Parsing …

Top-down parsing

- Regular languages
  - The weakest formal languages widely used
  - Many applications

- Lexical Analysis
- VLSI design → minimimum DFA is useful
- Scene understanding – components, relationships.
  - Reconstructing a crime event.
- Etc

But,

Consider the language:

$$\{(^i)^i \mid i \geq 0\}$$

()
(())
((( )))
⋮

((1+2) * 3)

if then
if then
if then

fi
fi
fi

Palindromes, Etc.

CFGs

- Not all strings of tokens are programs . . .
- . . . parser must distinguish between valid and invalid strings of tokens

- We need
  - A language for describing valid strings of tokens
  - A method for distinguishing valid from invalid strings of tokens

CFGs

- Programming languages have recursive structure

- An EXPR is
  if EXPR then EXPR else EXPR fi
  while EXPR loop EXPR pool
  ...

- Context-free grammars are a natural notation for this recursive structure

CFGs

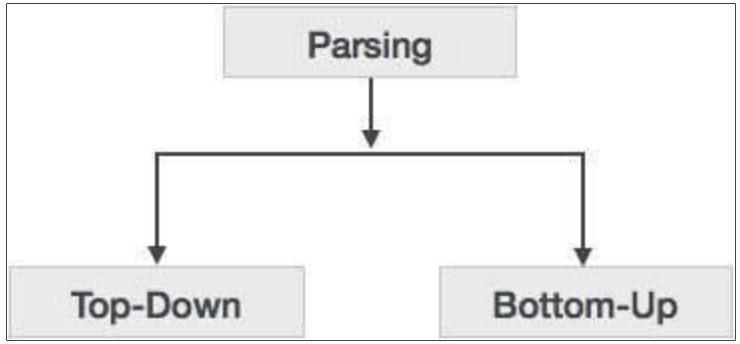
Which of the strings are in the language of the given CFG?

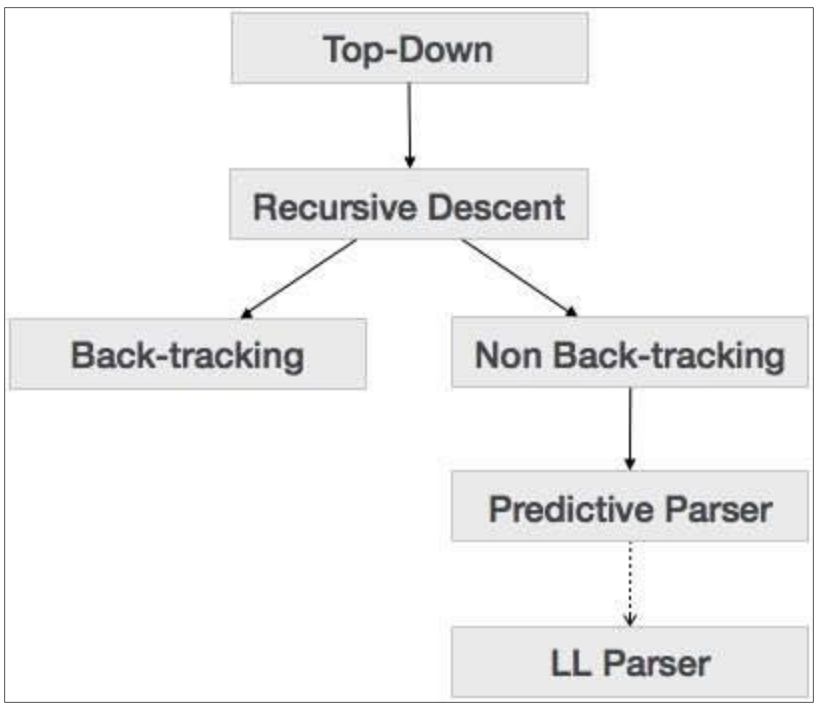
- ☐ abcba
- ☐ acca
- ☐ aba
- ☐ abcbcba

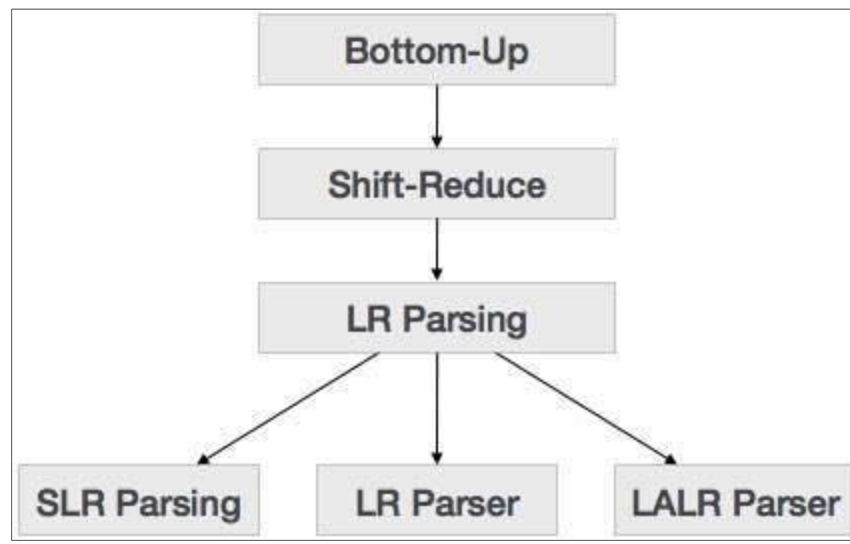
$$
\begin{aligned}
S &\rightarrow aXa \\
X &\rightarrow \varepsilon \\
&\mid\ bY \\
Y &\rightarrow \varepsilon \\
&\mid\ cXc
\end{aligned}
$$

Derivations

- We are not just interested in whether $s \in L(G)$
  - We need a parse tree for s
- A derivation defines a parse tree
  - But one parse tree may have many derivations
- Left-most and right-most derivations are important in parser implementation


Parsing
Top-Down
Bottom-Up


Top-Down
Recursive Descent
Back-tracking
Non Back-tracking
Predictive Parser
LL Parser


Bottom-Up
Shift-Reduce
LR Parsing
SLR Parsing
LR Parser
LALR Parser

Top-Down Parsing

Top-down Parsing

- Tree is built from root to leaves
- Depth first (preorder)
- Leftmost derivations are used
- General technique requires backtracking
- Recursive algorithms seems alright, but are quite expensive
  - So, we may look ahead in the input string for k characters, do some preprocessing like building some tables, etc (*and thus choose the right production to be used*) and thus may be the need for backtracking can be eliminated.

4.4.1 Recursive-Descent Parsing

Order A productions, and choose according to this order.

```
void A() {
    Choose an A-production, A → X1 X2 ··· Xk;
    for ( i = 1 to k ) {
        if ( Xi is a nonterminal )
            call procedure Xi();
        else if ( Xi equals the current input symbol a )
            advance the input to the next symbol;
        else /* an error has occurred */;
    }
}
```

Break the loop and go for the next A production. You need to retract in reading the input string.

Figure 4.13: A typical procedure for a nonterminal in a top-down parser

For each variable we write a function like this.
First we call S( ); /* S is the start symbol */

Example 4.29 : Consider the grammar

$$
\begin{array}{lcl}
S & \rightarrow & c\ A\ d \\
A & \rightarrow & a\ b\ \mid\ a
\end{array}
$$

$\boxed{w = cad}$

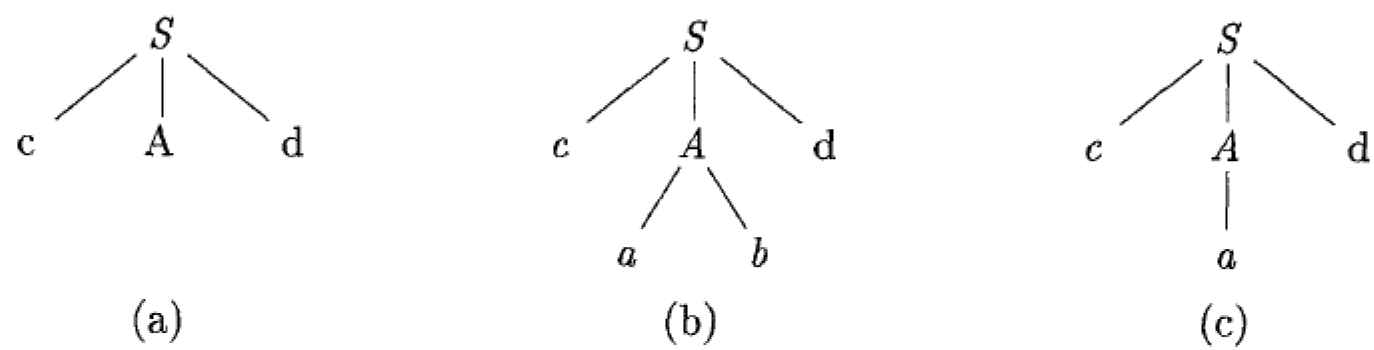


Figure 4.14: Steps in a top-down parse

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

$$\begin{array}{c} E \\ | \\ T \\ | \\ int \end{array}$$

Mismatch: int does not match (
Backtrack …

$(\; int_5 \;)$

$\uparrow$

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

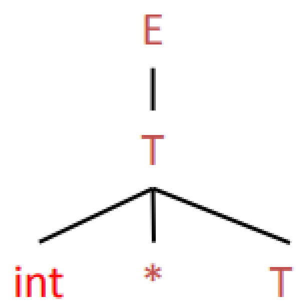
E
|
T

$(\; int_5 \;)$

↑

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$



Mismatch: int does not match (
Backtrack …

(int_5)

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

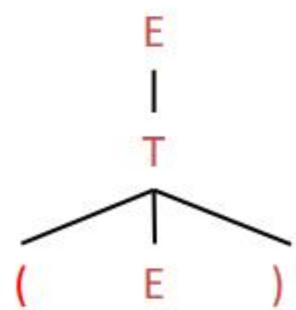
$$\begin{array}{c} E \\ | \\ T \end{array}$$

$(\; int_5 \;)$

$\uparrow$

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

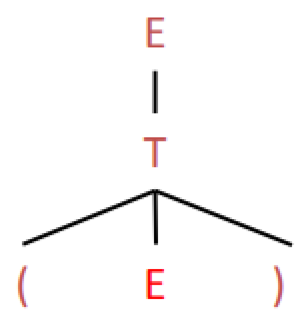


Match! Advance input.

(int_5)

$E \rightarrow T \mid T + E$

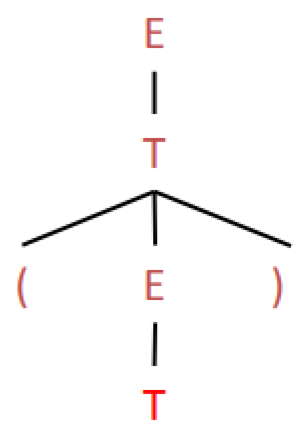
$T \rightarrow int \mid int * T \mid (E)$



$(\, int_5 \,)$

$E \rightarrow T \mid T + E$

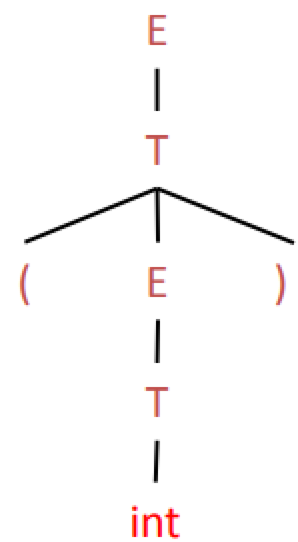
$T \rightarrow \text{int} \mid \text{int} * T \mid (E)$



(int_5)

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

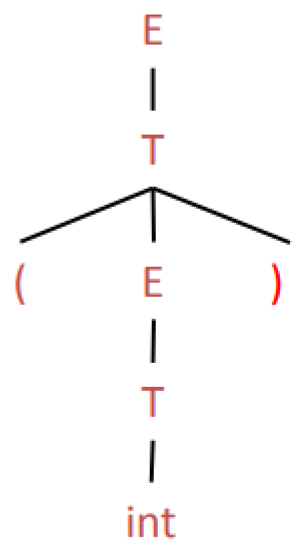


Match! Advance input.

$(\; int_5 \;)$

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$

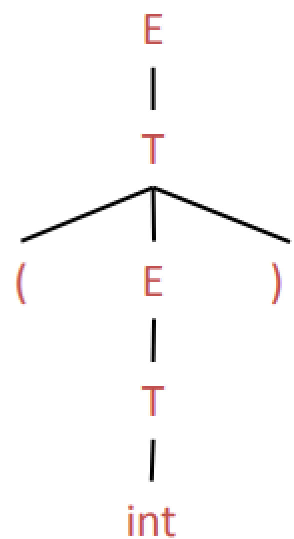


Match! Advance input.

$(\; int_5 \;)$

$E \rightarrow T \mid T + E$

$T \rightarrow int \mid int * T \mid (E)$



End of input, accept.

$(\; int_5 \;)$

Predictive Parsing

- We want to predict the correct production to be used (at a given stage of parsing).
- We do not want backtracking.
- These are called LL(1)
- A **parse table** is built which gives us the production to be used (in a given stage of parsing).

Parse table

The parse table tells a top-down parser which productions might possibly be applicable to the current input token.

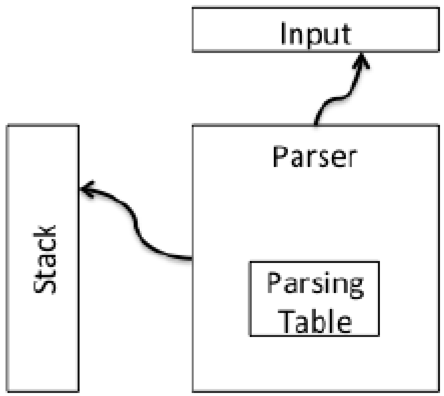
So if we are doing top-down parsing, possibly with backtracking, the parse table limits the search — we only try productions that can actually lead (eventually) to production of the current input symbol.

In the best case — with predictive parsing — the parse table is deterministic, eliminating the need for backtracking.

> So, if A is the current leftmost variable, then the parse table tells us which A-productions can be used to produce the current input token. (Or, if A is nullable, whether it would be a good idea to derive ϵ from A.)

www.d.umn.edu/~hudson/5641/l12m.pdf

LL(1) Parsing Algorithm




```
Initial configuration: Stack = S, Input = w$,
where, S = start symbol, $ = end of file marker
repeat {
   let X be the top stack symbol;
   let a be the next input symbol /*may be $*/;
   if X is a terminal symbol or $ then
        if X == a then {
             pop X from Stack;
             remove a from input;
        } else ERROR();
   else  /* X is a non-terminal symbol */
         if M[X,a] == X → Y_1 Y_2... Y_k then {
               pop X from Stack;
               push Y_k, Y_{k-1}, ..., Y_1 onto Stack;
                       (Y_1 on top)
         }
} until Stack has emptied;
```

Recognition by empty stack.

FIRST(α)

In order to conveniently specify the parse table for a grammar, we define two auxiliary functions: FIRST and FOLLOW.

For every string α over $V \cup \Sigma$, FIRST(α) is the set consisting of

- all terminals a s.t.

$$\alpha \stackrel{*}{\Rightarrow} a\beta$$

  for some string β over $V \cup \Sigma$, along with

- ϵ, if

$$\alpha \stackrel{*}{\Rightarrow} \epsilon\,.$$

Remember this. There is a scope for confusion here.

ϵ is in FIRST(α), if $\alpha \stackrel{*}{\Rightarrow} \epsilon$.
That is, Entire sentential form α can vanish

Example

$$S \rightarrow XSa \mid Yc$$
$$X \rightarrow aY \mid YY$$
$$Y \rightarrow bSa \mid cX \mid \epsilon$$

FIRST(S) = {a, b, c}

$\mathrm{FIRST}(X) = \{a, b, c, \epsilon\}$ so $\mathrm{FIRST}(XSa) = \{a, b, c\}$

$\mathrm{FIRST}(Y) = \{b, c, \epsilon\}$ so $\mathrm{FIRST}(Yc) = \{b, c\}$

$\mathrm{FIRST}(a) = \{a\} = \mathrm{FIRST}(aY)$ $\qquad$ $\mathrm{FIRST}(cX) = \{c\}$

$\mathrm{FIRST}(YY) = \{b, c, \epsilon\}$ $\qquad$ $\mathrm{FIRST}(\epsilon) = \{\epsilon\}$

$\mathrm{FIRST}(bSa) = \{b\}$

- **Find nullable variables first. It helps a lot.**

- if α begins with a terminal a then $\mathrm{FIRST}(\alpha) = \{a\}$
- If $\alpha = X_1 X_2 \cdots X_n$ then $\mathrm{FIRST}(\alpha)$ is found as follows.
- If α begins with a terminal a, then $\mathrm{FIRST}(\alpha) = \{a\}$, else

{continued in the next slide}

FIRST($X_1X_2 \cdots X_n$)

- Add to $\text{FIRST}(X_1X_2\cdots X_n)$ all non-ϵ symbols of $\text{FIRST}(X_1)$.
- Also add the non-ϵ symbols of $\text{FIRST}(X_2)$, if ϵ is in $\text{FIRST}(X_1)$;

  the non-ϵ symbols of $\text{FIRST}(X_3)$, if ϵ is in $\text{FIRST}(X_1)$ and $\text{FIRST}(X_2)$; and so on.

- Finally, add ϵ to $\text{FIRST}(X_1X_2\cdots X_n)$ if, for all i, ϵ is in $\text{FIRST}(X_i)$.

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\tag{4.28}
$$

Variable	FIRST
F	{ (, id }
T	FIRST(FT') = FIRST(F) = { (, id } /* F is not nullable */
E	FIRST(TE') = FIRST(T) = { (, id } /* T is not nullable */
E'	{ +, ε }
T'	{ *, ε }

FOLLOW(X)

For every variable A, **FOLLOW**(A) is the set consisting of

- all terminals a s.t.

$$S \stackrel{*}{\Rightarrow} \alpha A a \beta$$

  for some strings α, β over $V \cup \Sigma$, along with

- \$, if

$$S \stackrel{*}{\Rightarrow} \alpha A$$

  for some string α over $V \cup \Sigma$.

(Note: Here we have assumed that S *is the start symbol!*)

So, \$ is in FOLLOW(S) always.

In practice, we act as if the input string is always terminated with the special token \$. (This helps explain the fact that we include \$ in **FOLLOW**(A) iff A appears at the end of some sentential form.) Therefore, . . .

Example

$$\begin{aligned} S &\to XSa \mid Yc \\ X &\to aY \mid YY \\ Y &\to bSa \mid cX \mid \epsilon \end{aligned}$$

$a \in \text{FOLLOW}(S)$ since $S \overset{*}{\Rightarrow} XSa$

$c \notin \text{FOLLOW}(S)$

$b \notin \text{FOLLOW}(S)$

$\$ \in \text{FOLLOW}(S)$ since $S \overset{*}{\Rightarrow} S$

$a, b, c \in \text{FOLLOW}(X)$

since $S \overset{*}{\Rightarrow} XXSaa$ and $\text{FIRST}(X) = \{a, b, c, \epsilon\}$

$\$ \notin \text{FOLLOW}(X)$

since every sentential form (except S) ends with a or c

$a, b, c \in \text{FOLLOW}(Y)$

since $S \overset{*}{\Rightarrow} aYXSaa$ and $\text{FIRST}(X) = \{a, b, c, \epsilon\}$

$\$ \notin \text{FOLLOW}(Y)$

To compute $\text{FOLLOW}(A)$ for all nonterminals A, apply the following rules until nothing can be added to any FOLLOW set.

1. Place \$ in $\text{FOLLOW}(S)$, where S is the start symbol, and \$ is the input right endmarker.

2. If there is a production $A \rightarrow \alpha B \beta$, then everything in $\text{FIRST}(\beta)$ except ϵ is in $\text{FOLLOW}(B)$.

3. If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$, where $\beta \stackrel{*}{\Rightarrow} \epsilon$, then everything in $\text{FOLLOW}(A)$ is in $\text{FOLLOW}(B)$.

We can also say this, as: $\text{FIRST}(\beta)$ contains ϵ

Note, $ is never in FIRST of anything. ε is never in FOLLOW of anything.
FOLLOW(S) always contains $.
FIRST of something may contain ε, or may not contain.

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \textbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$

	FOLLOW
E	$,)
E’	
T	
T’	
F	

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$

	FOLLOW
E	$,)
E’	$,)
T	
T’	
F	

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$
4. $FIRST(E') - \{\epsilon\} \subseteq FOLLOW(T)$

	FOLLOW
E	$,)
E’	$,)
T	+
T’	
F	

$FIRST(E') = \{+, \epsilon\}$

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$
4. $FIRST(E') - \{\epsilon\} \subseteq FOLLOW(T)$
5. Since E’ is nullable, FOLLOW(E)
   is in FOLLOW(T)

	FOLLOW
E	$,)
E’	$,)
T	+, $,)
T’	
F	

$FIRST(E') = \{+, \epsilon\}$

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$
4. $FIRST(E') - \{\epsilon\} \subseteq FOLLOW(T)$
5. Since E' is nullable, FOLLOW(E) is in FOLLOW(T)
6. 3rd Production ⇒ FOLLOW (T) is in FOLLOW(T')

	FOLLOW
E	$,)
E'	$,)
T	+, $,)
T'	+, $,)
F	

$FIRST(E') = \{+, \epsilon\}$

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array} \tag{4.28}
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$
4. $FIRST(E') - \{\epsilon\} \ \subseteq FOLLOW(T)$
5. Since E' is nullable, FOLLOW(E) is in FOLLOW(T)
6. 3rd Production ⇒ FOLLOW (T) is in FOLLOW(T')
7. $FIRST(T') - \{\epsilon\} \in \ FOLLOW(F)$

	FOLLOW
E	$,)
E'	$,)
T	+, $,)
T'	+, $,)
F	*

$FIRST(E') \ = \ \{+, \epsilon\}, FIRST(T') \ = \ \{*, \epsilon\}$

$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E'\ |\ \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T'\ |\ \epsilon \\
F & \rightarrow & (\ E\ )\ |\ \mathbf{id}
\end{array}
\qquad (4.28)
$$

1. $\$ \in FOLLOW(E)$.
2. $F \rightarrow (E) \quad \Rightarrow \quad) \in \ FOLLOW(E)$
3. $FOLLOW(E) \subseteq FOLLOW(E')$
4. $FIRST(E') - \{\epsilon\} \ \subseteq FOLLOW(T)$
5. Since E' is nullable, FOLLOW(E) is in FOLLOW(T)
6. 3rd Production ==> FOLLOW (T) is in FOLLOW(T')
7. $FIRST(T')- \{\epsilon\} \in \ FOLLOW(F)$
8. Since T' is nullable, FOLLOW(T) is in FOLLOW(F)

	FOLLOW
E	$,)
E'	$,)
T	+, $,)
T'	+, $,)
F	*, +, $,)

$FIRST(E') \ = \ \{+, \epsilon\}, FIRST(T') \ = \ \{*, \epsilon\}$

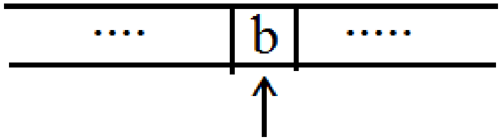
$$
\begin{array}{lcl}
E & \rightarrow & T\ E' \\
E' & \rightarrow & +\ T\ E' \mid \epsilon \\
T & \rightarrow & F\ T' \\
T' & \rightarrow & *\ F\ T' \mid \epsilon \\
F & \rightarrow & (\ E\ ) \mid \mathbf{id}
\end{array}
\qquad (4.28)
$$

$$\text{FOLLOW}(E) = \text{FOLLOW}(E') = \{), \$\}.$$

$$\text{FOLLOW}(T) = \text{FOLLOW}(T') = \{+,), \$\}.$$

$$\text{FOLLOW}(F) = \{+, *,), \$\}.$$

Specification of parse table



Current input character

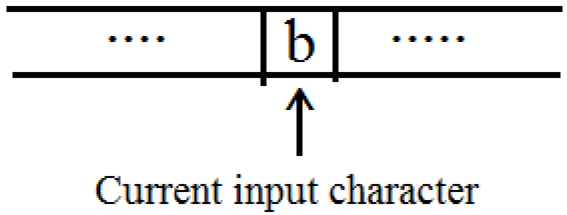
For leftmost variable A and current input token b, the *applicable productions* are

- all productions $A \rightarrow \alpha$ s.t. $b \in \mathrm{FIRST}(\alpha)$, and
- all productions $A \rightarrow \alpha$ s.t. $\epsilon \in \mathrm{FIRST}(\alpha)$ and $b \in \mathrm{FOLLOW}(A)$.

ϵ is in $\mathrm{FIRST}(\alpha)$, if $\alpha \overset{*}{\Rightarrow} \epsilon$.
That is, Entire sentential form α can vanish

For leftmost variable A and current input token \$, the *applicable productions* are

- all productions $A \rightarrow \alpha$ s.t. $\epsilon \in \mathrm{FIRST}(\alpha)$ and $\$ \in \mathrm{FOLLOW}(A)$.



More concise characterization of the applicable productions

For leftmost variable A and current input token $b \in \Sigma \cup \{\$\}$, the applicable productions are

- all productions $A \to \alpha$ s.t. $b \in \text{FIRST}(\alpha)$, and
- all productions $A \to \alpha$ s.t. $\epsilon \in \text{FIRST}(\alpha)$ and $b \in \text{FOLLOW}(A)$.

Let's construct the parse table for the grammar we've been looking at.

$$
\begin{aligned}
S &\rightarrow XSa \mid Yc \\
X &\rightarrow aY \mid YY \\
Y &\rightarrow bSa \mid cX \mid \epsilon
\end{aligned}
$$

$\text{FIRST}(XSa) = \{a, b, c\}$ $\text{FOLLOW}(S) = \{a, \$\}$

$\text{FIRST}(Yc) = \{b, c\}$ $\text{FOLLOW}(X) = \{a, b, c\}$

$\text{FIRST}(YY) = \{b, c, \epsilon\}$ $\text{FOLLOW}(Y) = \{a, b, c\}$

LEFTMOST VARIABLE	CURRENT INPUT TOKEN			
	a	b	c	$\$$
S	$S \rightarrow XSa$	$S \rightarrow XSa \mid Yc$	$S \rightarrow XSa \mid Yc$	none
X	$X \rightarrow aY \mid YY$	$X \rightarrow YY$	$X \rightarrow YY$	none
Y	$Y \rightarrow \epsilon$	$Y \rightarrow bSa \mid \epsilon$	$Y \rightarrow cX \mid \epsilon$	none

LEFTMOST VARIABLE	CURRENT INPUT TOKEN			
	a	b	c	$\$$
S	$S \to XSa$	$S \to XSa \mid Yc$	$S \to XSa \mid Yc$	none
X	$X \to aY \mid YY$	$X \to YY$	$X \to YY$	none
Y	$Y \to \epsilon$	$Y \to bSa \mid \epsilon$	$Y \to cX \mid \epsilon$	none

Input string: *aca*

Note: There is no need to add ***S'→S$*** *always. If we add this then an entry in the parse table with S' should exist! Of course the stack should be preloaded with $.*

STACK	CURRENT INPUT	PRODUCTION TO APPLY
$S\$$	$aca\$$	$S \to XSa$
$XSa\$$	$aca\$$	$X \to aY$ (backtrack $X \to YY$)
$aYSa\$$	$aca\$$	match a
$YSa\$$	$ca\$$	$Y \to \epsilon$ (backtrack $Y \to cX$)
$Sa\$$	$ca\$$	$S \to Yc$ (backtrack $S \to XSa$)
$Yca\$$	$ca\$$	$Y \to \epsilon$ (backtrack $Y \to cX$)
$ca\$$	$ca\$$	match c
$a\$$	$a\$$	match a
$\$$	$\$$	successful parse

LL(1) grammars, for predictive parsing

A grammar G is LL(1) if and only if whenever $A \rightarrow \alpha \mid \beta$ are two distinct productions of G, the following conditions hold:

1. For no terminal a do both α and β derive strings beginning with a.
2. At most one of α and β can derive the empty string.
3. If $\beta \stackrel{*}{\Rightarrow} \epsilon$, then α does not derive any string beginning with a terminal in $\text{FOLLOW}(A)$. Likewise, if $\alpha \stackrel{*}{\Rightarrow} \epsilon$, then β does not derive any string beginning with a terminal in $\text{FOLLOW}(A)$.

Definition A grammar is LL(1) if there is at most one production in the parsing table for each variable, token pair.

LL(1) grammars are without left recursion and are left factored.

It is a good practice to eliminate left recursion, and doing left factoring, before building the parse table.

Left Recursion and its elimination

- A grammar is left recursive, if for a variable *A*, there is a derivation $A \overset{+}{\Rightarrow} A\alpha$
- Top-down parsing can fall into infinite loop because of this
  - Hence these type of derivation should **not** be allowed.

Immediate left-recursion

$$A \to A\alpha_1 \mid A\alpha_2 \mid \cdots \mid A\alpha_m \mid \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$$

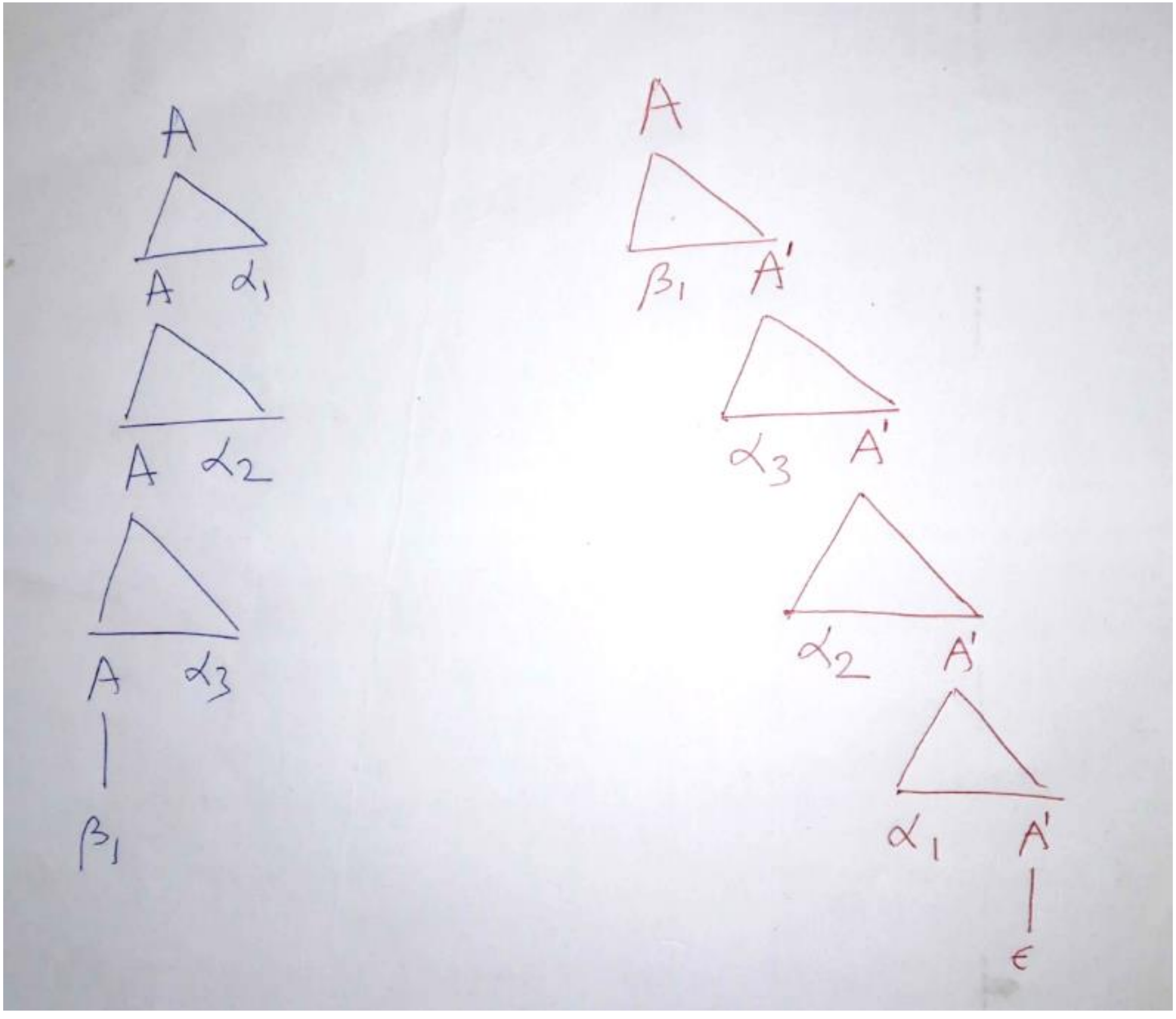
Replace these by the following

$$A \to \beta_1 A' \mid \beta_2 A' \mid \cdots \mid \beta_n A'$$
$$A' \to \alpha_1 A' \mid \alpha_2 A' \mid \cdots \mid \alpha_m A' \mid \epsilon$$

$$\boxed{\begin{array}{lcl} E & \to & E + T \mid E - T \mid T \\ T & \to & T * F \mid T \,/\, F \mid F \\ F & \to & (\,E\,) \mid \mathbf{id} \end{array}}$$

Can be replaced by

$$E \to TE'$$
$$E' \to +TE' \mid -TE' \mid \epsilon$$
$$T \to FT'$$
$$T' \to *FT' \mid /FT' \mid \epsilon$$
$$F \to (E) \mid id$$


A
A
α1
A
α2
A
α3
β1
A
β1
A'
α3
A'
α2
A'
α1
A'
ϵ

But, several productions can lead to left-recursion

- $A_1 \to A_2\alpha;\ \ A_2 \to A_3\beta;\ \ A_3 \to A_1\gamma$

- Can give rise to $A_1 \Rightarrow\ A_1\gamma\beta\alpha$

- This happened because of the third production $A_3 \to A_1\gamma$
- If we order variables $A_1, A_2, \dots,$
- A production $A_i \to A_j\gamma\ where\ j \leq i$ is problematic. So clean-up these!

Algorithm 4.19: Eliminating left recursion.

INPUT: Grammar G with no cycles or ϵ-productions.

OUTPUT: An equivalent grammar with no left recursion.

METHOD: Apply the algorithm in Fig. 4.11 to G. Note that the resulting non-left-recursive grammar may have ϵ-productions. □

Cyclic , if $A \overset{+}{\Rightarrow} A$
Remove unit productions it removes this too ☺

```
1)  arrange the nonterminals in some order A_1, A_2, ..., A_n.
2)  for ( each i from 1 to n ) {
3)      for ( each j from 1 to i-1 ) {
4)          replace each production of the form A_i → A_j γ by the
                productions A_i → δ_1 γ | δ_2 γ | ··· | δ_k γ, where
                A_j → δ_1 | δ_2 | ··· | δ_k are all current A_j-productions
5)      }
6)      eliminate the immediate left recursion among the A_i-productions
7)  }
```

Figure 4.11: Algorithm to eliminate left recursion from a grammar

$A \longrightarrow BA$
$B \longrightarrow \epsilon$

Can cause left recursion.

That's why one has to remove ϵ and unit productions.

Consider the grammar

$$\begin{aligned} S &\rightarrow SX \mid SSb \mid XS \mid a \\ X &\rightarrow Xb \mid Sa \mid b \end{aligned}$$

Let's order the variables S, X:

The first time through we simply eliminate immediate left recursion in S-productions, yielding

$$\begin{aligned} S &\rightarrow XSS' \mid aS' \\ S' &\rightarrow XS' \mid SbS' \mid \epsilon \\ X &\rightarrow Xb \mid Sa \mid b \end{aligned}$$

So at this point we have grammar

$$
\begin{aligned}
S &\rightarrow XSS' \mid aS' \\
S' &\rightarrow XS' \mid SbS' \mid \epsilon \\
X &\rightarrow Xb \mid Sa \mid b
\end{aligned}
$$

and the next obligation is to replace the production

$$X \rightarrow Sa$$

with the productions

$$X \rightarrow XSS'a \mid aS'a\,.$$

We then eliminate immediate left recursion among

$$X \rightarrow XSS'a \mid aS'a \mid Xb \mid b\,.$$

Eliminating immediate left recursion among

$$X \rightarrow XSS'a \mid Xb \mid b \mid aS'a$$

yields

$$\begin{aligned} X &\rightarrow bX' \mid aS'aX' \\ X' &\rightarrow SS'aX' \mid bX' \mid \epsilon \end{aligned}$$

So the final result is

$$\begin{aligned} S &\rightarrow XSS' \mid aS' \\ S' &\rightarrow XS' \mid SbS' \mid \epsilon \\ X &\rightarrow bX' \mid aS'aX' \\ X' &\rightarrow SS'aX' \mid bX' \mid \epsilon \end{aligned}$$

Left Factoring

- Left factoring is required when two or more grammar rule choices share a common prefix string, as in the rule

$$A \rightarrow \alpha\ \beta \mid \alpha\ \gamma$$

- Which one to use when we want to replace A?
- If wrong choice is made we need to backtrack.
- So, let us postpone the choice making moment.

– replace all of the A-productions $A \to \alpha\beta_1 \mid \alpha\beta_2 \mid \cdots \mid \alpha\beta_n \mid \gamma$, where γ represents all alternatives that do not begin with α, by

$$\begin{aligned} A &\to \alpha A' \mid \gamma \\ A' &\to \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n \end{aligned}$$

Here A' is a new nonterminal. Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix.

Example 4.22: The following grammar abstracts the "dangling-else" problem:

$$\begin{aligned} S &\to i\ E\ t\ S \mid i\ E\ t\ S\ e\ S \mid a \\ E &\to b \end{aligned} \tag{4.23}$$

Here, i, t, and e stand for **if**, **then**, and **else**; E and S stand for "conditional expression" and "statement." Left-factored, this grammar becomes:

$$\begin{aligned} S &\to i\ E\ t\ S\ S' \mid a \\ S' &\to e\ S \mid \epsilon \\ E &\to b \end{aligned} \tag{4.24}$$

$$
\begin{aligned}
S &\to AaS \mid b \\
A &\to c \mid d \mid B \\
B &\to AgC \mid AhC \mid DgC \mid DhC \\
C &\to c \mid d \mid D \\
D &\to eBf
\end{aligned}
$$

Is this grammar left-recursive?

Let's eliminate left recursion, with variable ordering S, A, B, C, D.

1. There's no immediate left recursion among S-productions.

2a. There are no productions from A whose rhs begins with S.

2b. There's no immediate left recursion among A-productions.

3a. There are no productions from B whose rhs begins with S.

3b. There are two productions from B whose rhs begins with A.

We replace

$$B \to AgC \mid AhC$$

with what?

After eliminating left recursion and after doing left factoring …

$$
\begin{aligned}
S &\rightarrow AaS \mid b \\
A &\rightarrow cB' \mid dB' \mid eCDB'fDB' \\
B' &\rightarrow DB' \mid \epsilon \\
C &\rightarrow c \mid d \mid eCDB'f \\
D &\rightarrow gC \mid hC
\end{aligned}
$$

$$
\begin{aligned}
S &\rightarrow AaS \mid b \\
A &\rightarrow cB \mid dB \mid eCDBfDB \\
B &\rightarrow DB \mid \epsilon \\
C &\rightarrow c \mid d \mid eCDBf \\
D &\rightarrow gC \mid hC
\end{aligned}
$$

$\mathrm{FIRST}(AaS) = \{c, d, e\}$

$\mathrm{FIRST}(DB) = \{g, h\}$

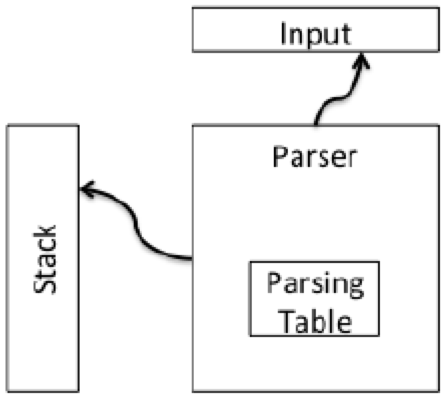
$\mathrm{FIRST}(\epsilon) = \{\epsilon\}$

$\mathrm{FOLLOW}(B) = \{a, f\}$

	CURRENT INPUT TOKEN								
VAR	a	b	c	d	e	f	g	h	$
S		b	AaS	AaS	AaS				
A			cB	dB	$eCDBfDB$				
B	ϵ					ϵ	DB	DB	
C			c	d	$eCDBf$				
D							gC	hC	

So, the grammar is LL(1)

LL(1) Parsing Algorithm




```
Initial configuration: Stack = S, Input = w$,
where, S = start symbol, $ = end of file marker
repeat {
   let X be the top stack symbol;
   let a be the next input symbol /*may be $*/;
   if X is a terminal symbol or $ then
        if X == a then {
             pop X from Stack;
             remove a from input;
        } else ERROR();
   else /* X is a non-terminal symbol */
        if M[X,a] == X → Y_1 Y_2... Y_k then {
             pop X from Stack;
             push Y_k, Y_(k-1), ..., Y_1 onto Stack;
                    (Y_1 on top)
        }
} until Stack has emptied;
```

Recognition by empty stack.

LL(1) Parsing Algorithm Example

Grammar

$S' \rightarrow S\$$
$S \rightarrow aAS \mid c$
$A \rightarrow ba \mid SB$
$B \rightarrow bA \mid S$

string: *acbbac*$

LL(1) Parsing Table

	a	b	c	$
S′	S′ → S$		S′ → S$	
S	S → aAS		S → c	
A	A → SB	A → ba	A → SB	
B	B → S	B → bA	B → S