Computer Vision

Fitting and Alignment

Dr. Mrinmoy Ghorai

Indian Institute of Information Technology
Sri City, Chittoor

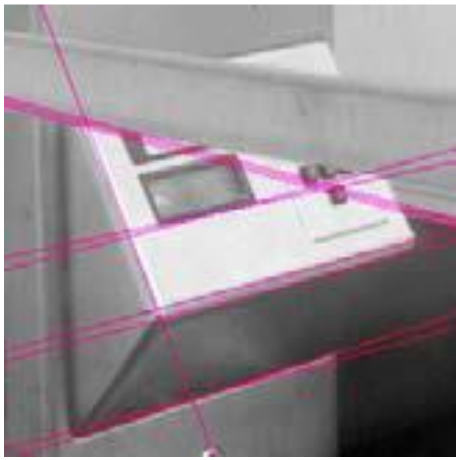
Fitting and Alignment

Fitting: find the parameters of a model that best fit the data

Alignment: find the parameters of the transformation that best align matched points

Fitting and alignment

- Choose a *parametric model* to represent a set of features



simple model: lines

simple model: circles

complicated model: face shape

complicated model: car

Source: K. Grauman

Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)

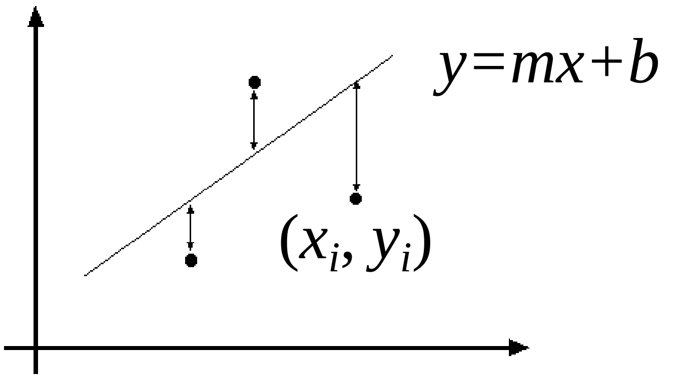
- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Simple example: Fitting a line

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize

$$\boxed{E = \sum_{i=1}^{n} (y_i - m x_i - b)^2}$$



Modified from S. Lazebnik

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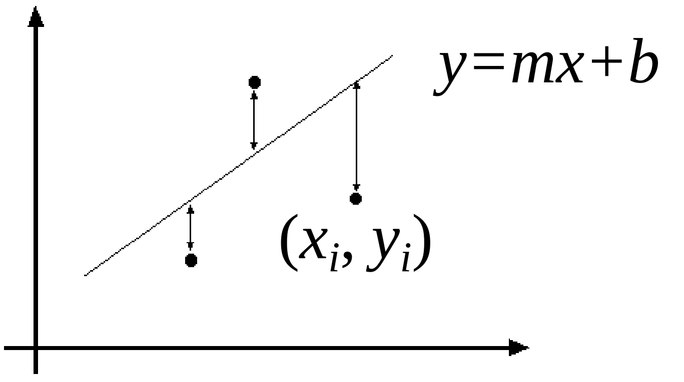

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2$$

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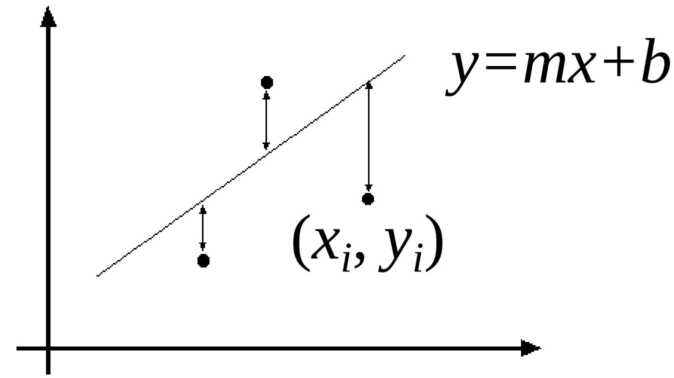

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2$$

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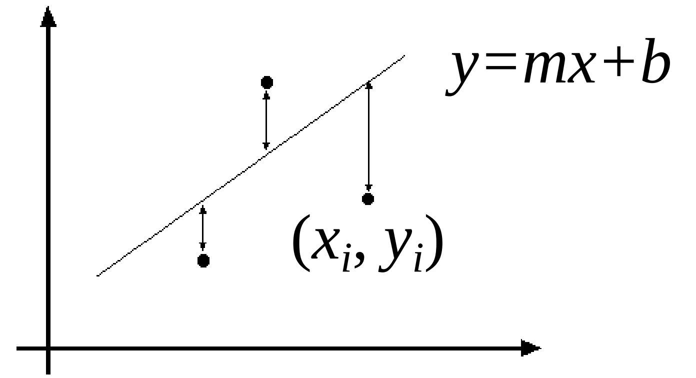

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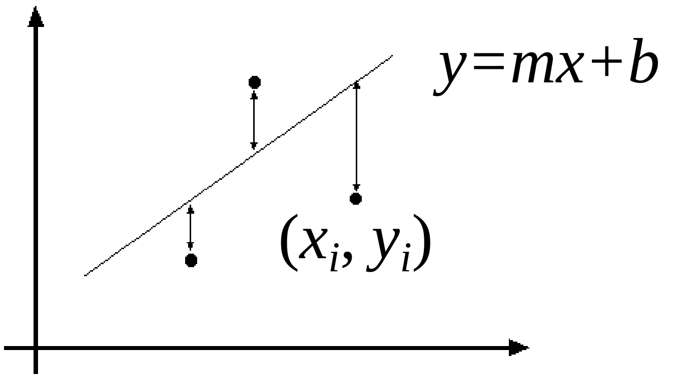

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$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

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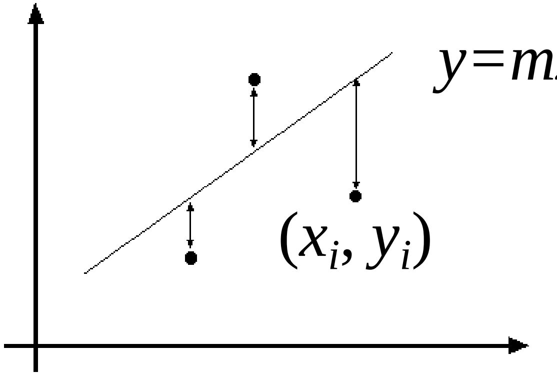

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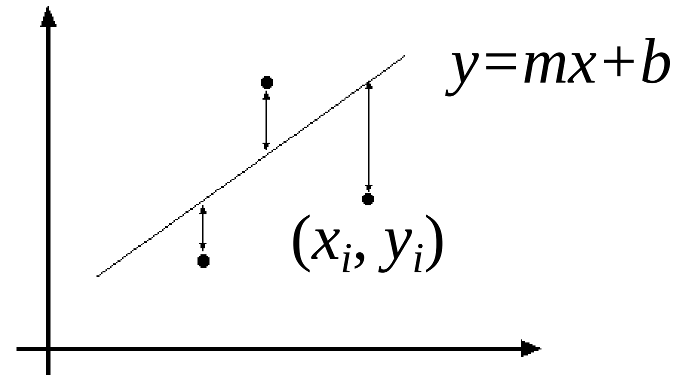

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$$\mathbf{A}^T\mathbf{Ap} = \mathbf{A}^T\mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{y}$$

Modified from S. Lazebnik

Hypothesize and test

- Hough Transform
- RANSAC Algorithm

1. Propose parameters
2. Score the given parameters
3. Choose from among the set of parameters
4. Possibly refine parameters using inliers

Hough Transform: Outline

1. Create a grid of parameter values

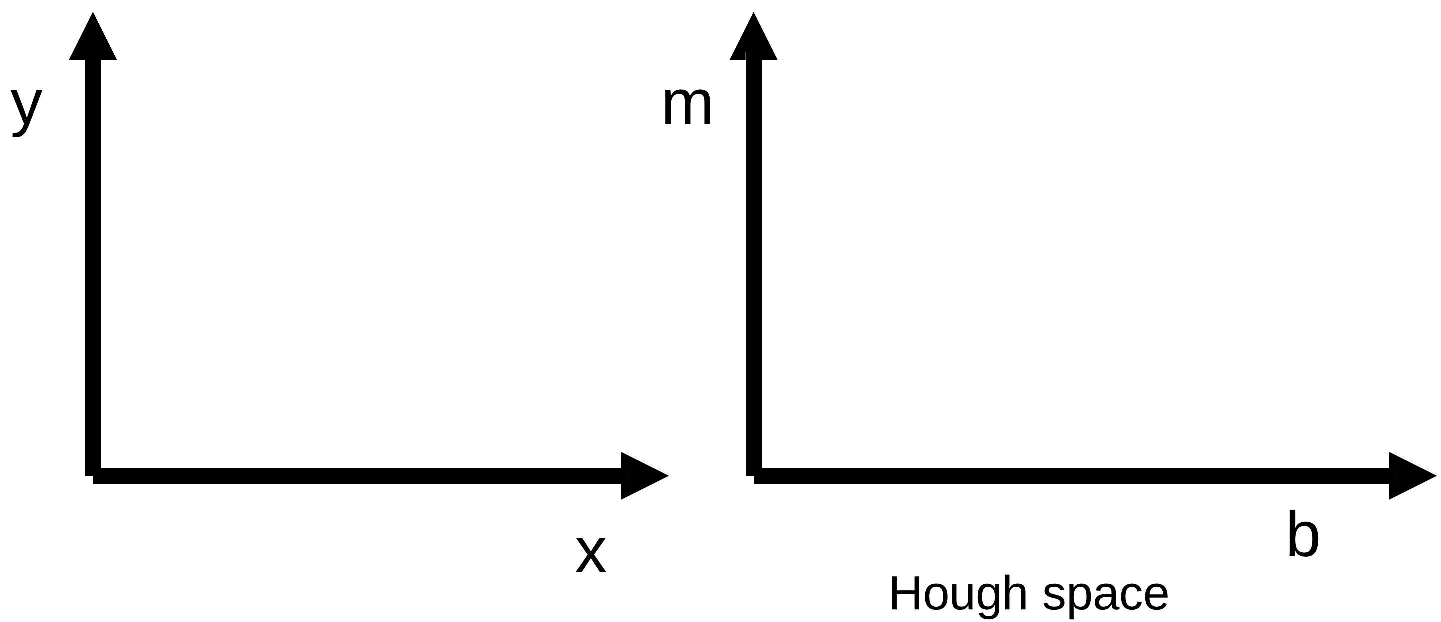
2. Each point votes for a set of parameters, incrementing those values in grid

3. Find maximum or local maxima in grid

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Given a set of points, find the curve or line that explains the data points best



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Slide from S. Savarese

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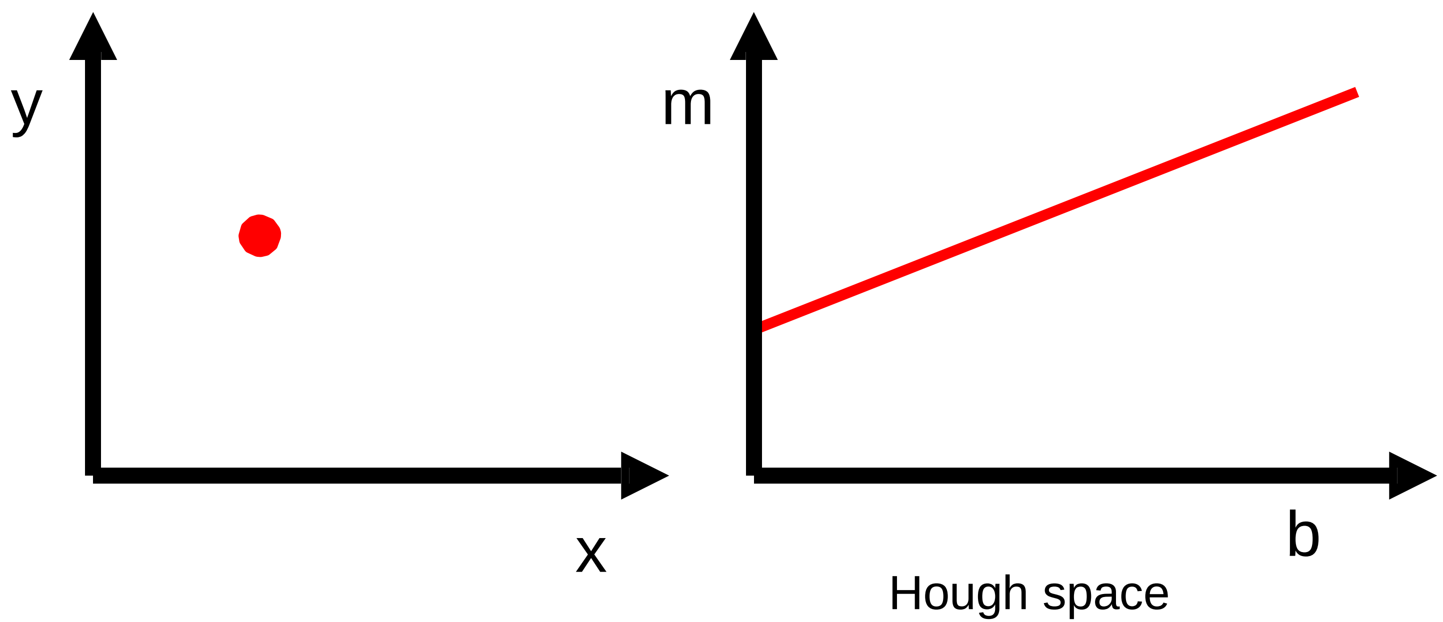

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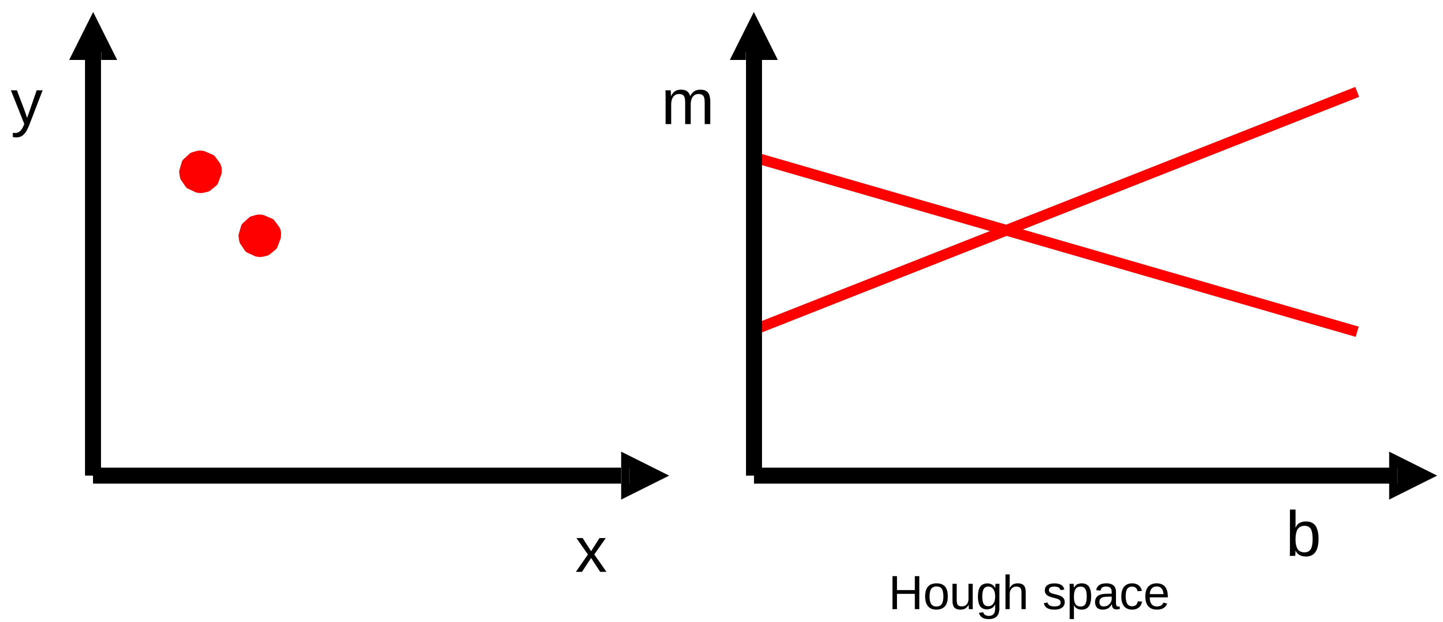

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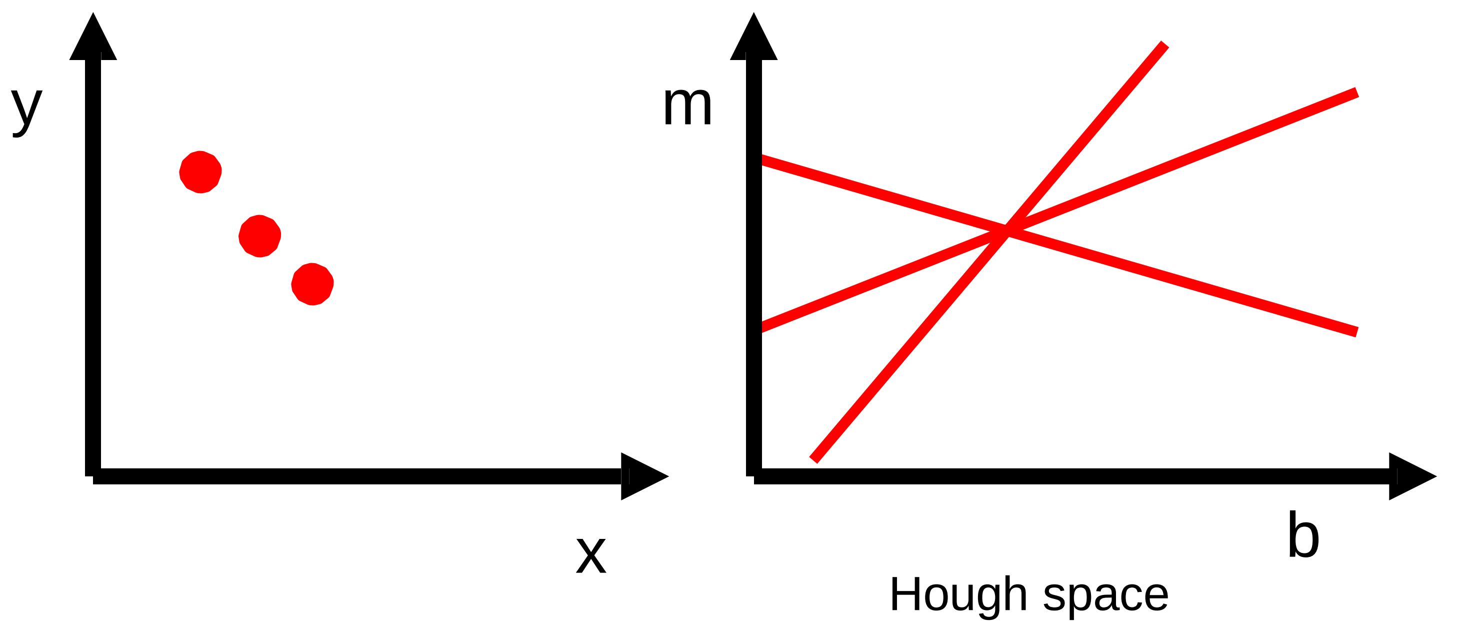

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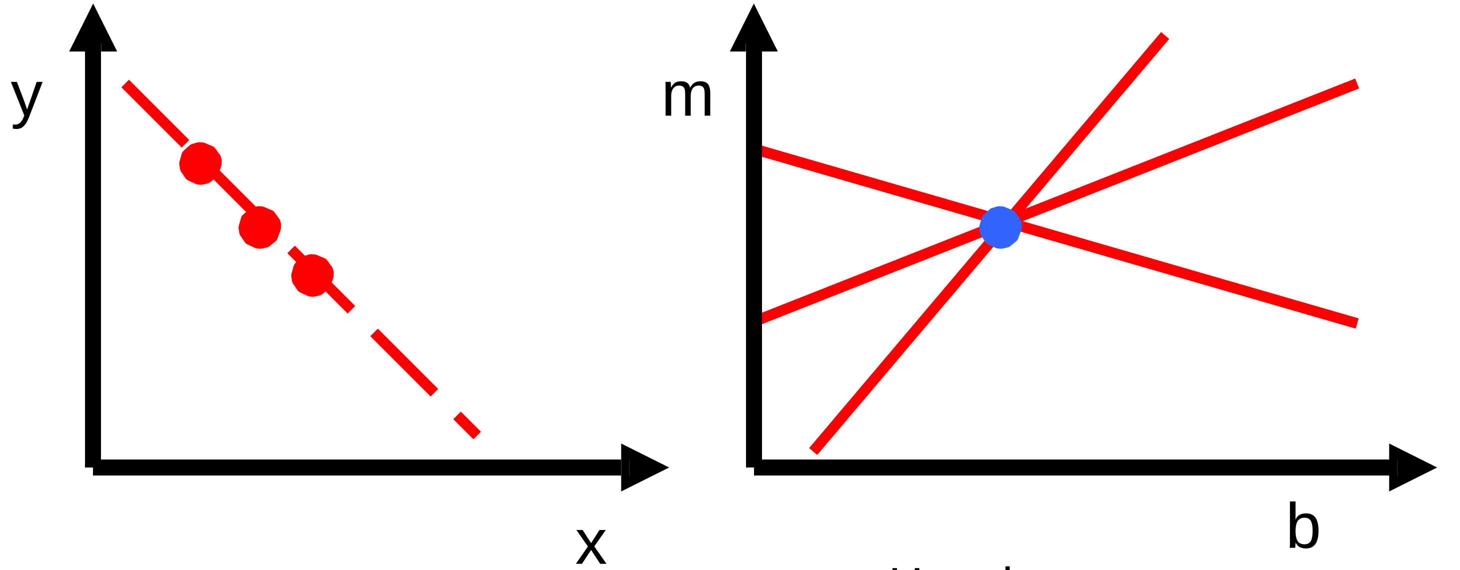

Hough space

$$y = m\,x + b$$

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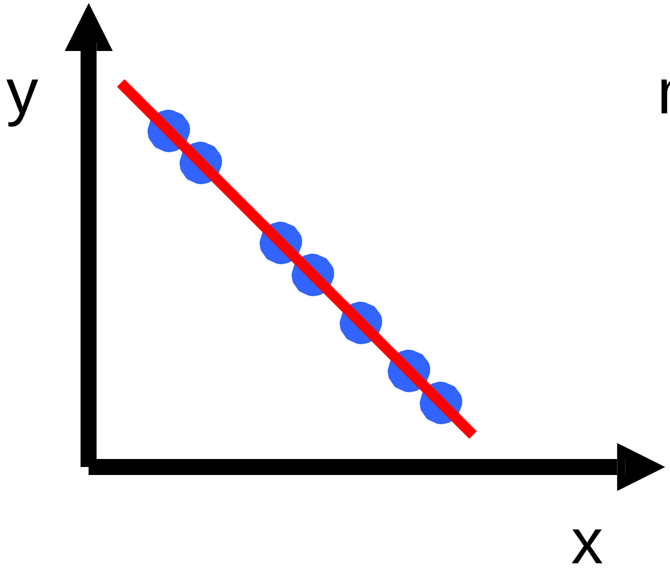
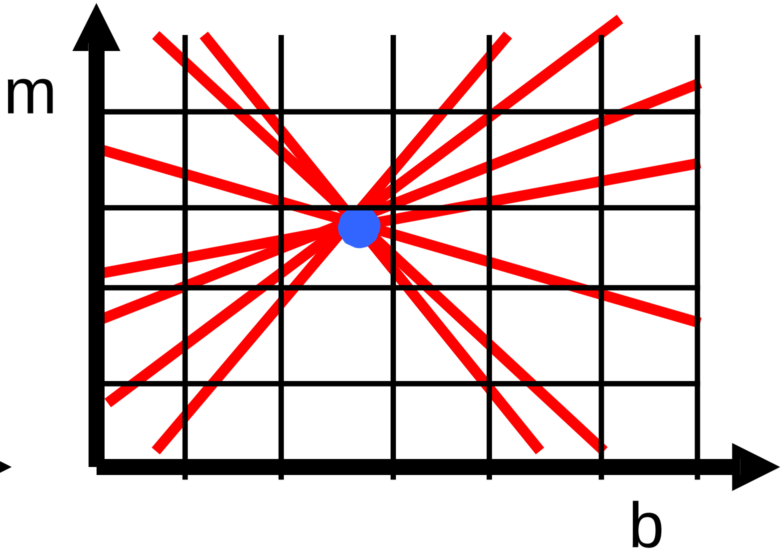
Hough transform





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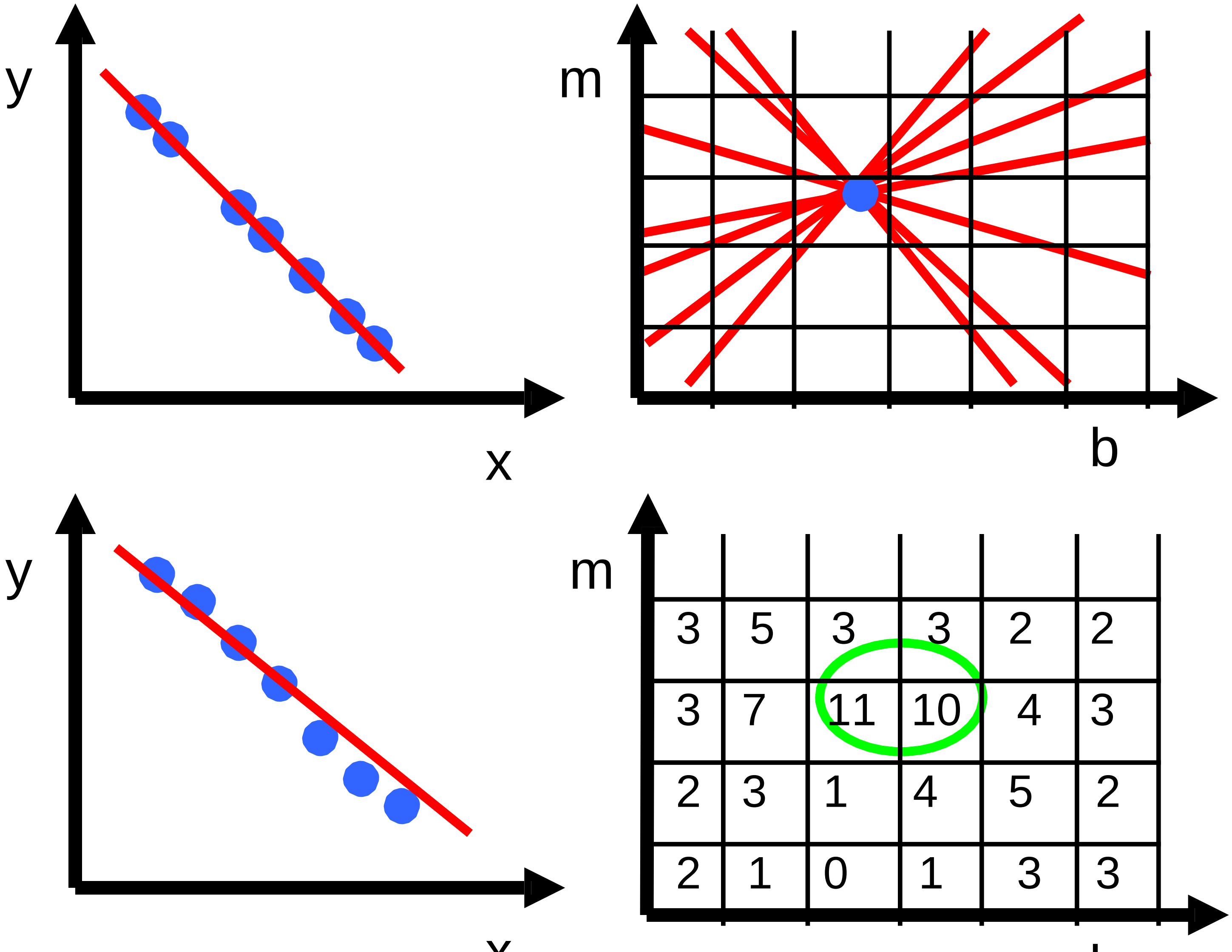

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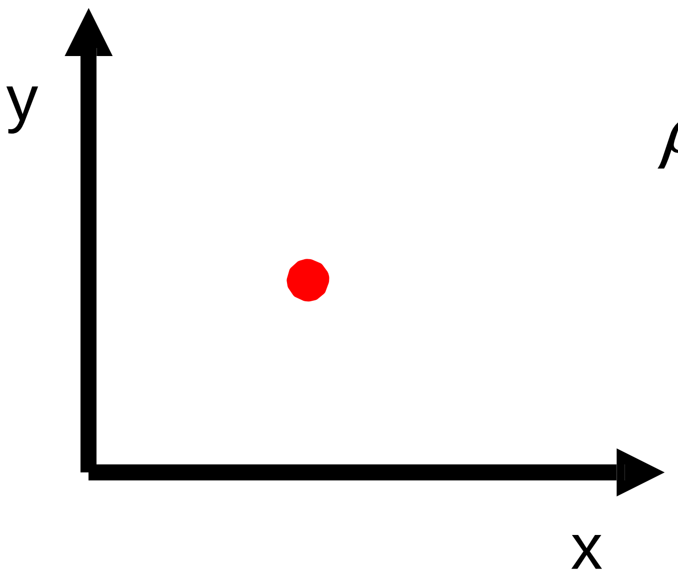
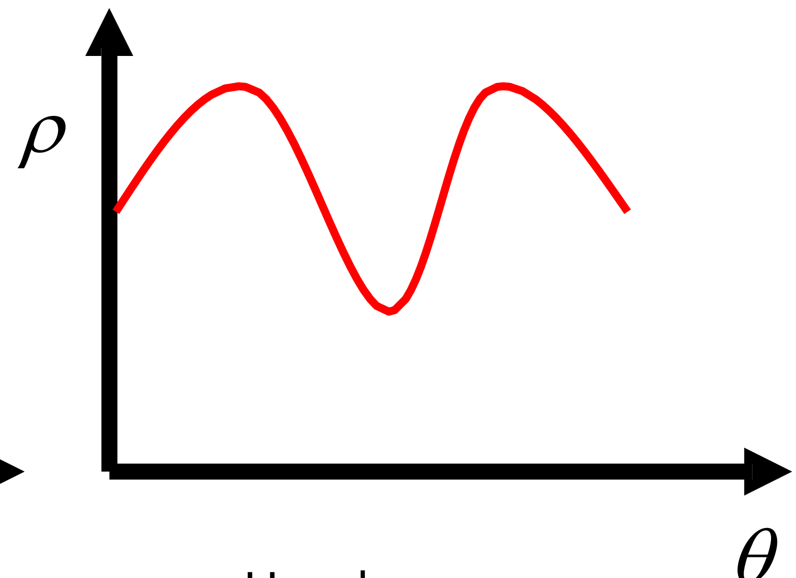
Use a polar representation for the parameter space





Hough space

$$x\cos\theta + y\sin\theta = \rho$$

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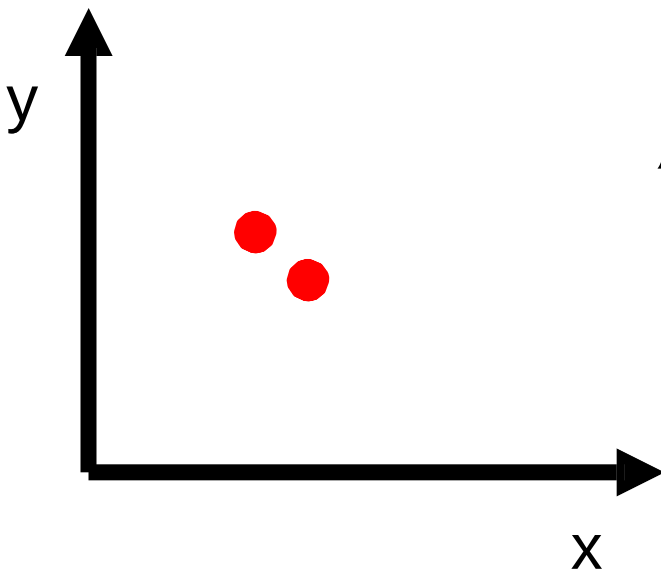

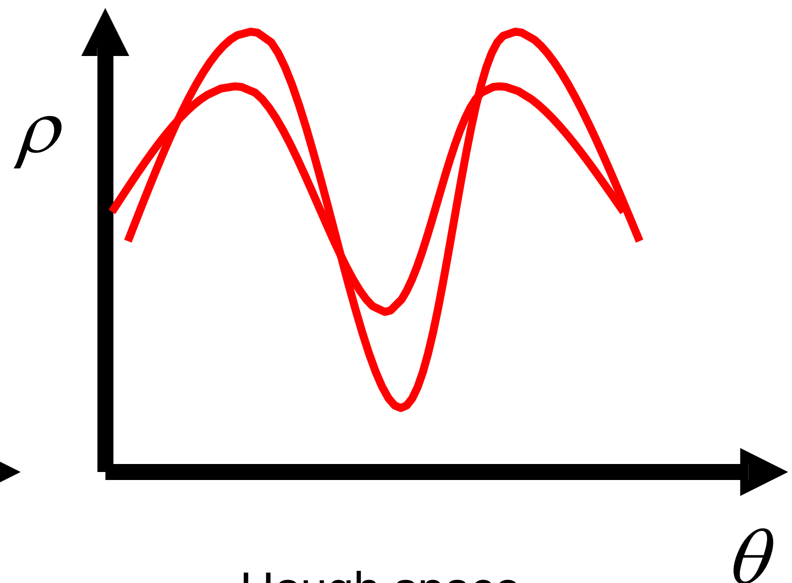

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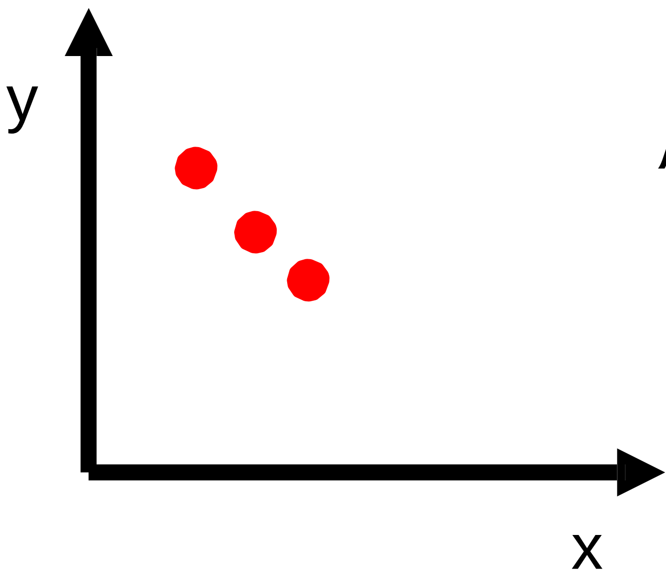

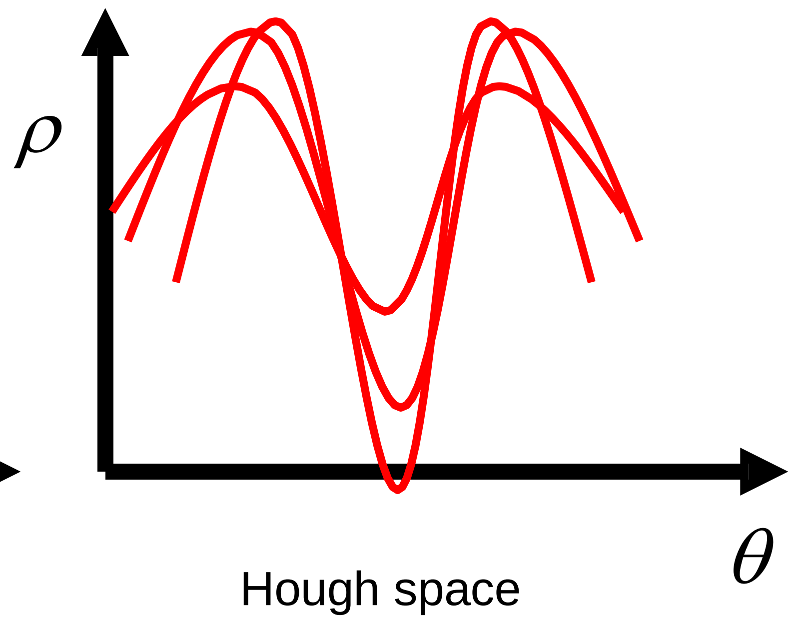

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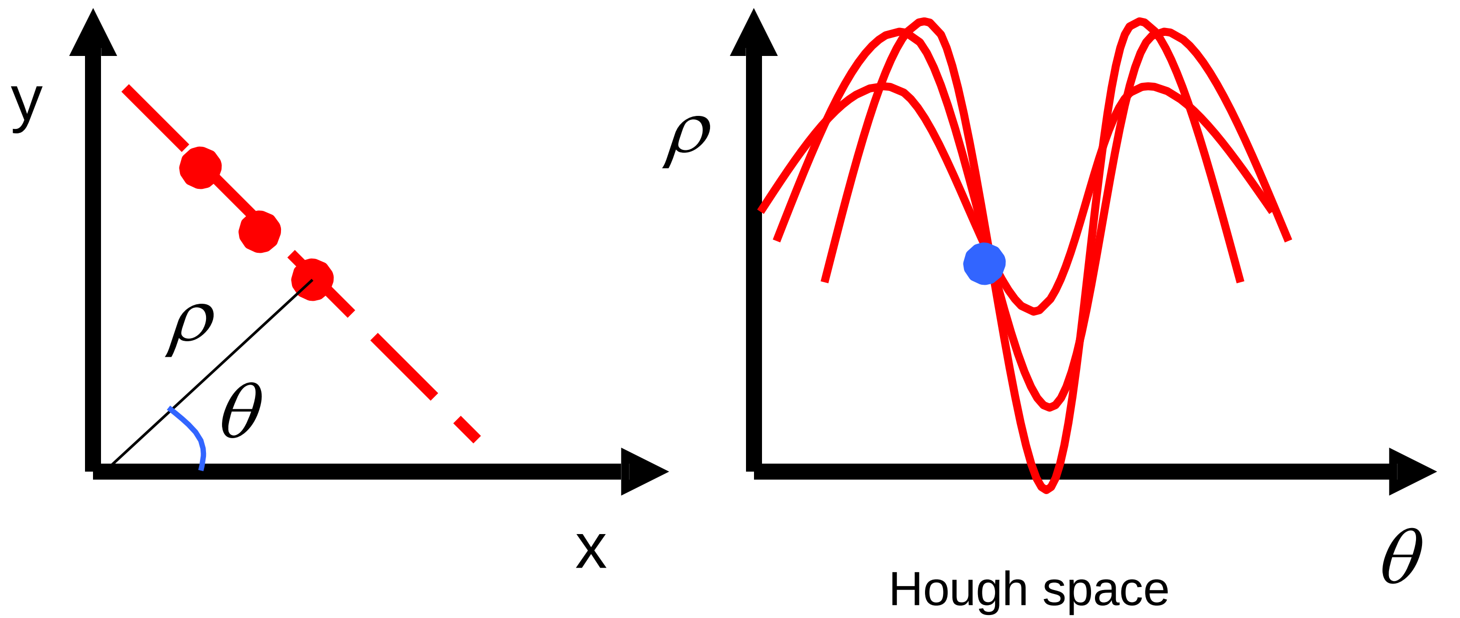

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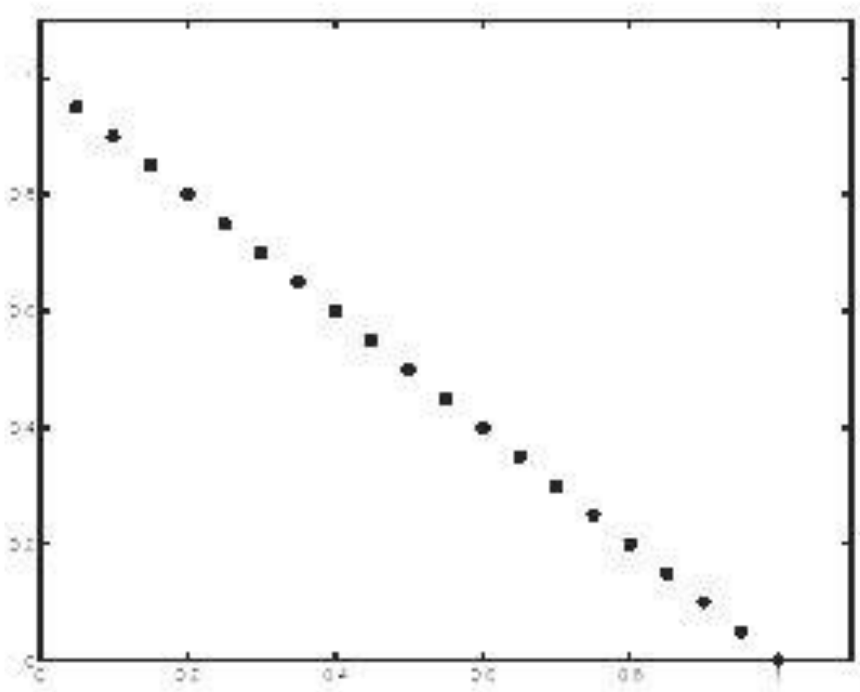
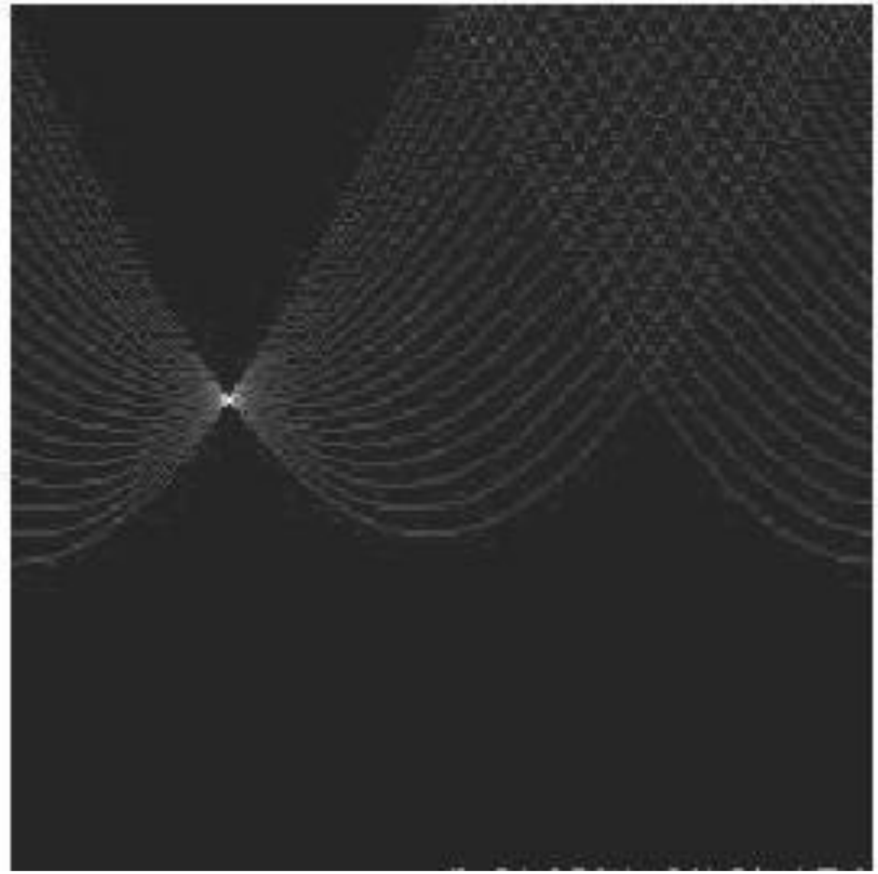
Hough transform - experiments



features

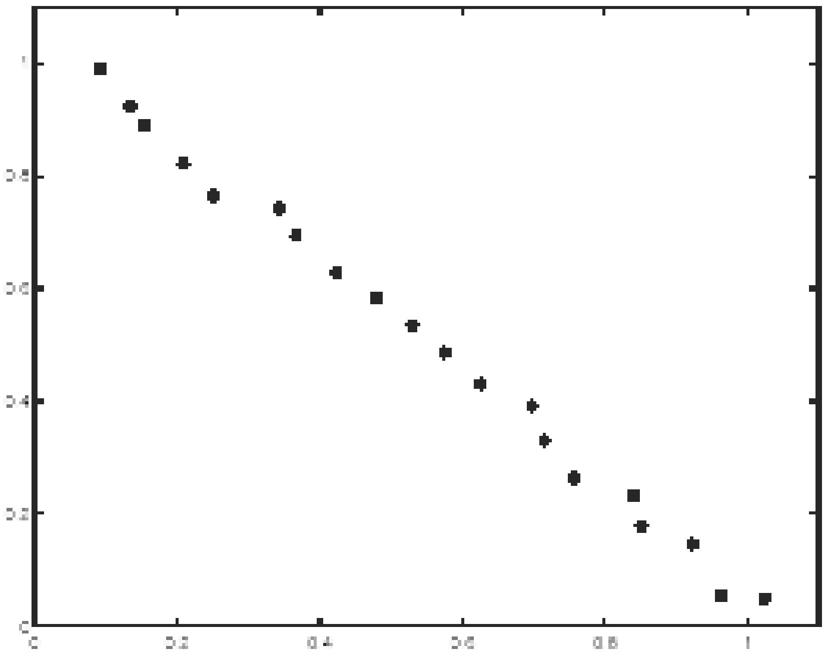


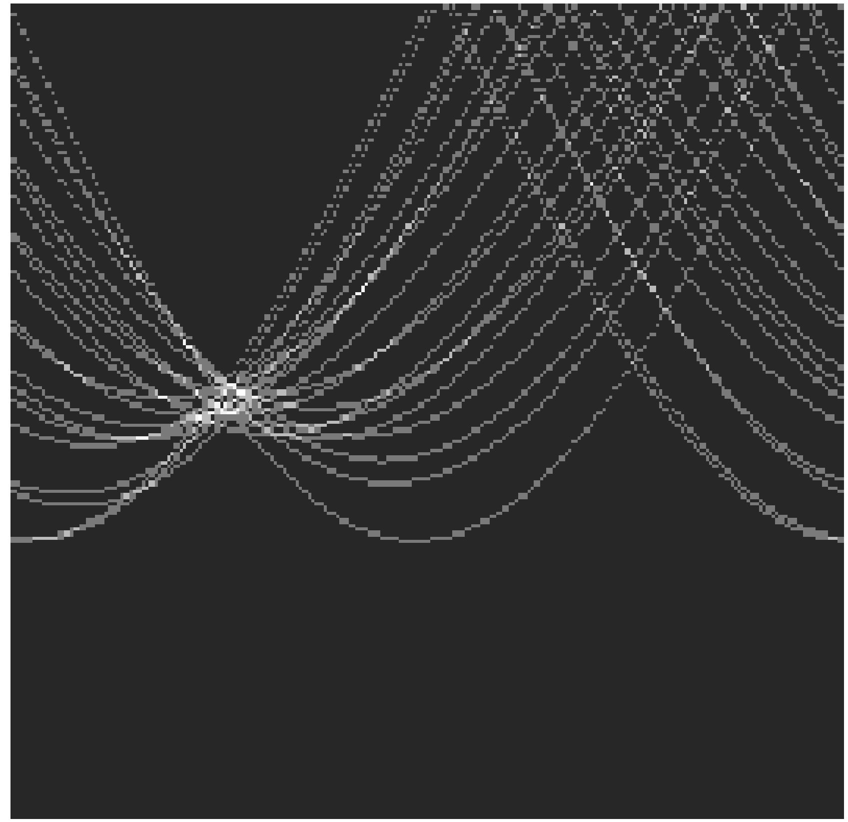
votes

Slide from S. Savarese

Hough transform - experiments

Noisy data





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Need to adjust grid size or smooth

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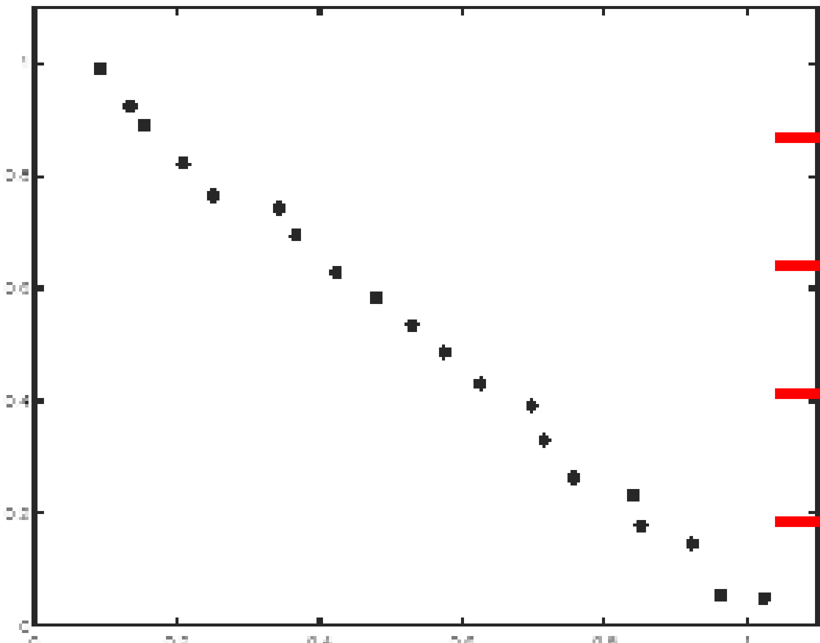

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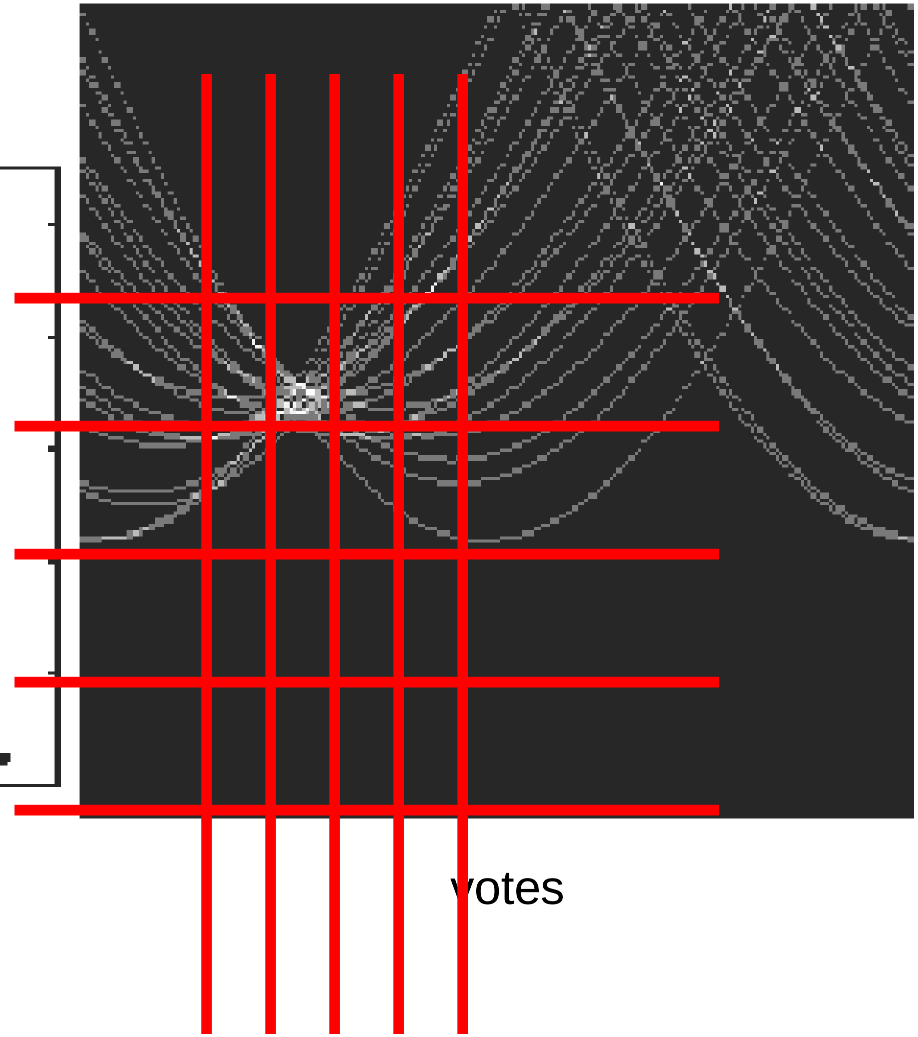

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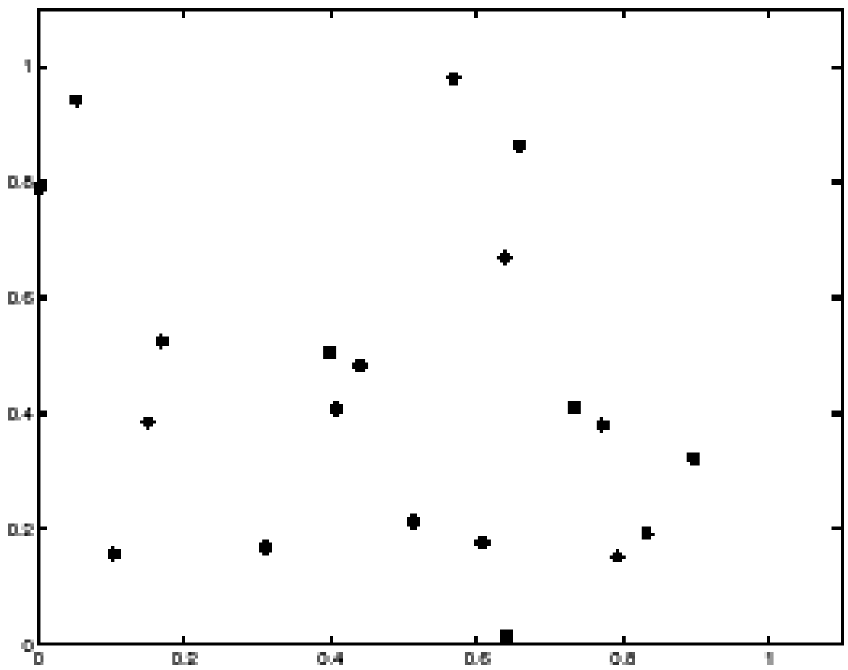

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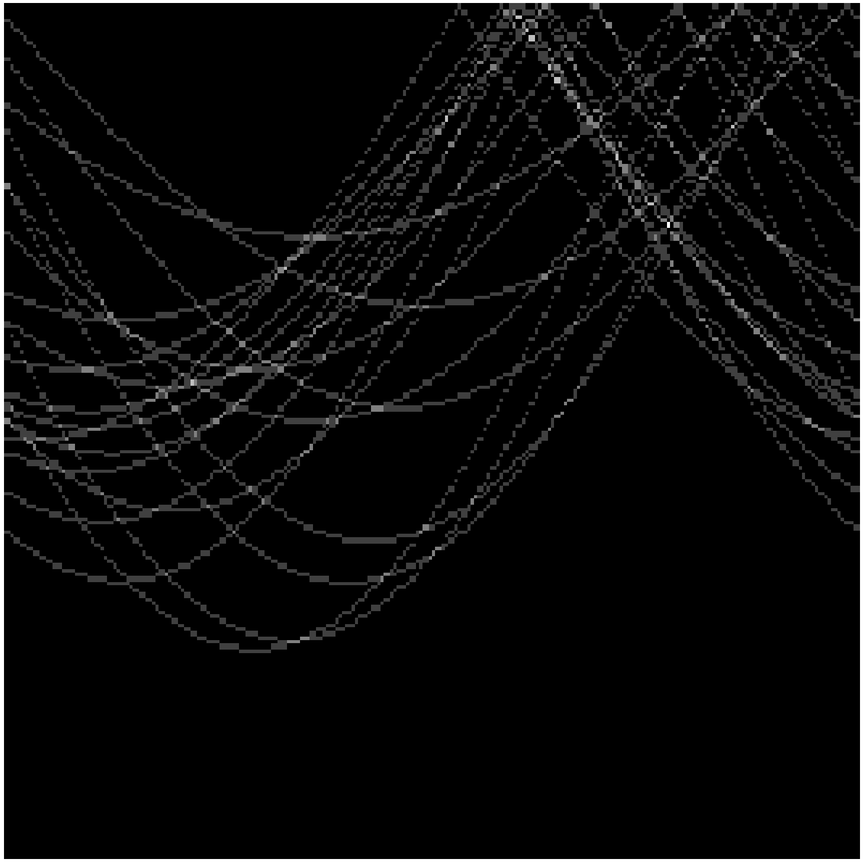


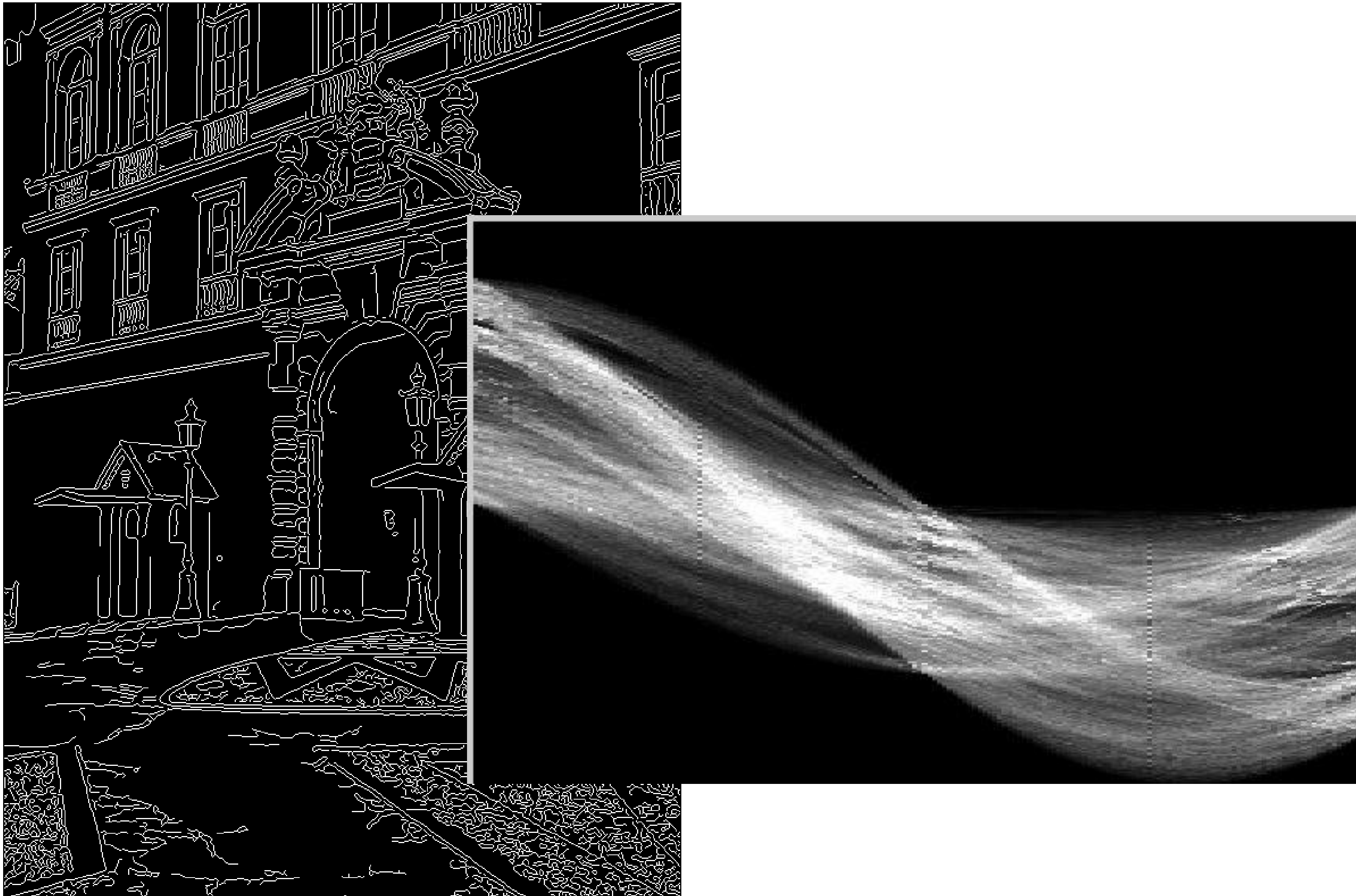
votes

Issue: spurious peaks due to uniform noise

Slide from S. Savarese

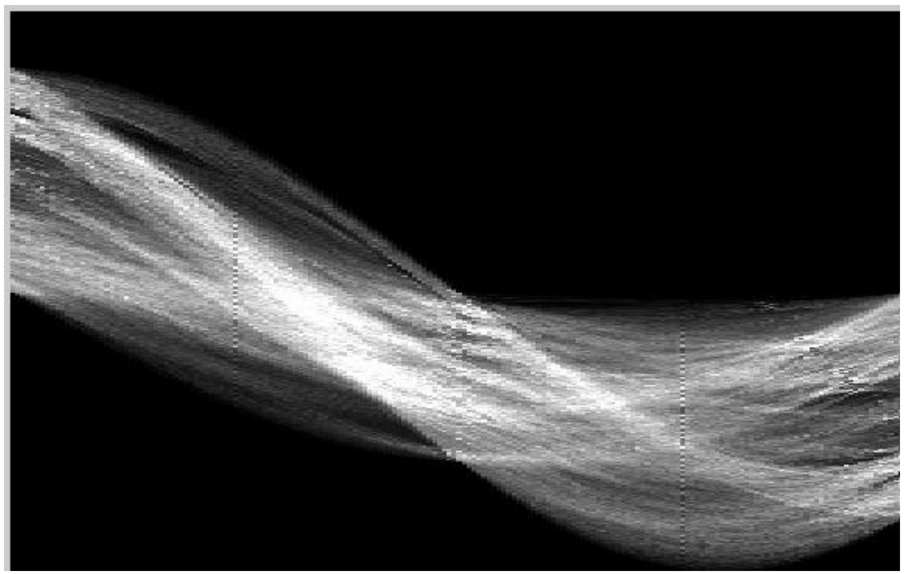
1. Image → Canny

2. Canny → Hough votes



3. Hough votes → Edges

Find peaks and post-process

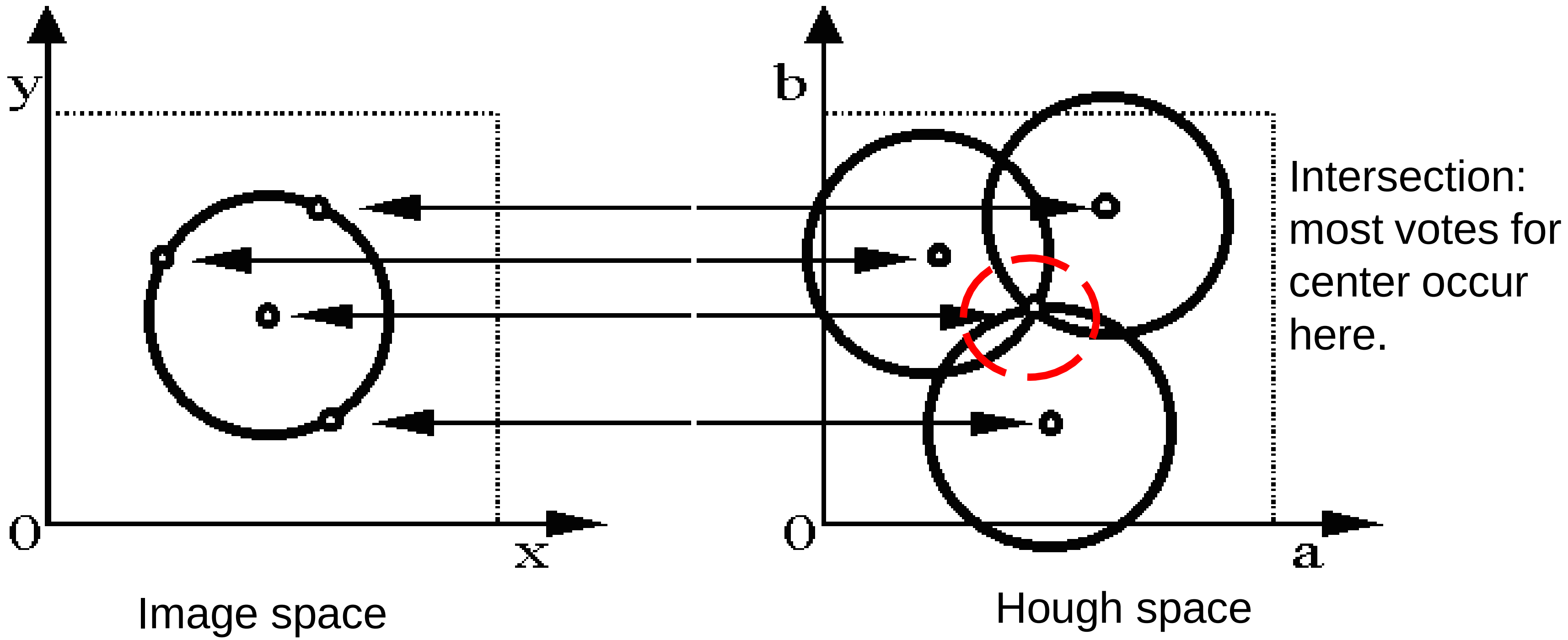


Hough transform for circles

- Circle: center (a,b) and radius r

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

- For a fixed radius r



Kristen Grauman

Hough transform conclusions

Good

- Robust to outliers: each point votes separately
- Fairly efficient (much faster than trying all sets of parameters)
- Provides multiple good fits

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- Some sensitivity to noise
- Bin size trades off between noise tolerance, precision, and speed/memory
  - Can be hard to find sweet spot
- Not suitable for more than a few parameters
  - grid size grows exponentially

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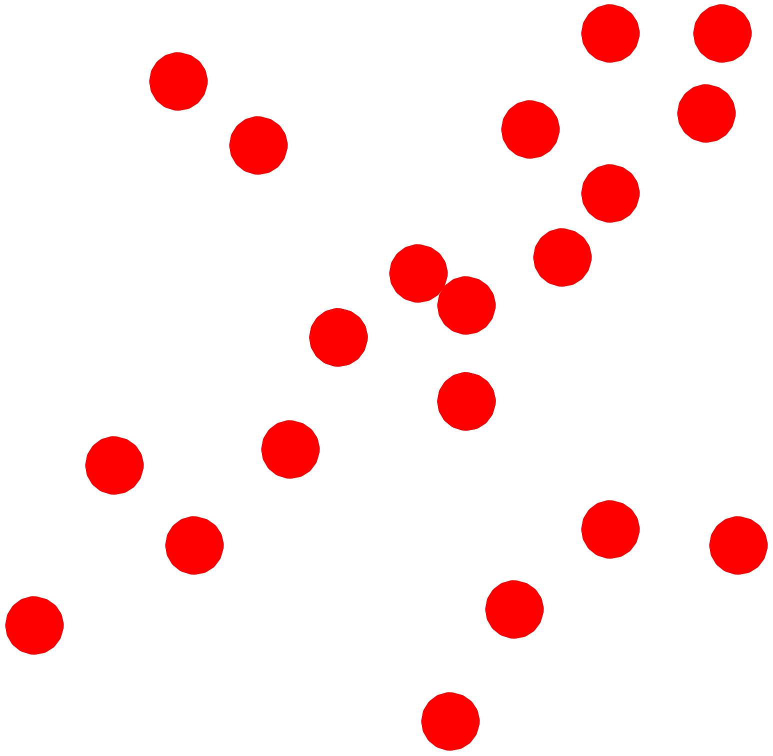
Common applications

- Line fitting (also circles, ellipses, etc.)
- Object instance recognition (parameters are position/scale/orientation)
- Object category recognition (parameters are position/scale)

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in ‘81.



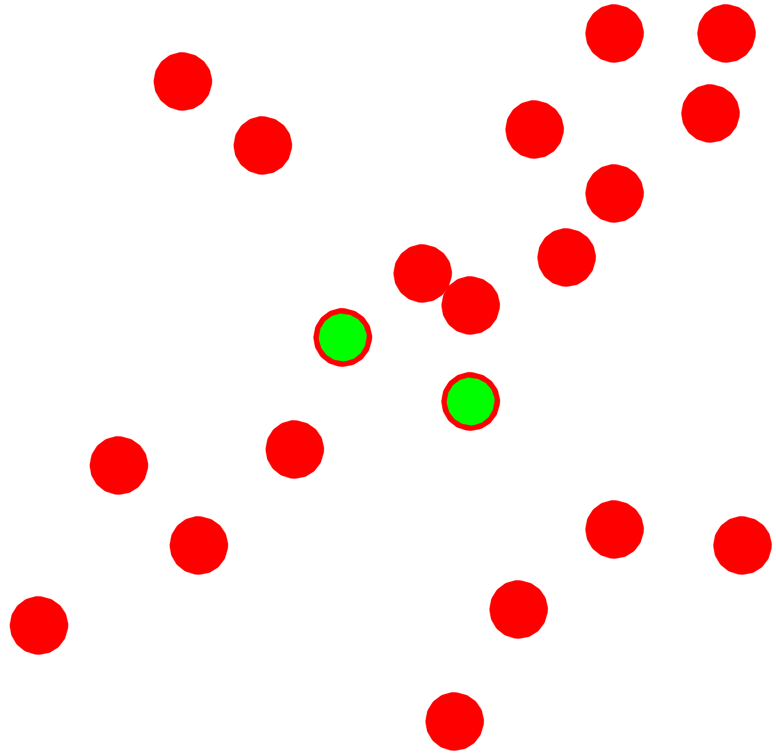
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



Algorithm:

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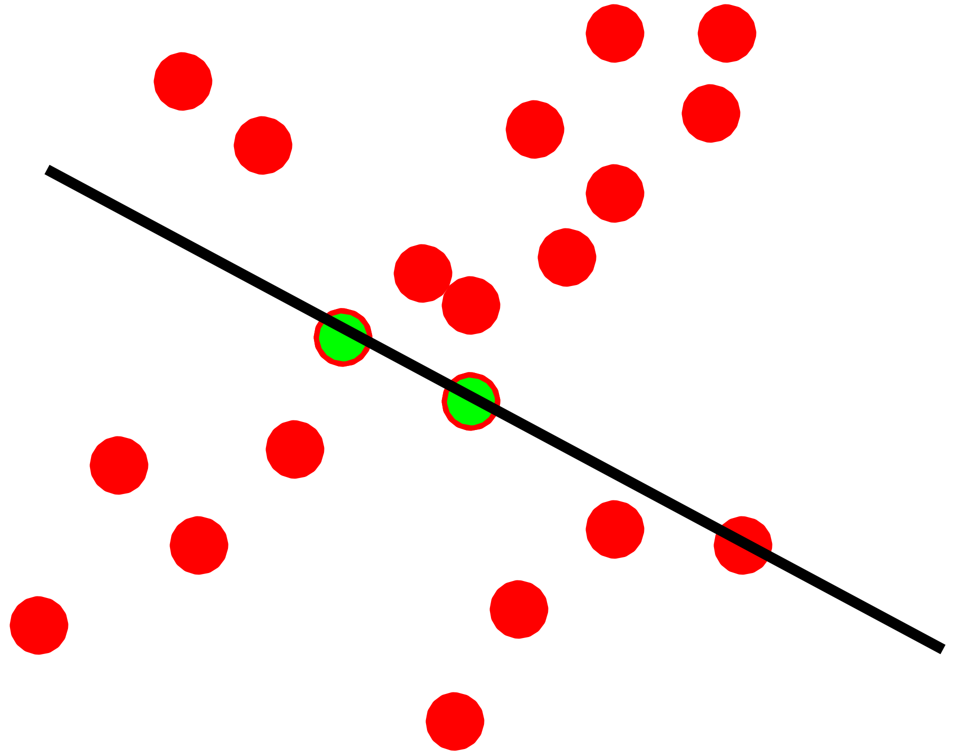
Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



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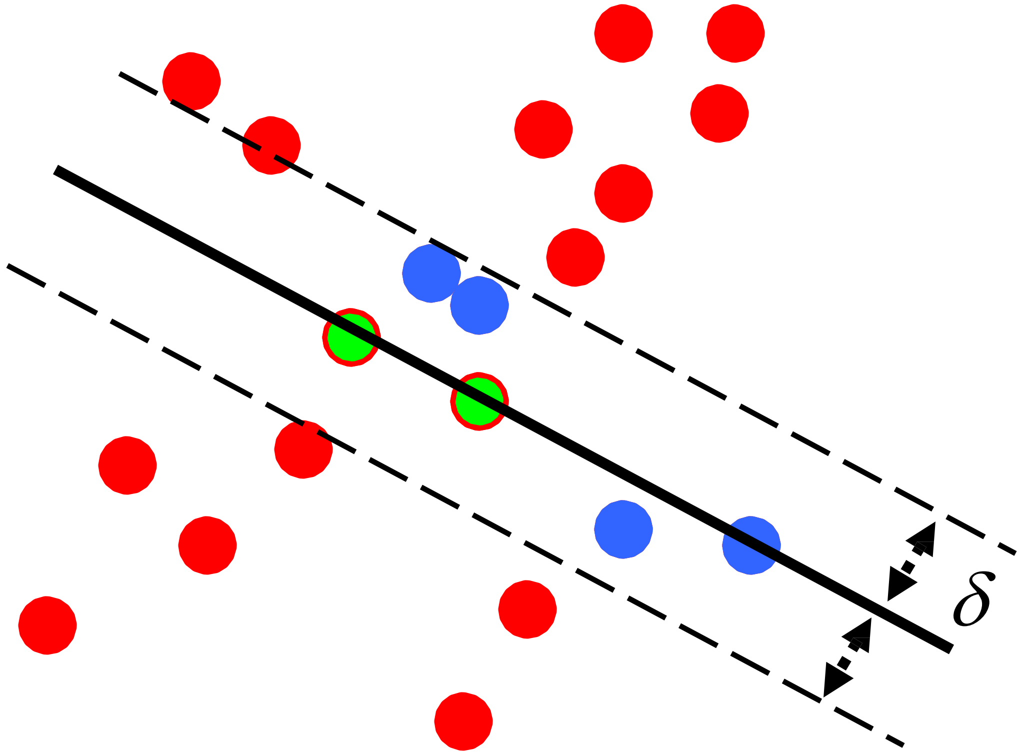

$N_I = 6$

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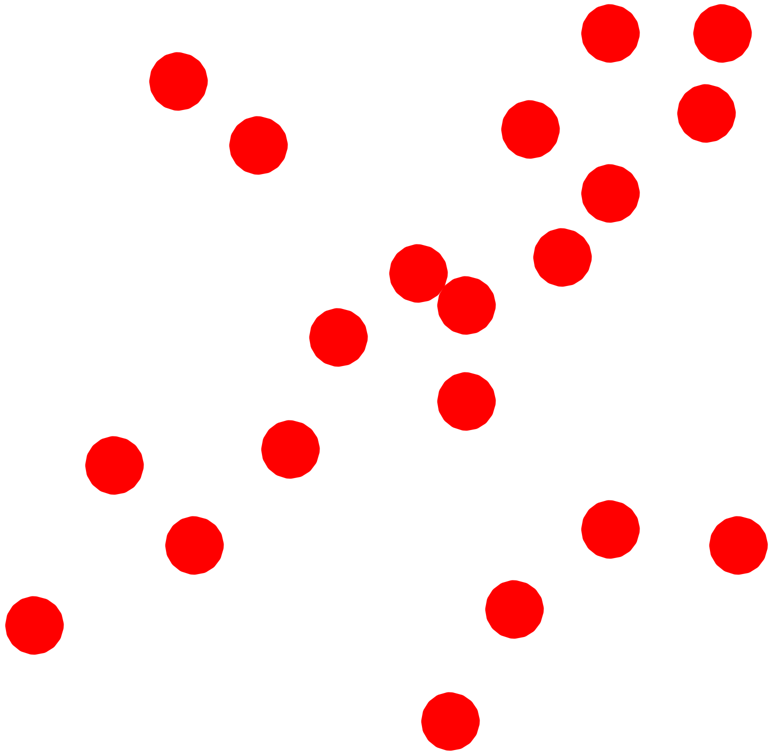

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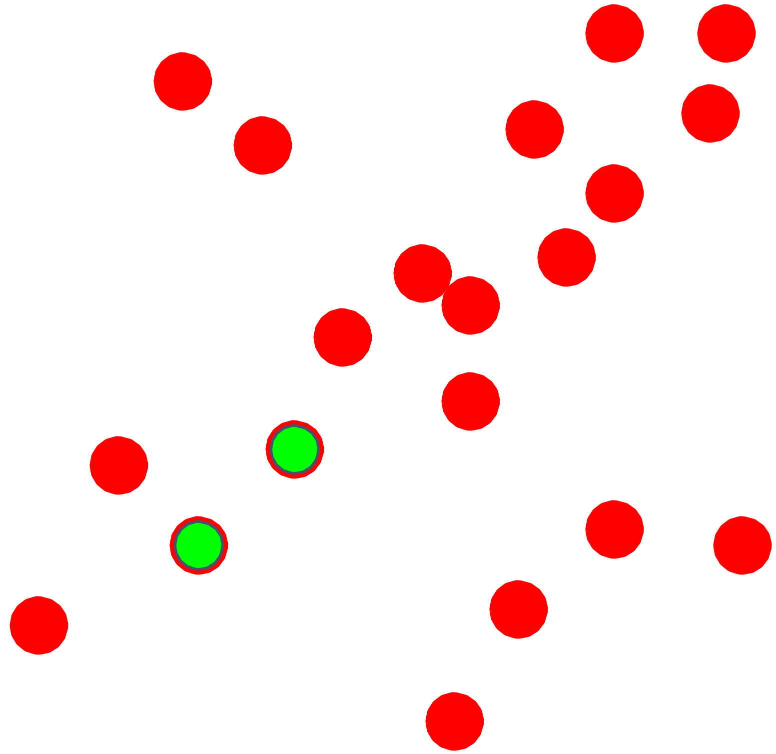

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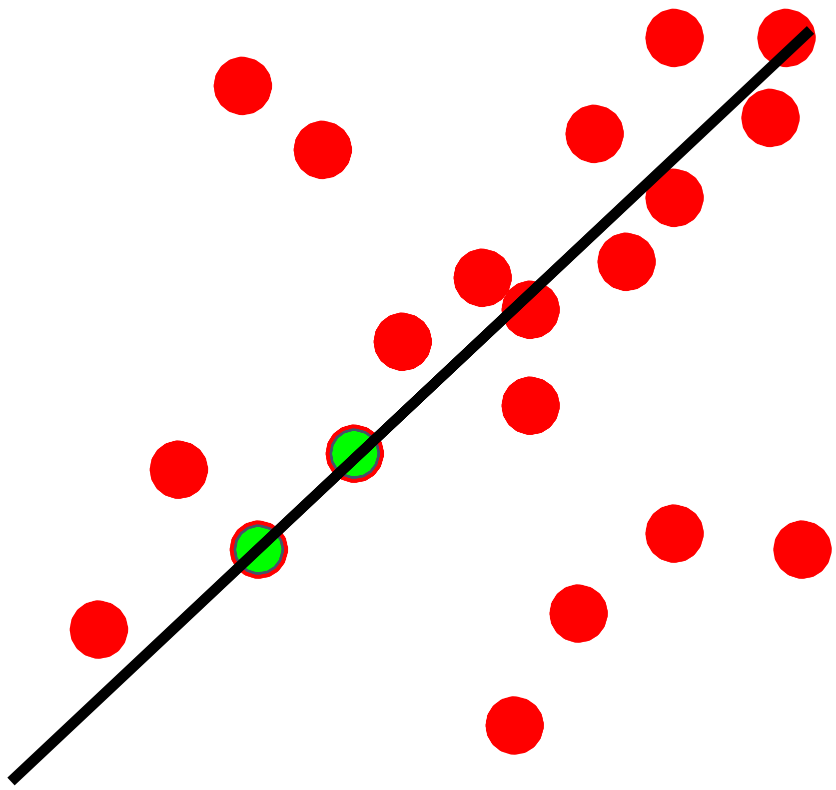

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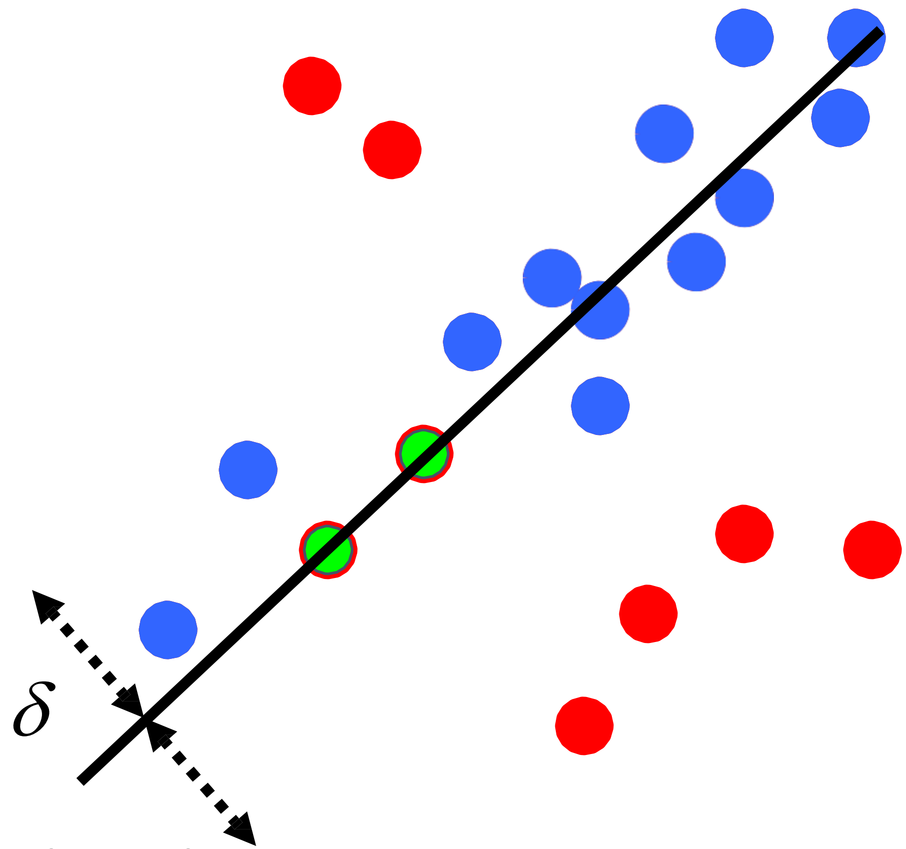
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC



$$N_I = 14$$

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How to choose parameters?

- Number of samples N
  - Choose N so that, with probability p, at least one random sample is free from outliers (e.g. p=0.99) (outlier ratio: e)
- Number of sampled points s
  - Minimum number needed to fit the model
- Distance threshold δ
  - Choose δ so that a good point is likely (e.g., prob=0.95) within threshold

modified from M. Pollefeys

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	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

modified from M. Pollefeys

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of objective function parameters than Hough transform
- Optimization parameters are easier to choose than Hough transform

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Common applications

- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)

Things to remember

- Least Squares Fit
  - closed form solution
  - robust to noise
  - not robust to outliers
- Hough transform
  - robust to noise and outliers
  - can fit multiple models
  - only works for a few parameters (1-4 typically)
- RANSAC
  - robust to noise and outliers
  - works with a moderate number of parameters (e.g, 1-8)

Acknowledgements

- Thanks to the following researchers for making their teaching/research material online
  - Forsyth
  - Steve Seitz
  - Noah Snavely
  - J.B. Huang
  - Derek Hoiem
  - D. Lowe
  - A. Bobick
  - S. Lazebnik
  - K. Grauman
  - R. Zaleski
  - Leibe
  - And many more ……….