

Computer Vision

Neural Networks

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**Indian Institute of Information Technology
Sri City, Chittoor**



We have learned so far in this module

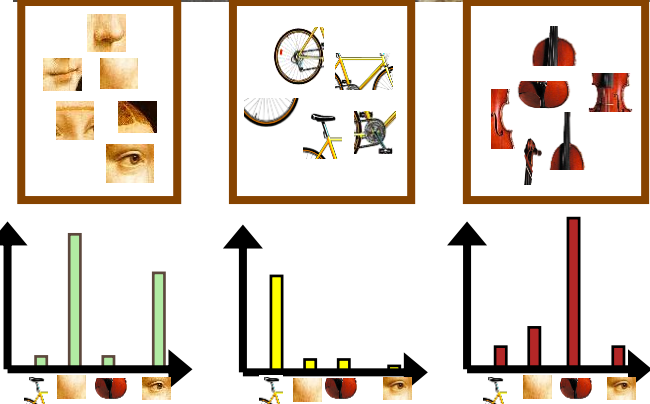
Image features and categorization

Choosing right features
Object, Scene, Action, etc.



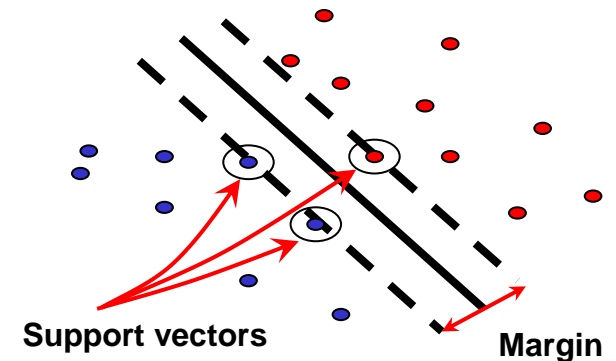
Bag-of-visual-words

Extract local features
Learn “visual vocabulary”
Quantize features using visual vocabulary
Represent by frequencies of “visual words”



Classifiers

Nearest neighbor, KNN, Linear classifier,
SVM, Non-linear SVM, Multi-class SVM,
Softmax classifier



Previous Class

Two key components in context of the image classification

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Mapping the raw image pixels/features to class scores

(e.g. a linear function)

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2. A loss function:

Measures the goodness of parameter values in terms of how well it performs over the training data
(e.g. Softmax/SVM)

Previous Class

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Hinge Loss

Max-margin loss

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Softmax Loss:

Cross-entropy loss

$$L_i = -\log \left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}} \right)$$

Today's class

Optimization

Gradient Descent & Back propagation

Perceptron

Update rule

Neural networks

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Strategy #3: Following the gradients:

There is no need to randomly search for a good direction: this direction is related to the **gradient** of the loss function.

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while True:
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Stochastic Gradient Descent (SGD):

Special case of MGD when mini-batch contains only a single example

Interpretation of the gradient

Interpretation. Derivatives indicate the rate of change of a function with respect to that variable surrounding an infinitesimally small region near a particular point:

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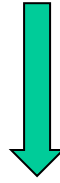
$$f(x, y) = \max(x, y) \quad \rightarrow \quad \frac{\partial f}{\partial x} = 1(x \geq y) \quad \frac{\partial f}{\partial y} = 1(y \geq x)$$

Compound expressions with chain rule

$$f(x, y, z) = (x + y)z$$

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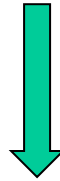
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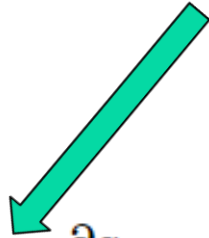
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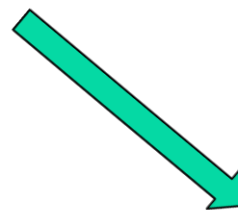
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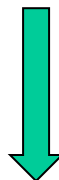
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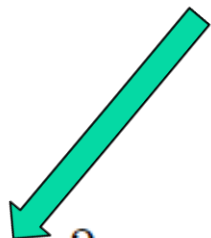
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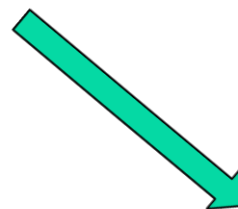
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Chain rule: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial x}$

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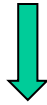
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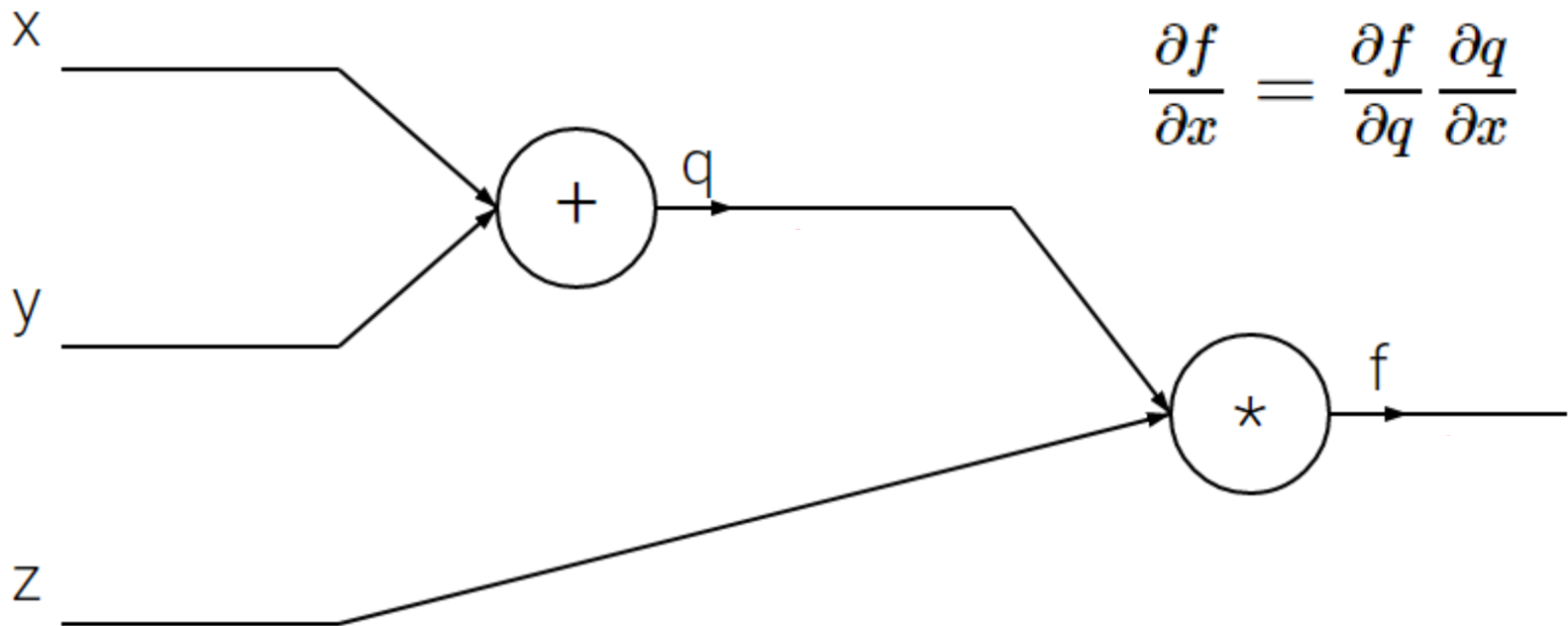


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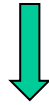
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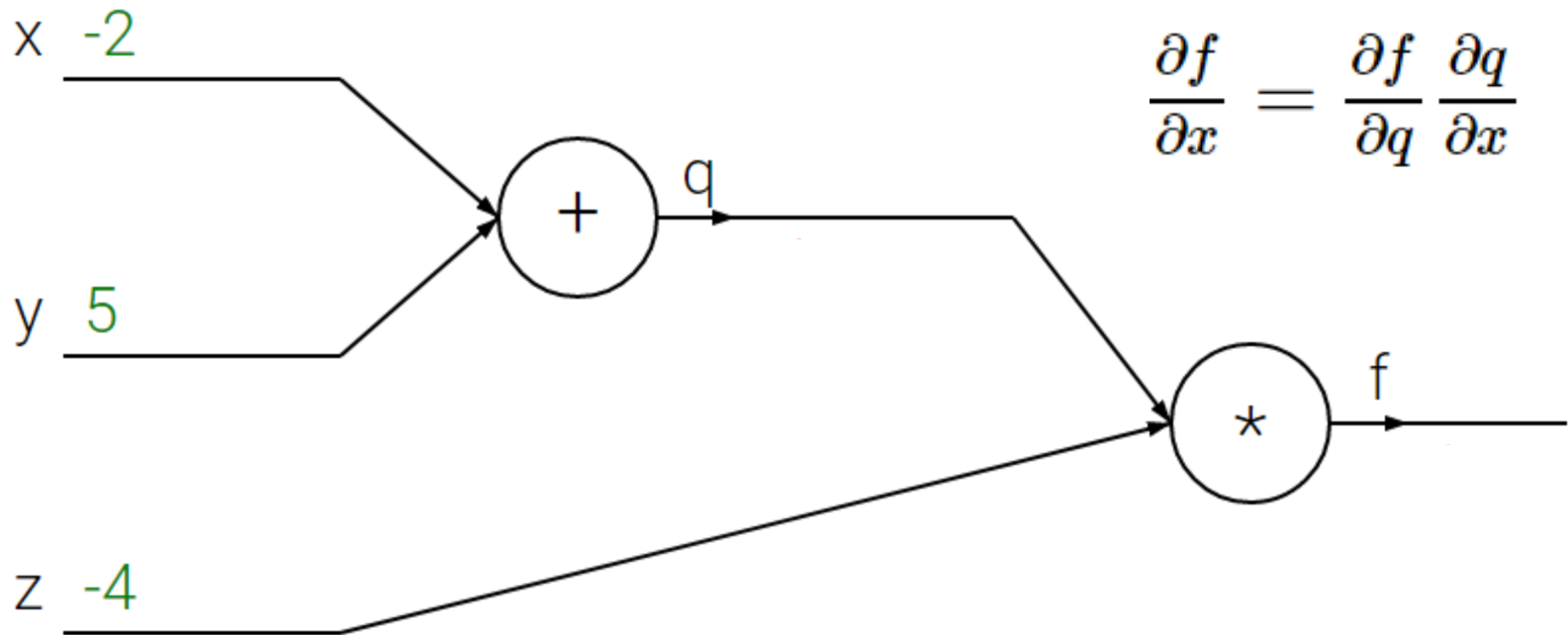


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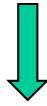
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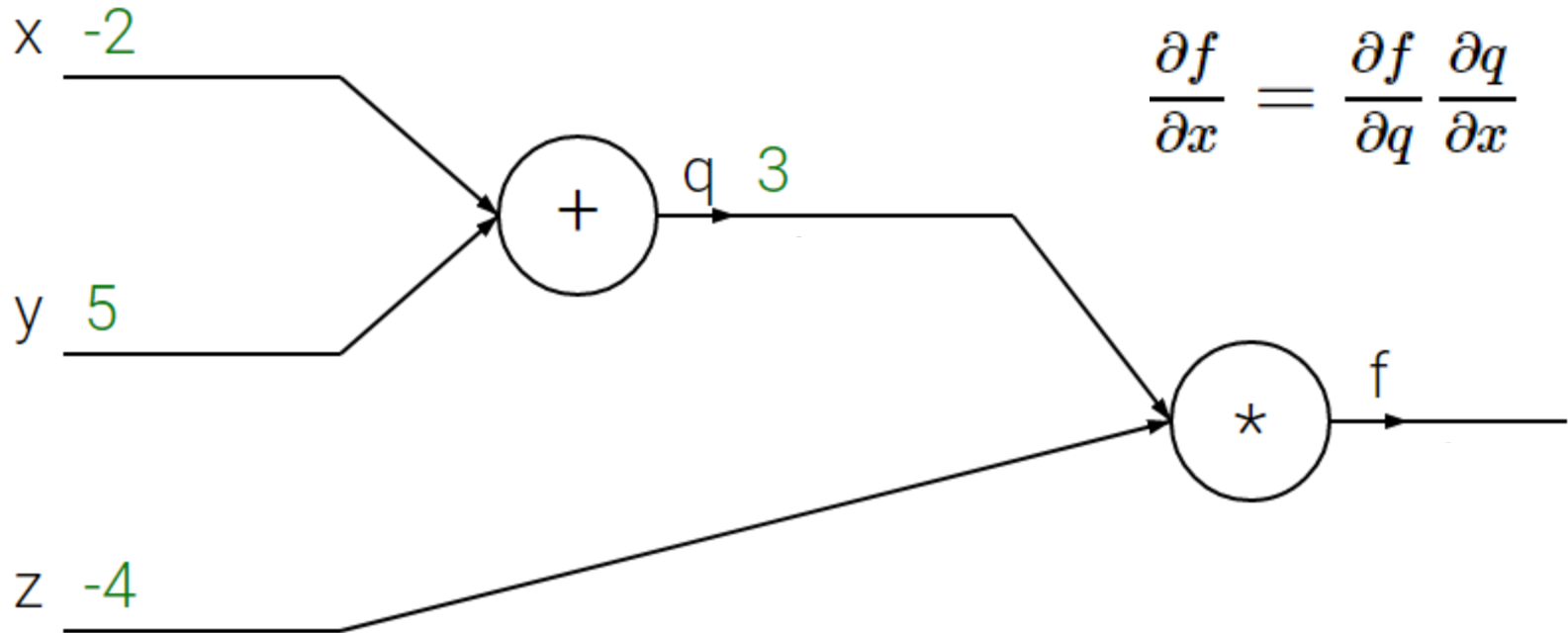


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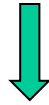
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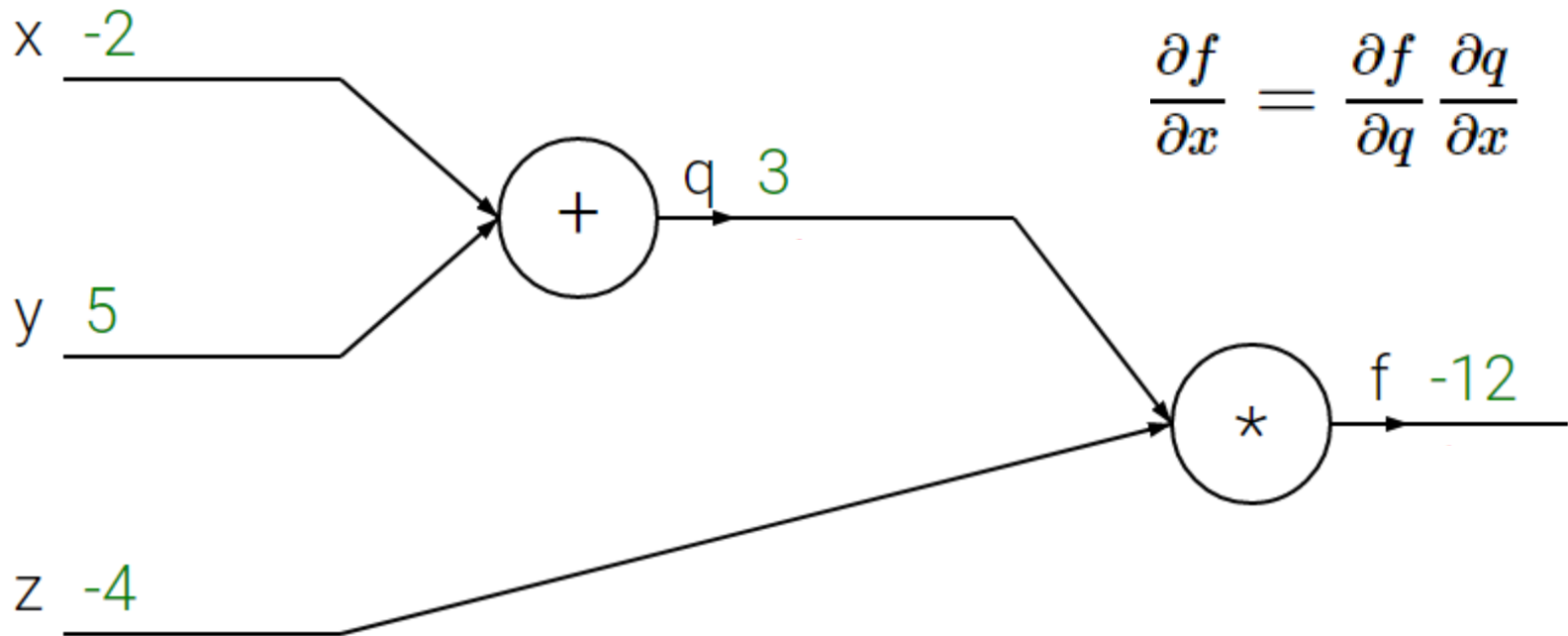


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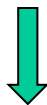
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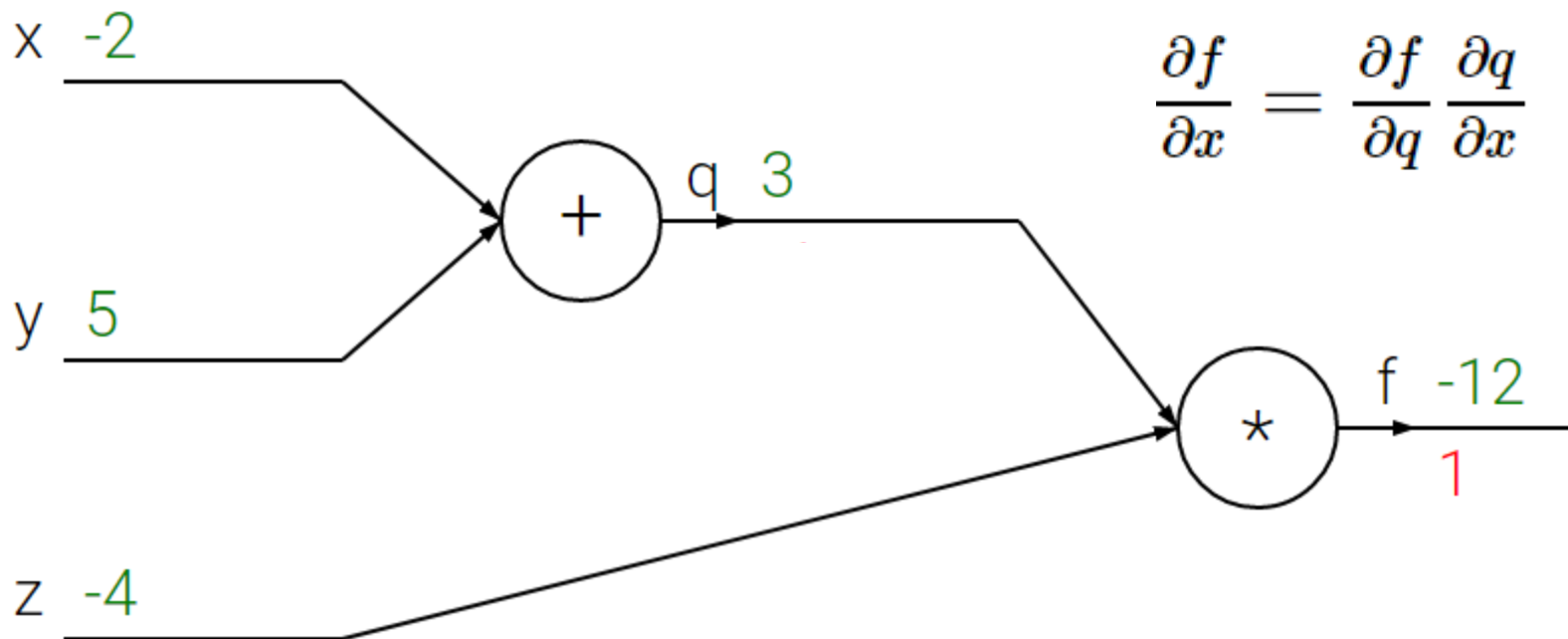


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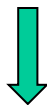
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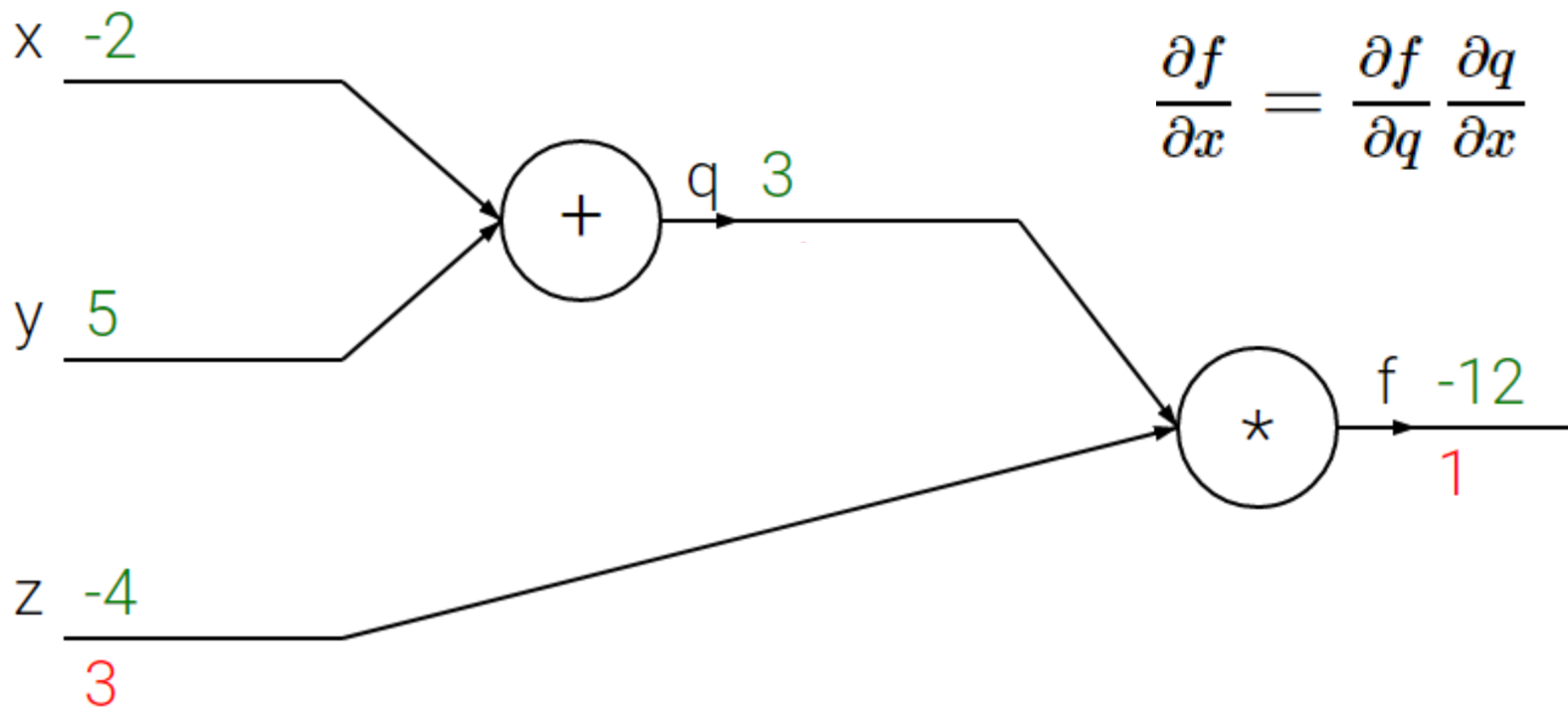


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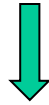
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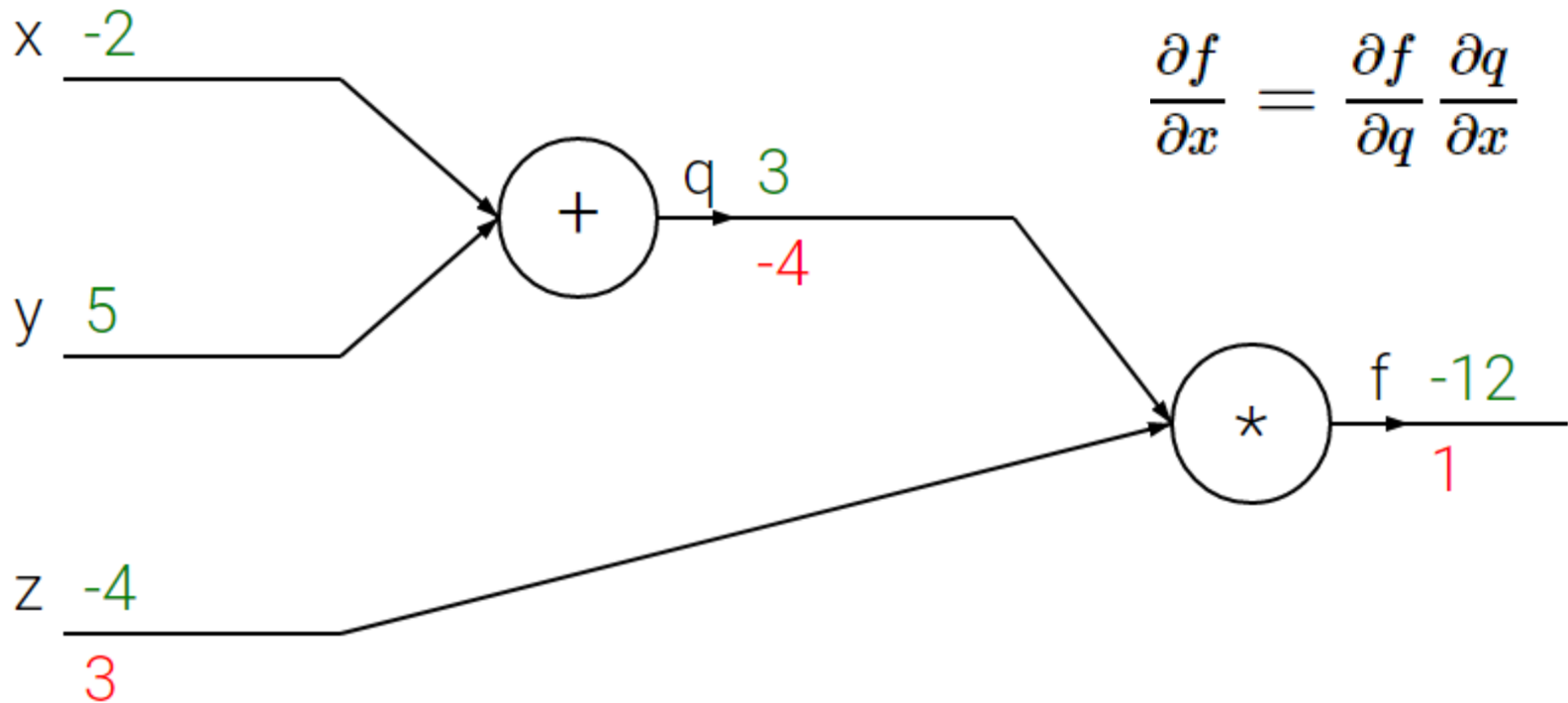


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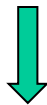
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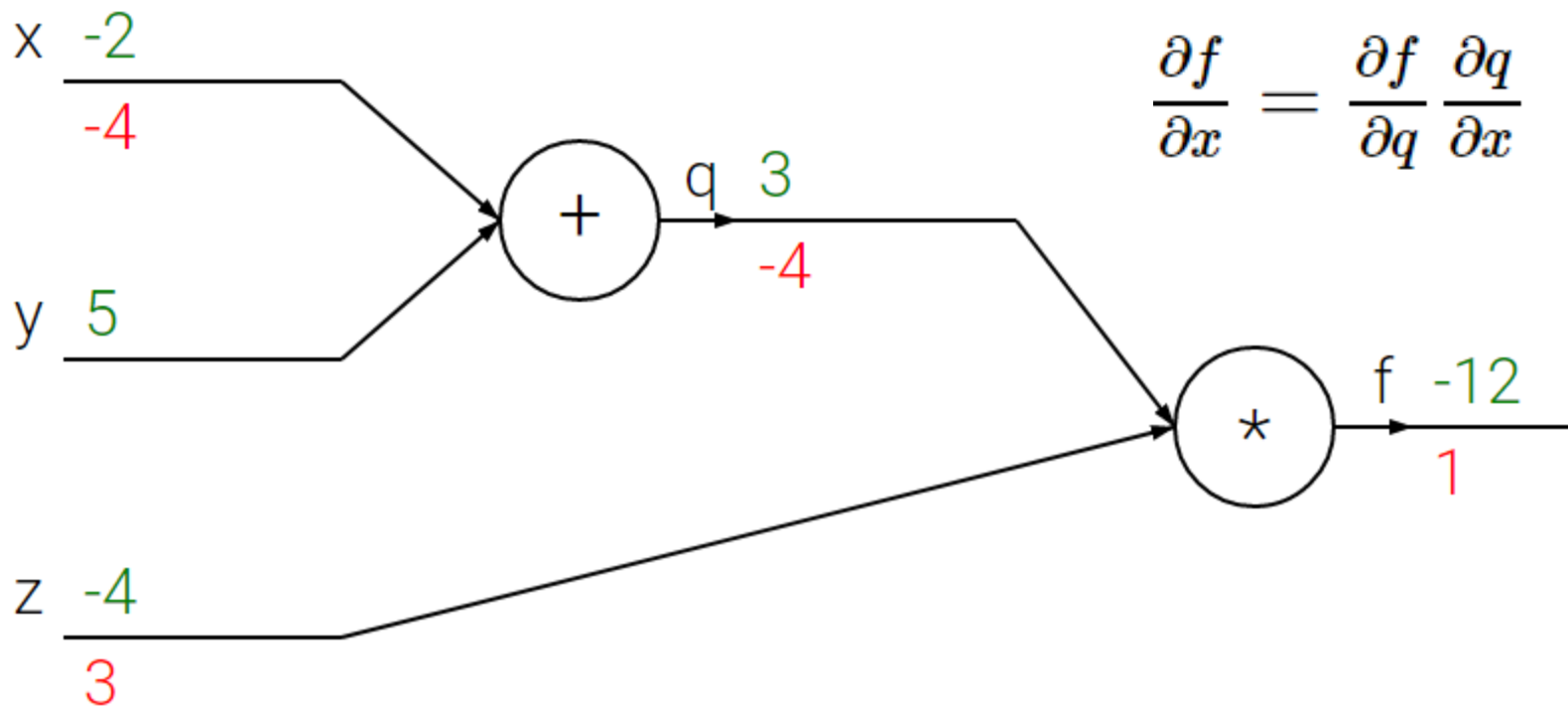


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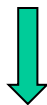
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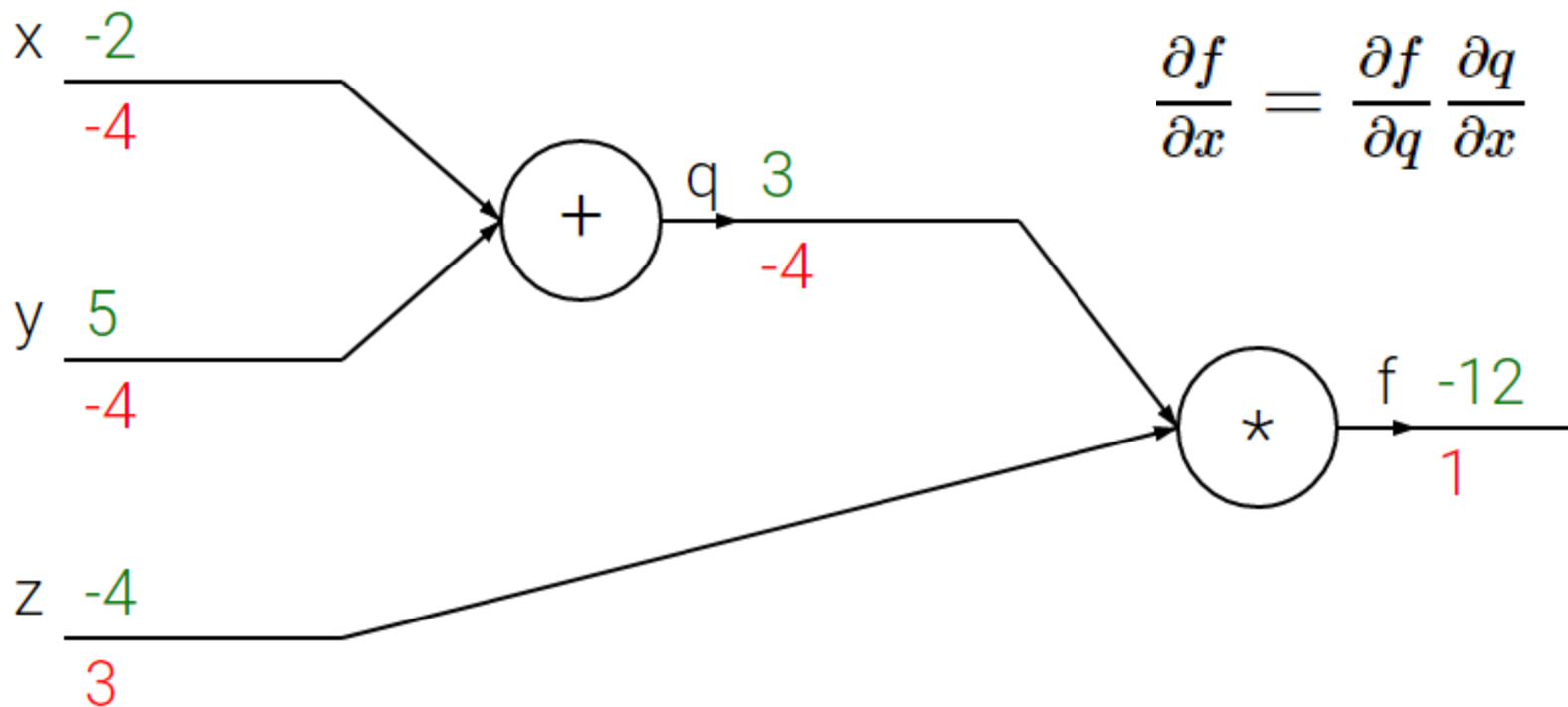


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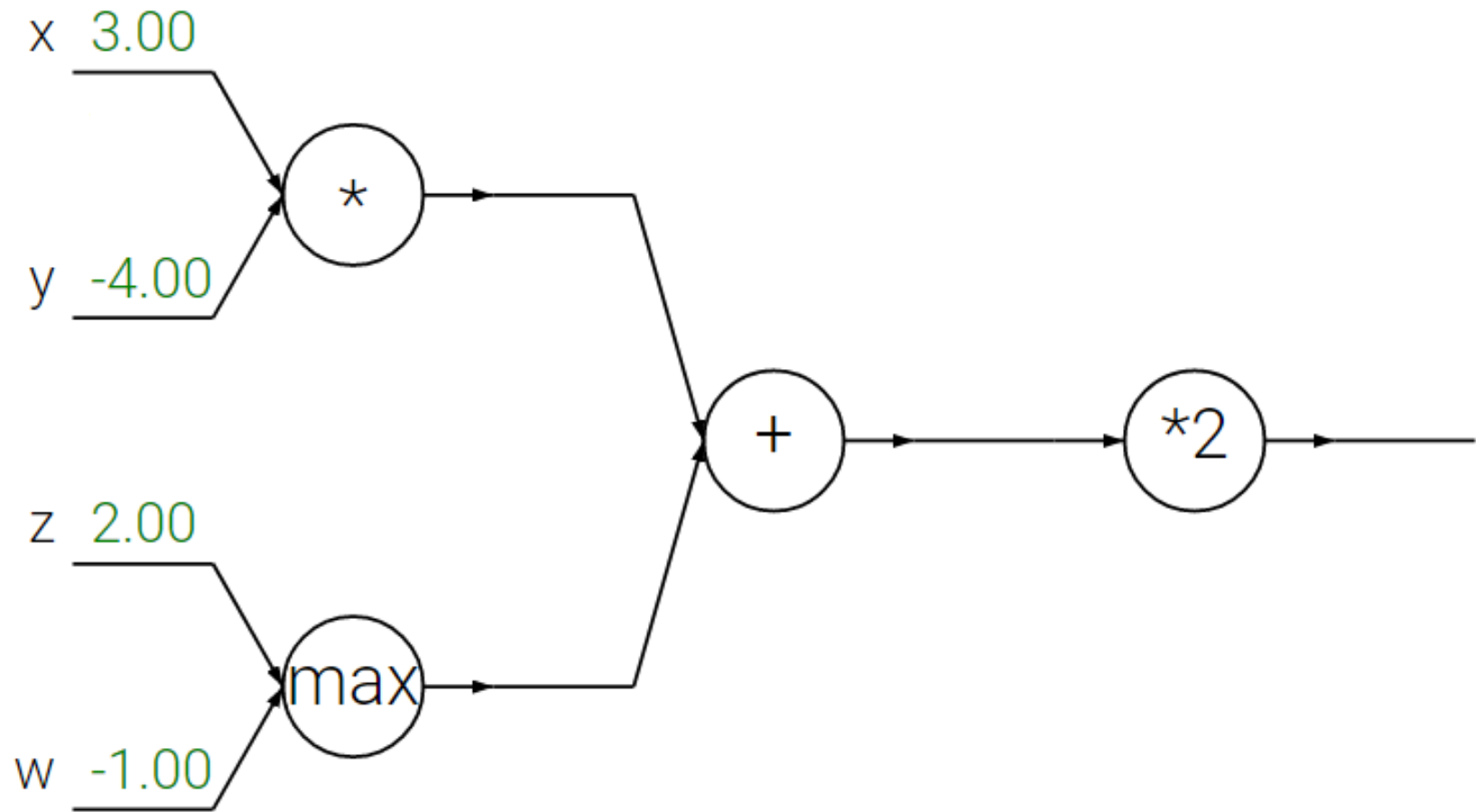
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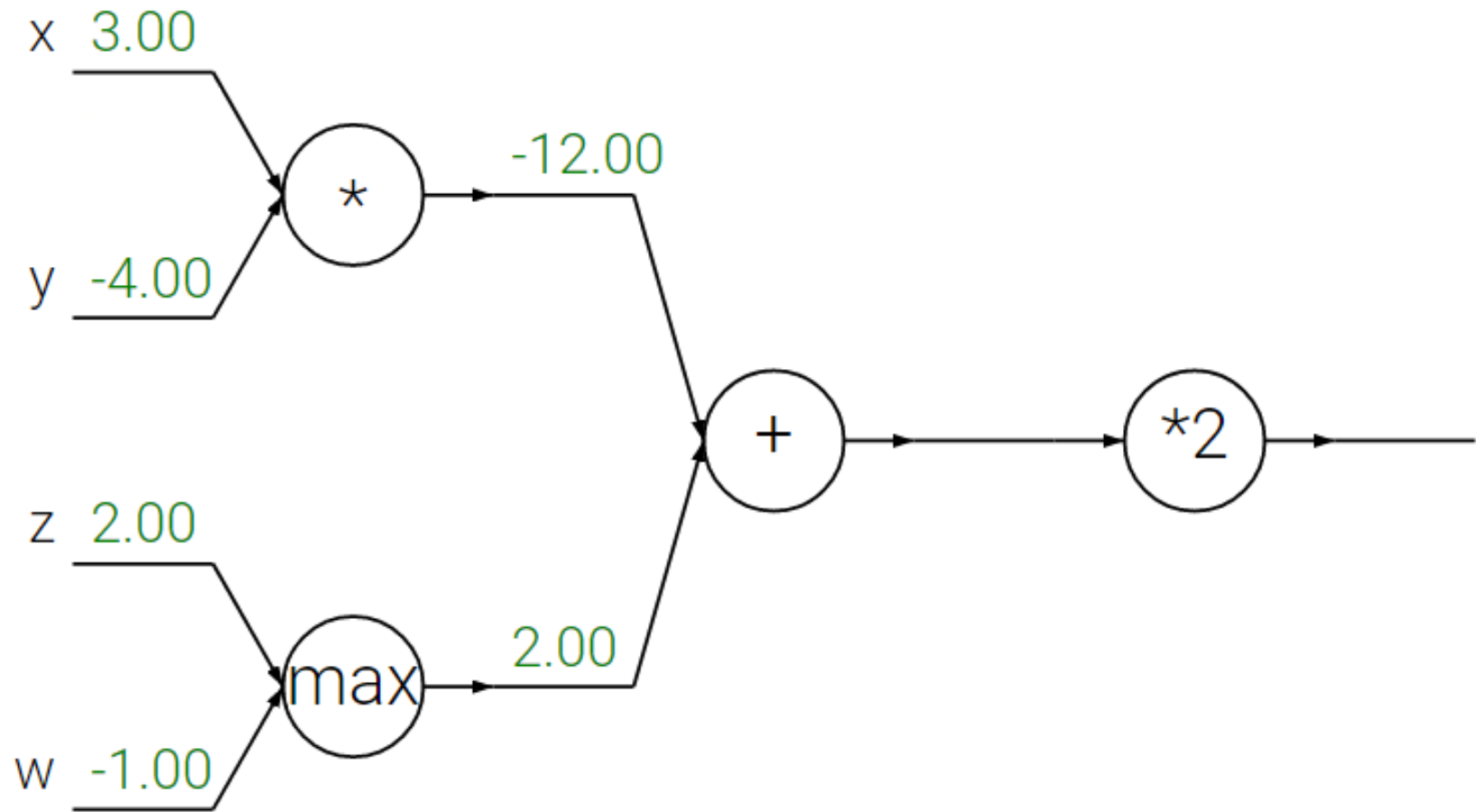
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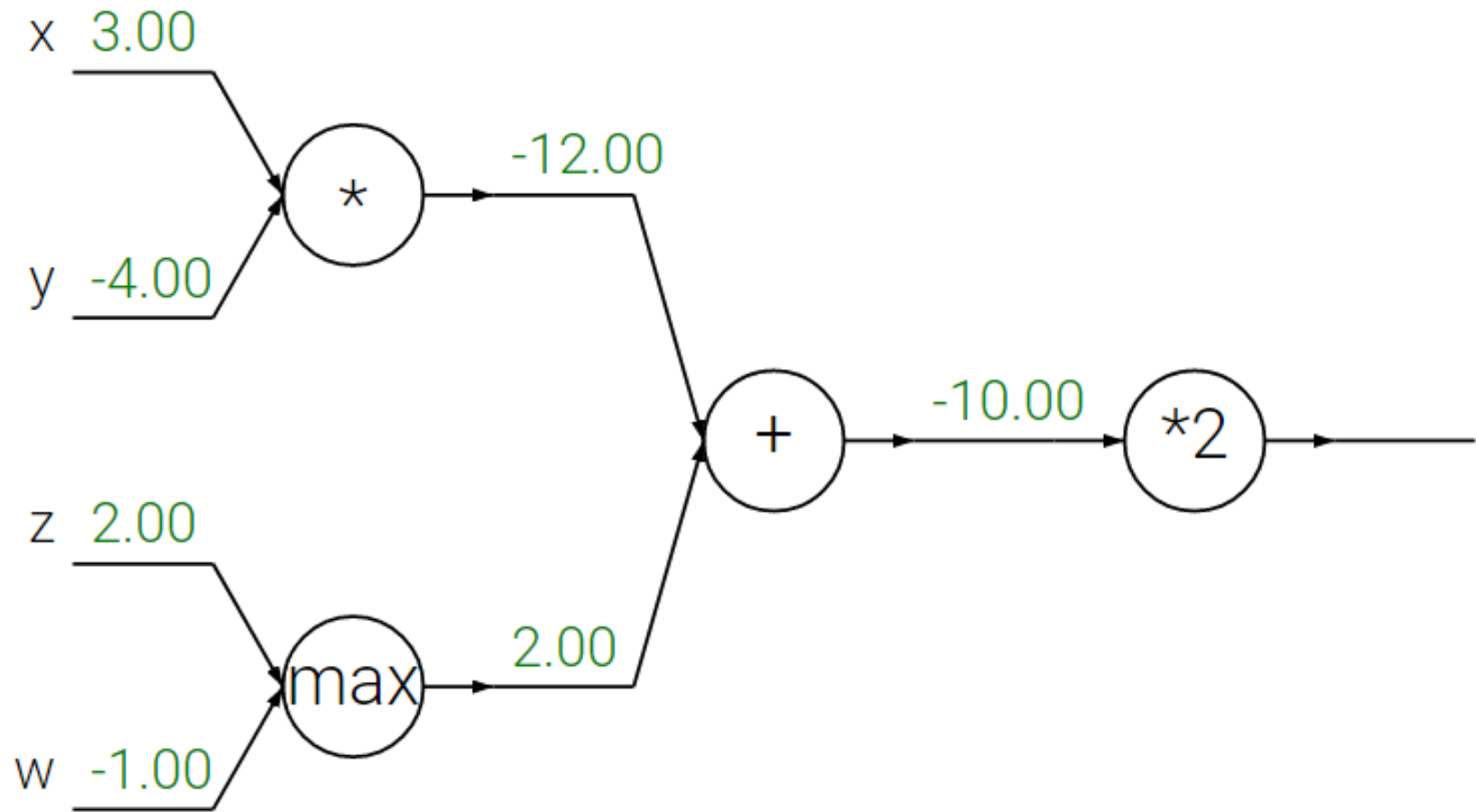
Forward and Backward Pass



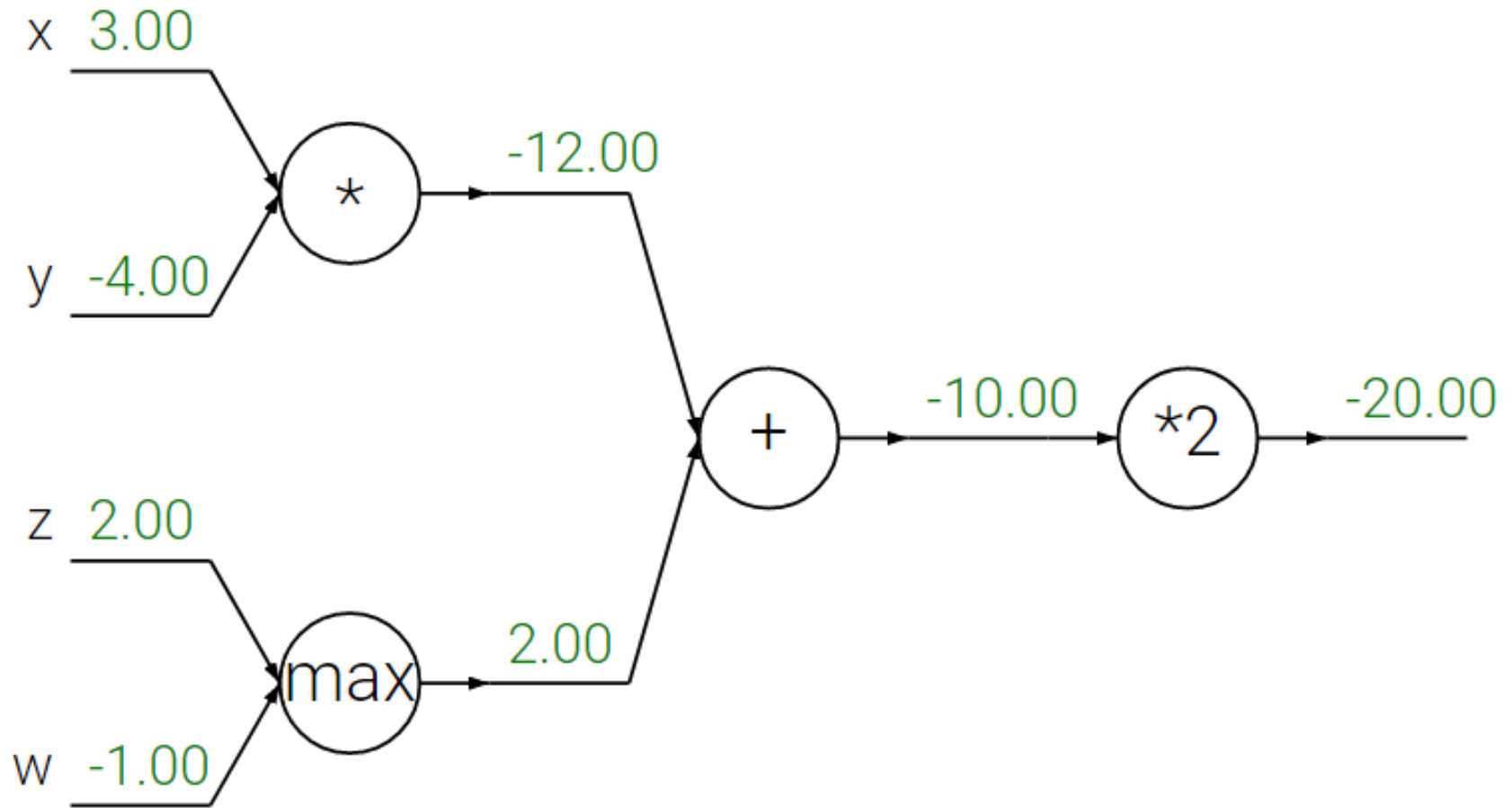
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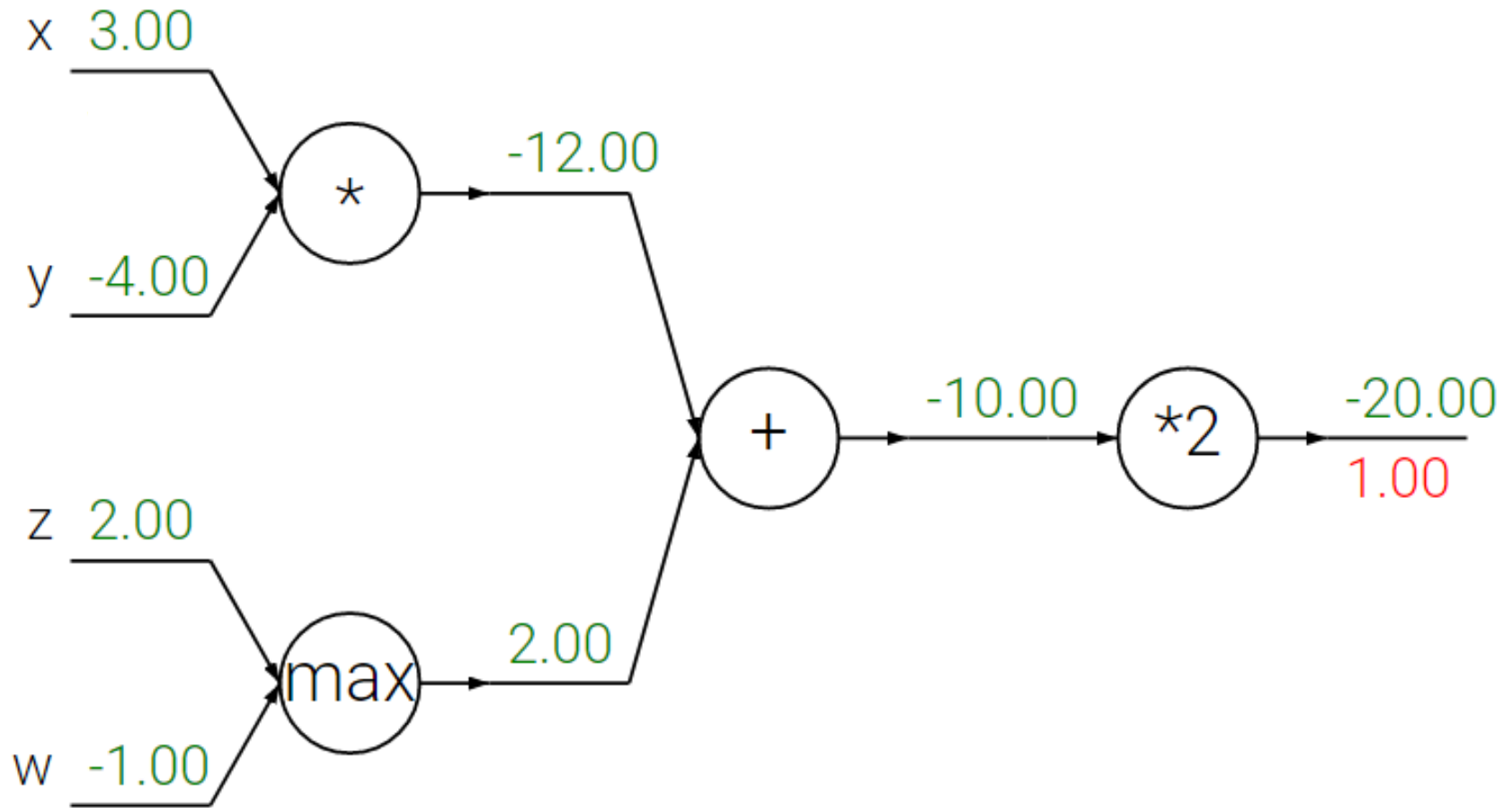
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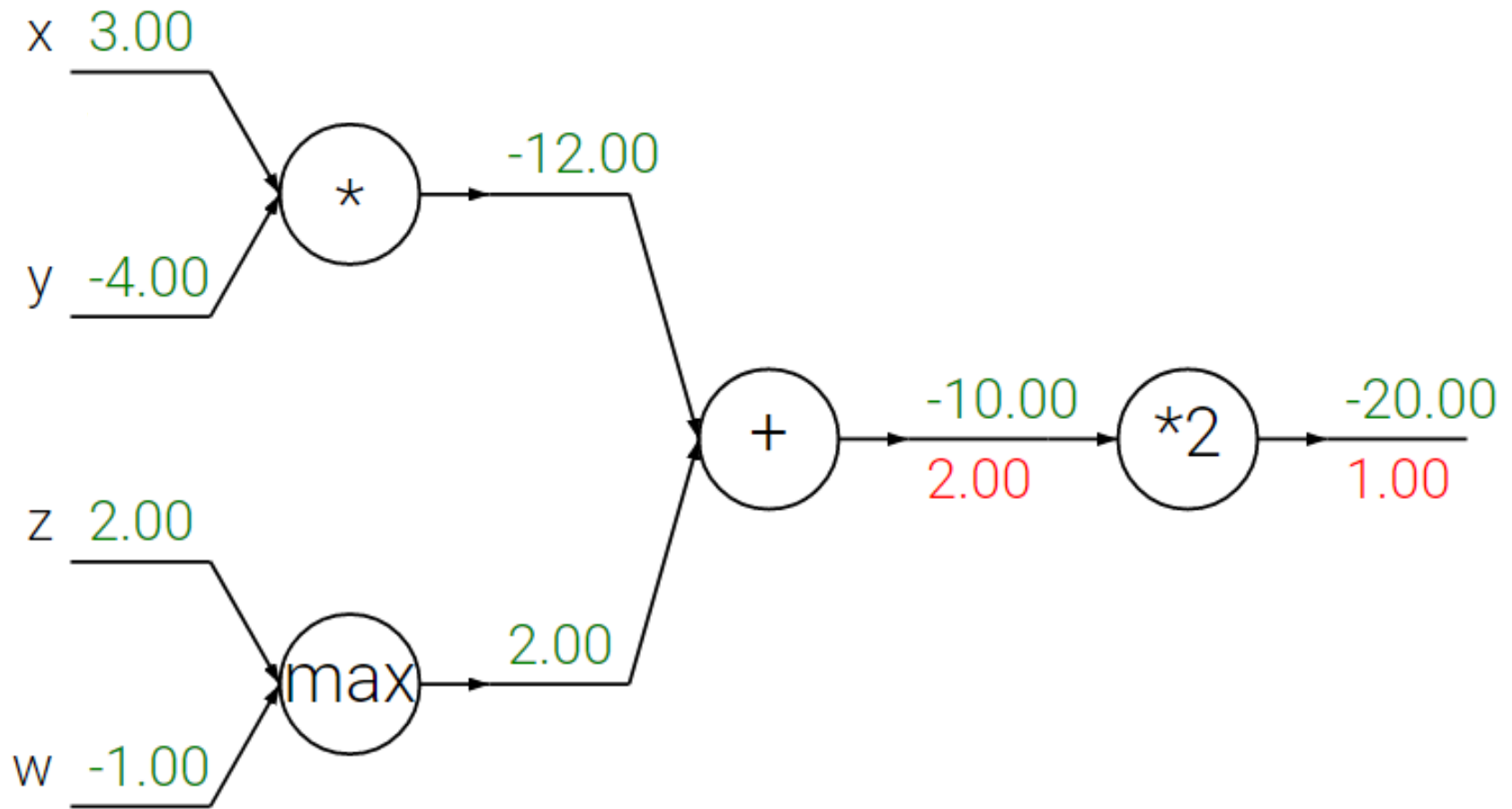
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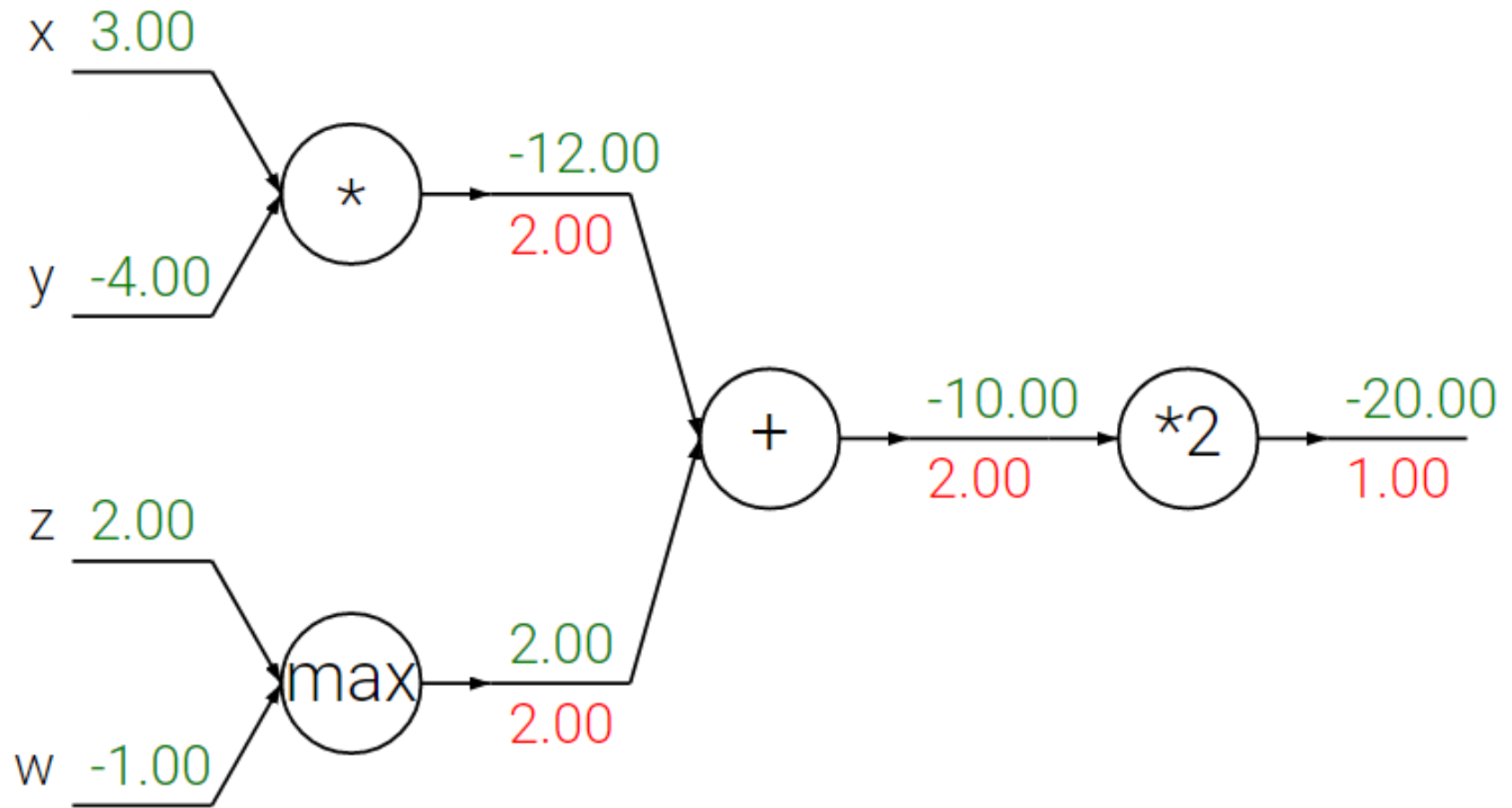
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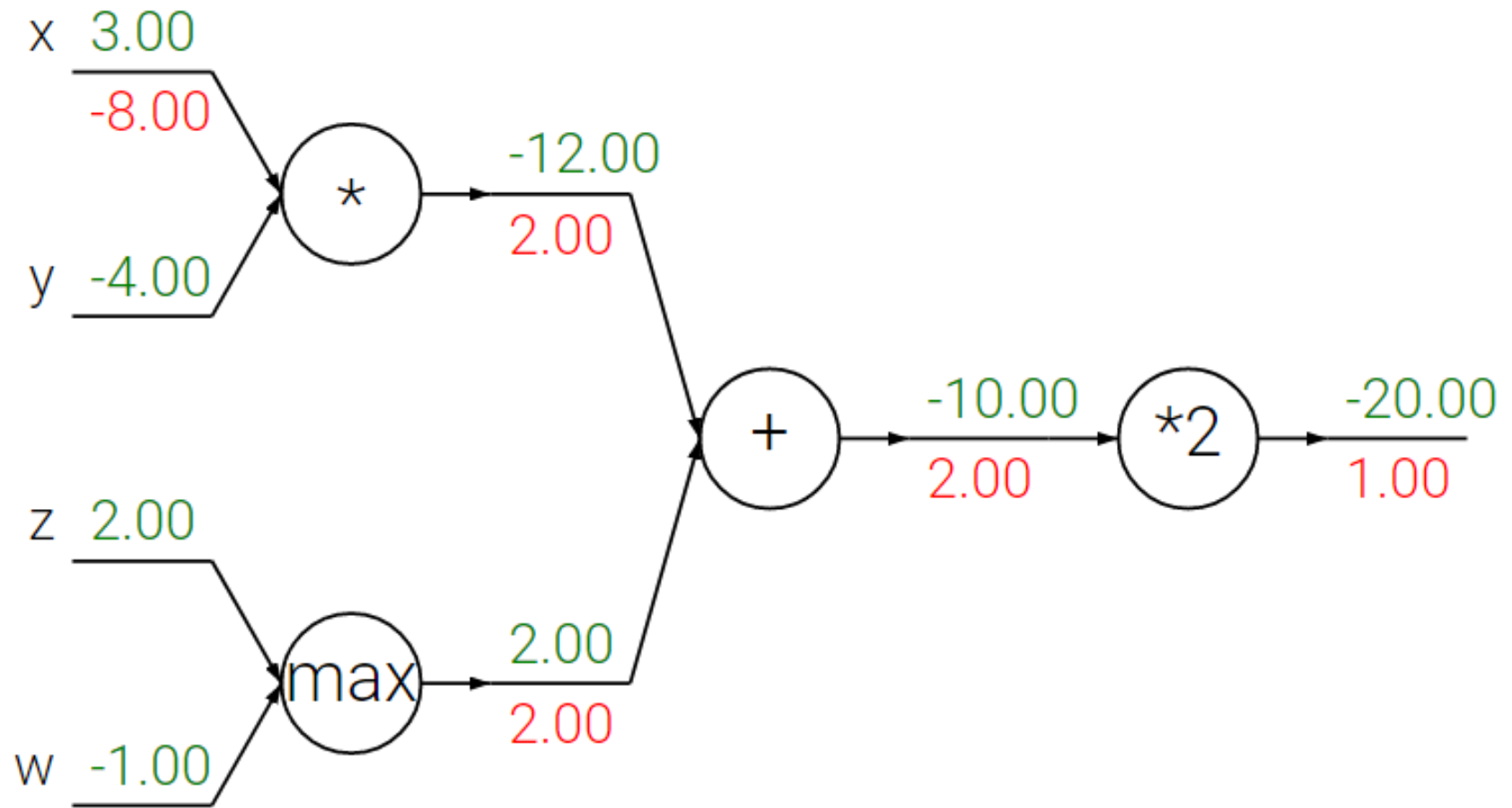
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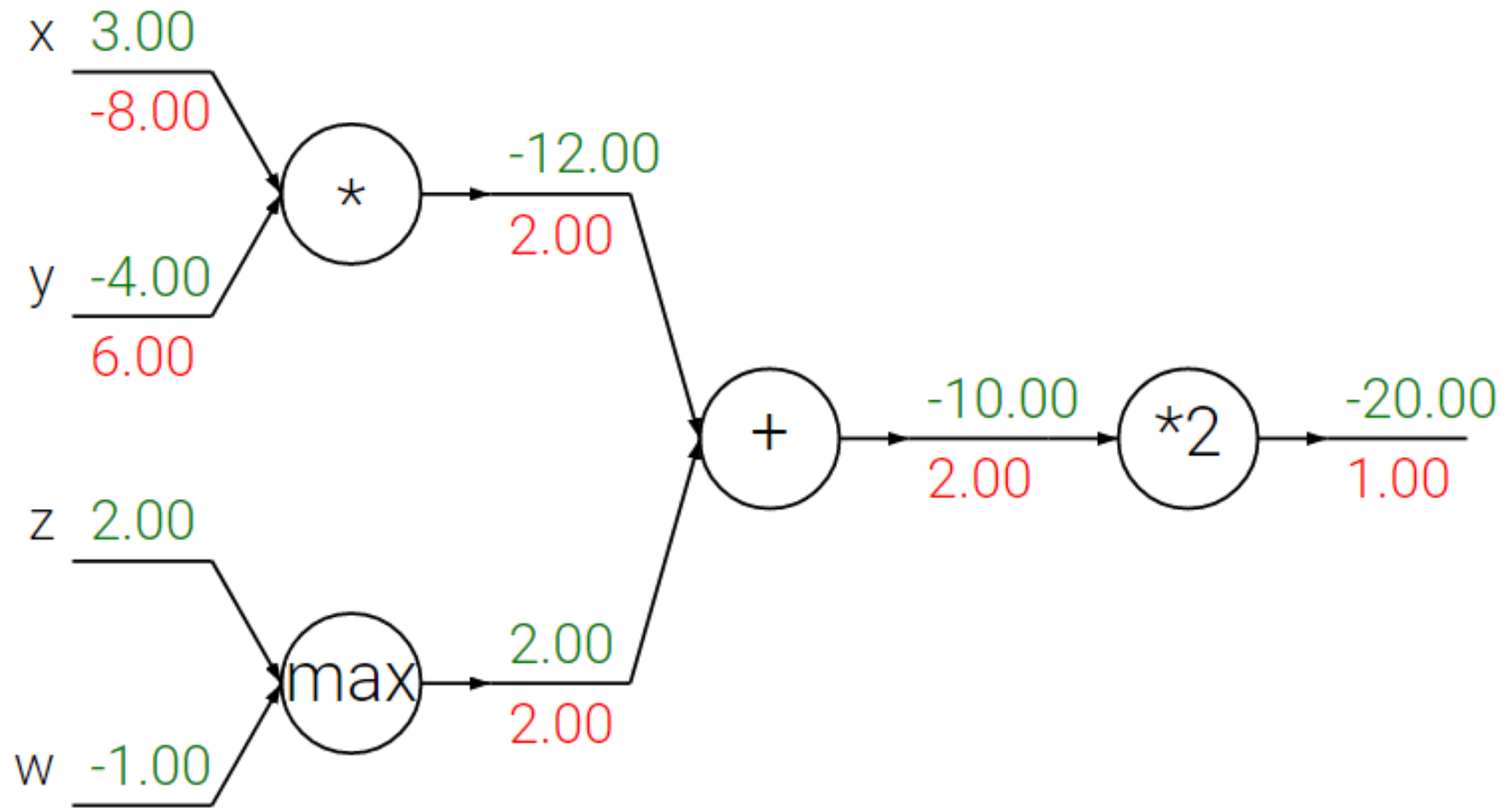
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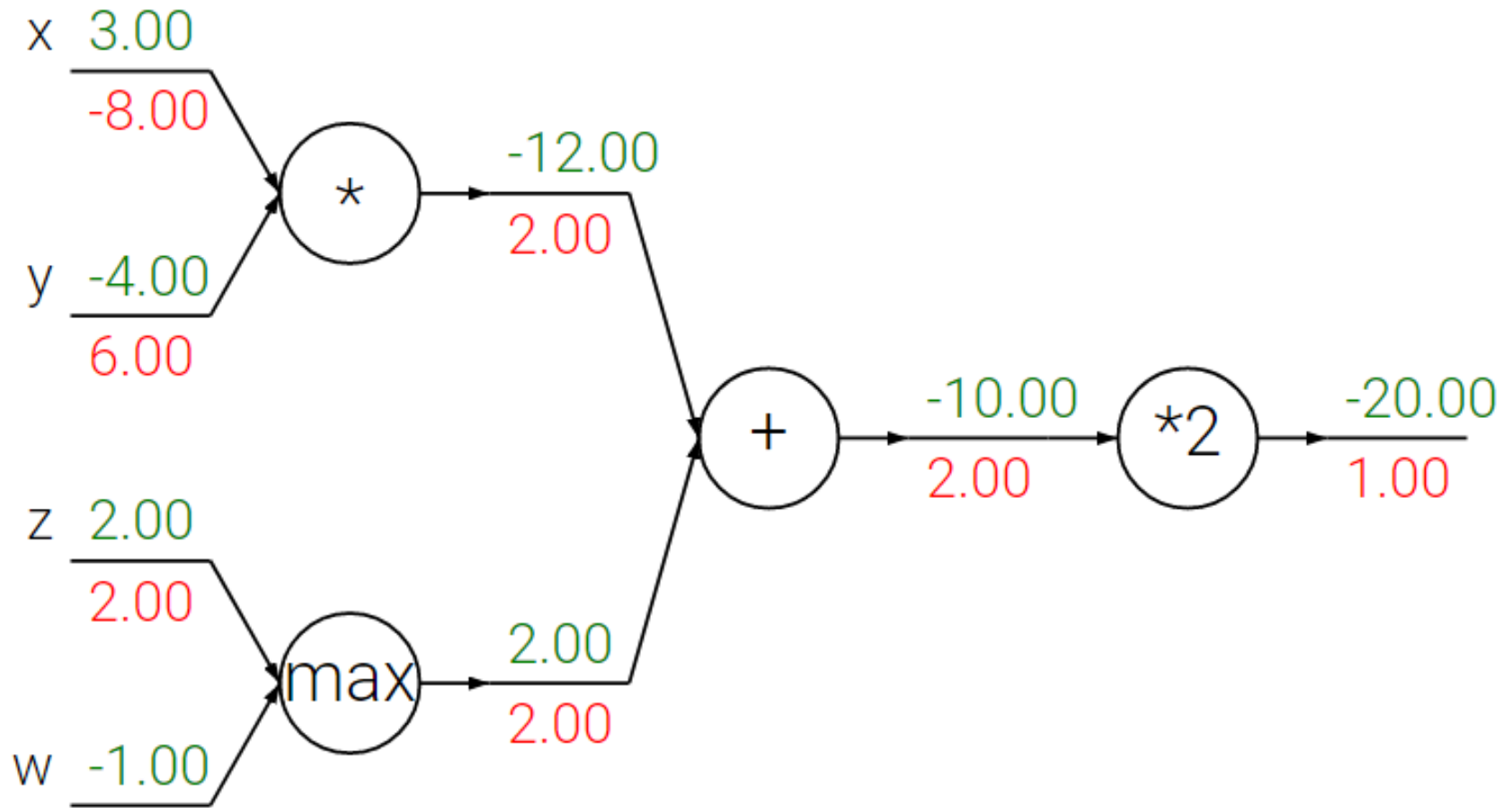
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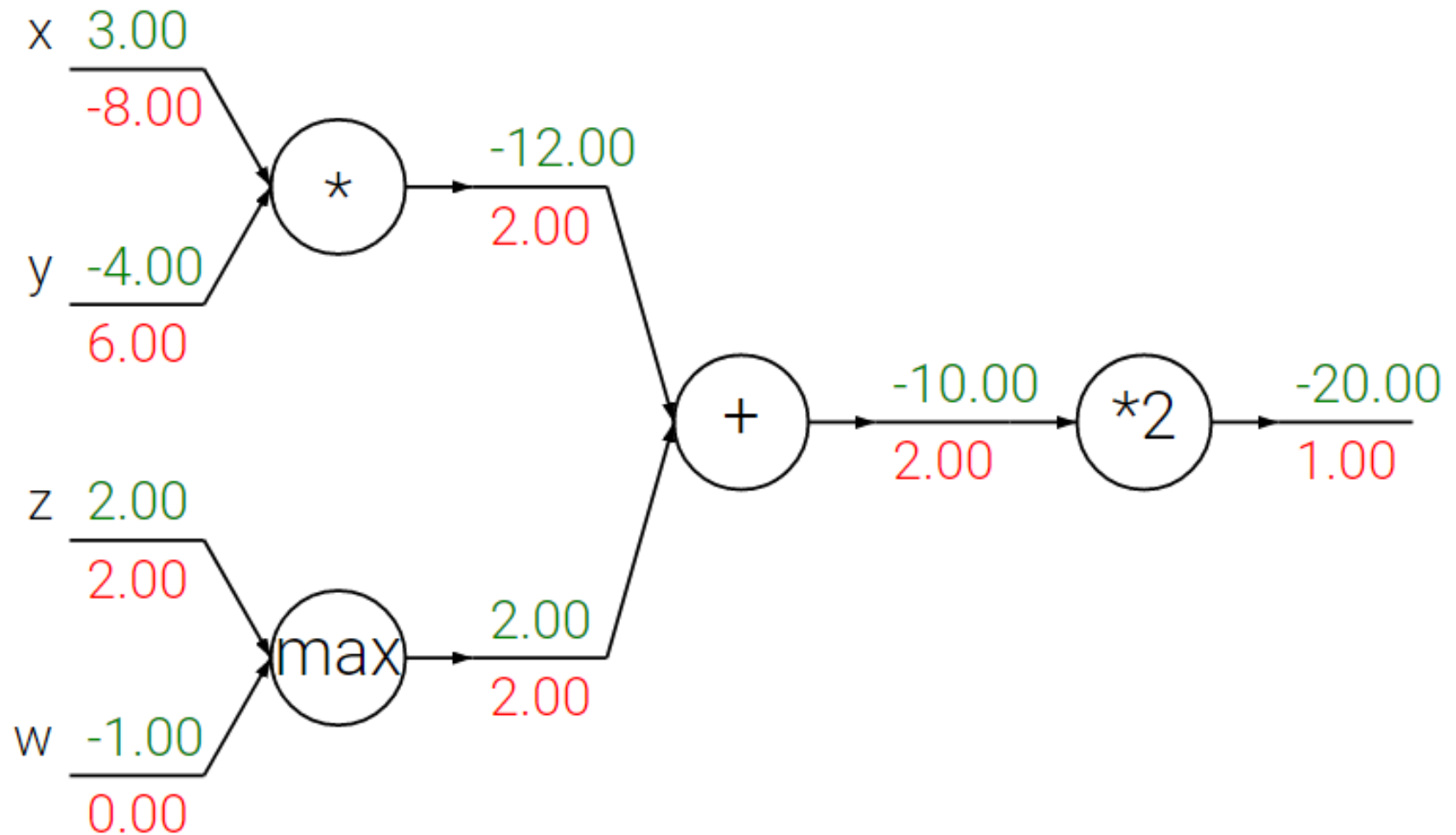
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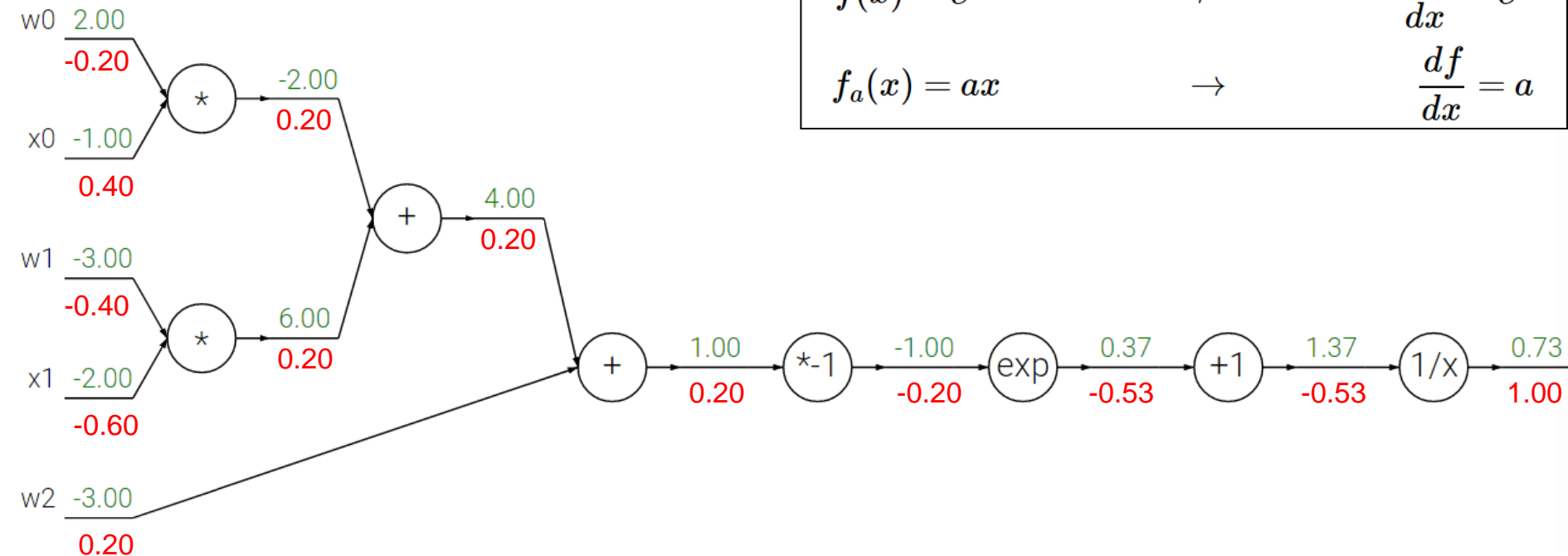
Forward and Backward Pass



Sigmoid example

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$f(x) = \frac{1}{x}$	\rightarrow	$\frac{df}{dx} = -1/x^2$
$f_c(x) = c + x$	\rightarrow	$\frac{df}{dx} = 1$
$f(x) = e^x$	\rightarrow	$\frac{df}{dx} = e^x$
$f_a(x) = ax$	\rightarrow	$\frac{df}{dx} = a$



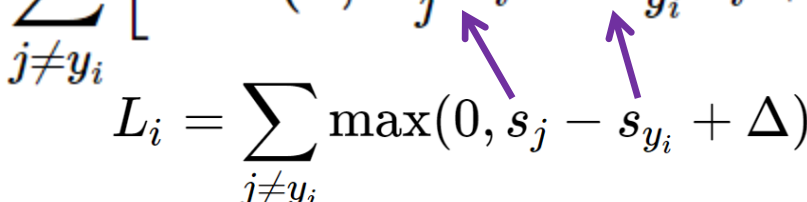
SVM Loss: Gradient

SVM loss function for a single datapoint (without regularization):

$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

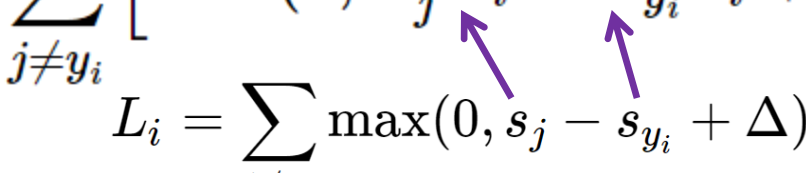
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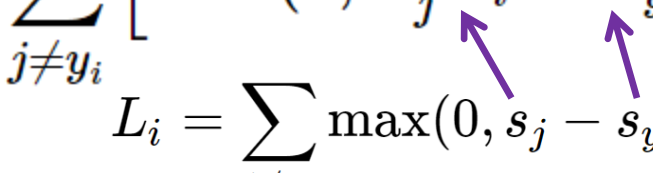
$$L_i = \sum_{j \neq y_i} \left[\max(0, w_j^T x_i - w_{y_i}^T x_i + \Delta) \right]$$

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Gradient w.r.t. w_{y_i} :

$$\nabla_{w_{y_i}} L_i = - \left(\sum_{j \neq y_i} 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) \right) x_i$$

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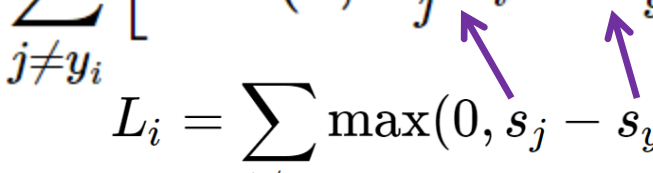
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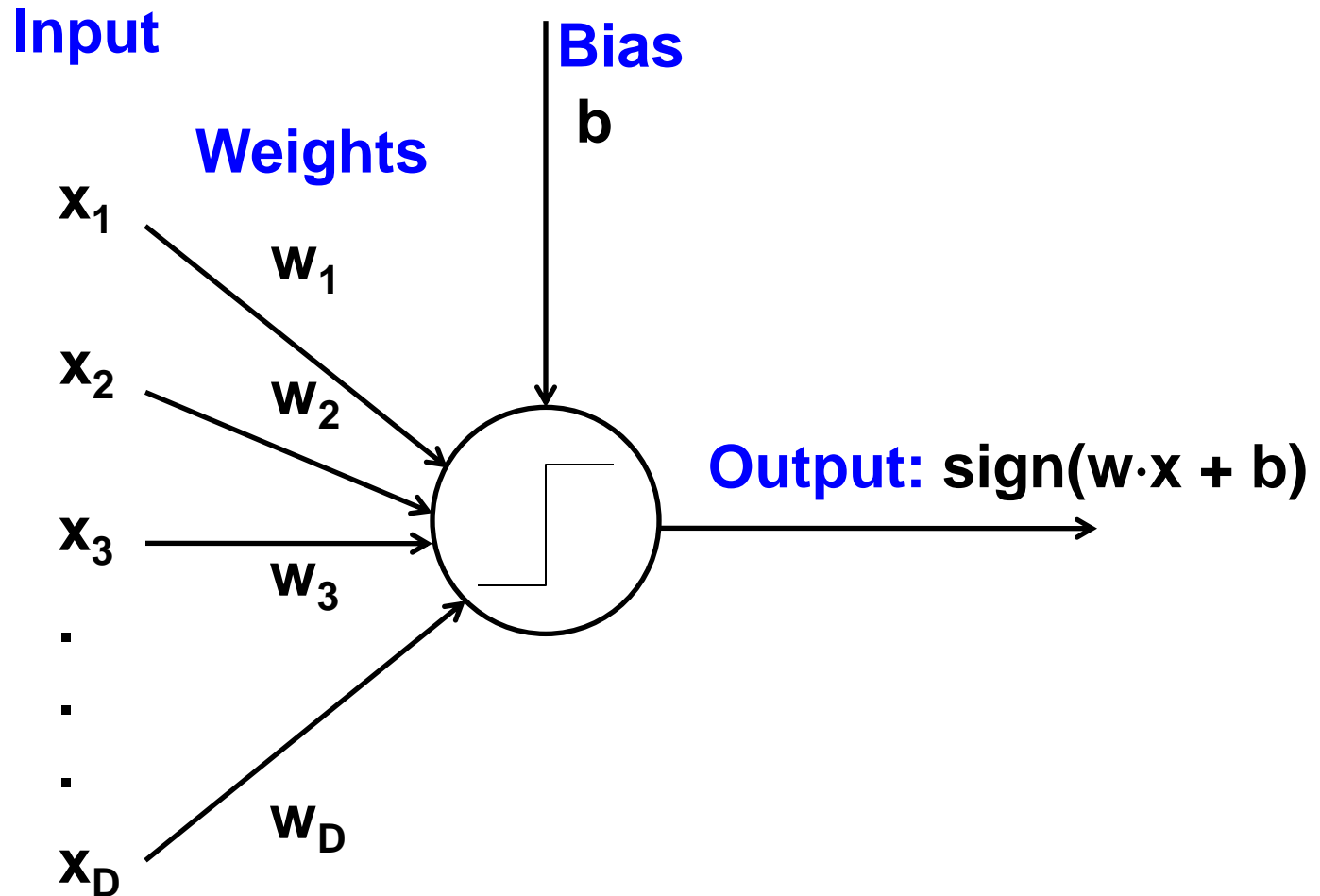
Count of the number of classes that didn't meet the desired margin

Gradient for the other rows where $j \neq y_i$:

$$\nabla_{w_j} L_i = 1(w_j^T x_i - w_{y_i}^T x_i + \Delta > 0) x_i$$

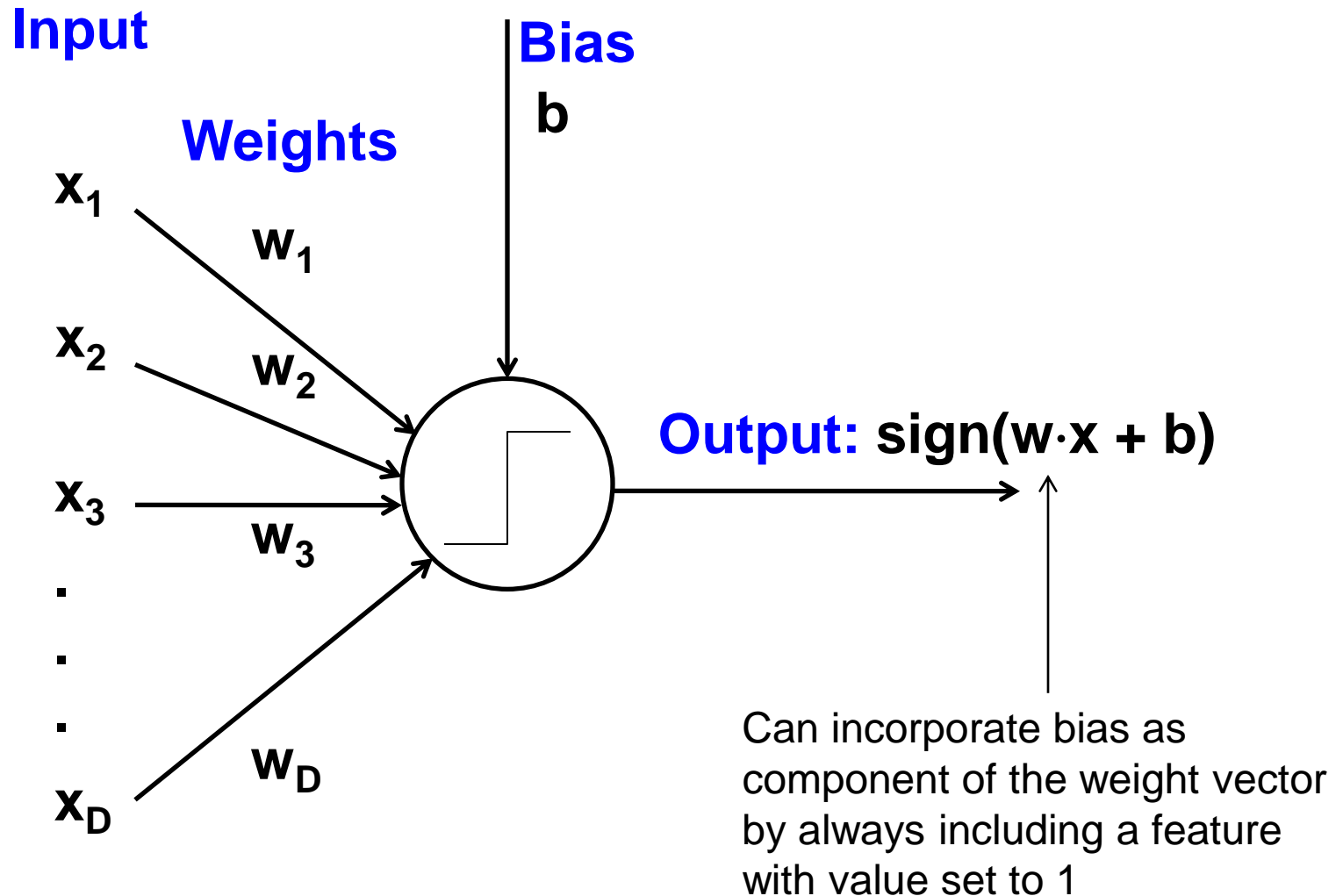
Perceptron

- Supervised learning of binary classifier



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Perceptron update rule

- Initialize weights randomly
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 $w_i \leftarrow w_i + \alpha(y - y')x_i$
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 - If $y = -1$ and $y' = 1$, w_i will be decreased if x_i is positive or increased if x_i is negative $\rightarrow \mathbf{w} \cdot \mathbf{x}$ will get smaller

Single neuron as a linear classifier

Binary Softmax classifier (*Logistic Regression*)

$$\sigma(\sum_i w_i x_i + b)$$

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Probability of the other class would be:

$$P(y_i = 0 \mid x_i; w) = 1 - P(y_i = 1 \mid x_i; w)$$

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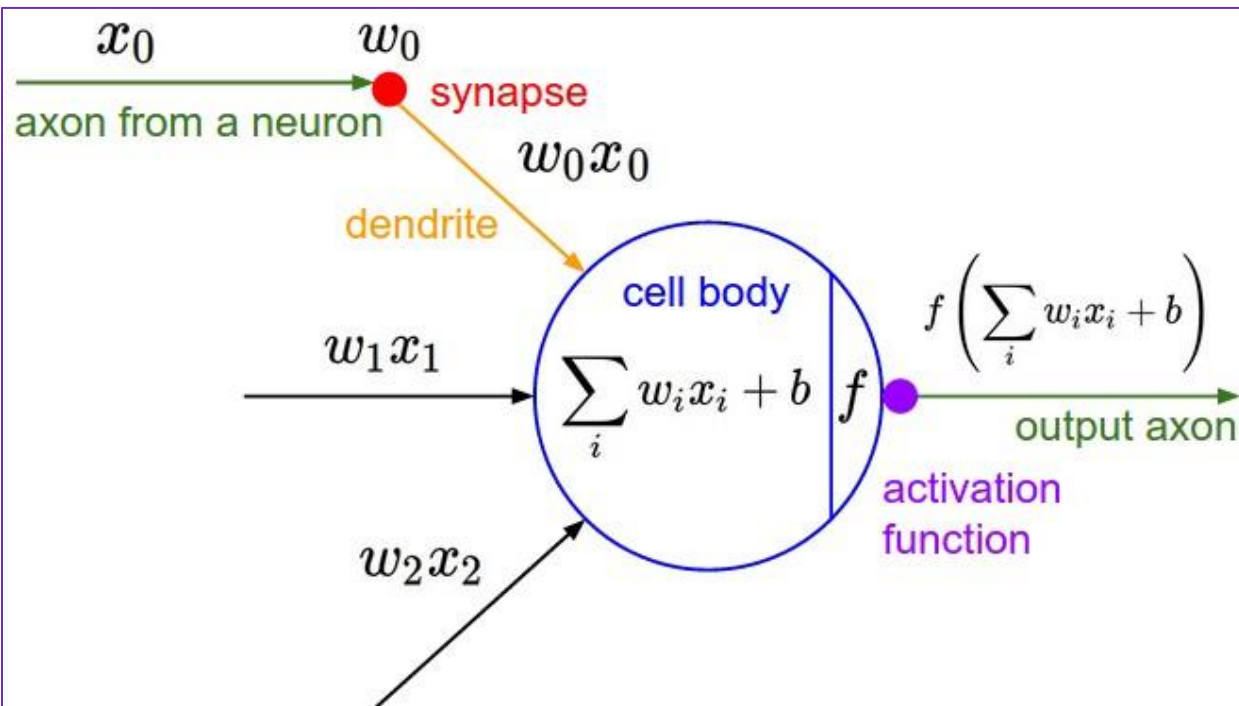
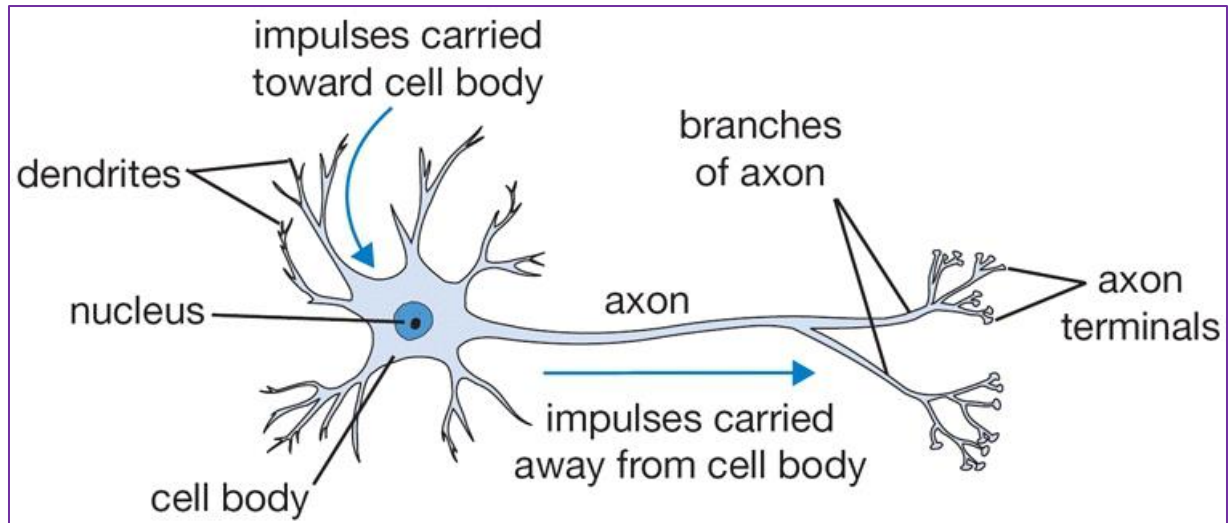
Probability of the other class would be:

$$P(y_i = 0 \mid x_i; w) = 1 - P(y_i = 1 \mid x_i; w)$$

Binary SVM classifier.

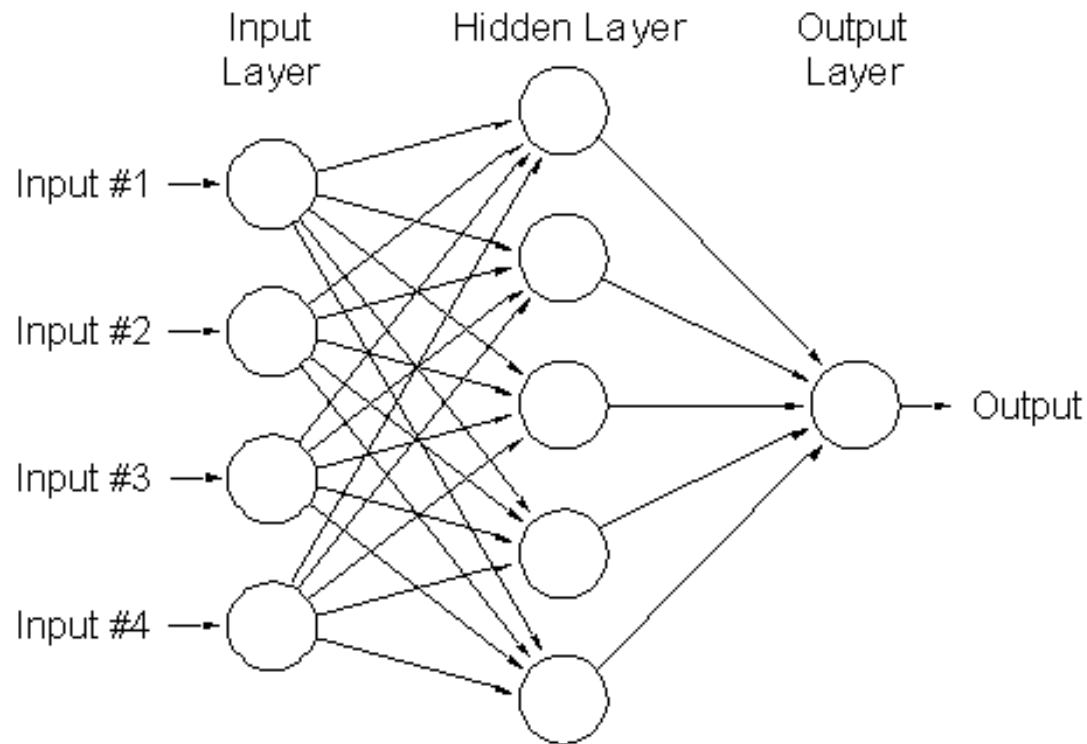
Alternatively, we could attach a max-margin hinge loss to the output of the neuron and train it to become a binary Support Vector Machine.

Loose inspiration: Human neurons



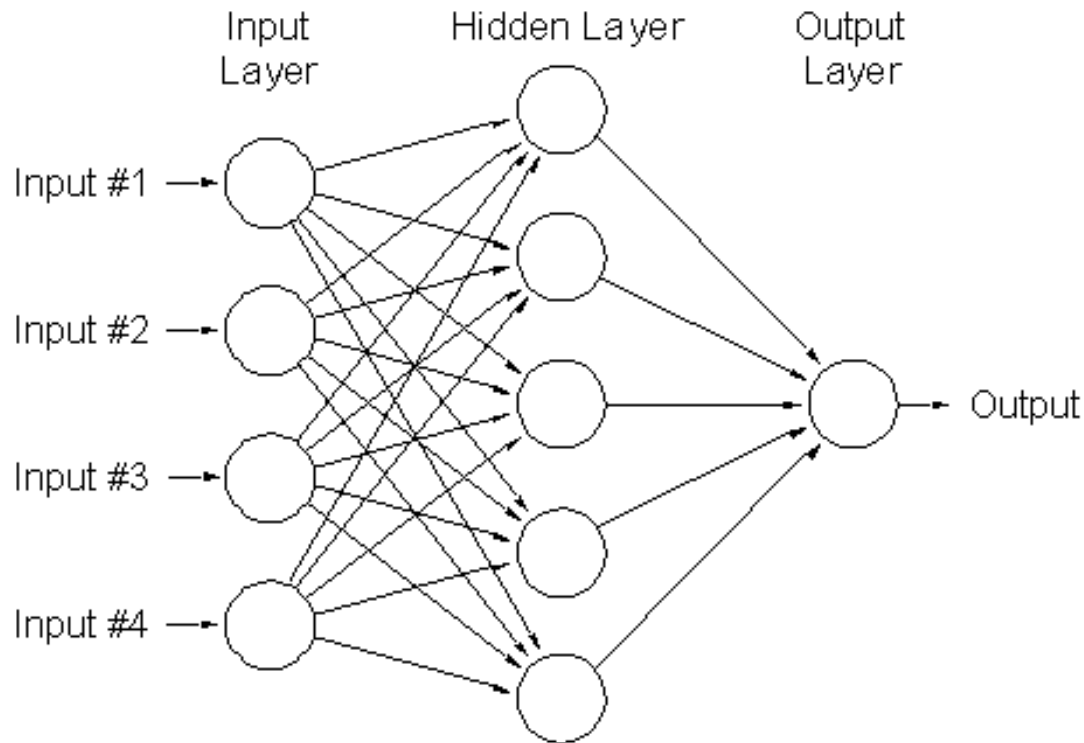
Multi-Layer Neural Networks

- Network with a hidden layer:



Multi-Layer Neural Networks

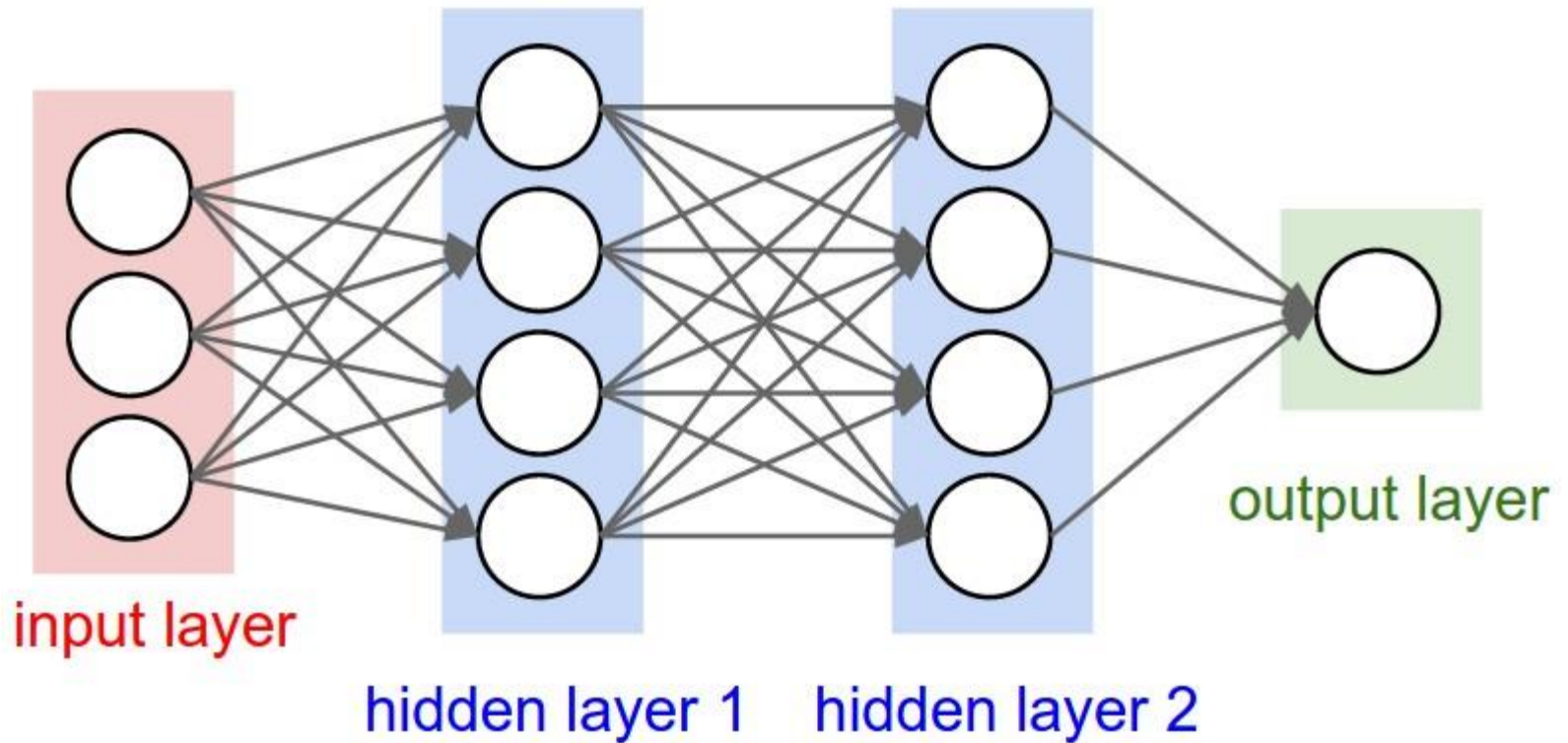
- Network with a hidden layer:



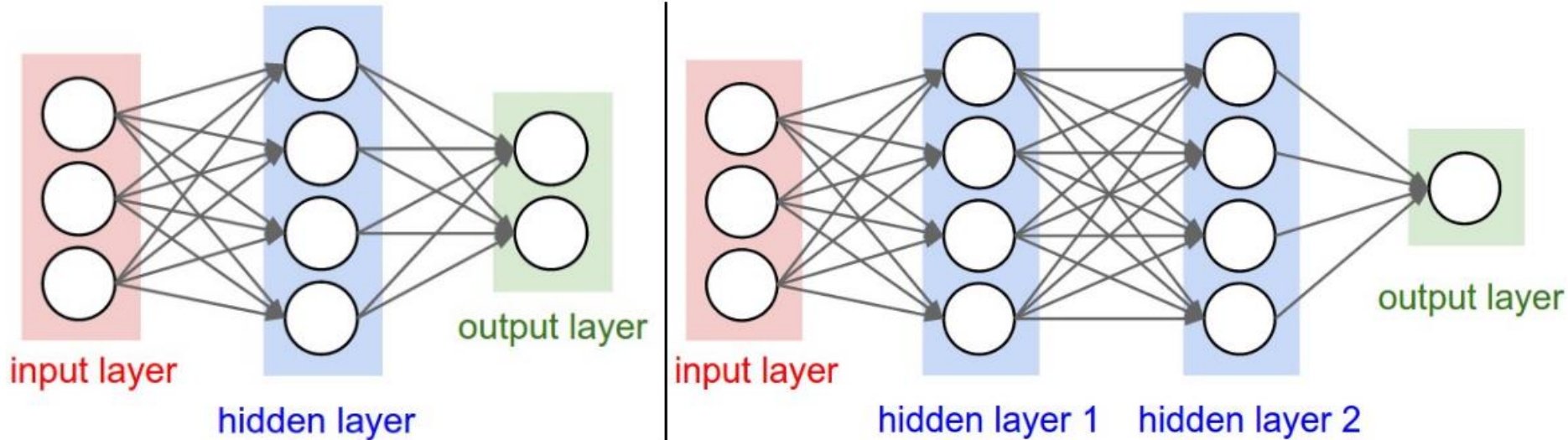
- Can represent nonlinear functions (provided each perceptron has a nonlinearity)

Multi-Layer Neural Networks

- Beyond a single hidden layer:



Sizing neural networks

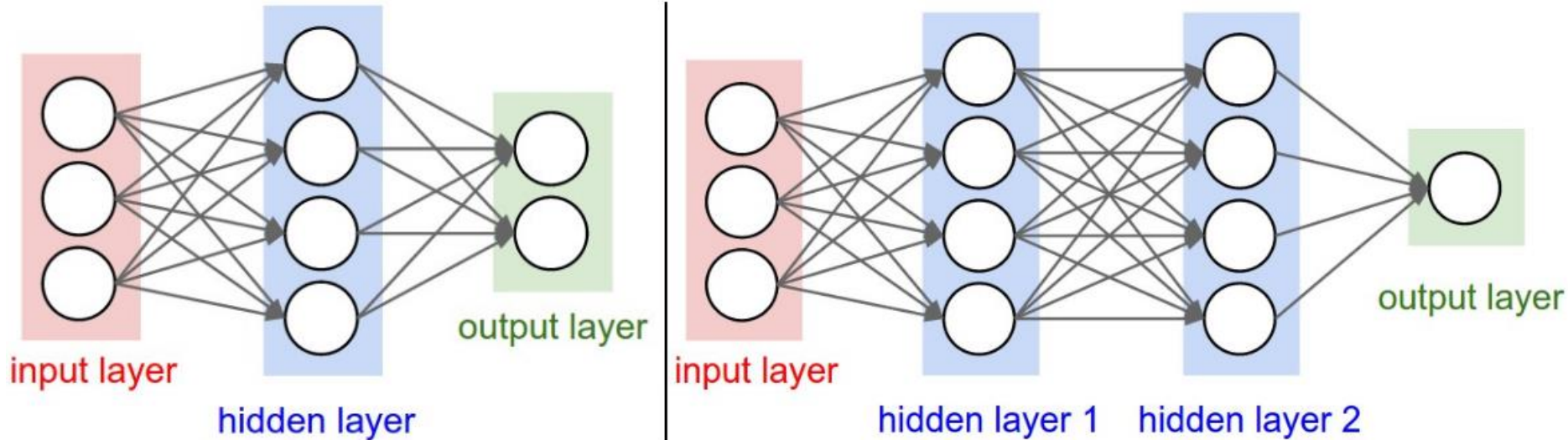


First network (left):

No. of neurons (not counting the inputs):

No. of learnable parameters:

Sizing neural networks

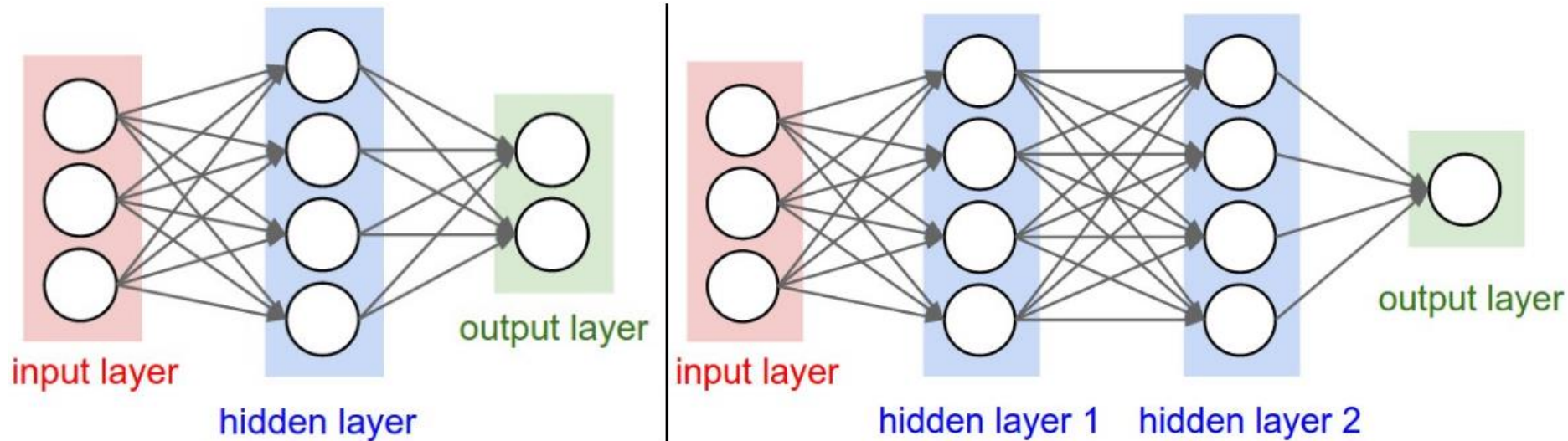


First network (left):

No. of neurons (not counting the inputs): $4 + 2 = 6$

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Sizing neural networks

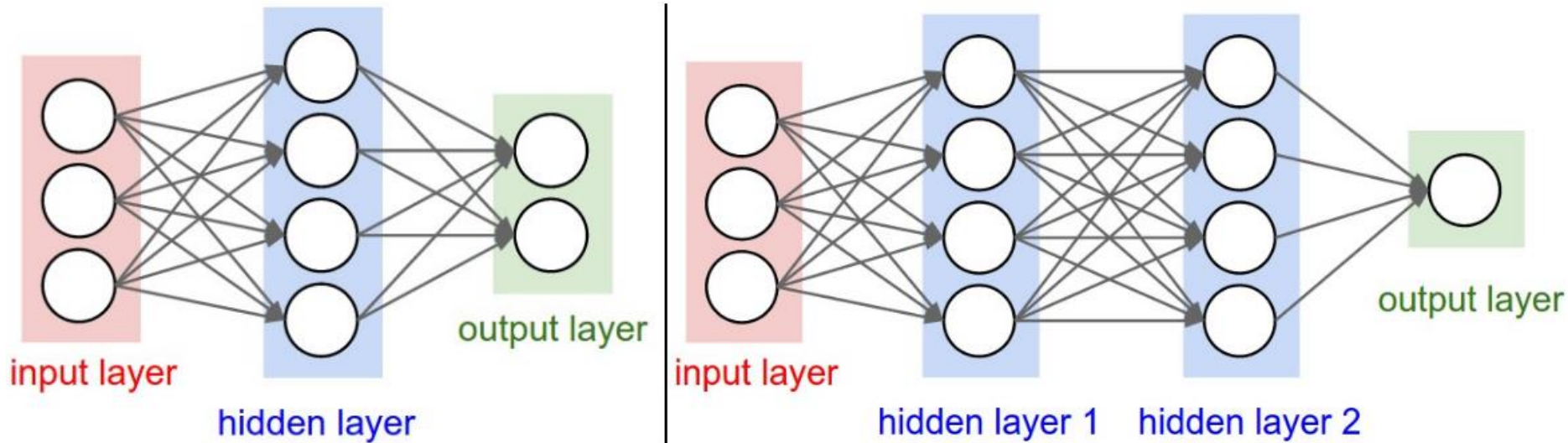


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Sizing neural networks



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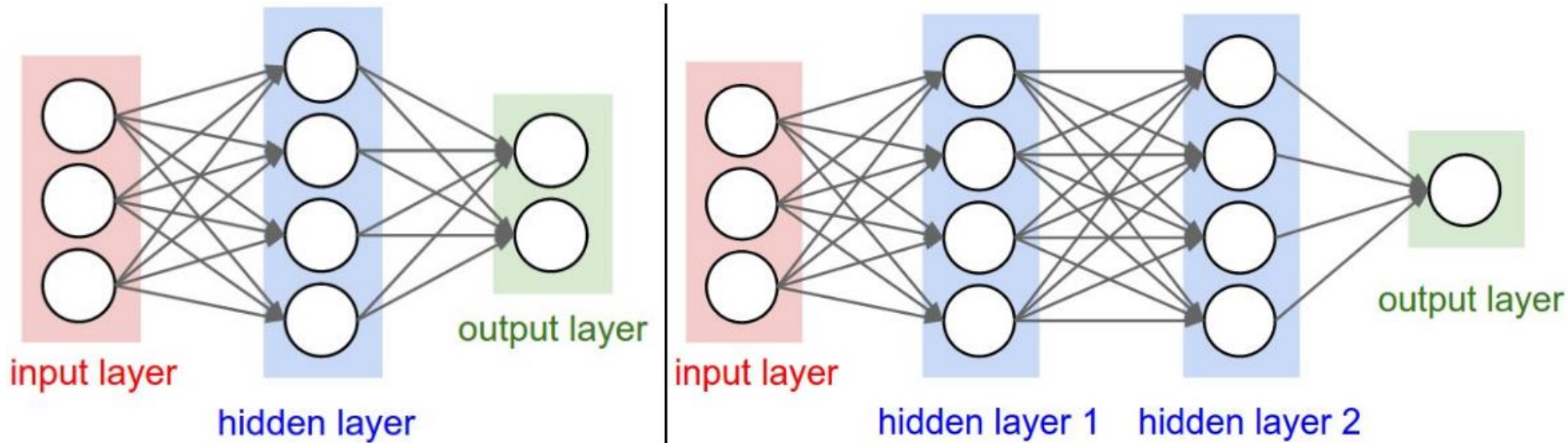
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Sizing neural networks



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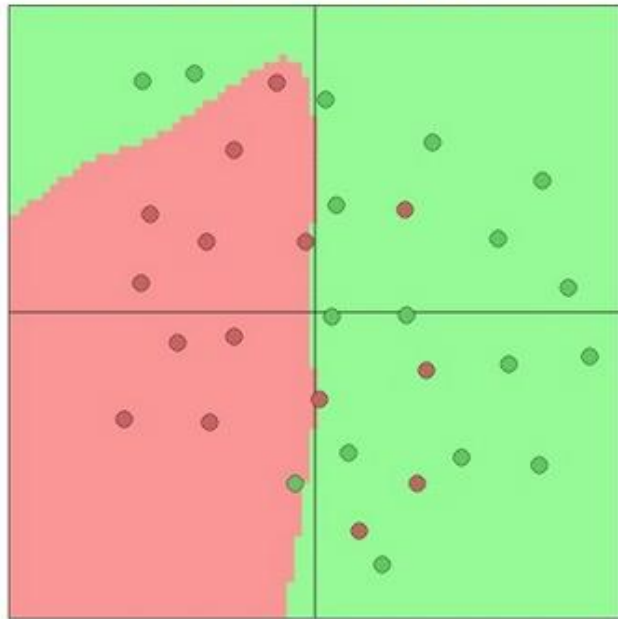
Second network (right):

No. of neurons (not counting the inputs): $4 + 4 + 1 = 9$

No. of learnable parameters: $[3 \times 4] + [4 \times 4] + [4 \times 1] = 32$ weights +
 $4 + 4 + 1 = 9$ biases = 41.

Multi-Layer Neural Networks

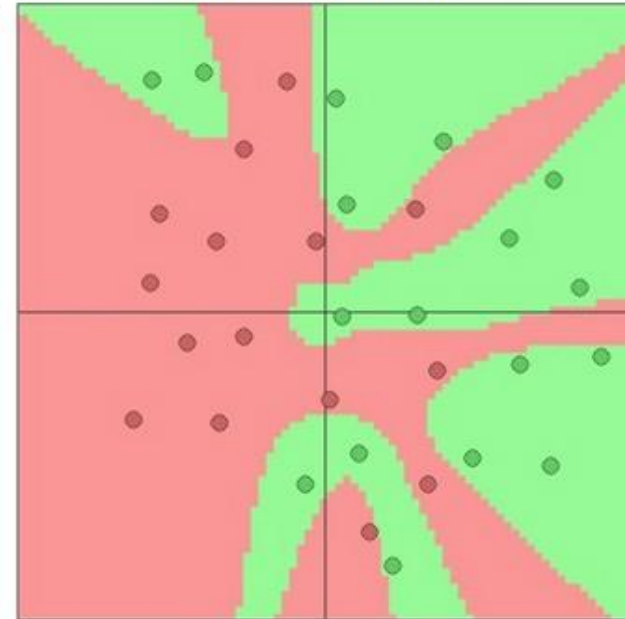
3 hidden neurons



6 hidden neurons



20 hidden neurons



Training of multi-layer networks

- Find network weights to minimize the error between true and estimated outputs of training examples:

$$E(\mathbf{w}) = \sum_{j=1}^N \left(y_j - f_{\mathbf{w}}(\mathbf{x}_j) \right)^2$$

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- Back-propagation**: gradients are computed in the direction from output to input layers and combined using chain rule

Neural networks: Pros and cons

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- Cons

- Hard to analyze theoretically (e.g., training is prone to local optima)
- Huge amount of training data, computing power may be required to get good performance
- The space of implementation choices are huge (network architectures, parameters)

Acknowledgements

Thanks to the following researchers for making their teaching/research material online

- Forsyth
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- K. Grauman
- R. Zaleski
- Antonio Torralba
- Rob Fergus
- Leibe
- And many more

Next Lecture

Convolutional Neural Networks

