Computer Vision

Light: Radiometry and Reflectance

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Today's Agenda

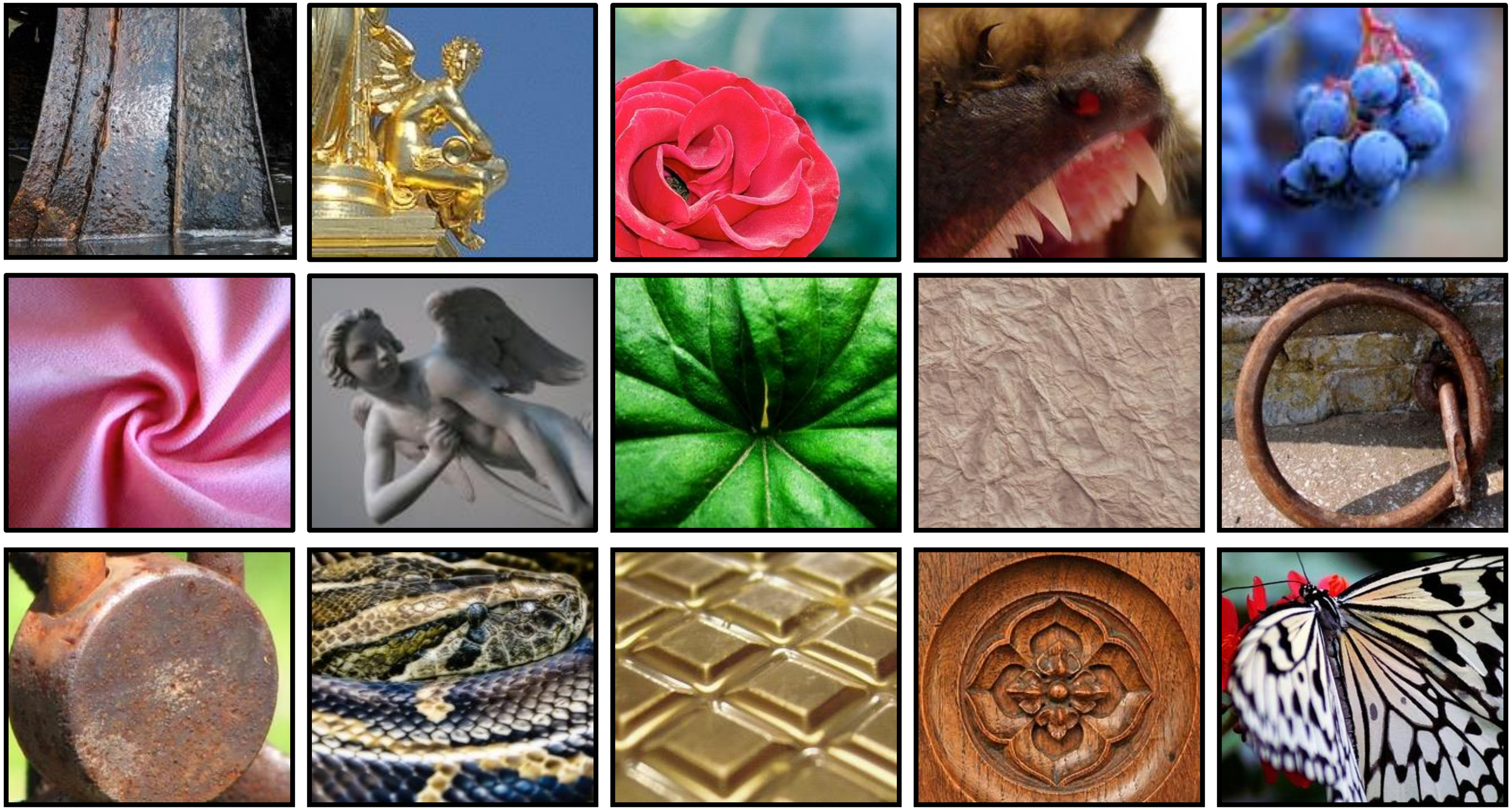
- Light
  - Radiometry
  - Reflectance

Why should we care?

- The **appearance** of objects is given by the way in which they reflect and transmit light.
- The **color** of objects is determined by the parts of the spectrum of (incident white) light that are reflected or transmitted without being absorbed.

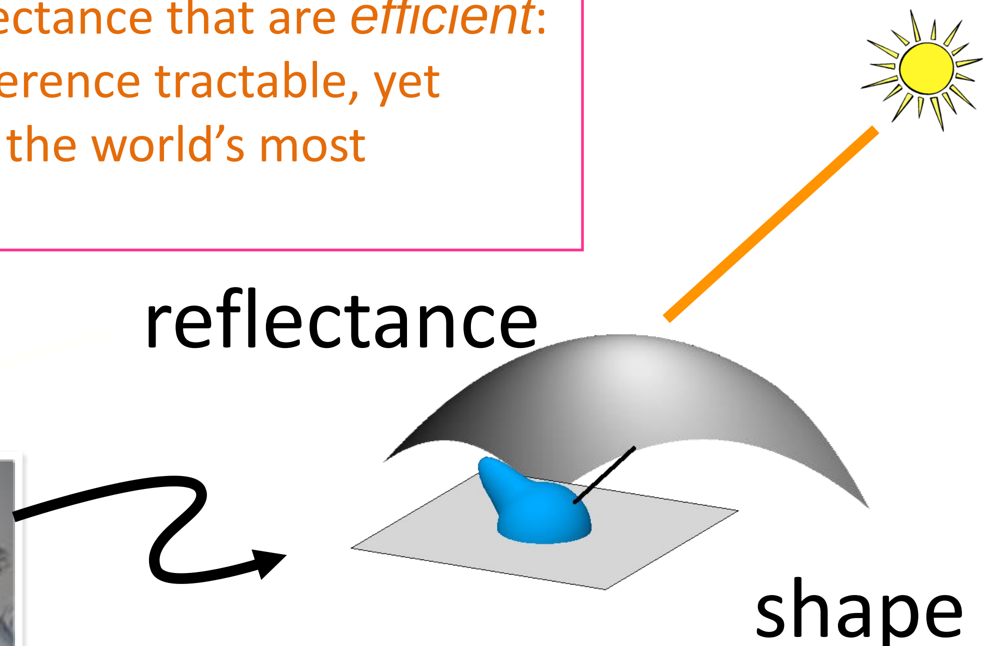
Source: **Wikipedia**

Appearance



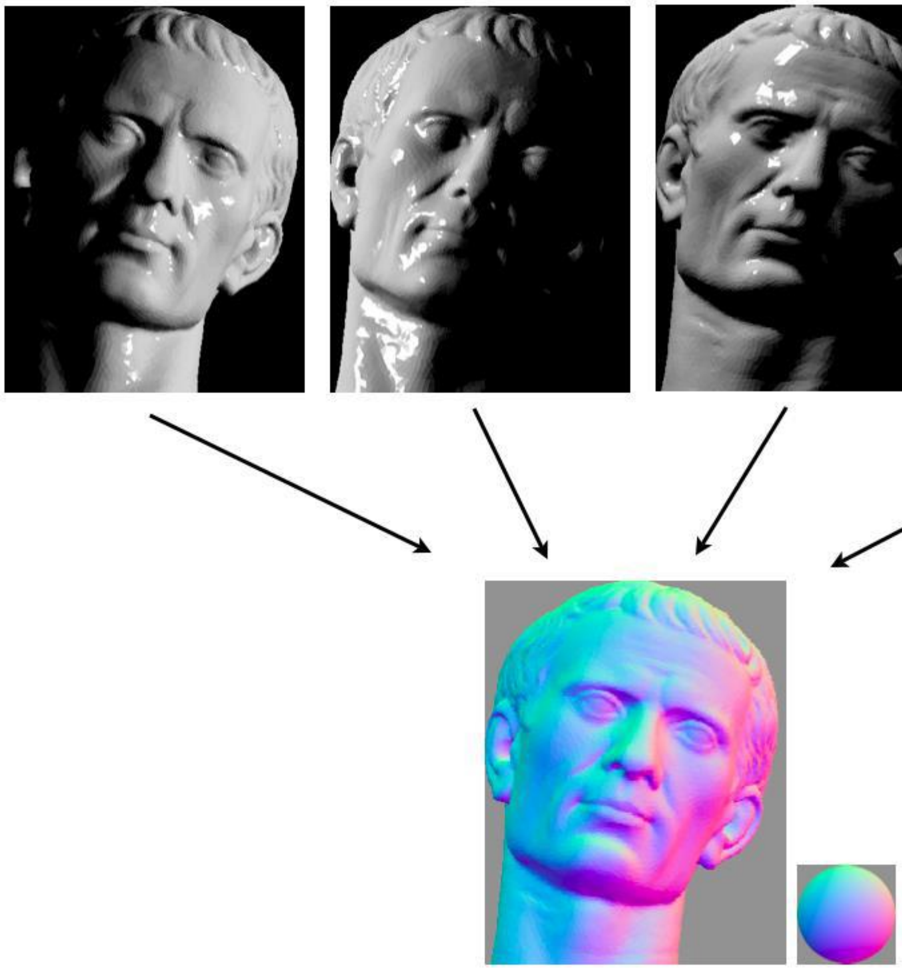
“Physics-based” computer vision (a.k.a “inverse optics”)

Our challenge: Invent computational representations of shape, lighting, and reflectance that are *efficient*: simple enough to make inference tractable, yet general enough to capture the world’s most important phenomena



$\mathbf{I} \Rightarrow$ shape, illumination, reflectance

Application: Photometric Stereo



Analysis under different lighting conditions to estimate a normal direction at each pixel.

- Why study the physics (optics) of the world?
- Lets see some pictures!

Source: **Wikipedia**

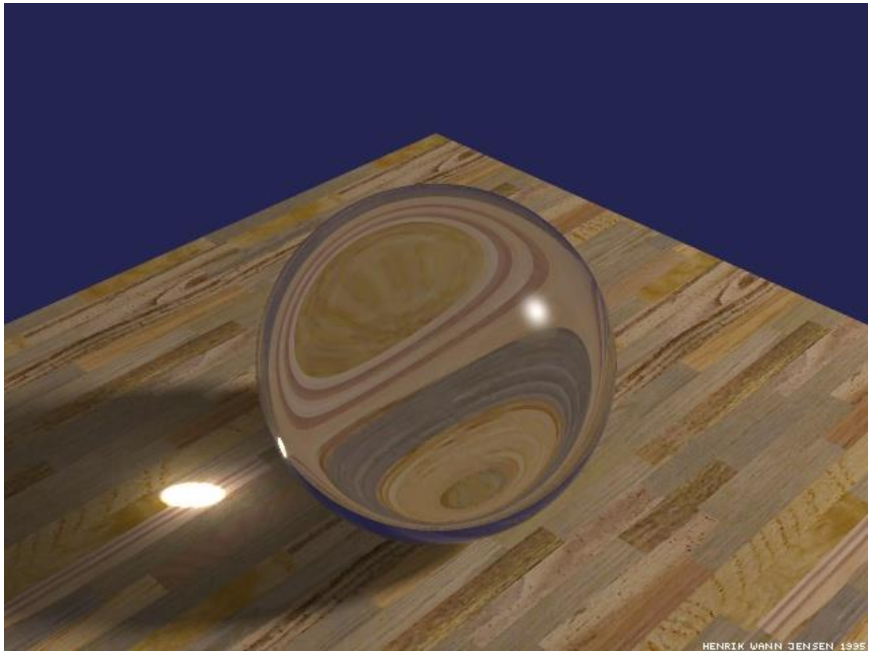
Light and Shadows

Light and Shadows

Reflections

Reflections

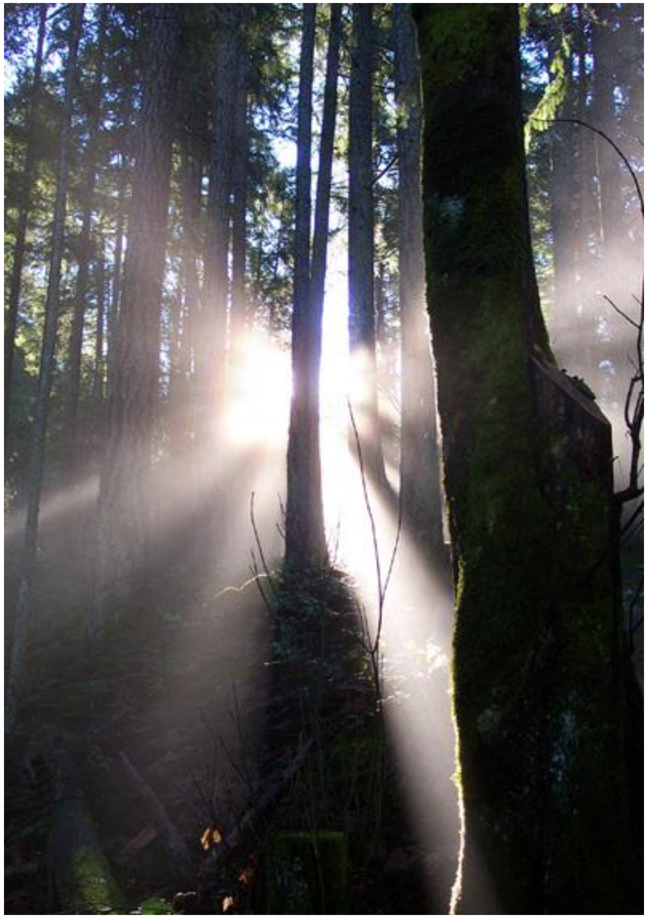
Refractions

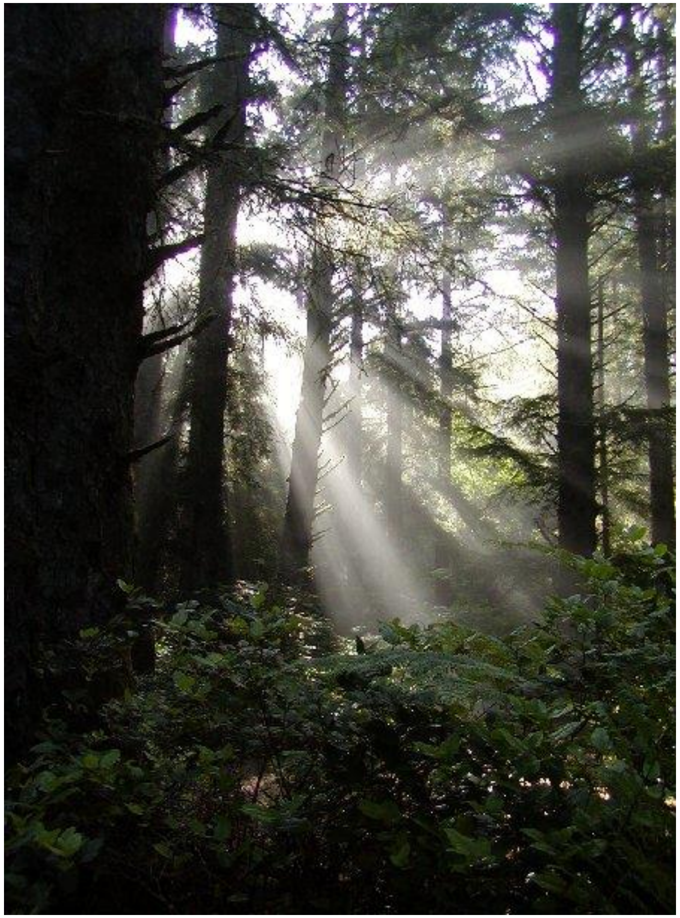


Refractions

Inter-reflections

Scattering





Scattering

More Complex Appearances

More Complex Appearances

More Complex Appearances



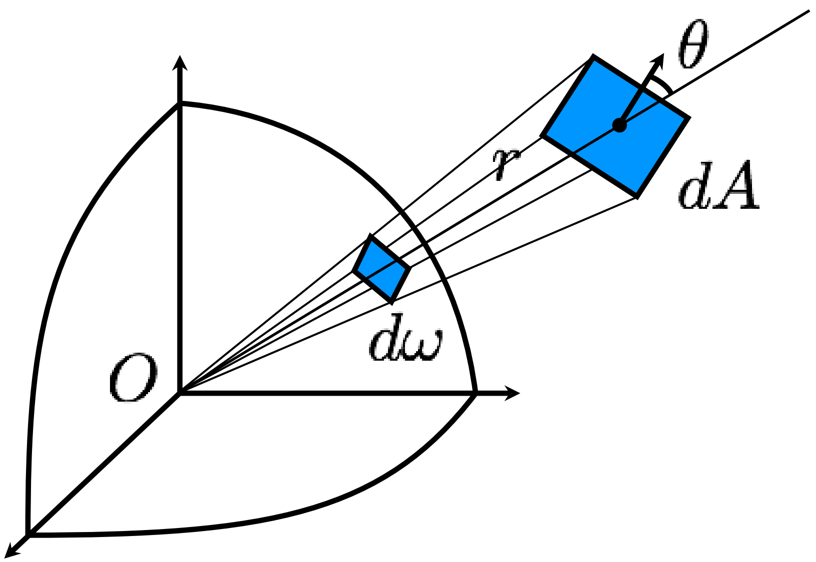






Measuring Light and Radiometry

- Solid angle: The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

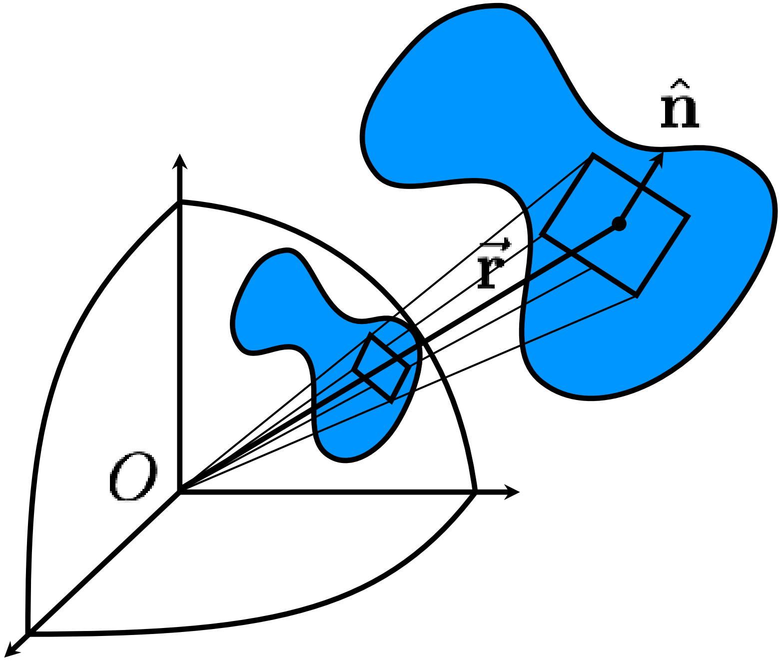
One can show:

$$\boxed{d\omega = \frac{dA\cos\theta}{r^2}}$$

Units: steradians [sr]

Measuring Light and Radiometry

- To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}} \; dS}{|\vec{\mathbf{r}}|^3}$$

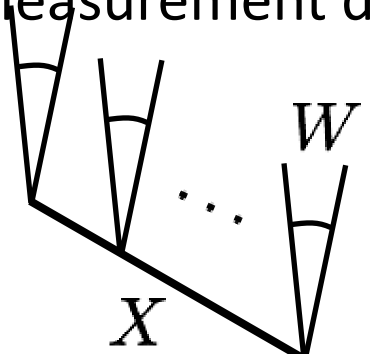
One can show:

$$\boxed{d\omega = \frac{dA \cos\theta}{r^2}}$$

Units: steradians [sr]

Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area |X|

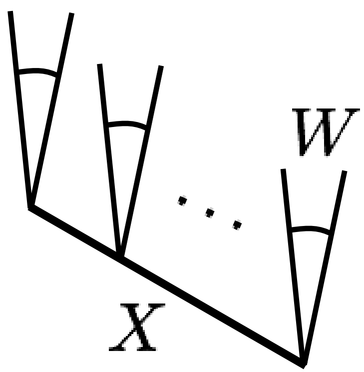


* shown in 2D for clarity; imagine three dimensions

radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]



$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

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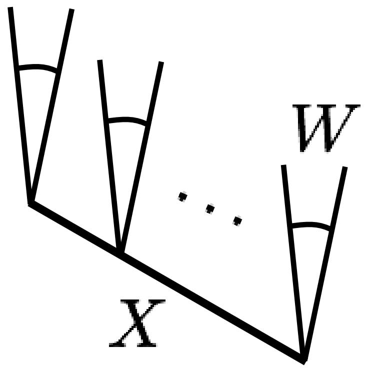
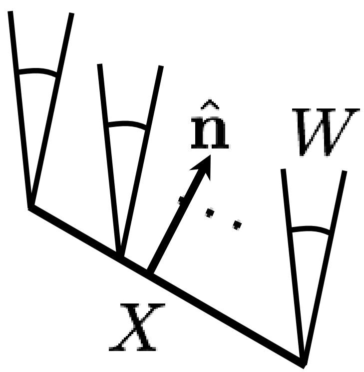


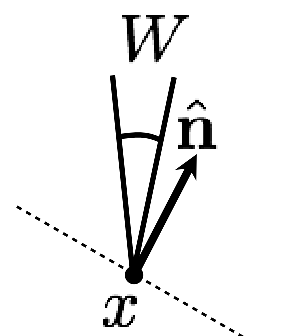
$$\lim_{X \longrightarrow x}$$

$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]
- Depends on sensor direction normal.



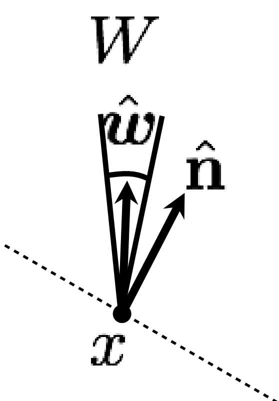


$$\frac{\Phi(W,X)}{|X|} \qquad \lim_{X \longrightarrow x} \qquad E_{\hat{\mathbf{n}}}(W,x)$$

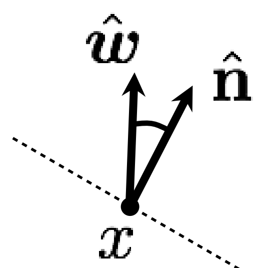
- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [W/(m^2·sr)]



$$\lim_{W \longrightarrow \hat{\omega}}$$



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\omega}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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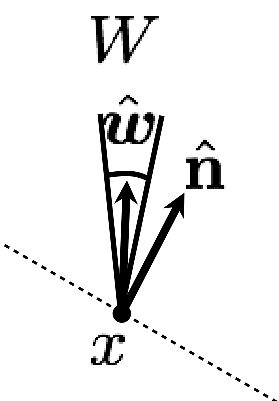
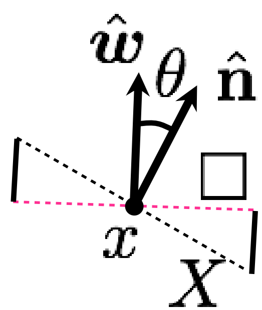


$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|}$$

$$\lim_{W \longrightarrow \hat{w}}$$



$$L_{\hat{\mathbf{n}}}(\hat{\omega}, x)$$

$$\cos\theta = \frac{\square/2}{|X|/2}$$

$$\longrightarrow \square = |X| \cos\theta$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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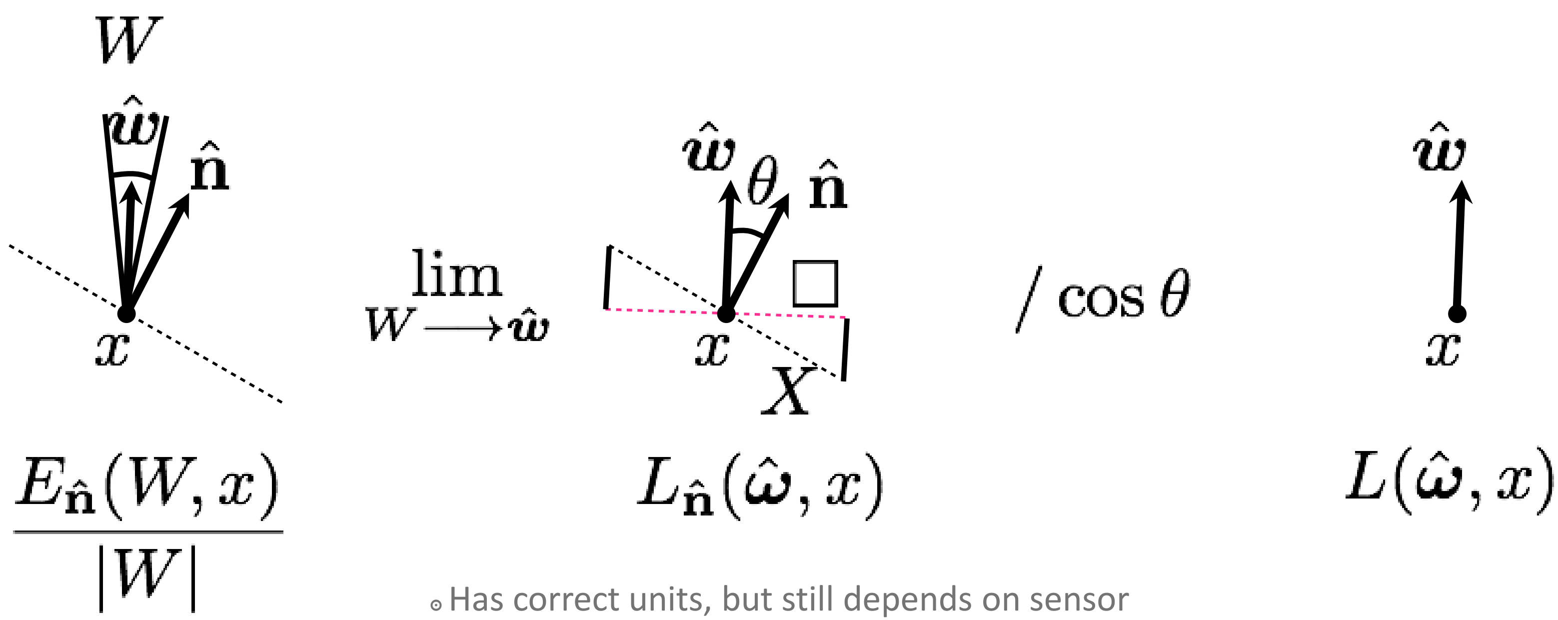

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
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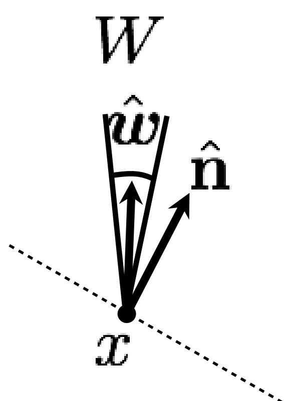
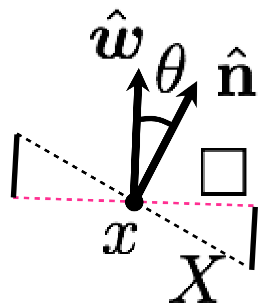
- Units: watts per steradian per square meter [W/(m^2·sr)]

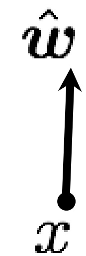
"foreshortened in the direction of travel"



$\lim_{W \to \hat{w}}$



$/ \cos\theta$

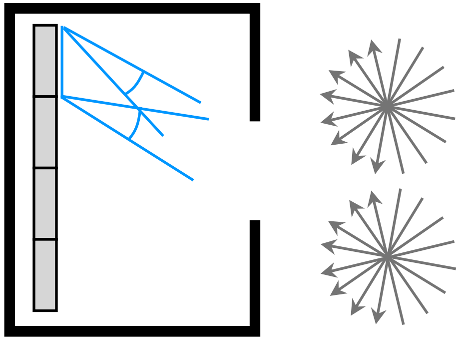


$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\omega}, x) \qquad L(\hat{\omega}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

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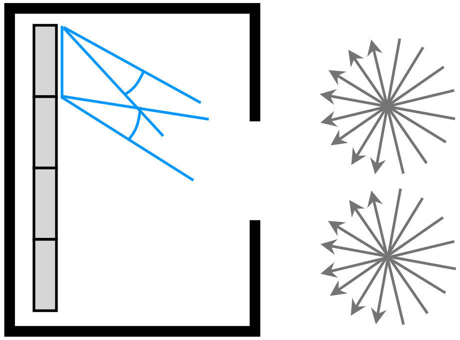
  - Allows computing the radiant flux measured by *any* finite sensor

$$\Phi(W, X) = \int_X \int_W L(\hat{\boldsymbol{\omega}}, x) \cos\theta d\boldsymbol{\omega} dA$$



Quantifying light: flux, irradiance, and radiance

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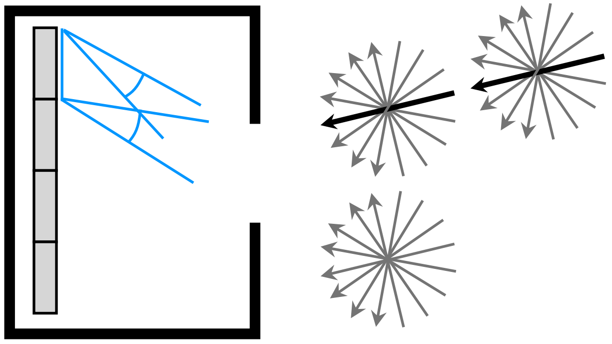
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta \, d\omega \, dA$$

  - Constant along a ray in free space

$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$



Quantifying light: flux, irradiance, and radiance

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  - Constant along a ray in free space

$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$

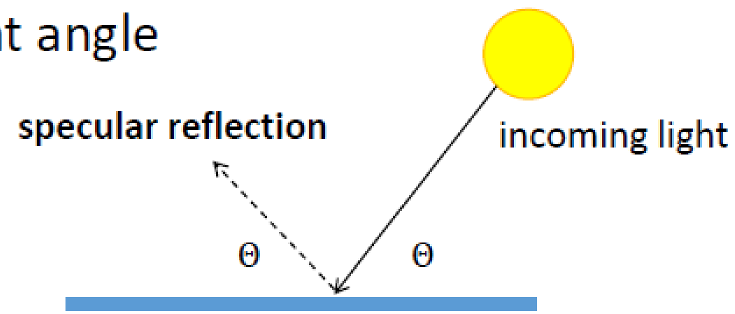
  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.

- "Processed" images (like PNG and JPEG) are not linear radiance measurements!!

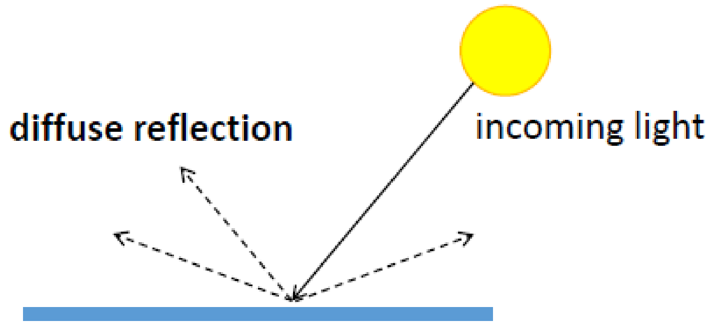
Reflectance and BRDF

Basic models of reflection

- Specular: light bounces off at the incident angle
  - E.g., mirror

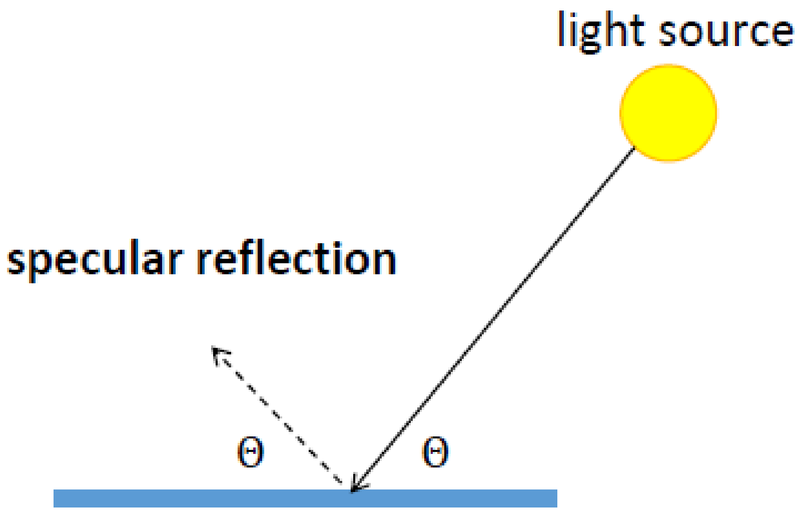


- Diffuse: light scatters in all directions
  - E.g., brick, cloth, rough wood



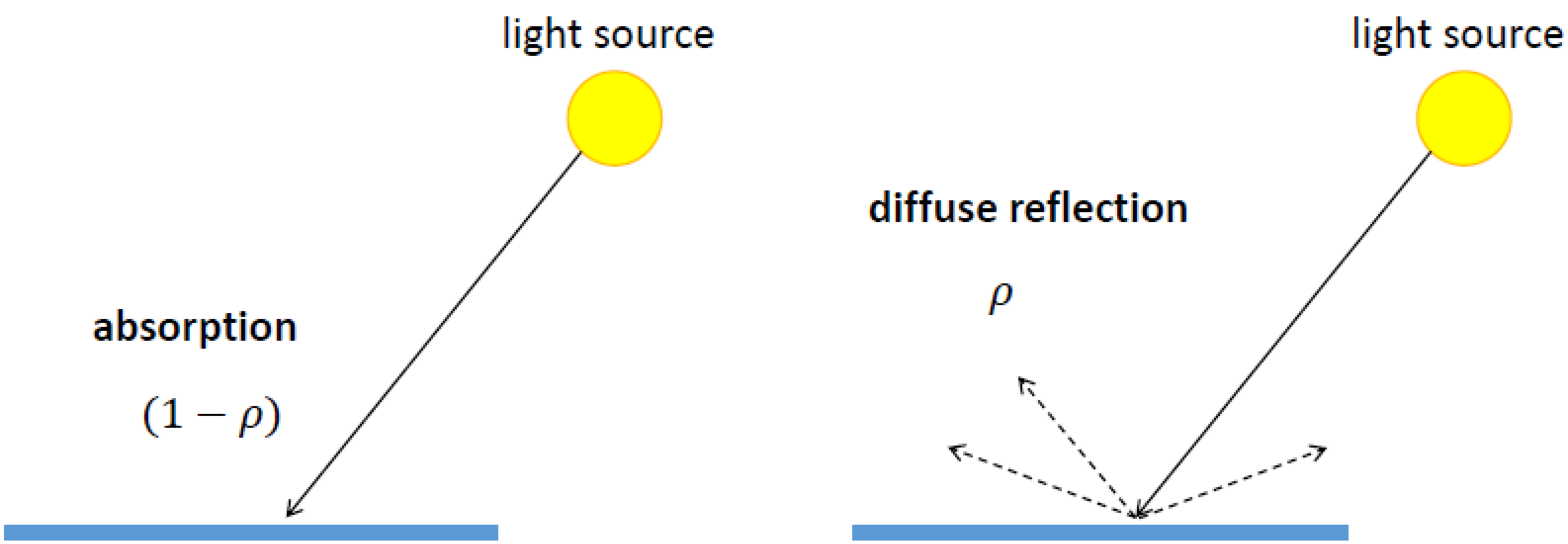
Specular Reflection

- Reflected direction depends on light orientation and surface normal
  - E.g., mirrors are fully specular



Lambertian reflectance model

- Some light is absorbed (function of albedo ρ)
- Remaining light is scattered (diffuse reflection)
- Examples: soft cloth, concrete, matte paints



Slide credit: Derek Hoiem

Most surfaces have both specular and diffuse components

- Specularity = spot where specular reflection dominates (typically reflects light source)

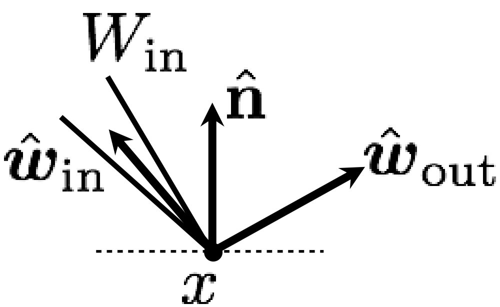
Typically, specular component is small

Slide credit: Derek Hoiem

Photo: northcountryhardwoodfloors.com

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

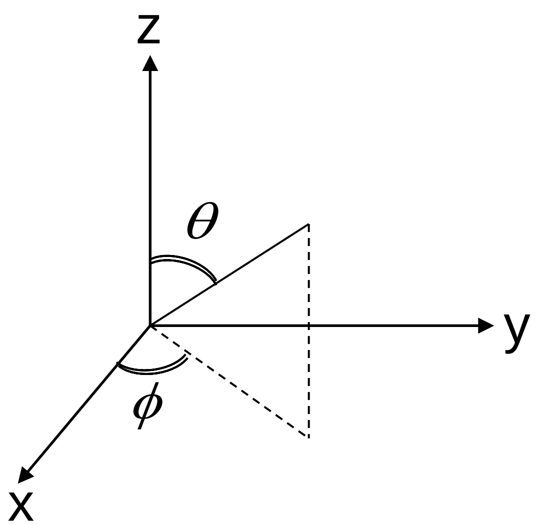


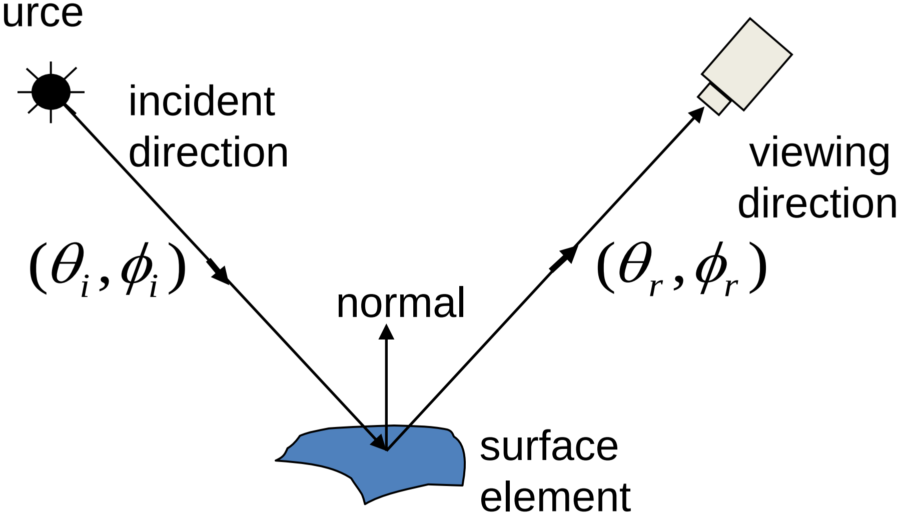
$$\lim_{W_{\text{in}} \to \hat{\boldsymbol{\omega}}_{\text{in}}} \qquad f_{x,\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{L^{\text{out}}(x, \hat{\boldsymbol{\omega}}_{\text{out}})}{E^{\text{in}}_{\hat{\mathbf{n}}}(W_{\text{in}}, x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

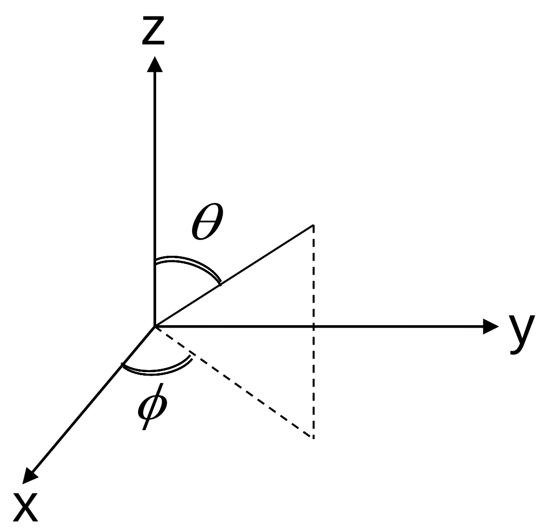
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

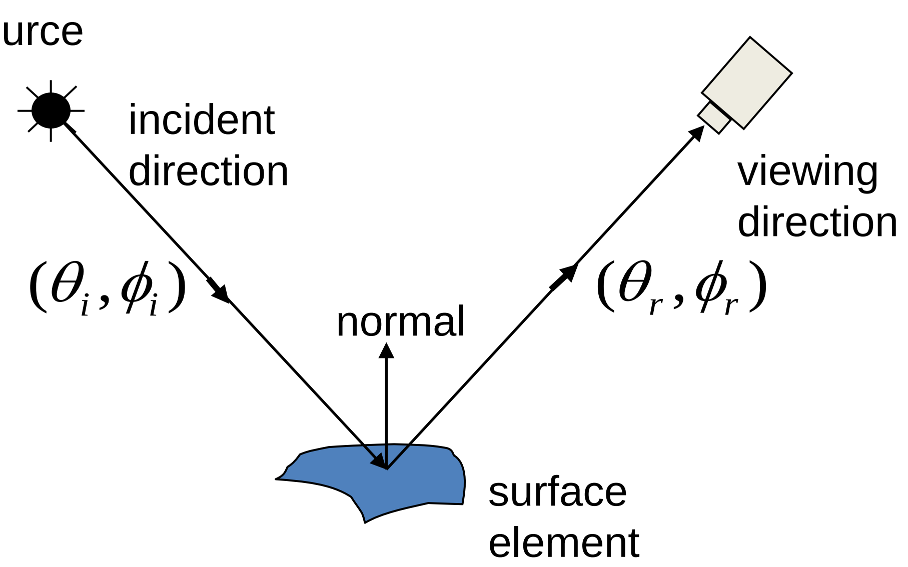
$$\boxed{\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}}$$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties

Important Properties of BRDFs



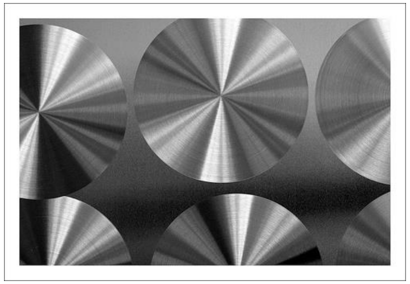


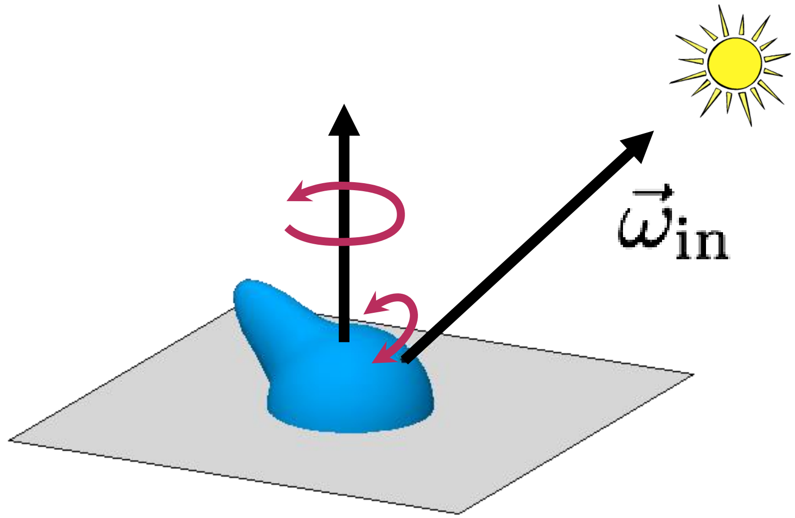
- Conservation of Energy:

$$\forall \hat{\omega}_{\text{in}}, \quad \int_{\Omega_{\text{out}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) \cos\theta_{\text{out}} d\hat{\omega}_{\text{out}} \leq 1$$

Why smaller
than or equal?

Common assumption: Isotropy





$$f_r(\vec{\omega}_{\text{in}}, \cdot)$$

BRDF does not change when surface is rotated about the normal.

[Matusik et al., 2003]

4D → 3D

$$\boxed{f_r(\cancel{\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

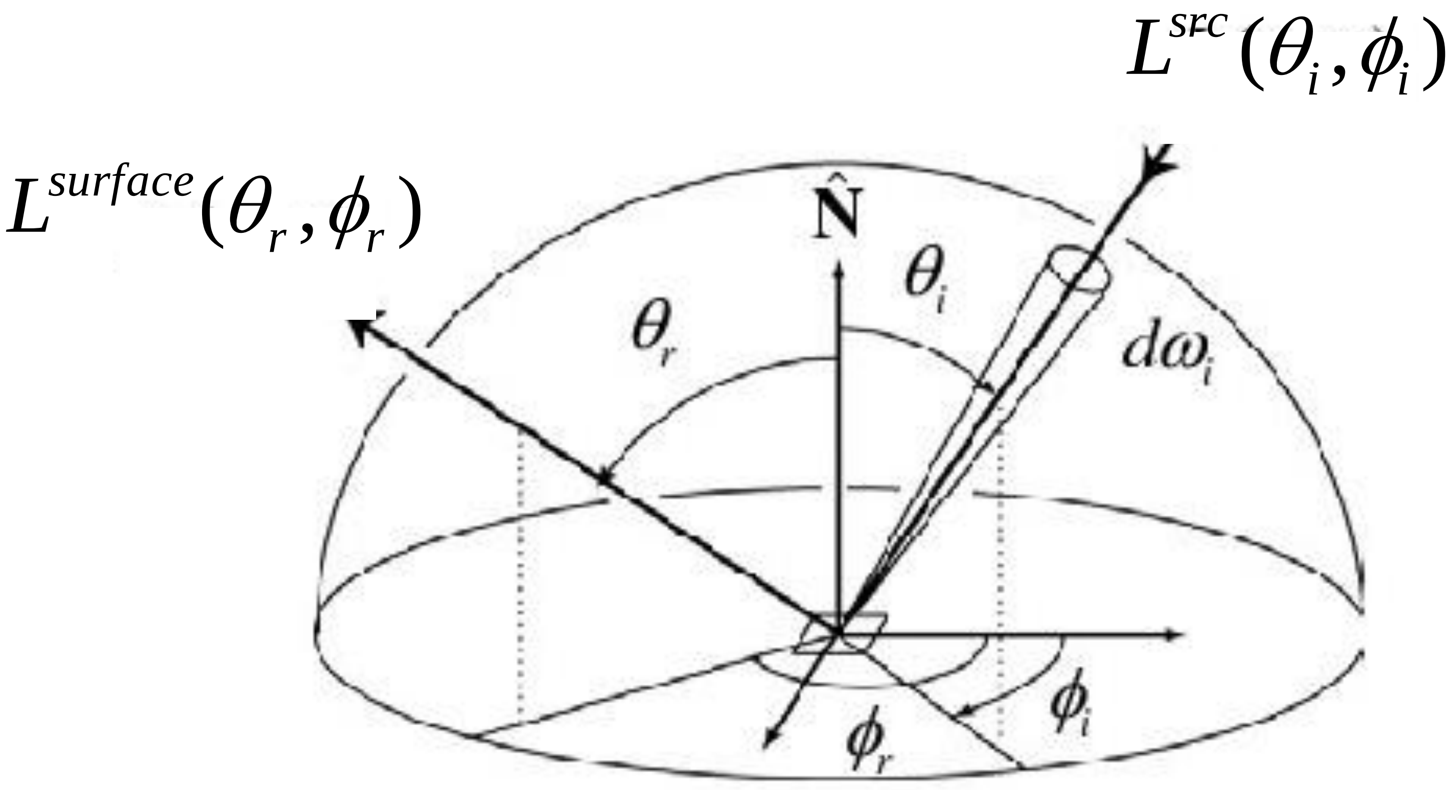
- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Reflectance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) \quad = \quad E^{surface}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) = \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) = \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\, \underline{\cos\theta_i d\omega_i}$$

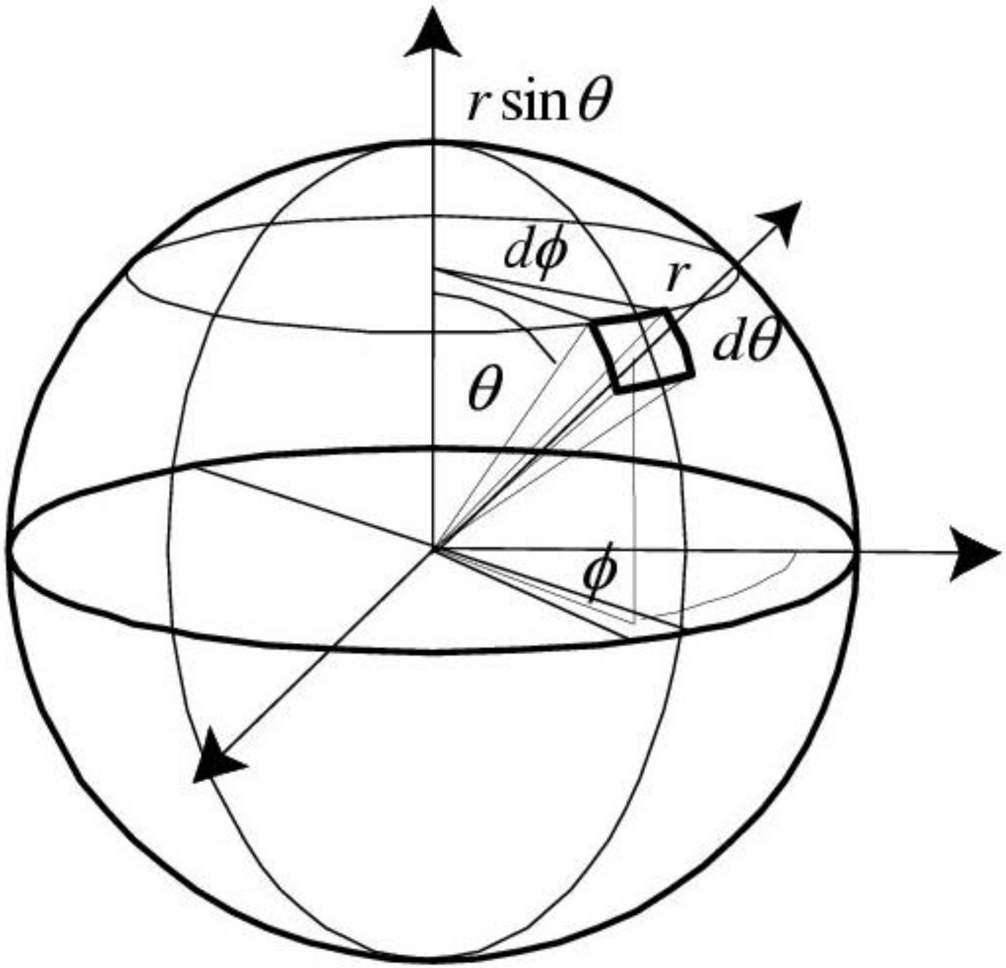
Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r,\phi_r) = \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{\sin\theta_i d\theta_i d\phi_i}$$

Differential Solid Angles

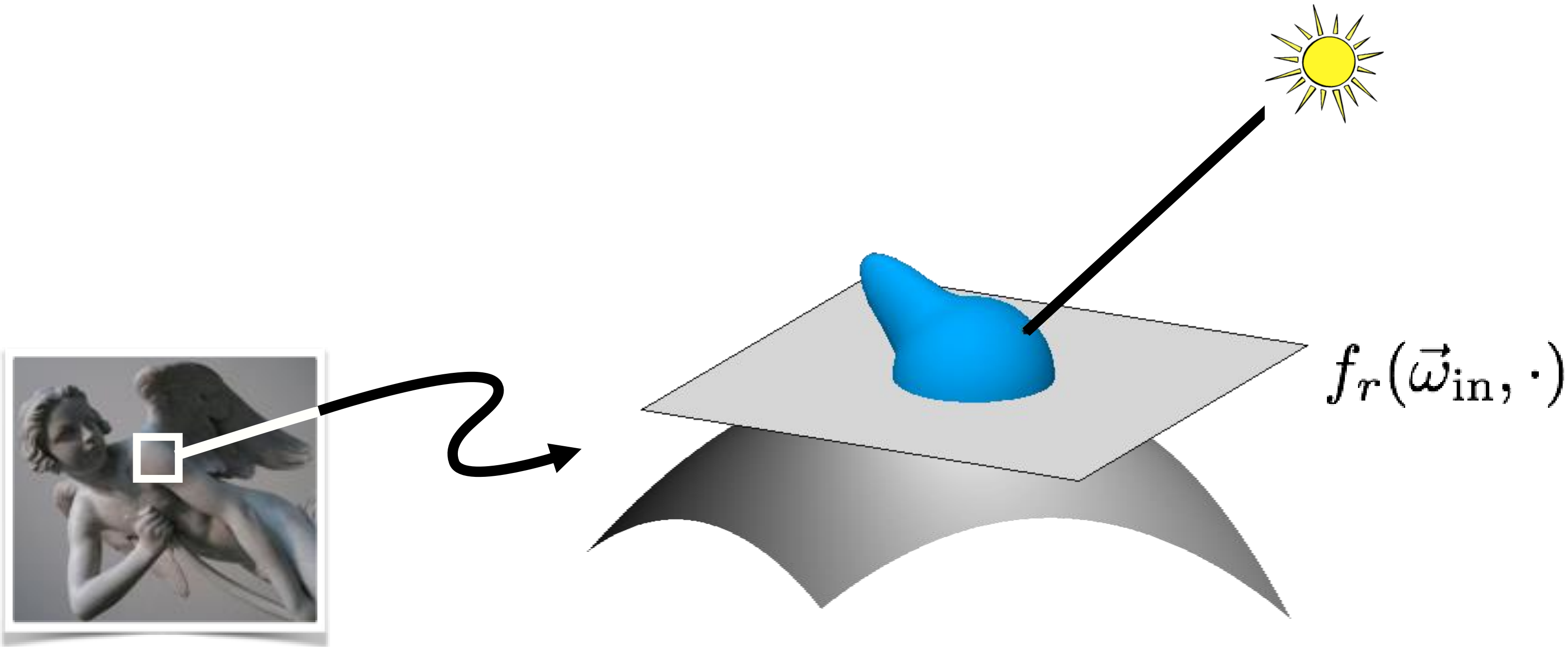


$$dA = (r\,d\theta)(r \sin\theta\, d\phi)$$
$$= r^2 \sin\theta\, d\theta\, d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\, d\theta\, d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\, d\theta\, d\phi = 4\pi$$

BRDF



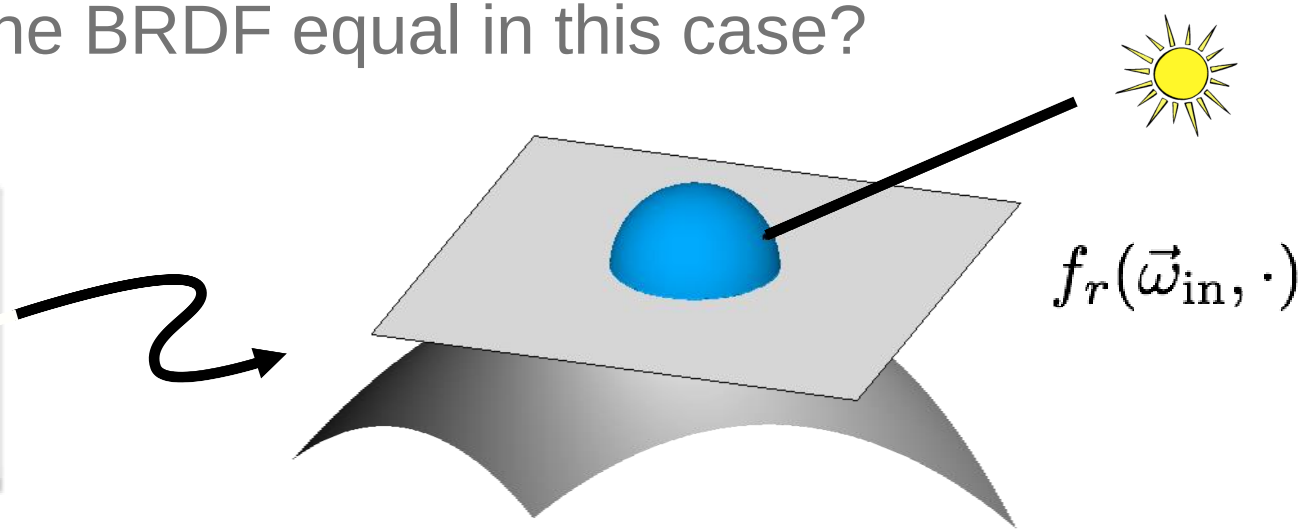
$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

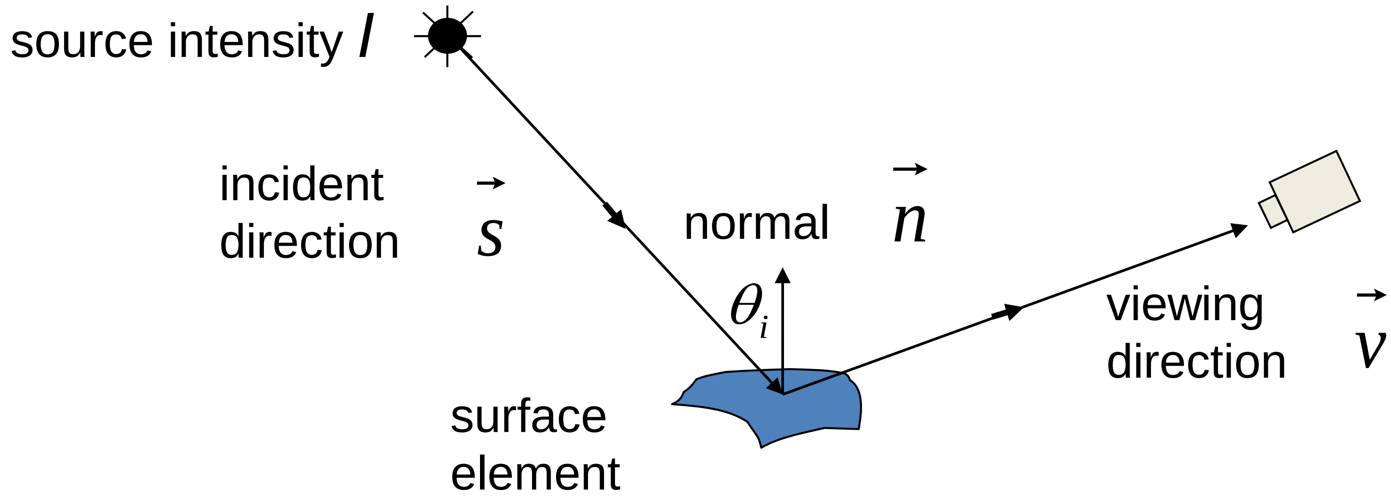
What does the BRDF equal in this case?



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF



- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)

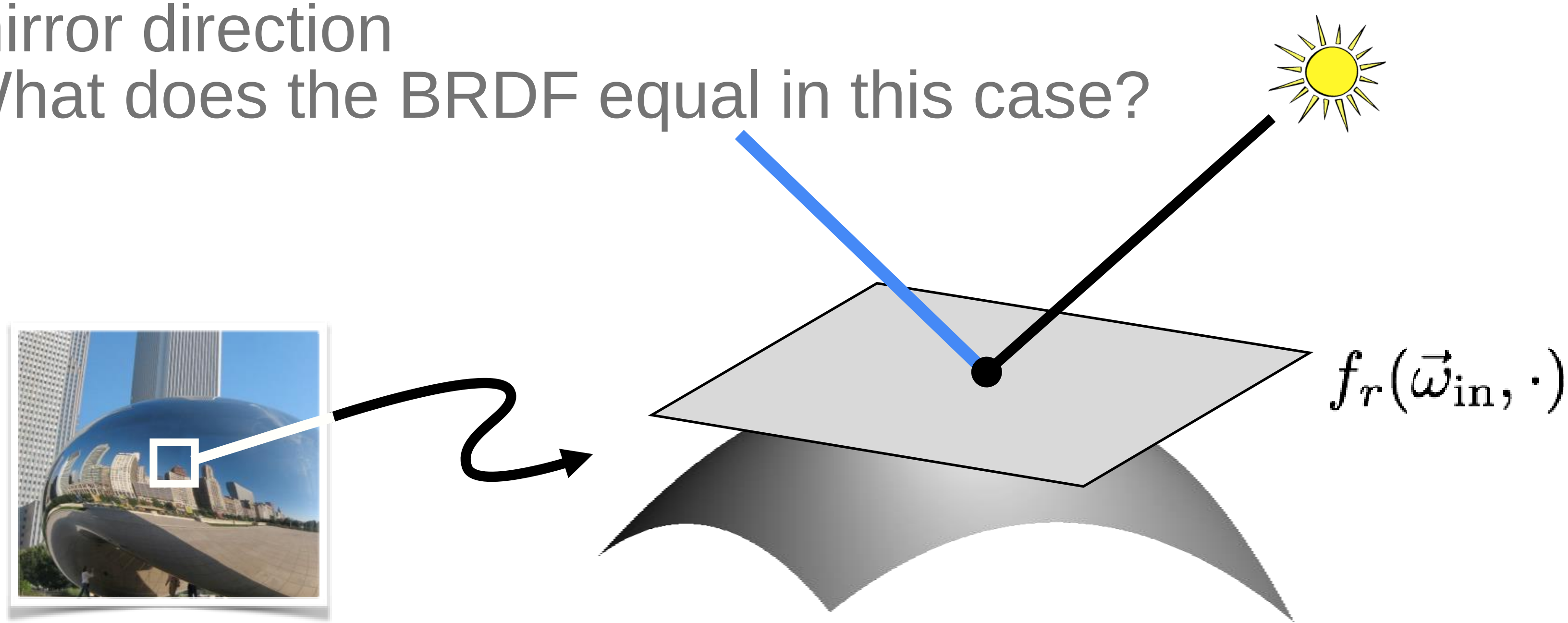
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ albedo

- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

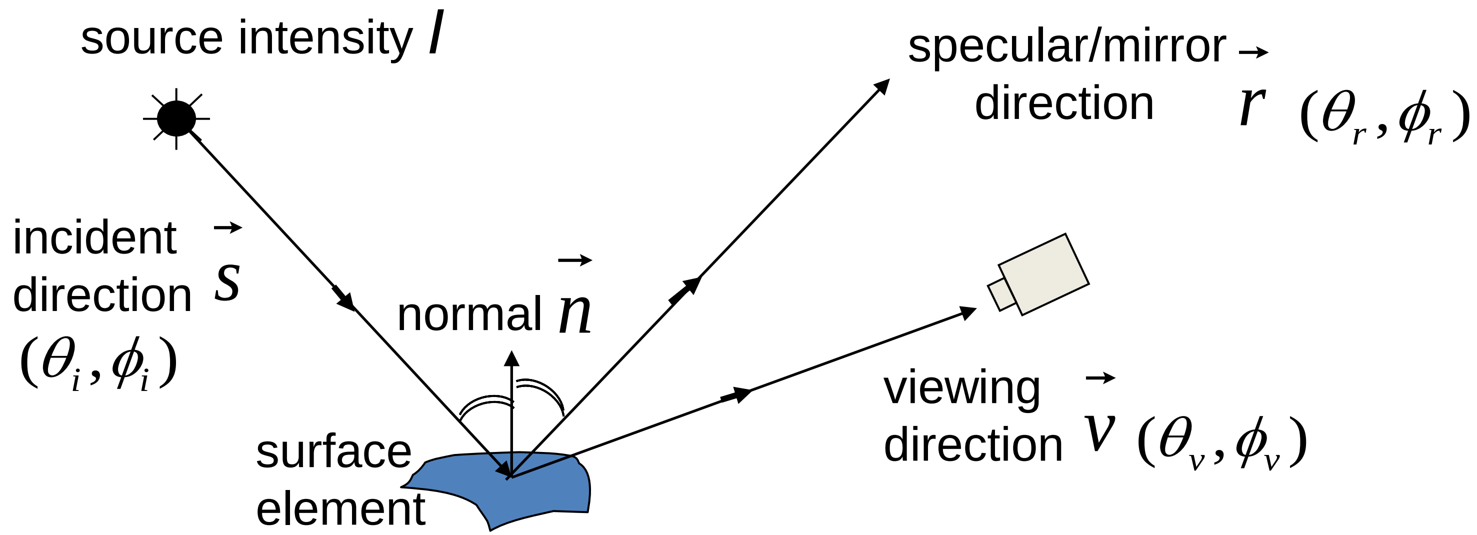
What does the BRDF equal in this case?



$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.
- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).
- Mirror BRDF is simply a double-delta function :

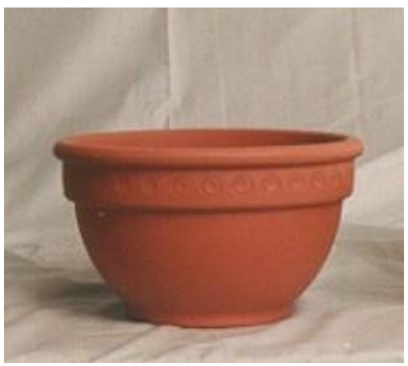
specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s\ \delta(\theta_i - \theta_v)\ \delta(\phi_i + \pi - \phi_v)$$

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



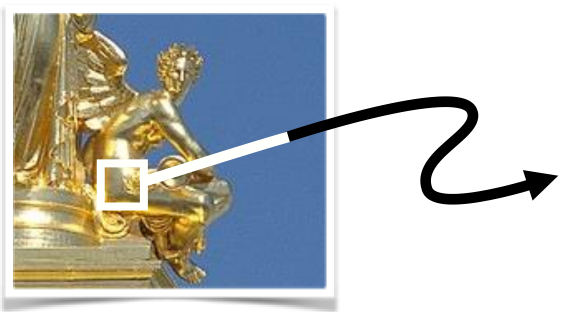
Surface Reflection:

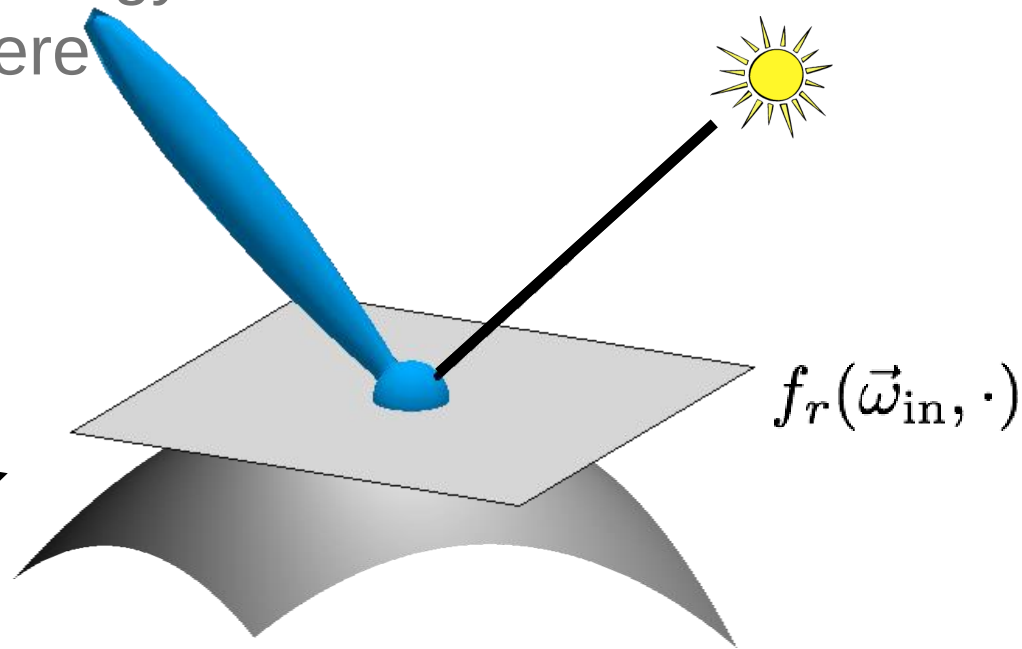
Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere





$$\boxed{f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})}$$

Bi-directional Reflectance Distribution Function (BRD

Thank you: Question?