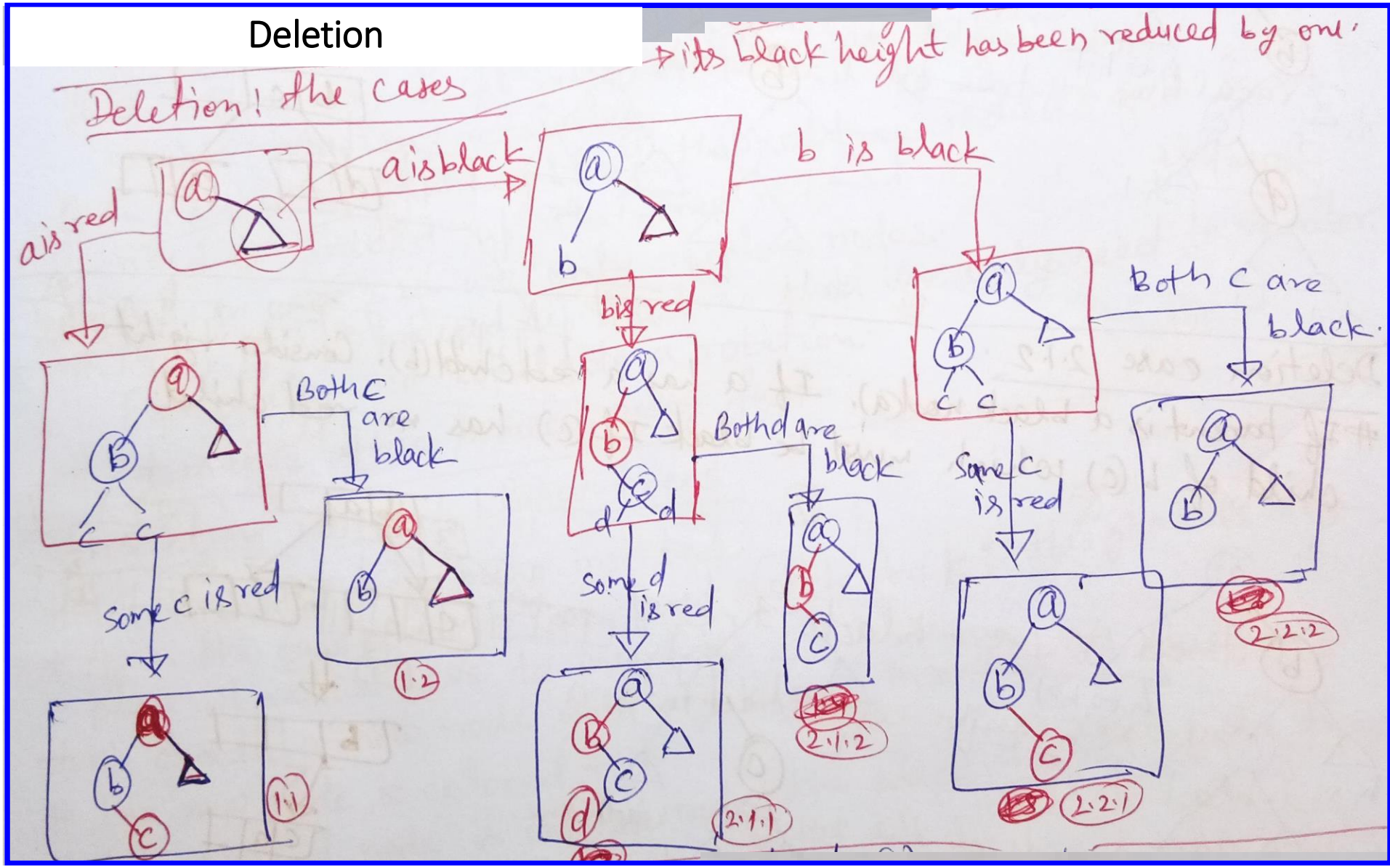




Hence we can assume that the node deleted is a black leaf.
Removing this reduces black depth of an external node by 1.
Hence, in a general step, we consider how to reorganize the tree when the black height of some subtree goes down from h to h-1.

Red-Black Trees

Deletion



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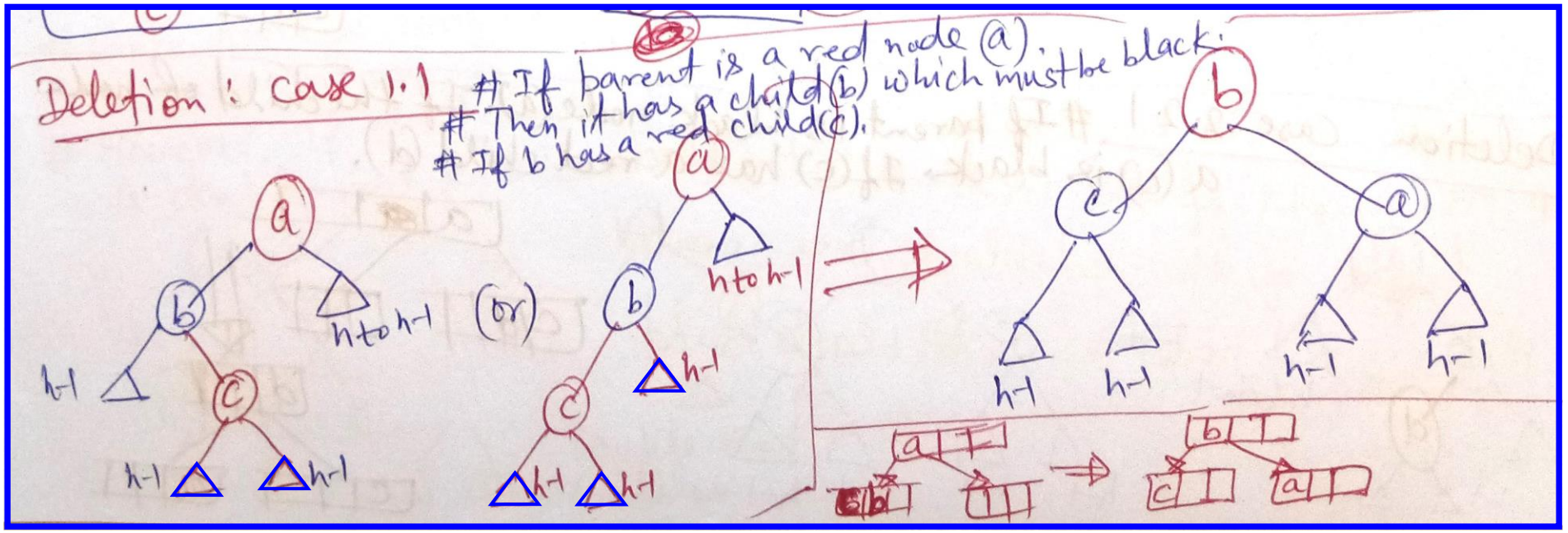

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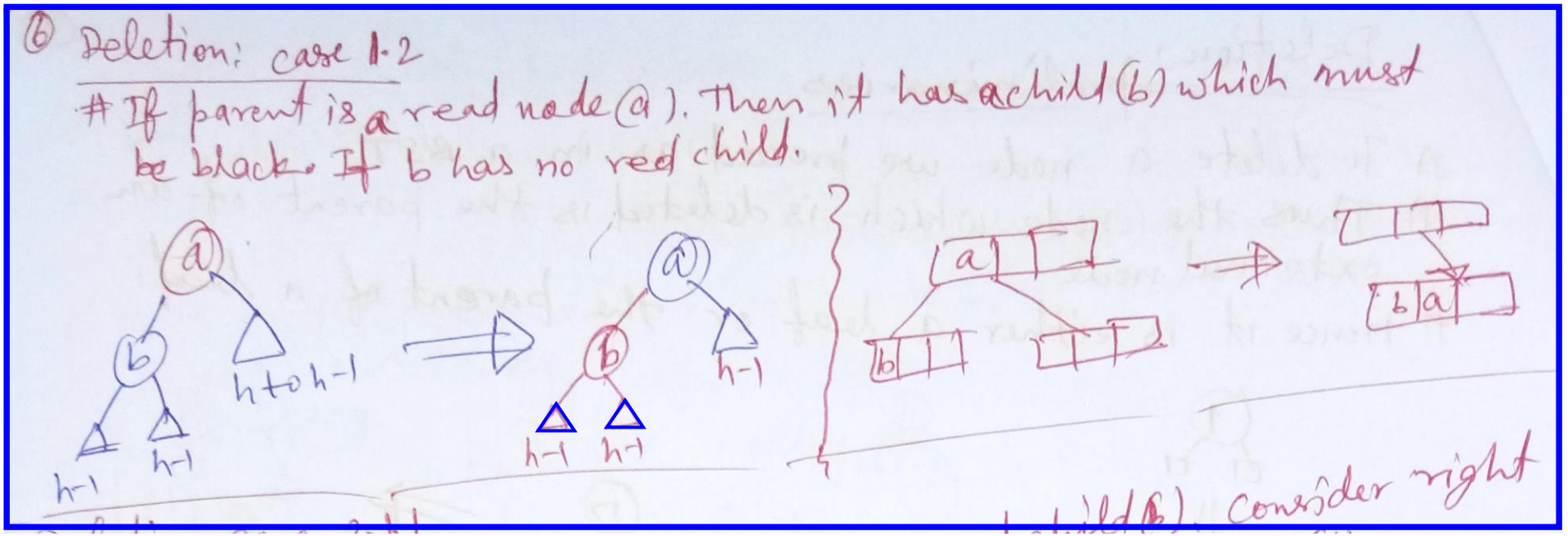

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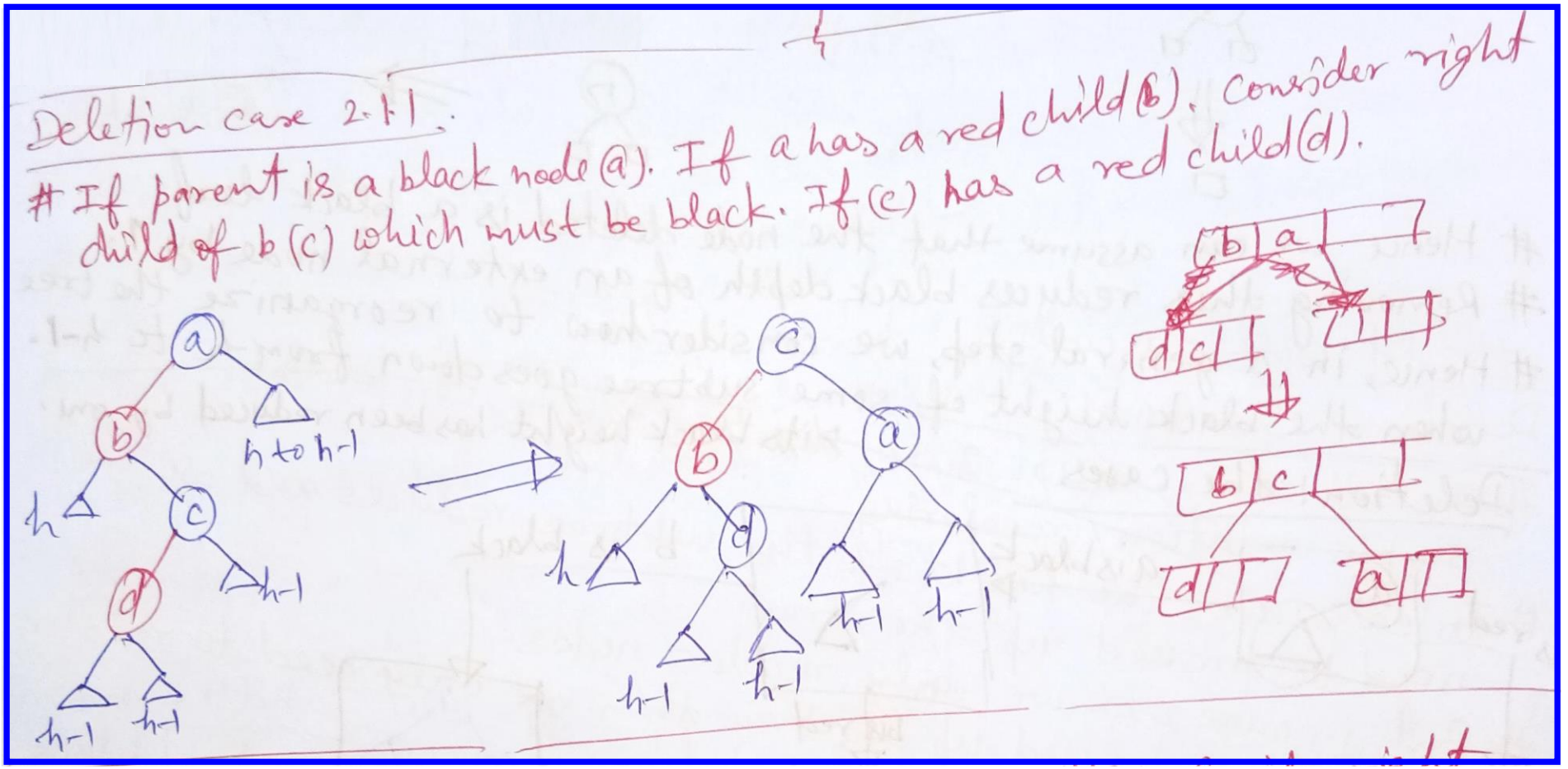

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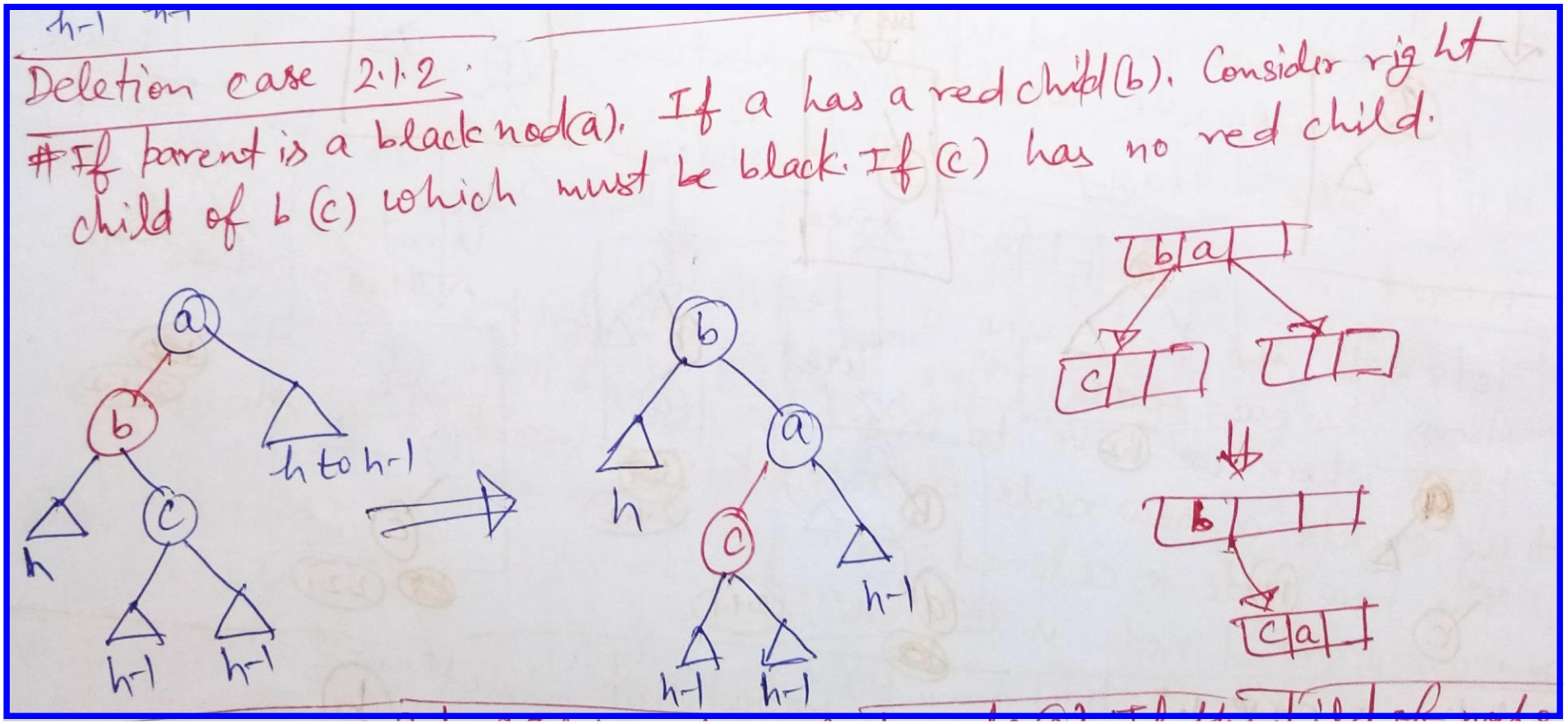

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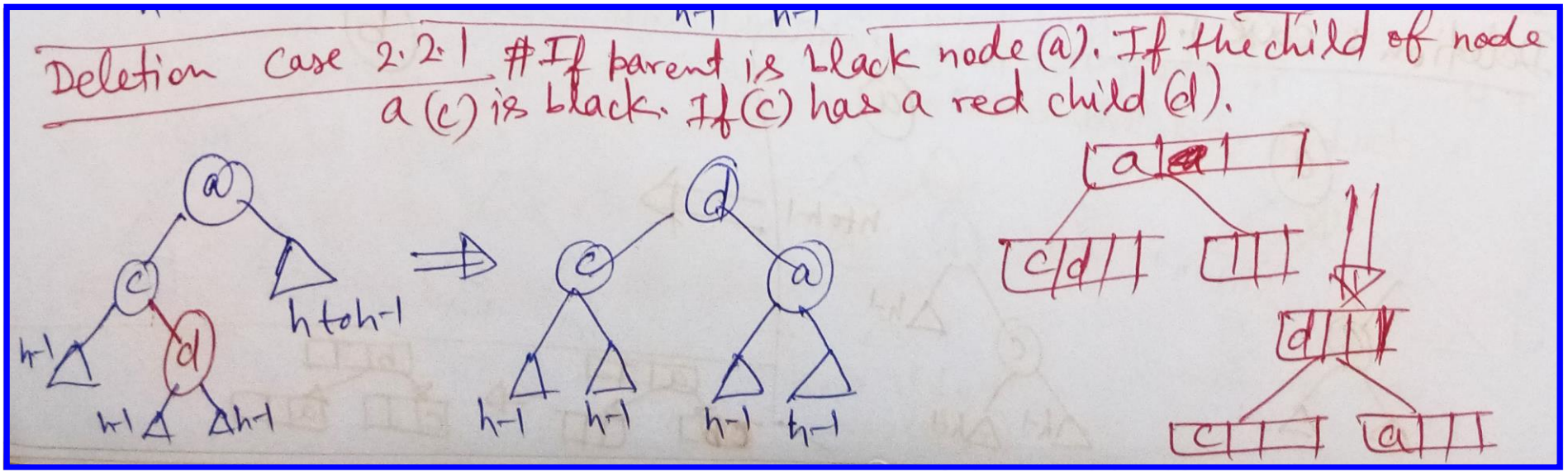
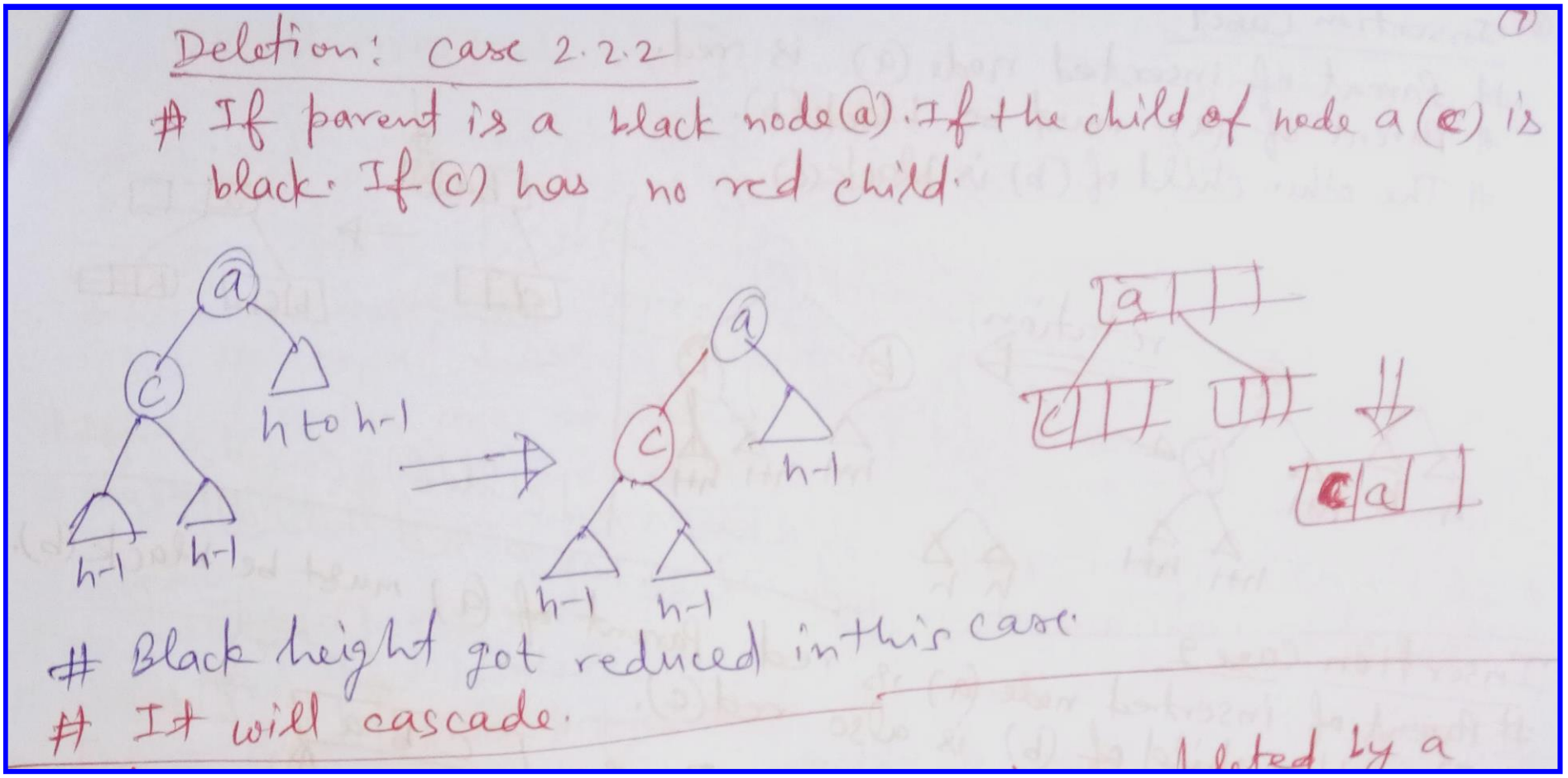


Red-Black Trees



Red-Black Trees

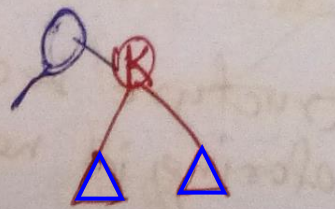
Deletion: Summary

- In all cases, except 2.2.2, deletion can be completed by a simple rotation/recoloring.
- In case 2.2.2, the height of the subtree reduces and so we need to proceed up the tree.
- But in case 2.2.2 we only recolor nodes.
- Thus, if we proceed up the tree then we only need to recolor. Eventually we would do a rotation.
- It makes this process very fast.

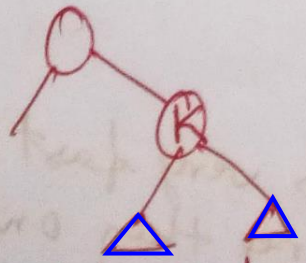
Red-Black Trees

Insertion in Red-black trees

Let K be the key being inserted.
As in the case of a BST we first search for K; this gives us the place where we have to insert K.
We create a new node with key k and insert it at this place.
The new node is colored red.
Since inserted node is colored red, the black height of the tree remains unchanged.
However, if the parent of inserted node is also red then we have a double red problem.

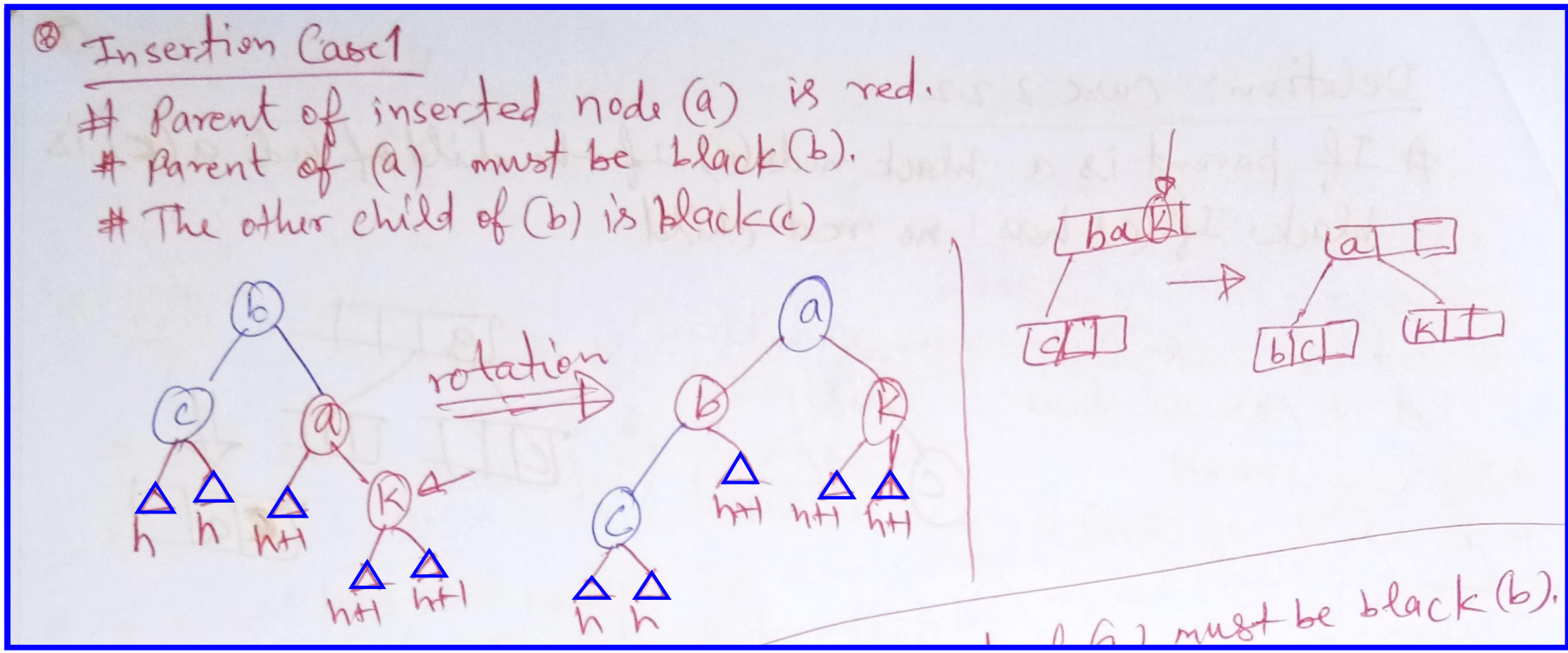


No problem



Double Red Problem

Red-Black Trees



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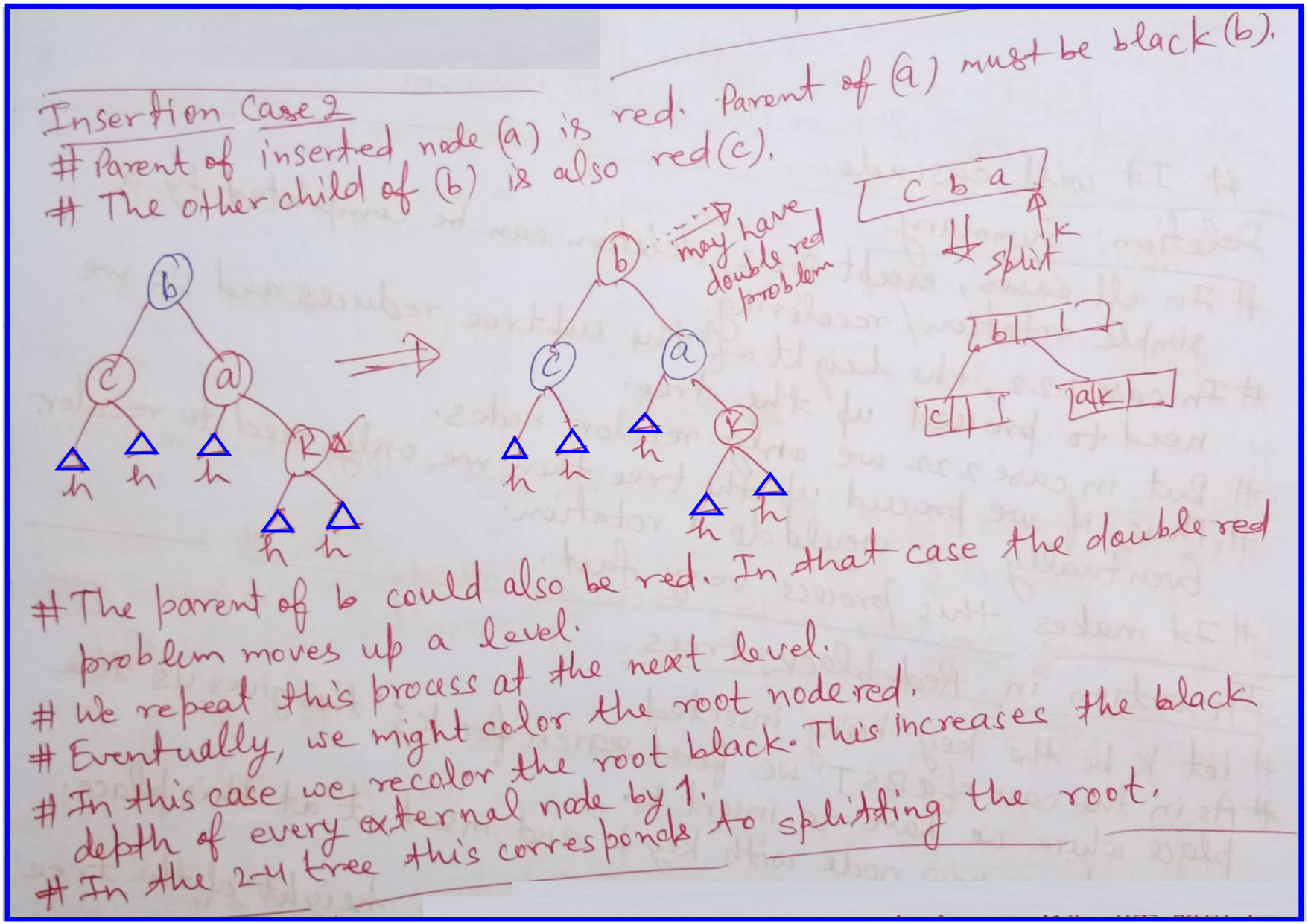

Red-Black Trees

Insertion and Deletion: Summary

- In both insertion and deletion we need to make at most one rotation.
- We might have to move up the tree but in doing so we only recolor nodes.
- Time taken is $O(\log n)$.
- Thus, red-black tree is very fast data structure because when we move up in the tree then only recoloring is needed.

This is great!

- SEARCH in an RBTree is immediately O(log(n)), since the depth of an RBTree is O(log(n)).

- What about INSERT/DELETE?
  - Turns out, you can INSERT and DELETE items from an RBTree in time O(log(n)), while maintaining the RBTree property.

INSERT/DELETE

- INSERT/DELETE for RBTrees
  - You should know what the “proxy for balance” property is and why it ensures approximate balance.

Conclusion: The best of both worlds

	Sorted Arrays	Linked Lists	Red Black Trees
Search	O(log(n)) 😀	O(n) 😕	O(log(n)) 😀
Delete	O(n) 😕	O(n) 😕	O(log(n)) 😀
Insert	O(n) 😕	O(1) 😀	O(log(n)) 😀

Recap

- Balanced binary trees are the best of both worlds!
- But we need to keep them balanced.
- **Red-Black Trees** do that for us.
  - We get O(log(n))-time INSERT/DELETE/SEARCH
  - Clever idea: have a proxy for balance