

Computer Vision

Single-View Metrology and Camera Properties

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Perspective and 3D Geometry

- **Camera models and Projective geometry**

- What's the **mapping between image and world** coordinates?

- **Projection Matrix and Camera calibration**

- What's the **projection matrix** between scene and image coordinates?
- How to **calibrate** the projection matrix?

- **Single view metrology and Camera properties**

- How can we measure the **size of 3D objects** in an image?
- What are the important **camera properties**?

- **Photo stitching**

- What's the **mapping from two images** taken **without camera translation**?

- **Epipolar Geometry and Stereo Vision**

- What's the **mapping from two images** taken **with camera translation**?

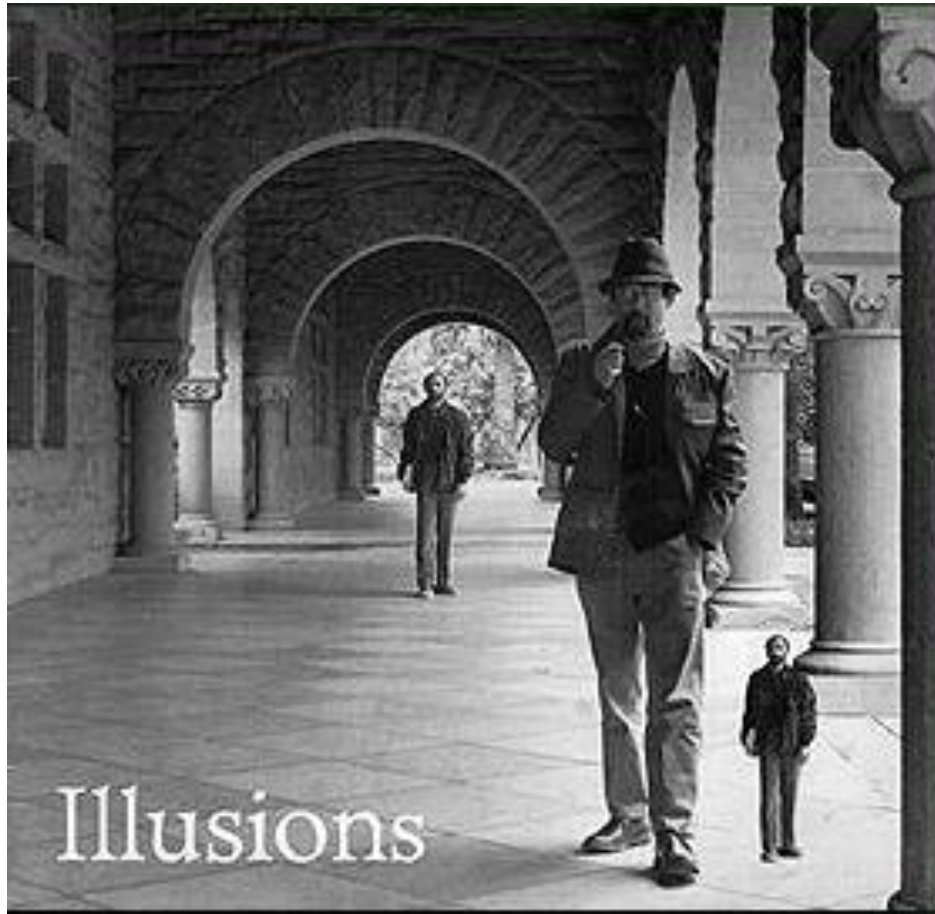
- **Structure from motion**

- How can we **recover 3D points from multiple images**?

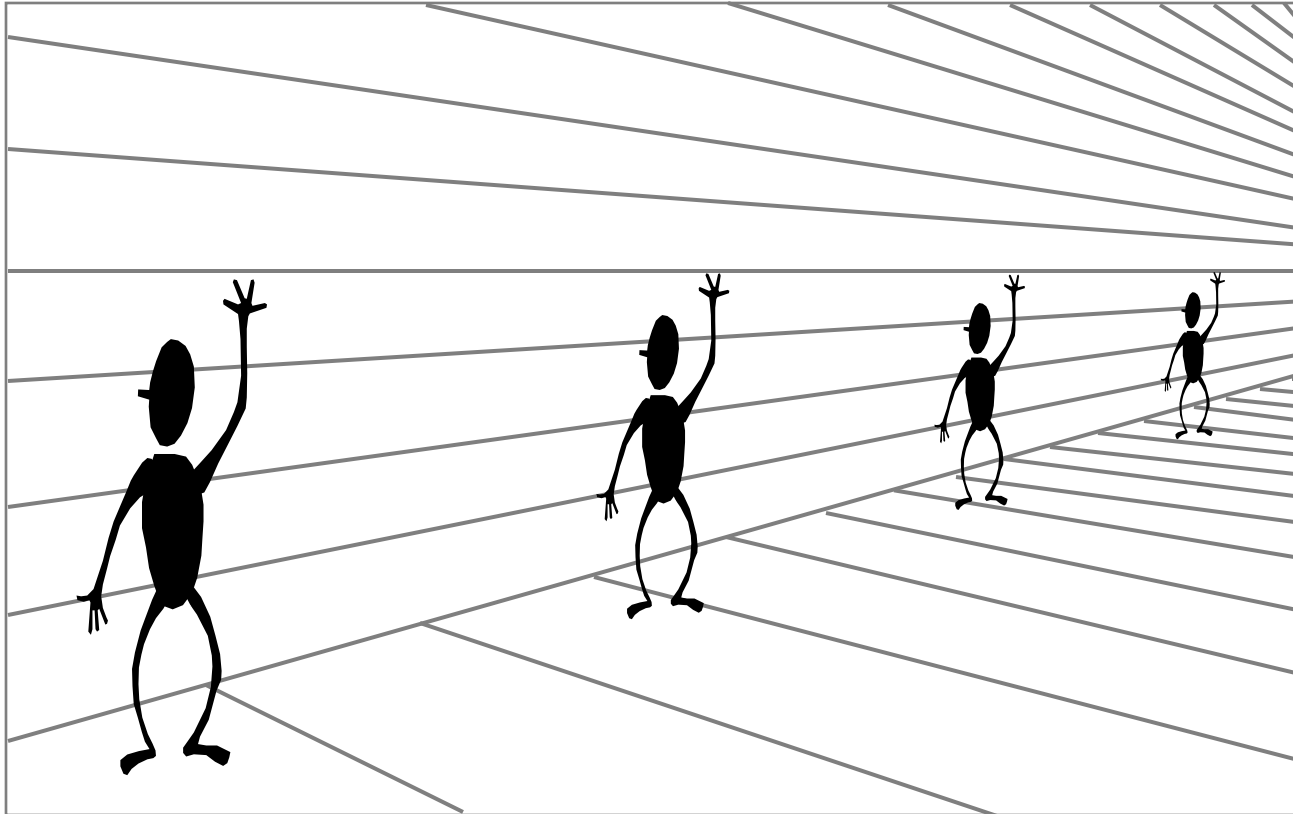
This class

- How can we measure the size of objects in the world from an image?
- What about other camera properties:
 - focal length,
 - field of view,
 - depth of field,
 - aperture,
 - f-number

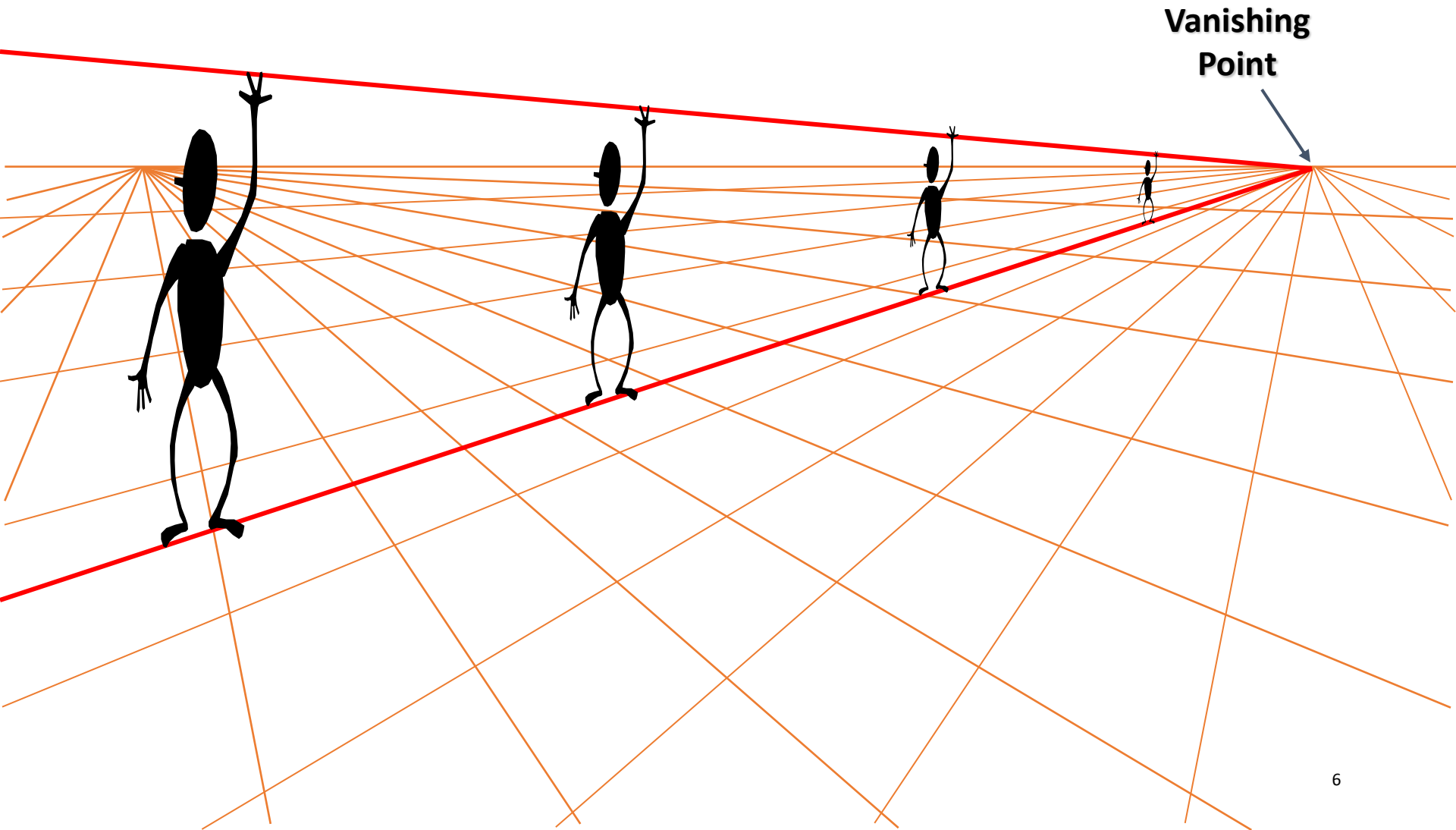
How can we measure the size of 3D objects from an image?



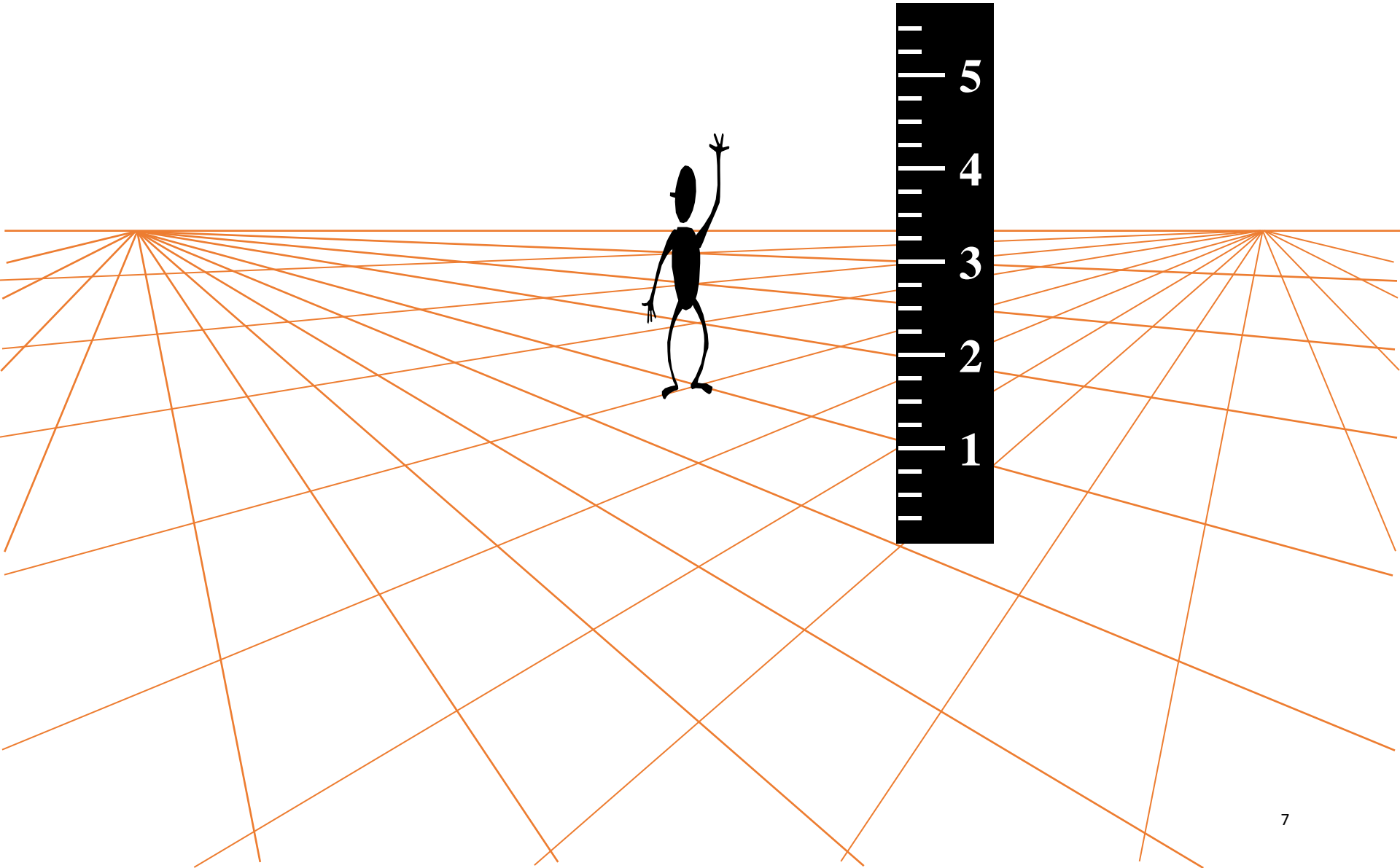
Perspective cues



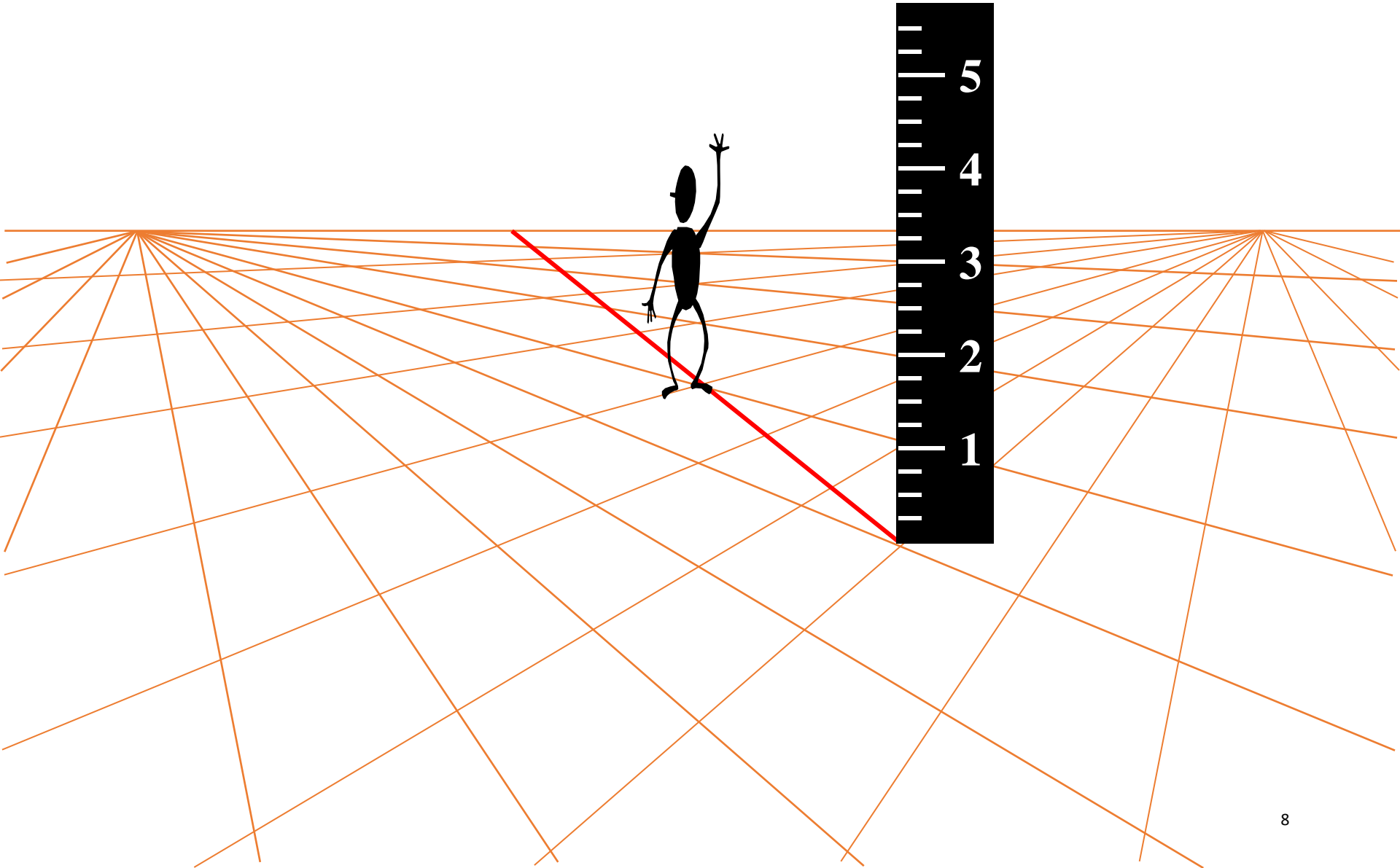
Comparing heights



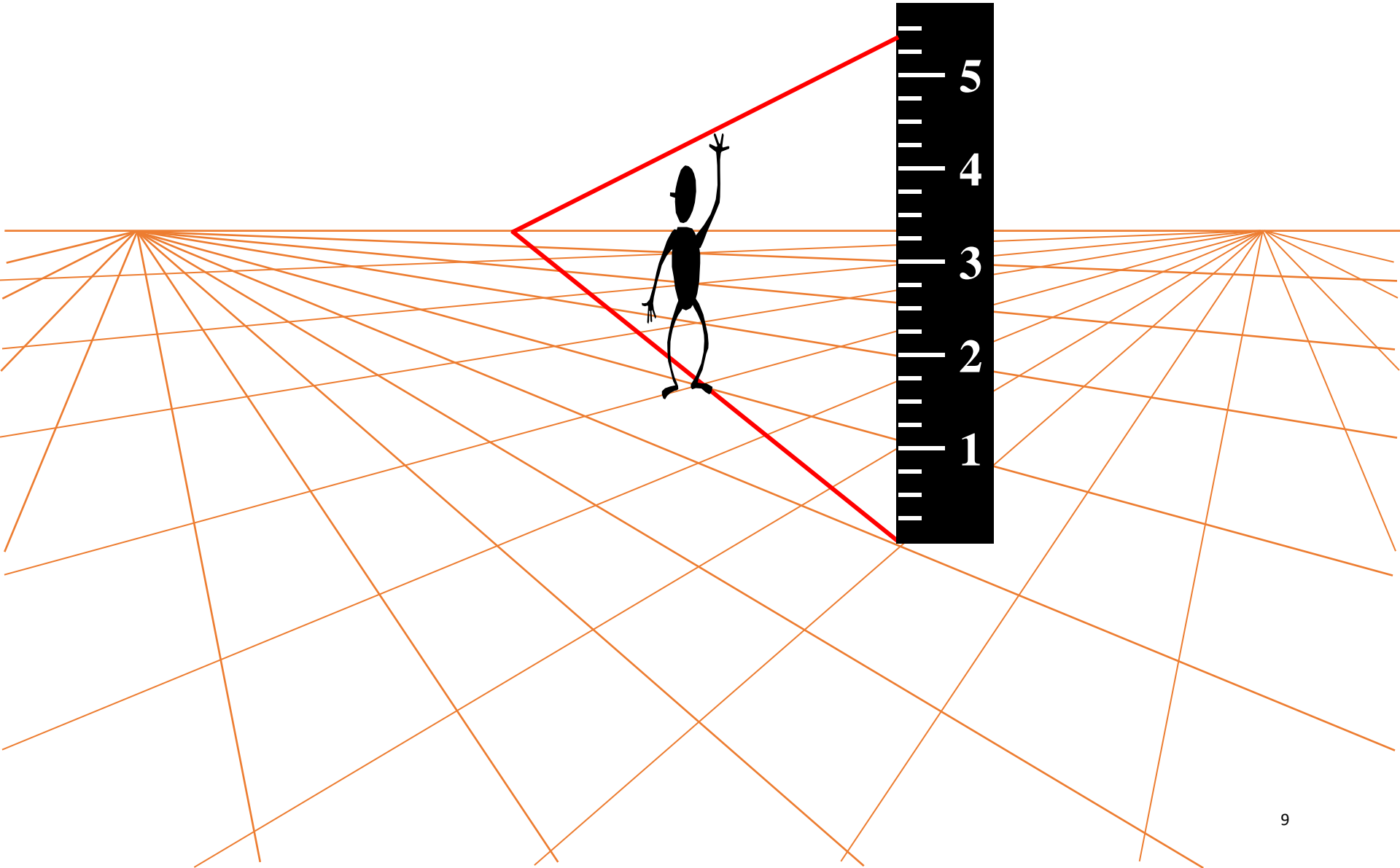
Measuring height



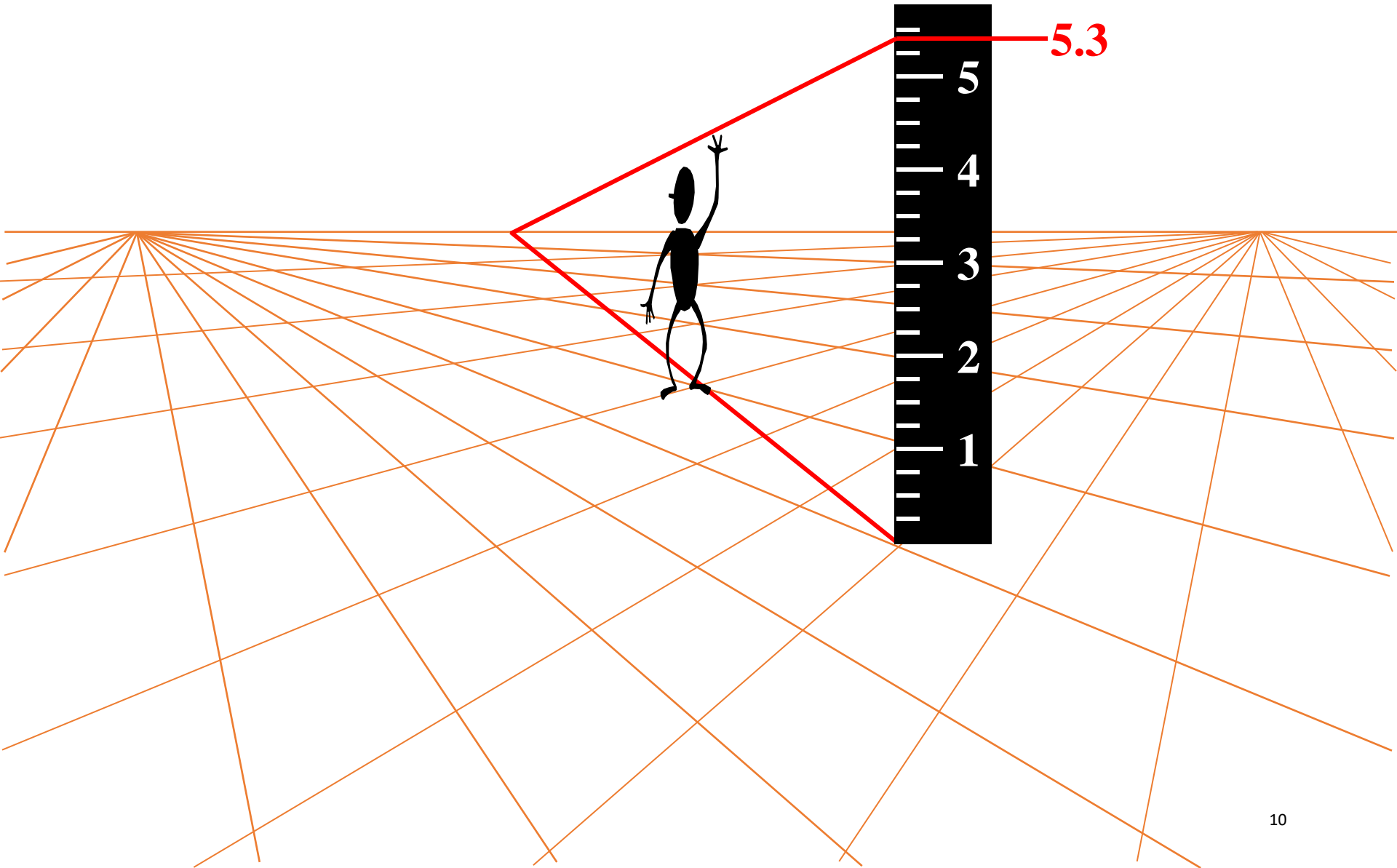
Measuring height



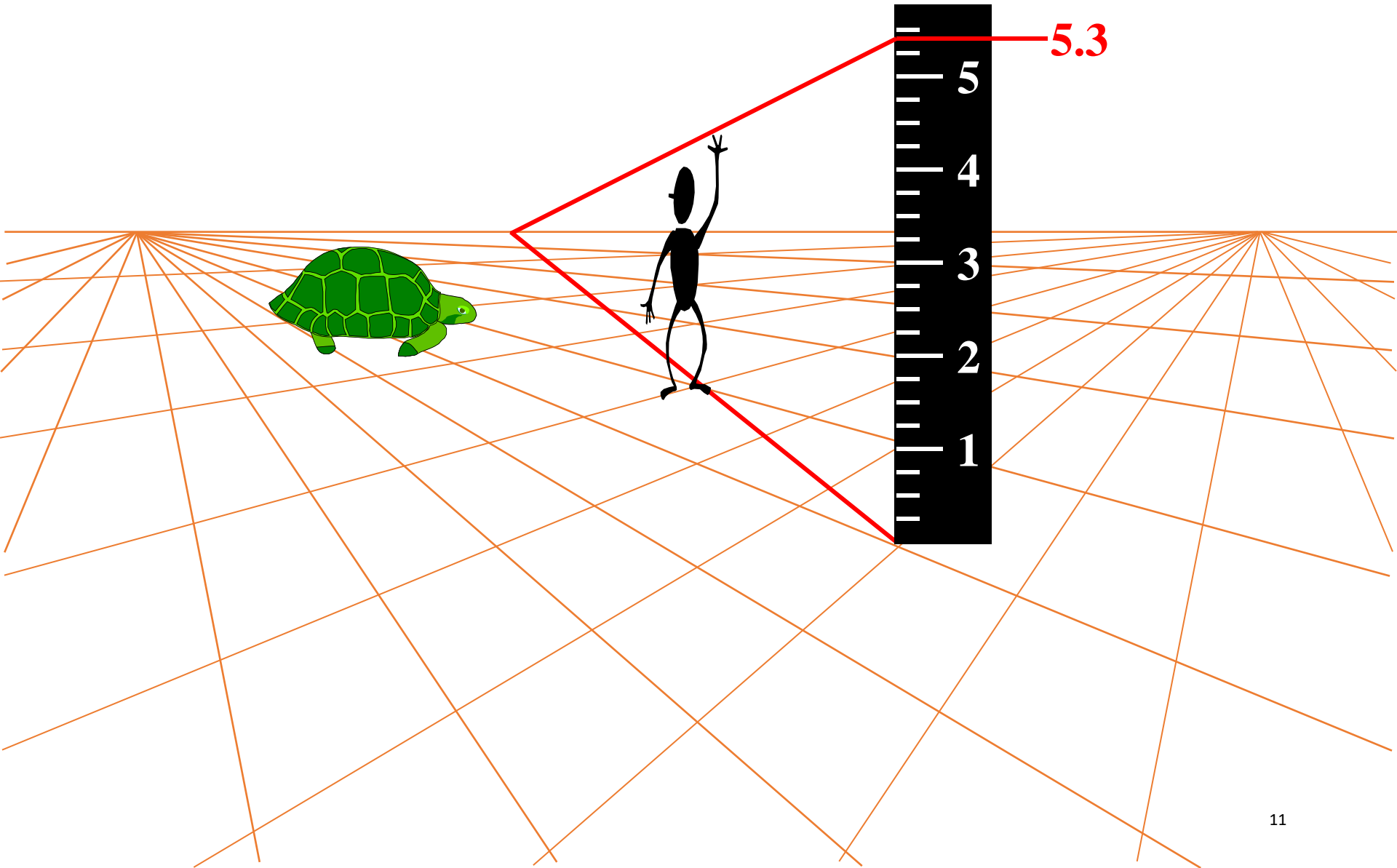
Measuring height



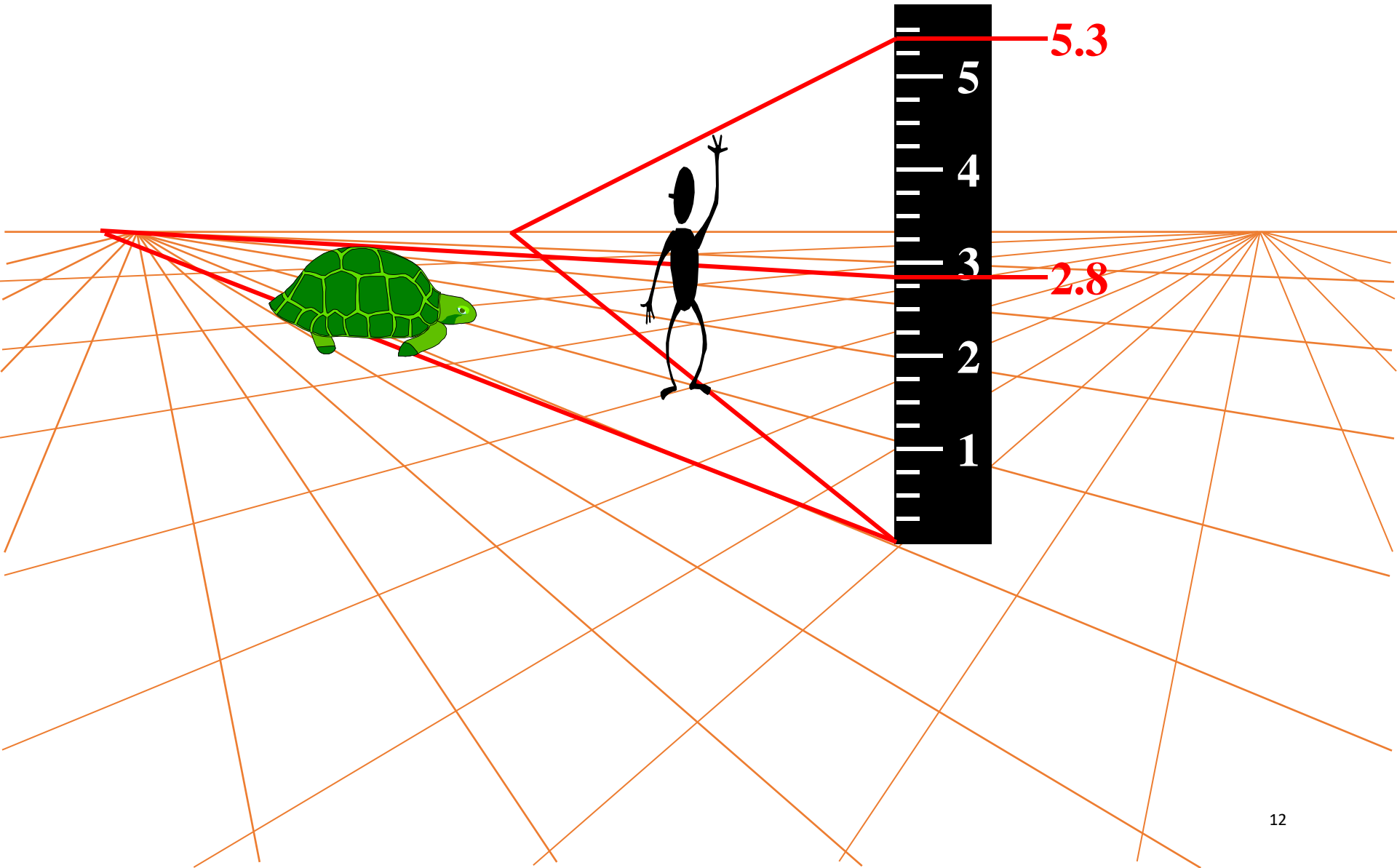
Measuring height



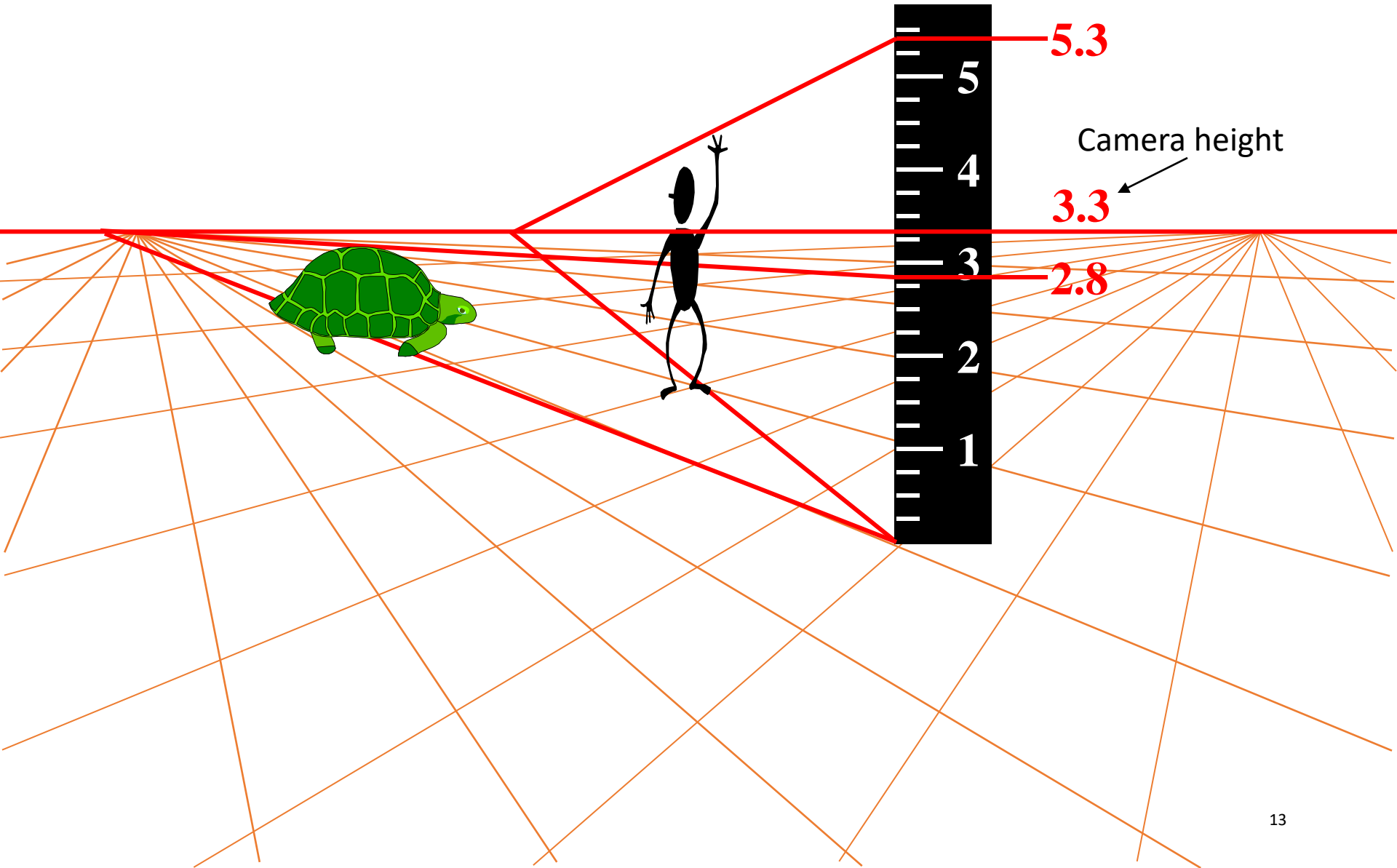
Measuring height



Measuring height



Measuring height



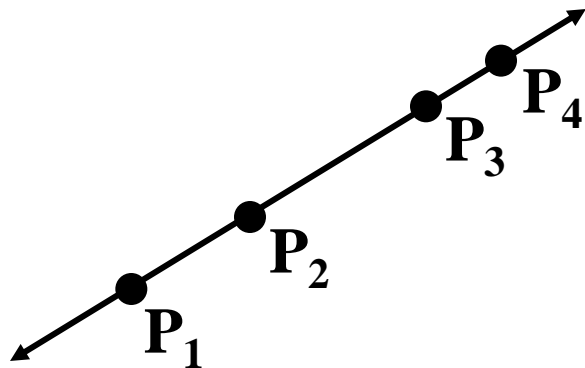
Which is higher – the camera or the man in the parachute?



The cross ratio

- A Projective Invariant
 - Does not change under projective transformations

The cross-ratio of 4 collinear points



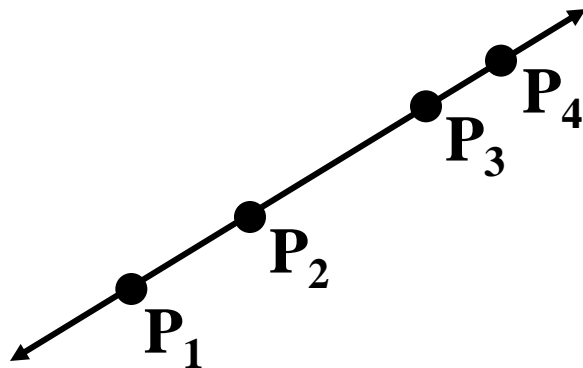
$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

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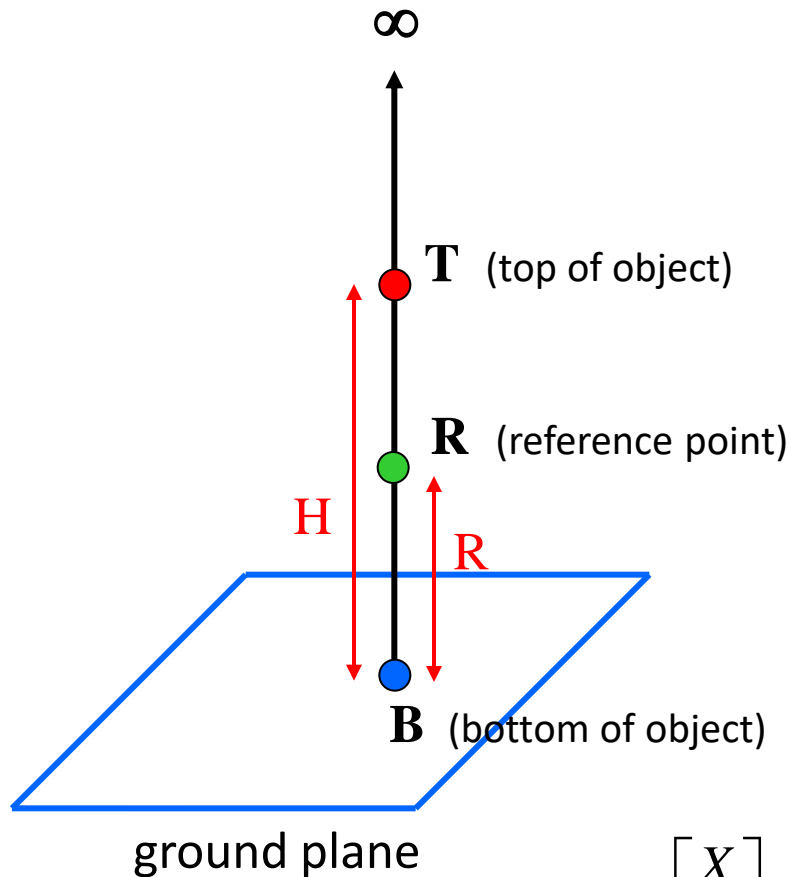
Can permute the point ordering

$$\frac{\|\mathbf{P}_1 - \mathbf{P}_3\| \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_1 - \mathbf{P}_2\| \|\mathbf{P}_4 - \mathbf{P}_3\|}$$

- $4! = 24$ different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

Measuring height



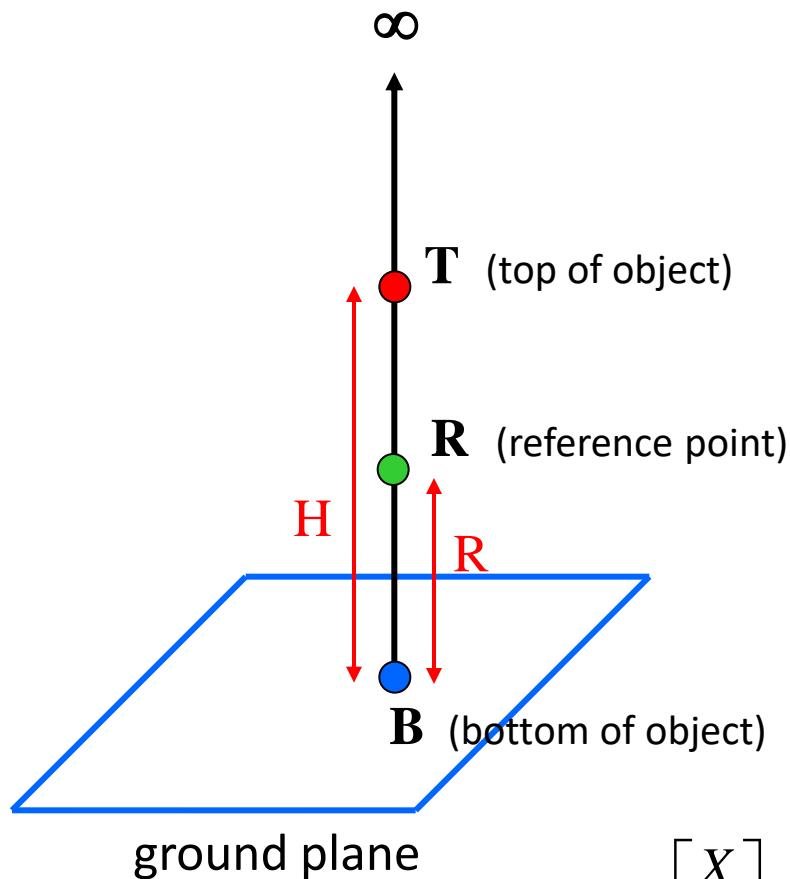
scene points represented as

$$\mathbf{P} = \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

image points as

$$\mathbf{p} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Measuring height



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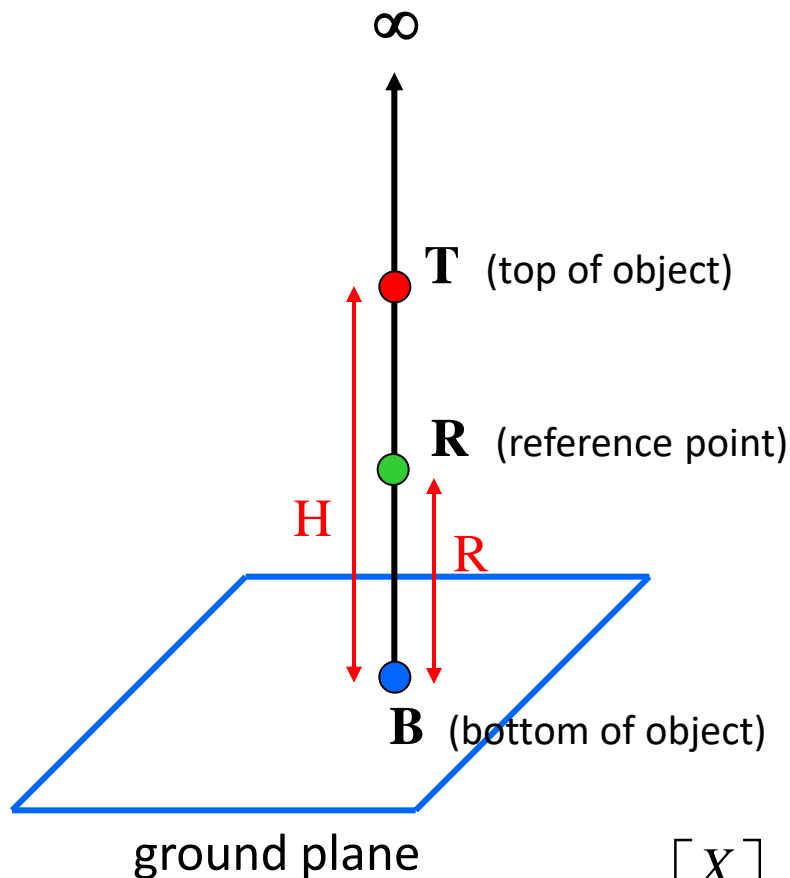
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$$\frac{\|\mathbf{B} - \mathbf{T}\| \|\infty - \mathbf{R}\|}{\|\mathbf{B} - \mathbf{R}\| \|\infty - \mathbf{T}\|}$$

scene cross ratio

Measuring height



scene points represented as

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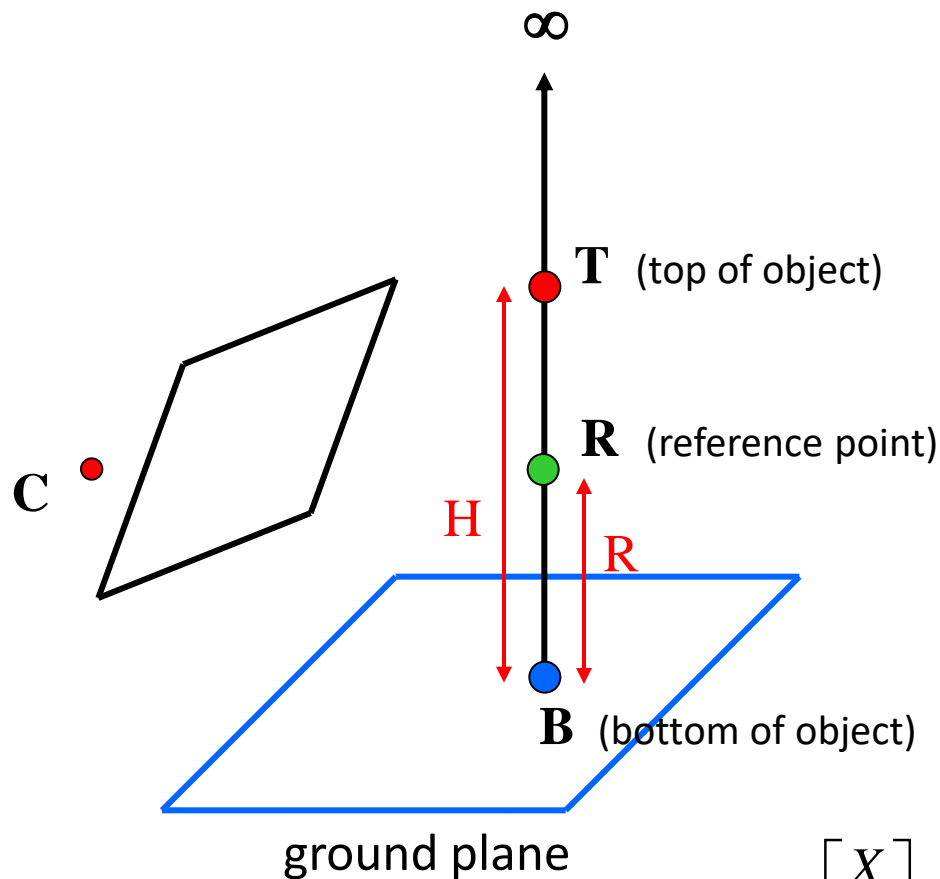
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scene cross ratio

Measuring height



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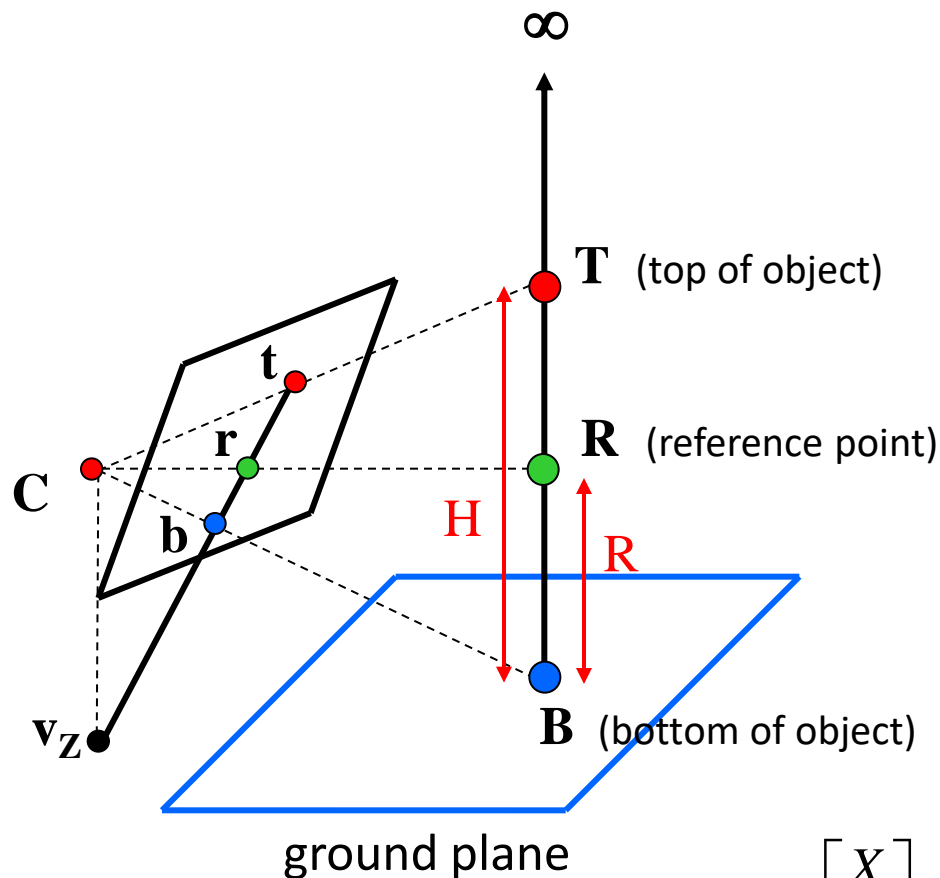
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scene cross ratio

Measuring height



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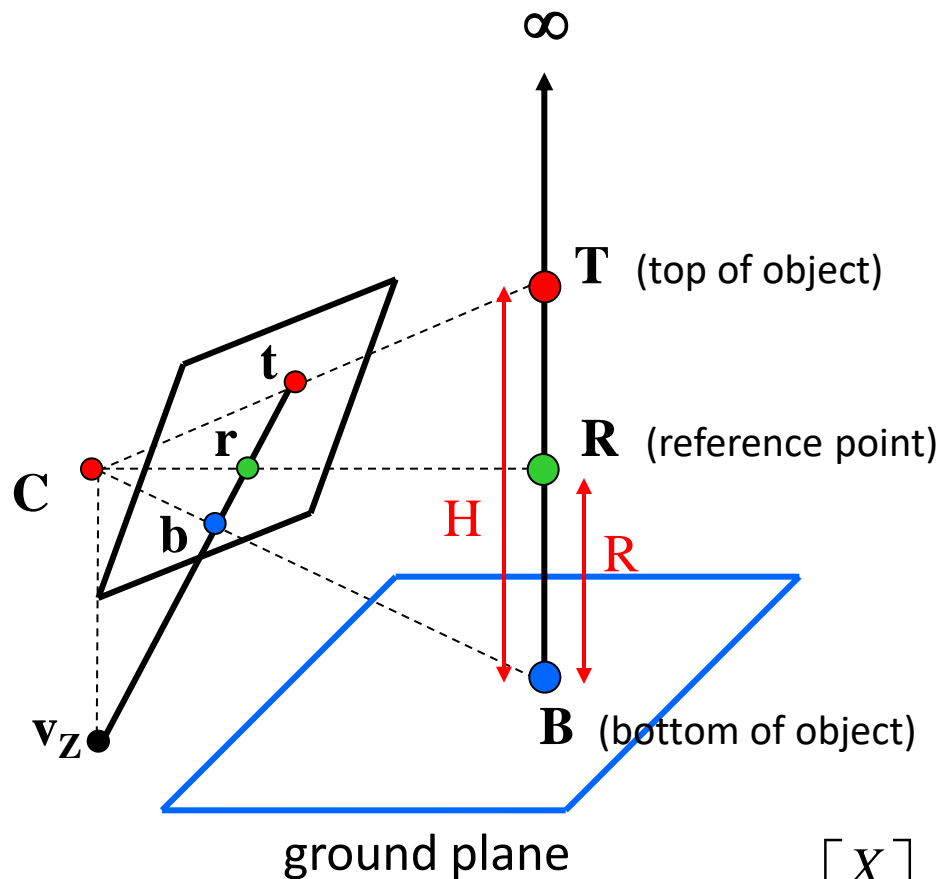
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scene cross ratio

Measuring height



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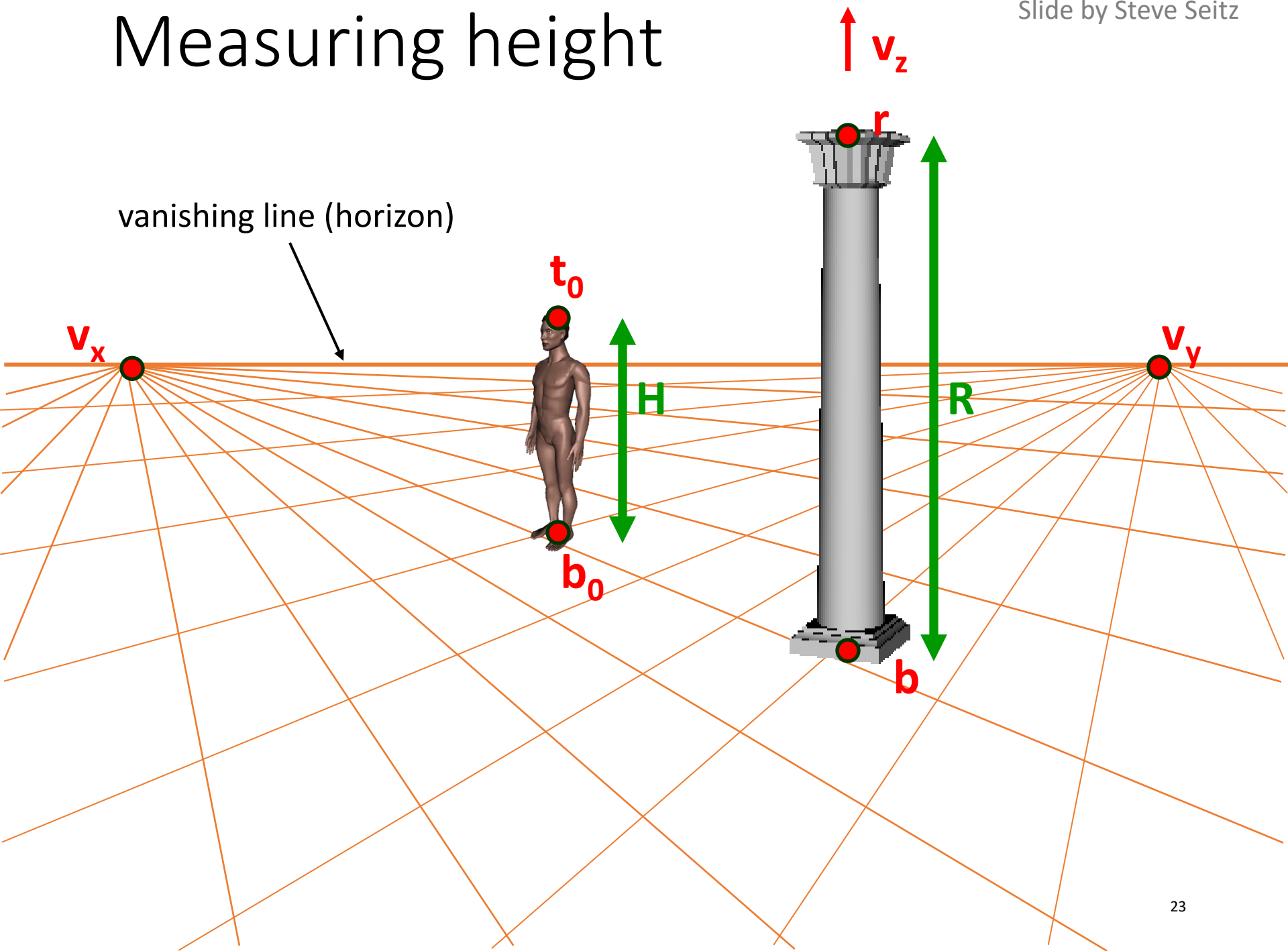
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scene cross ratio

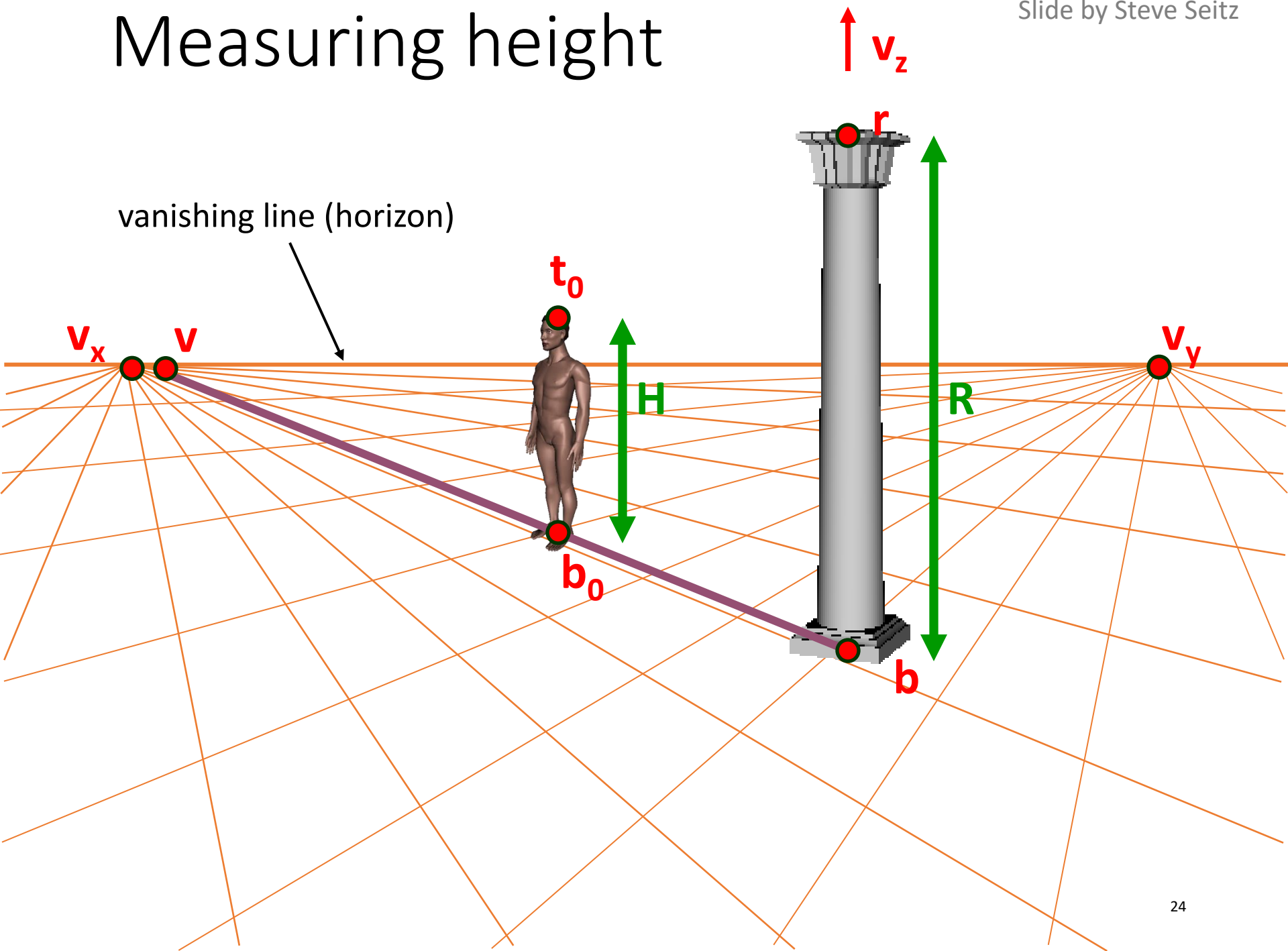
$$\frac{\|\mathbf{b} - \mathbf{t}\| \|\mathbf{v}_Z - \mathbf{r}\|}{\|\mathbf{b} - \mathbf{r}\| \|\mathbf{v}_Z - \mathbf{t}\|} = \frac{H}{R}$$

image cross ratio

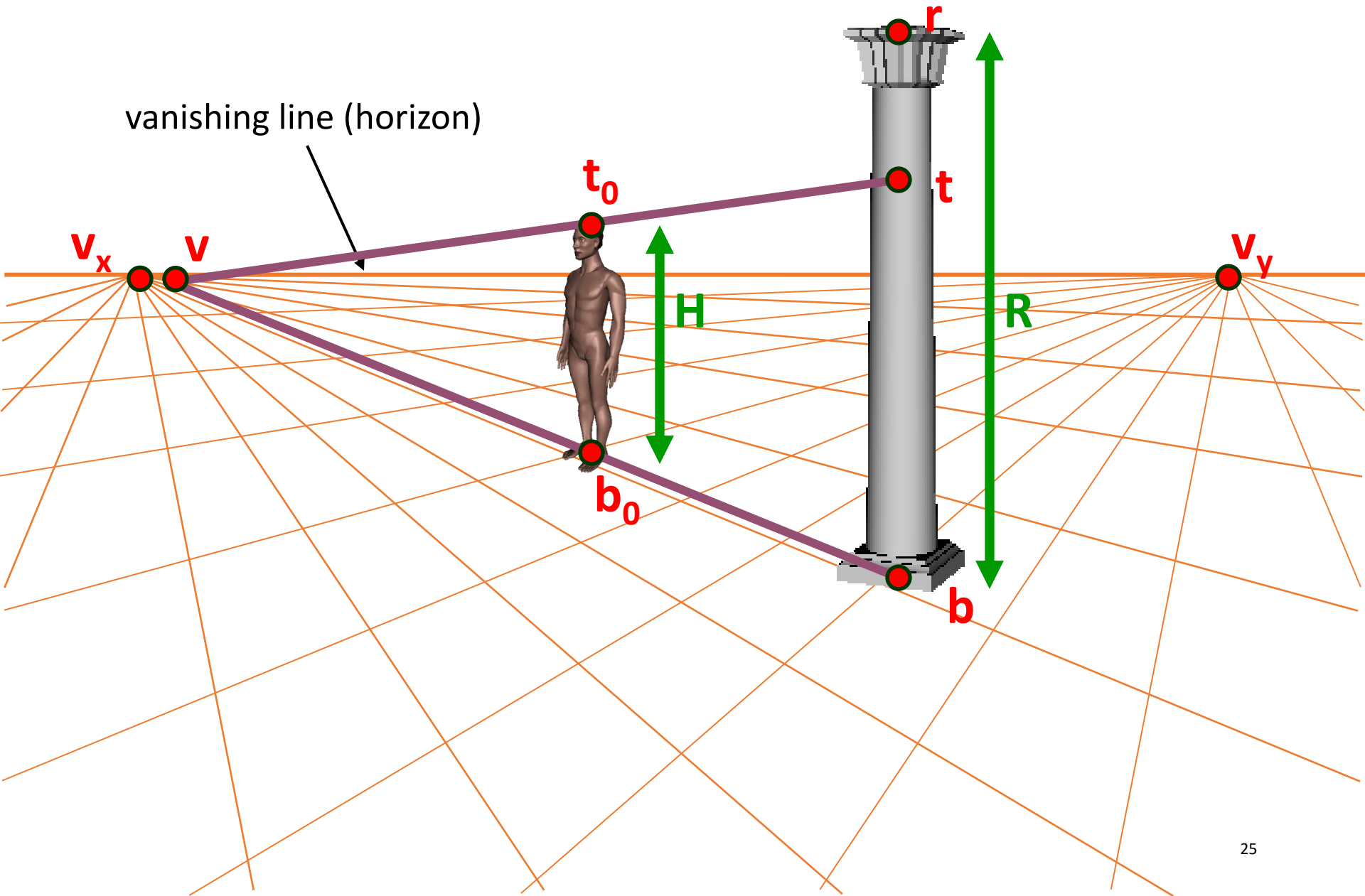
Measuring height



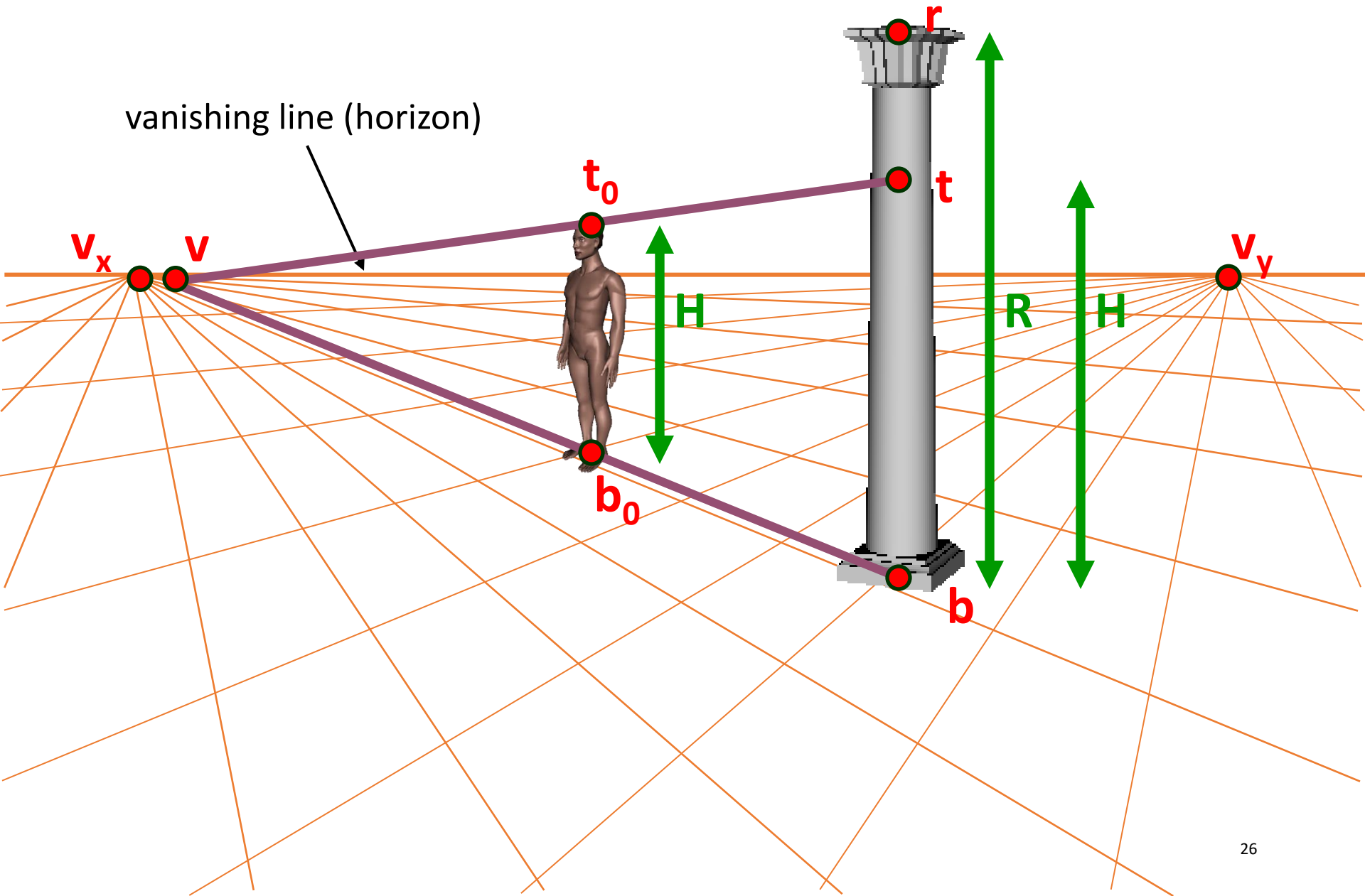
Measuring height



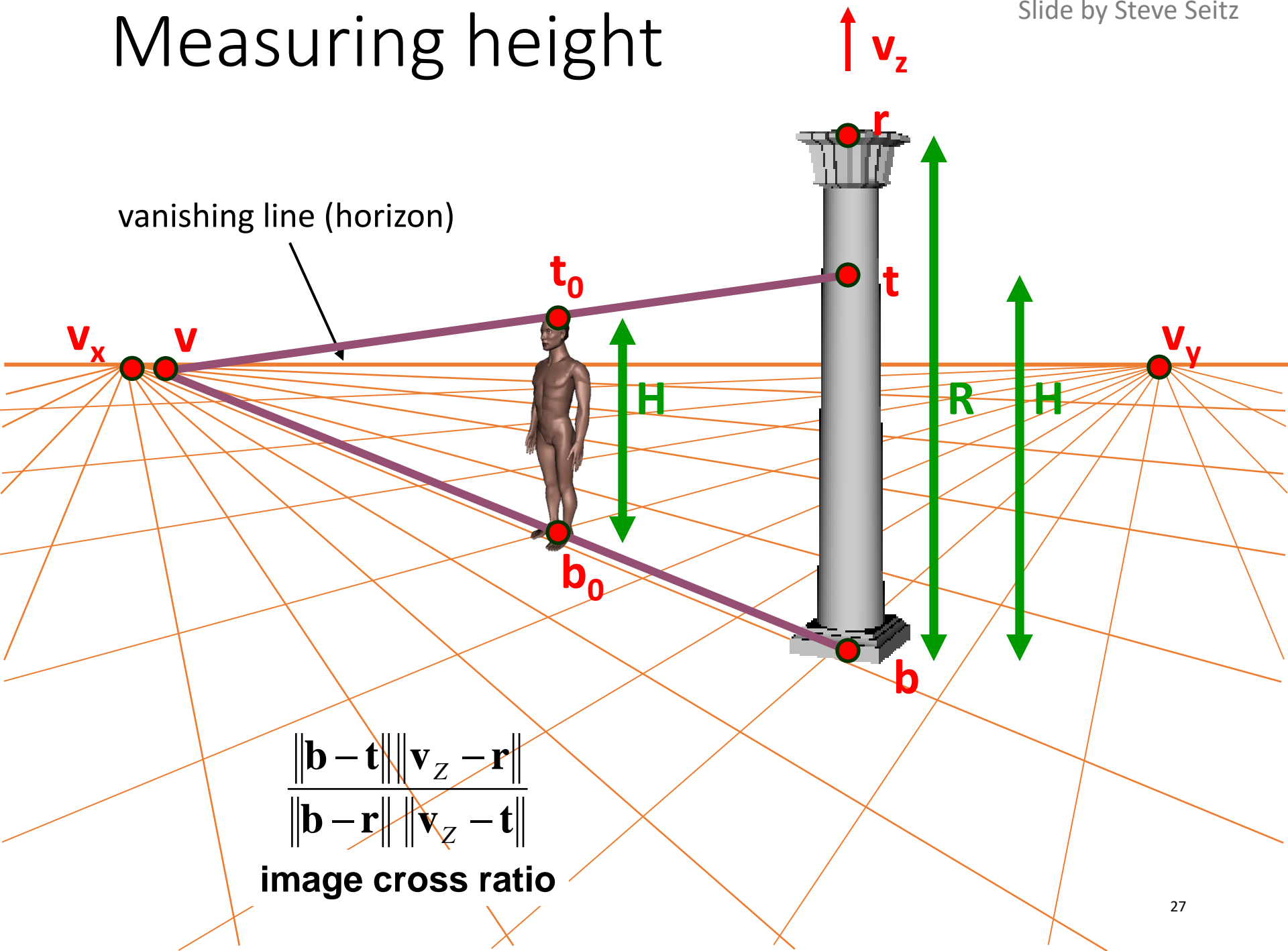
Measuring height



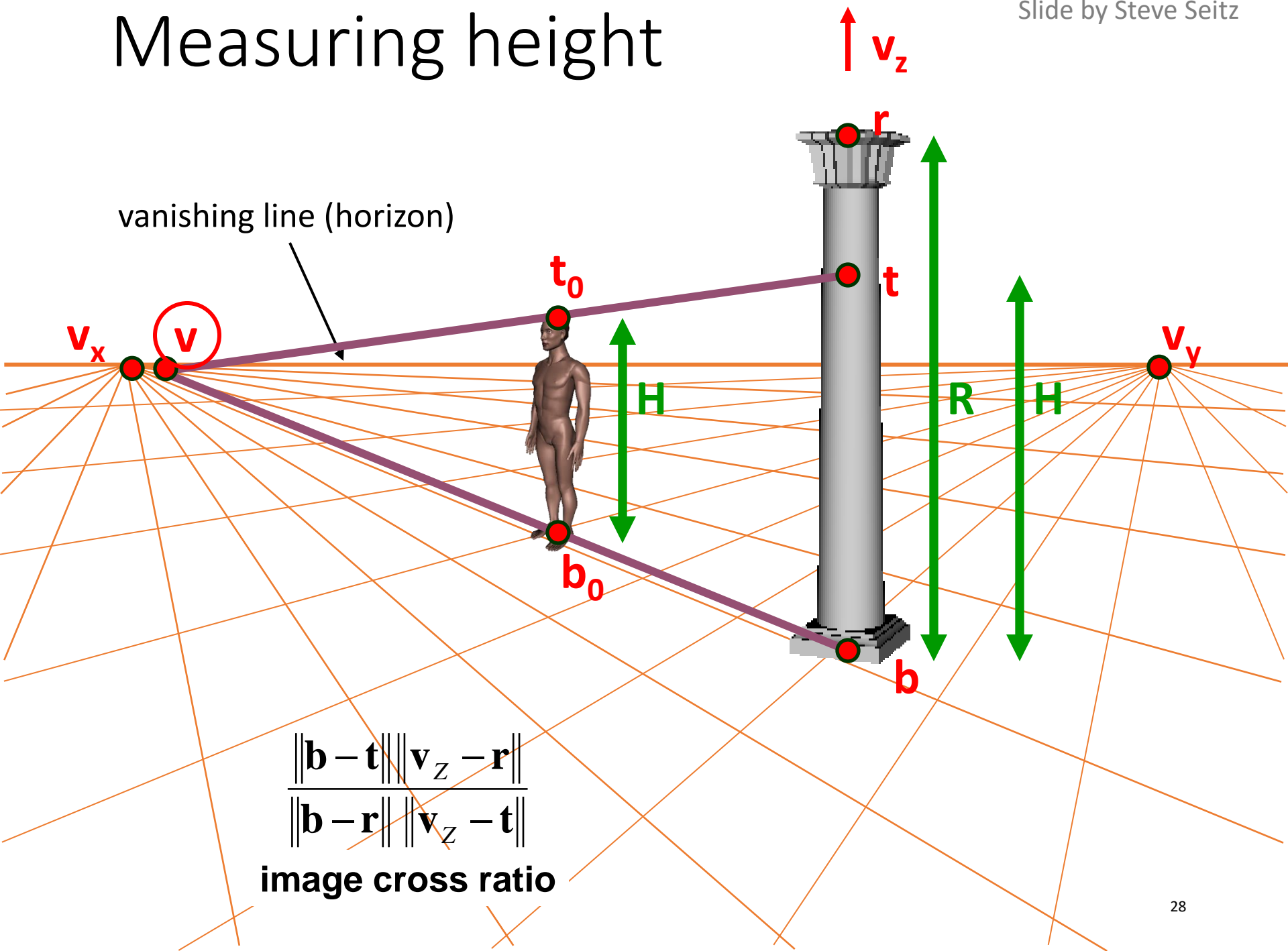
Measuring height



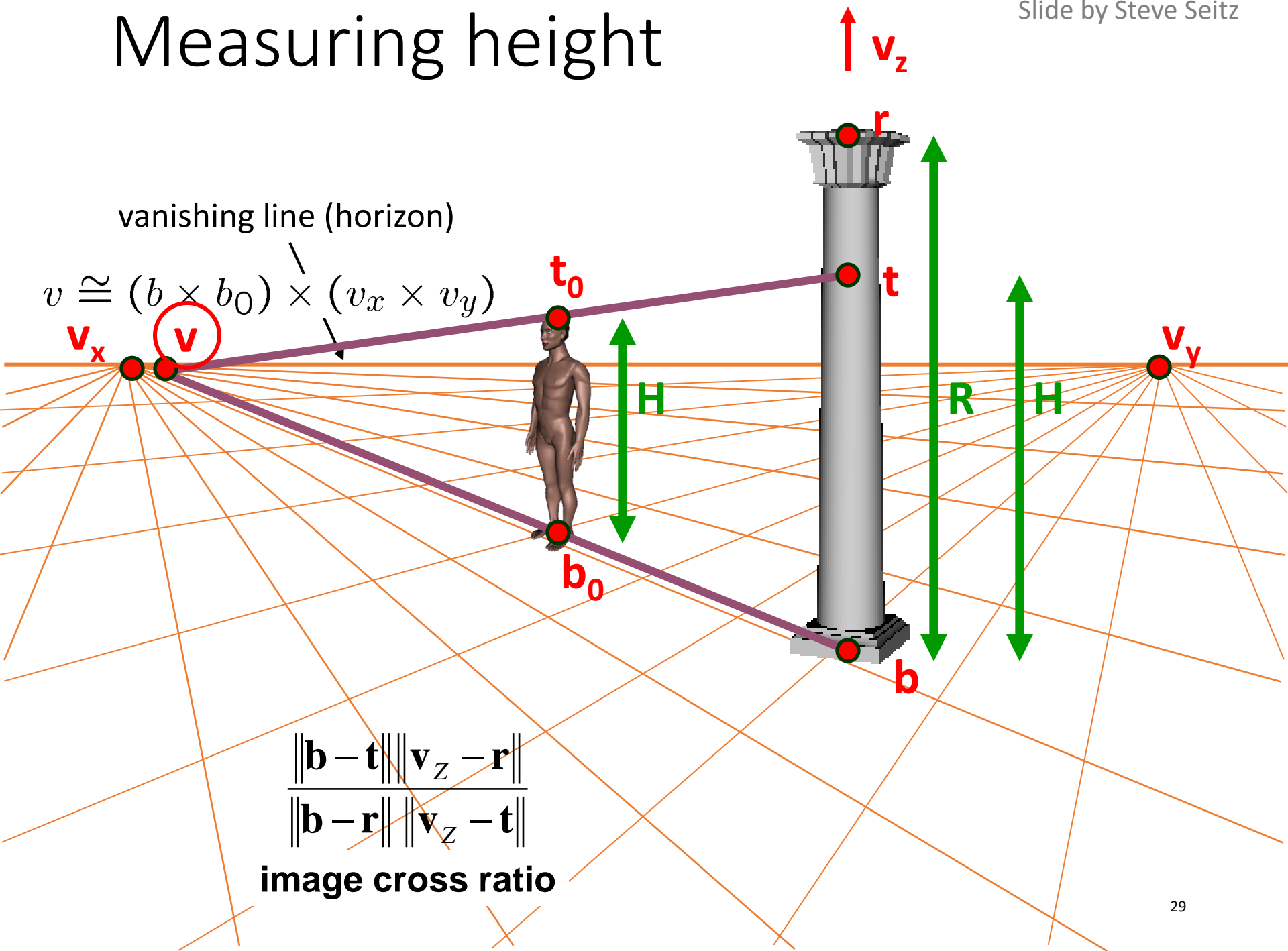
Measuring height



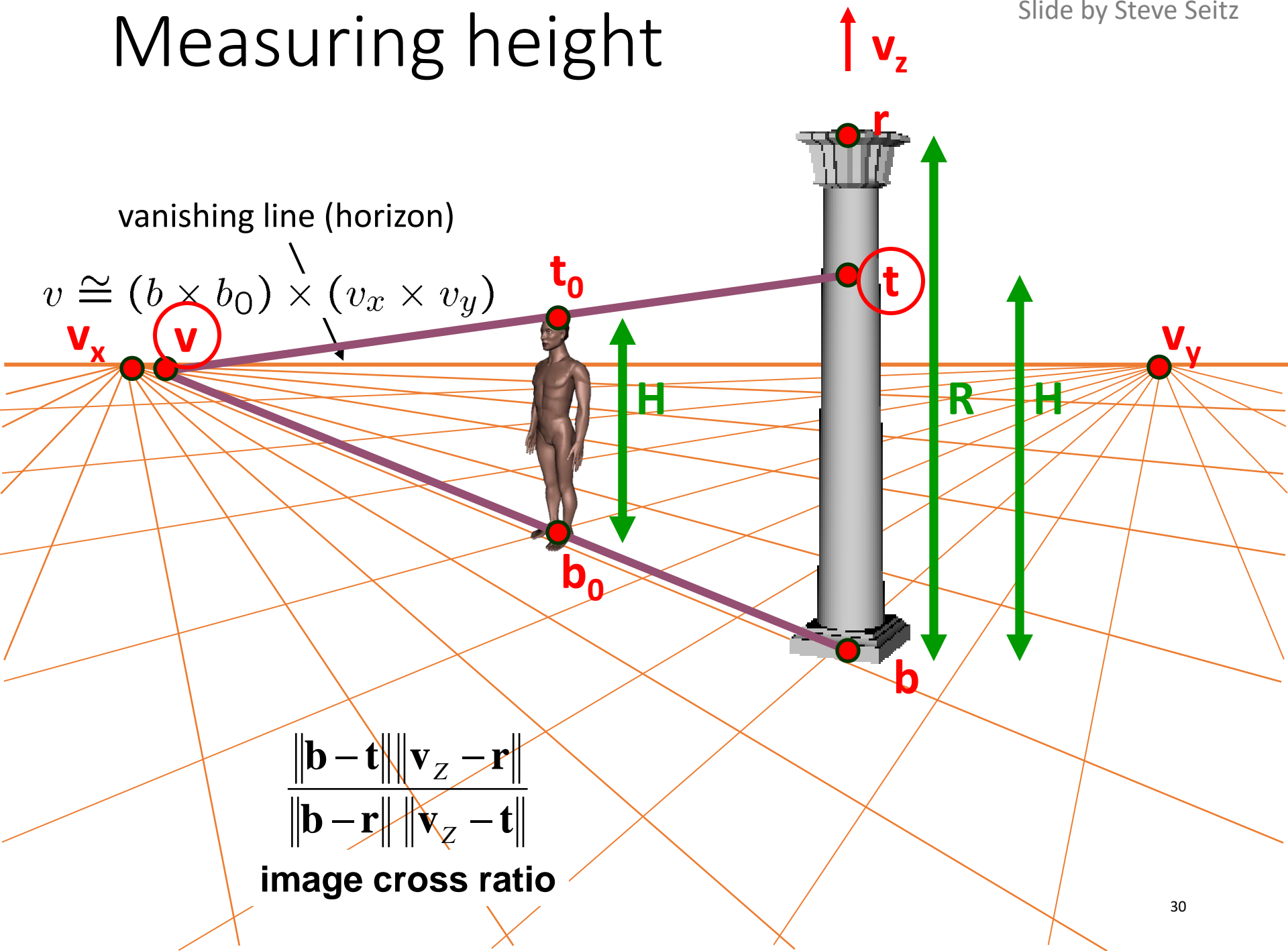
Measuring height



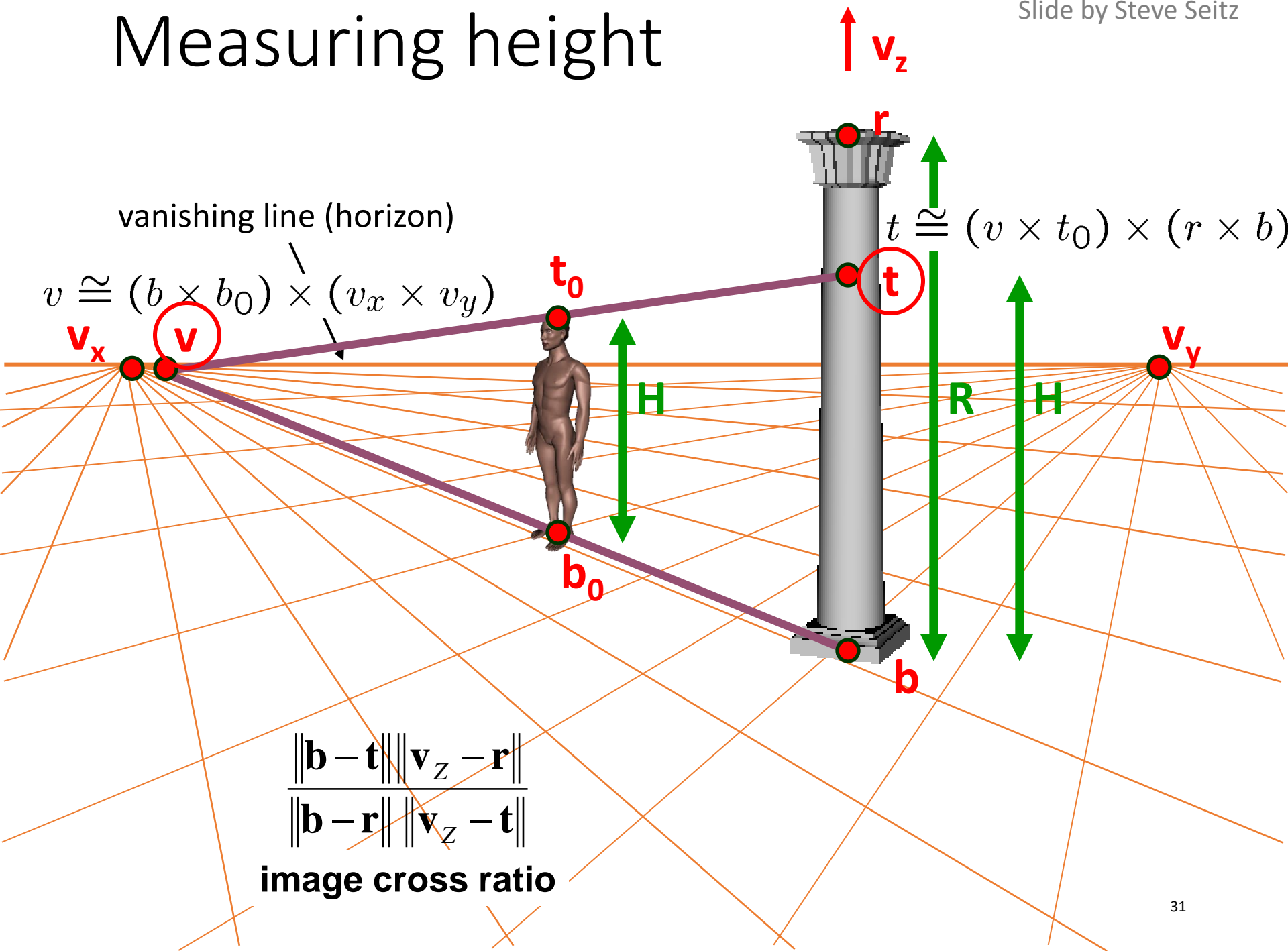
Measuring height



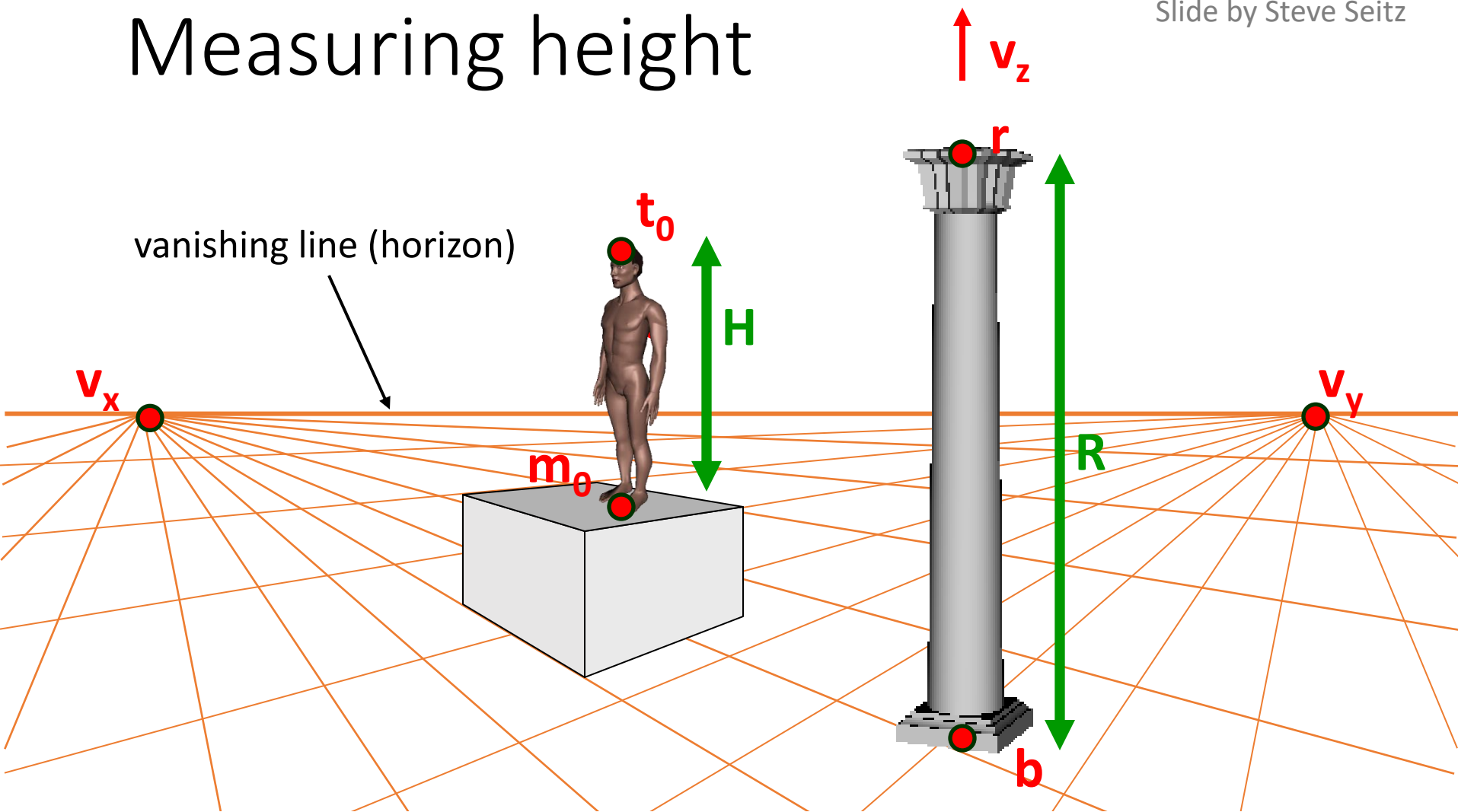
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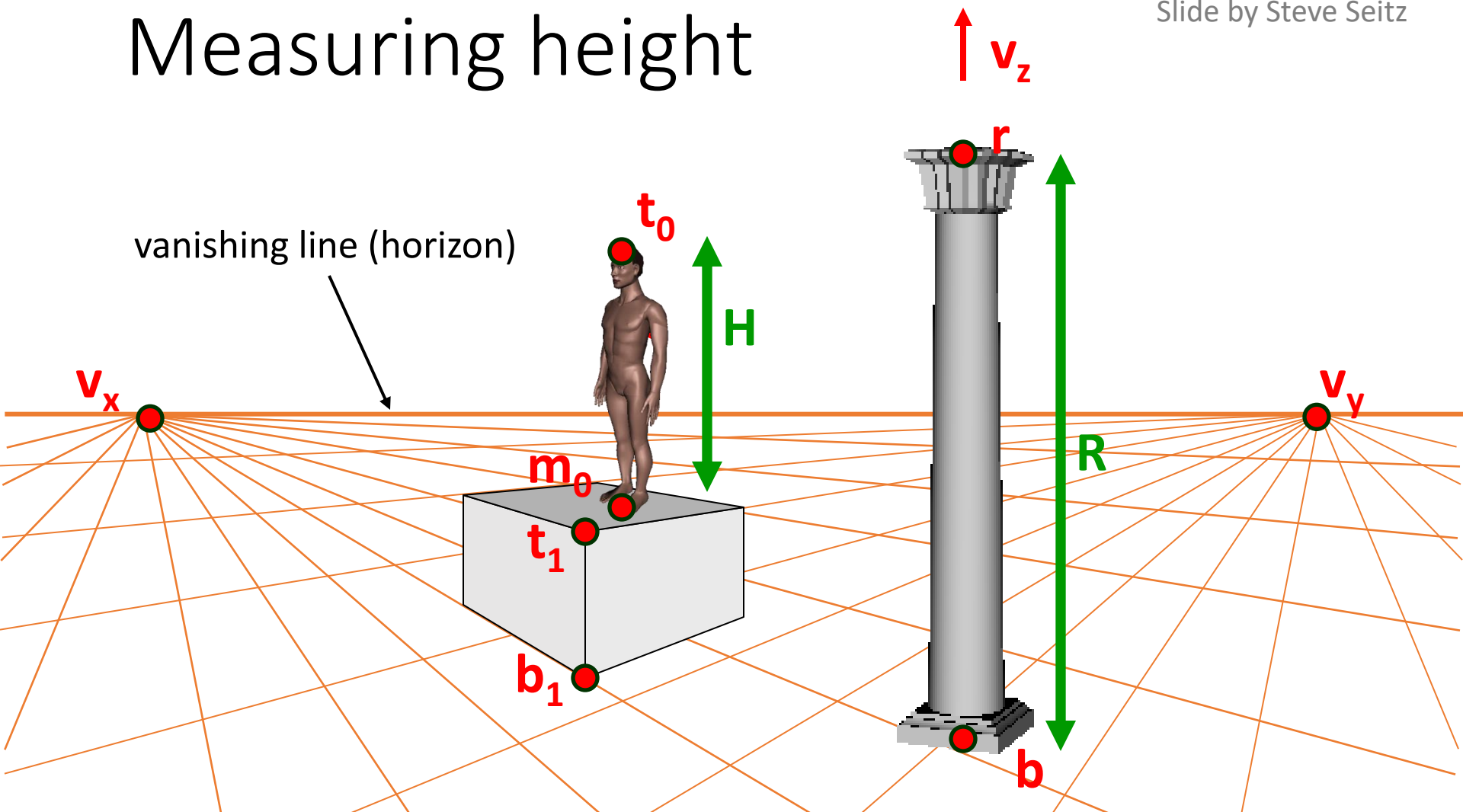
Measuring height



What if the point on the ground plane \mathbf{b}_0 is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find \mathbf{b}_0 as shown above

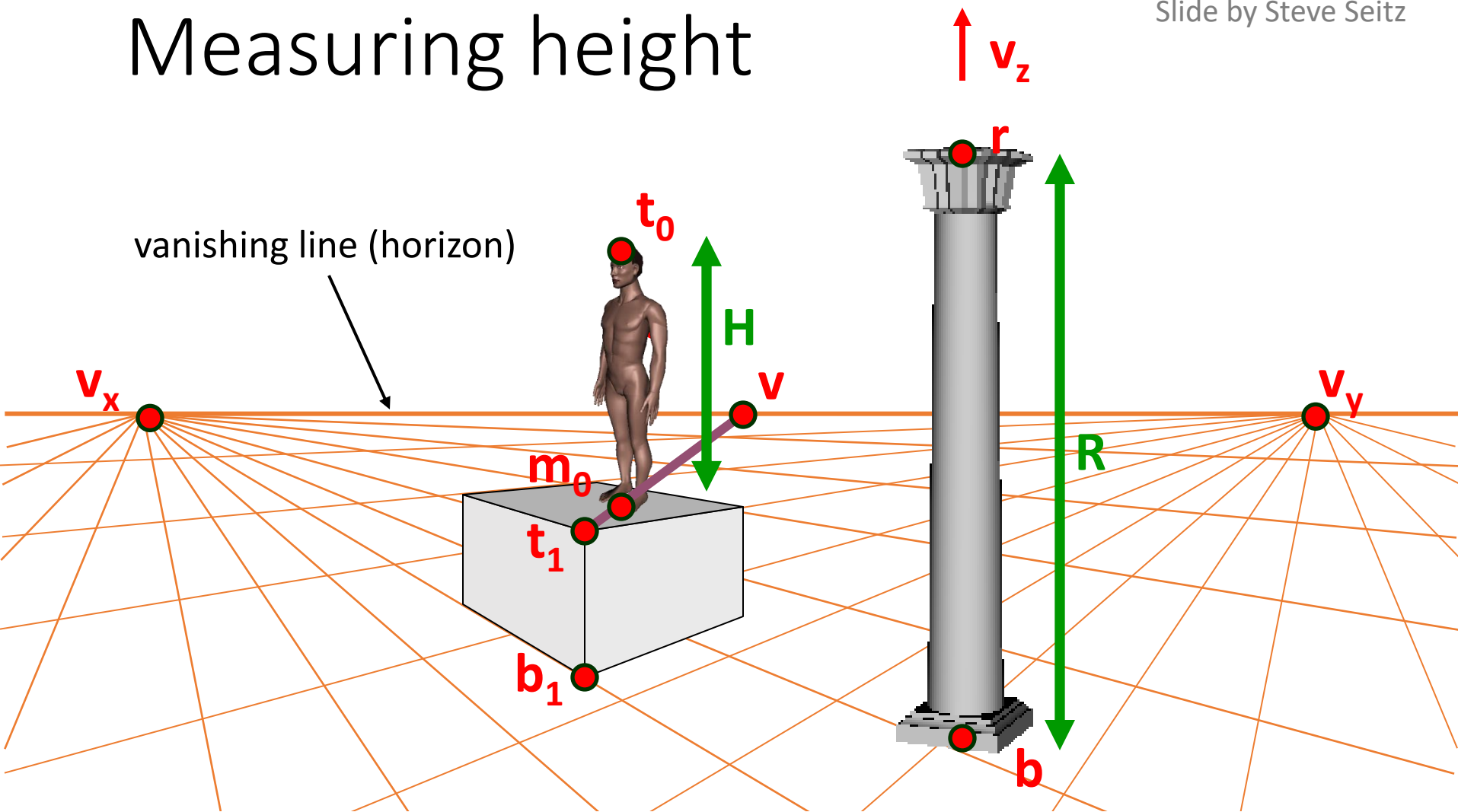
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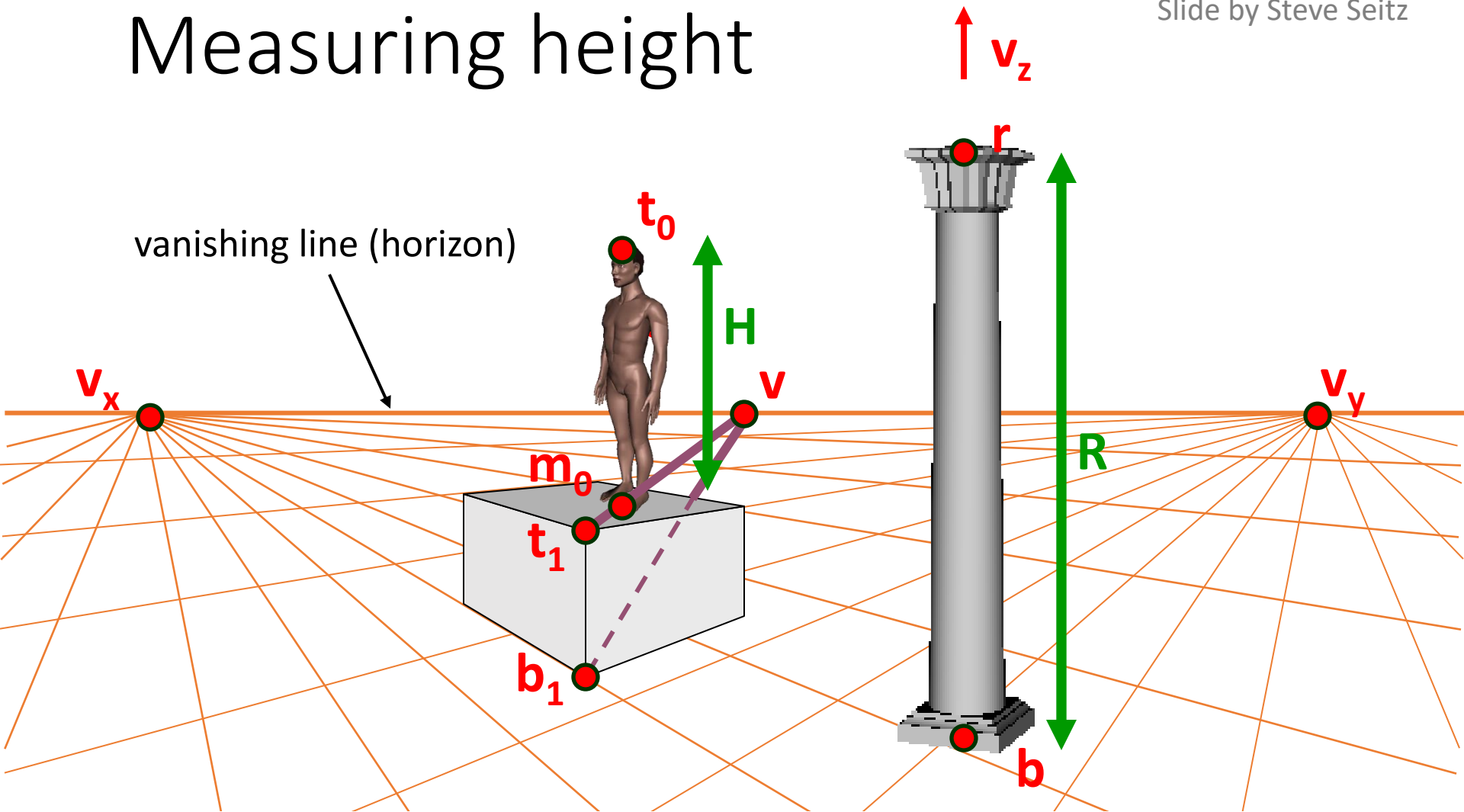
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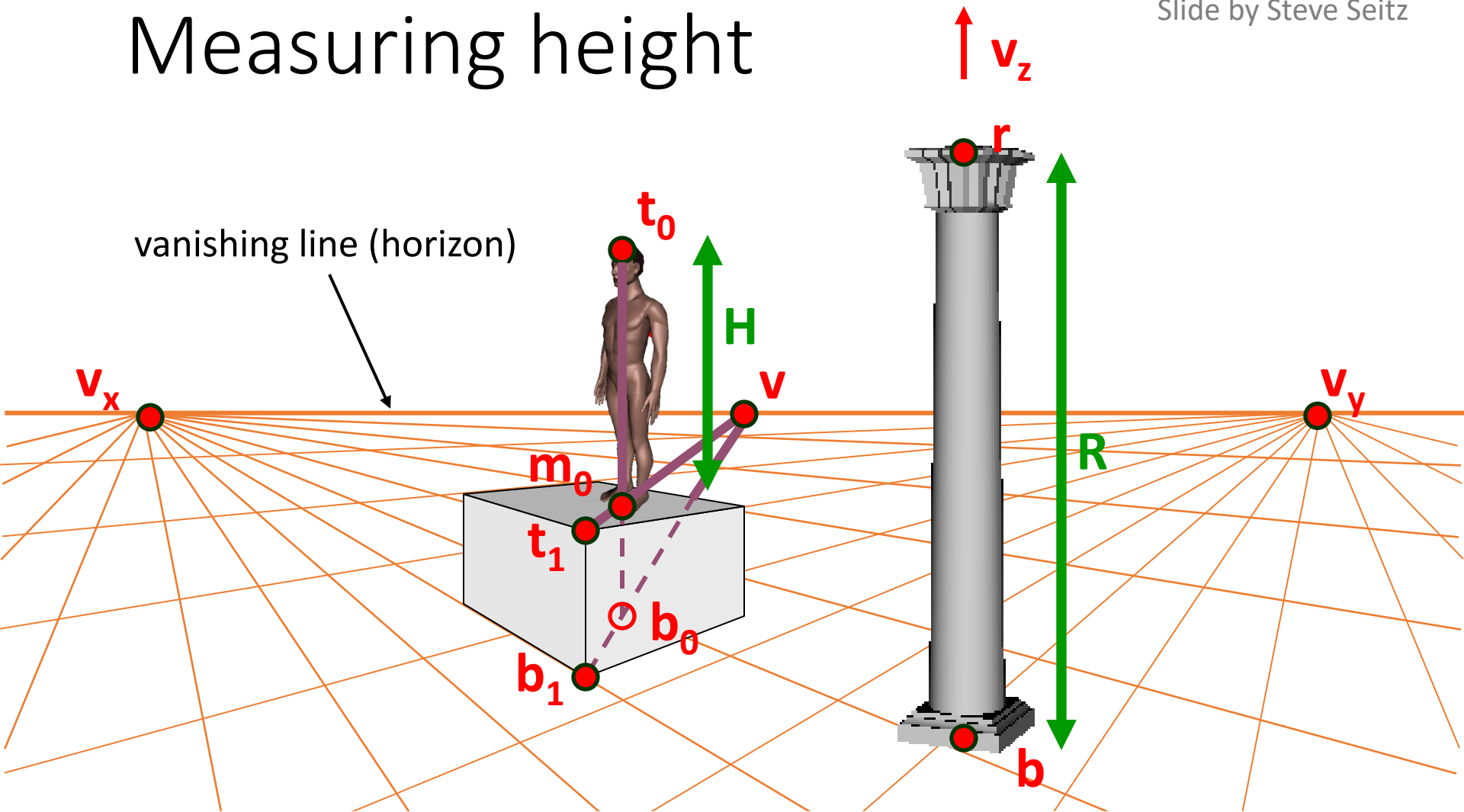
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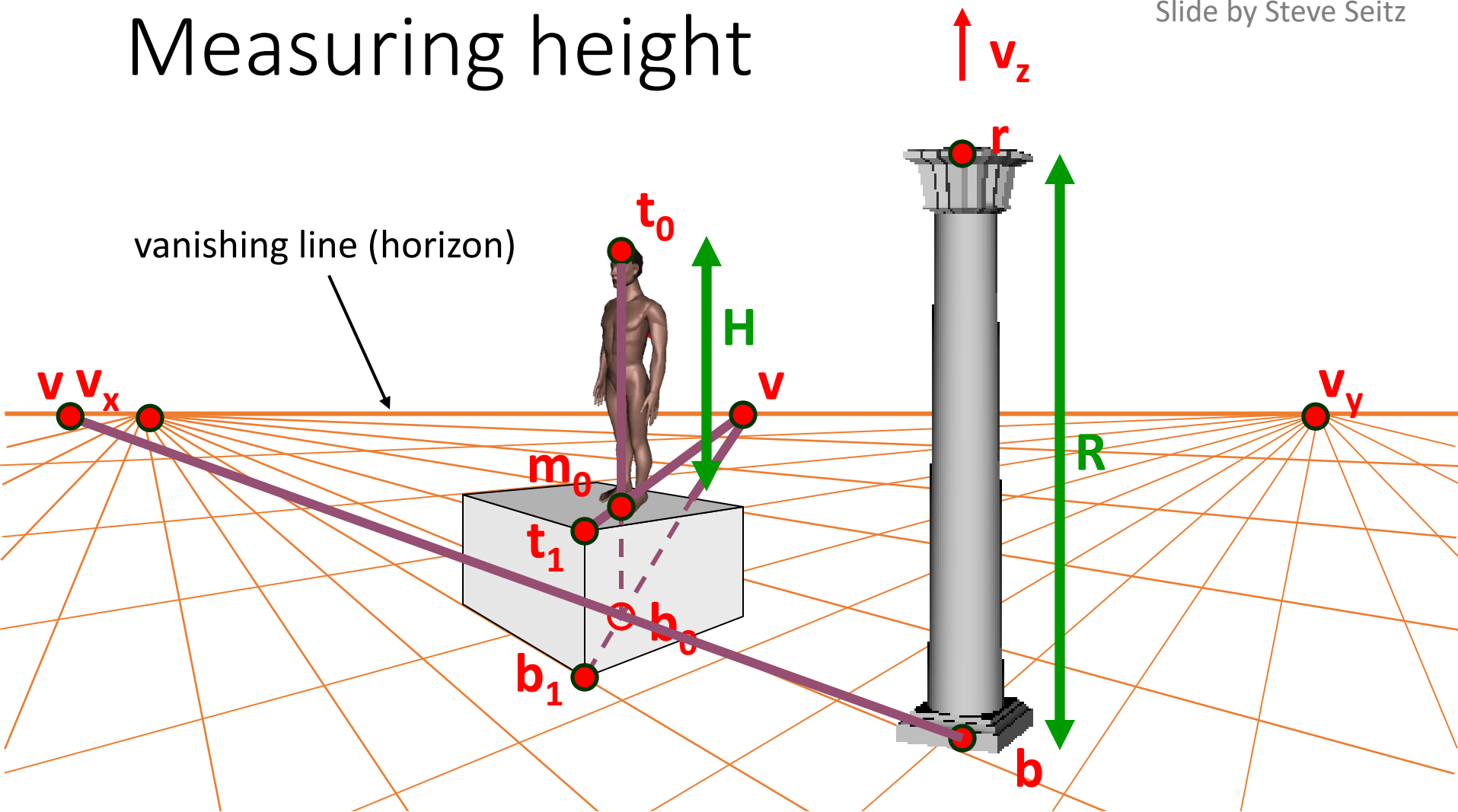
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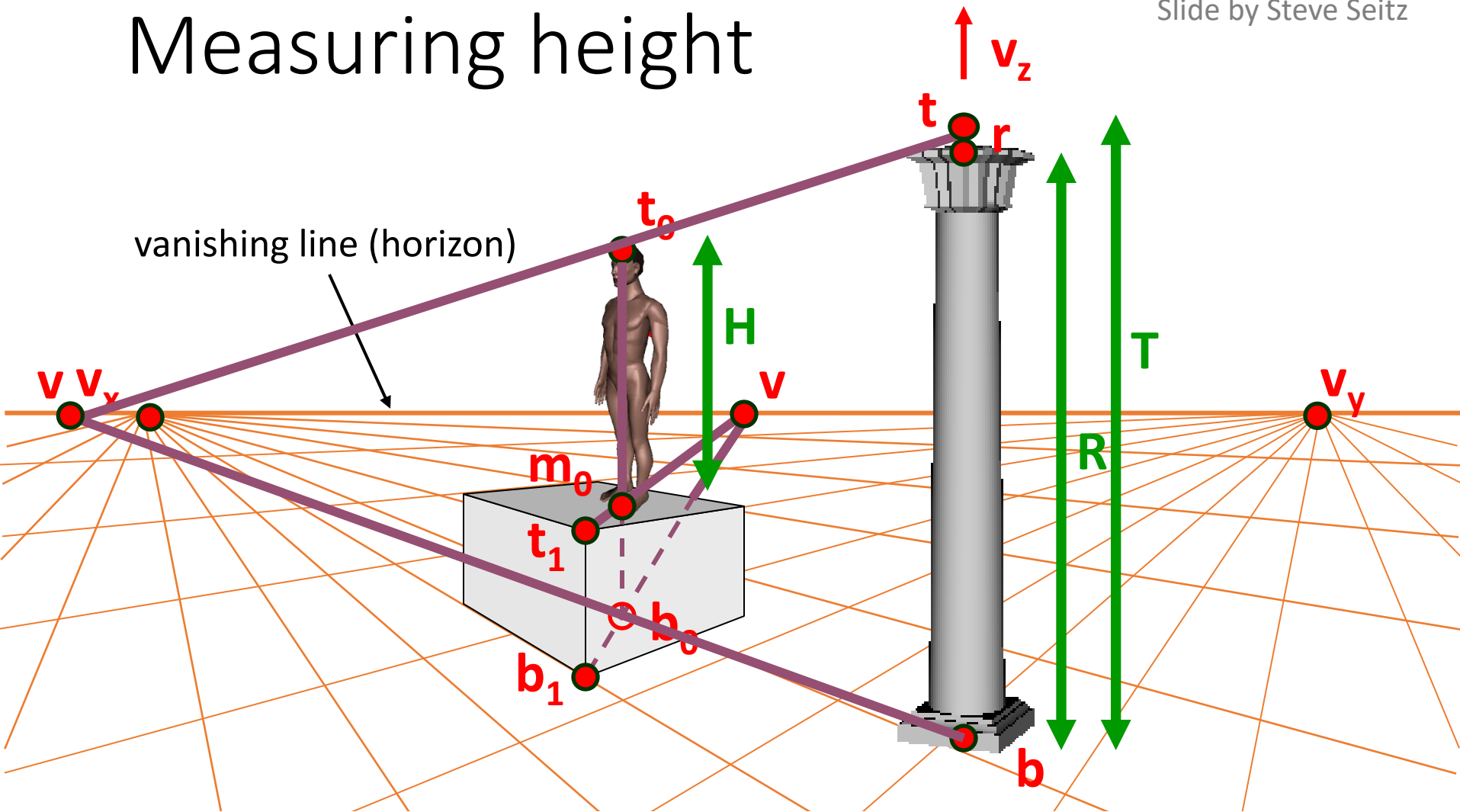
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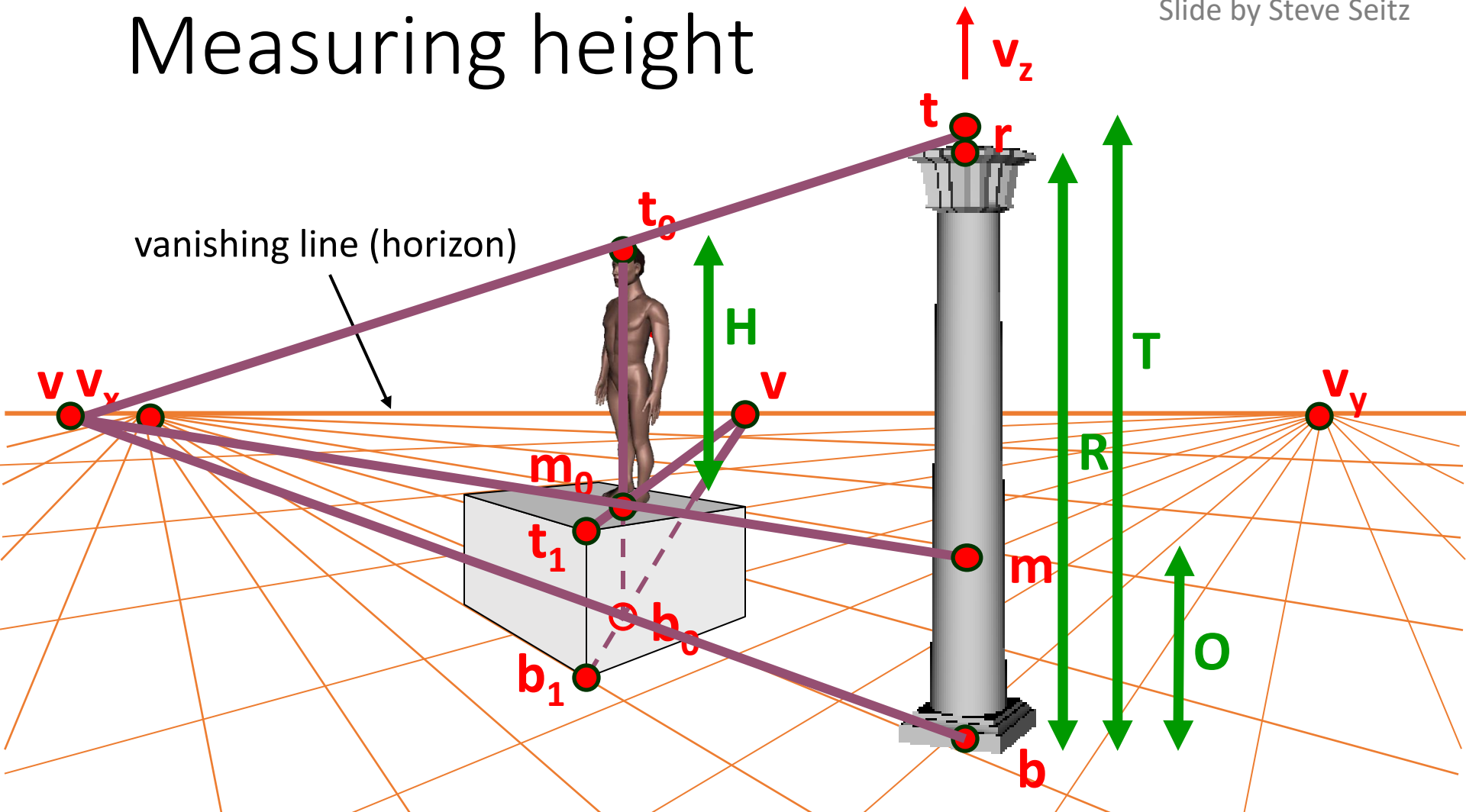
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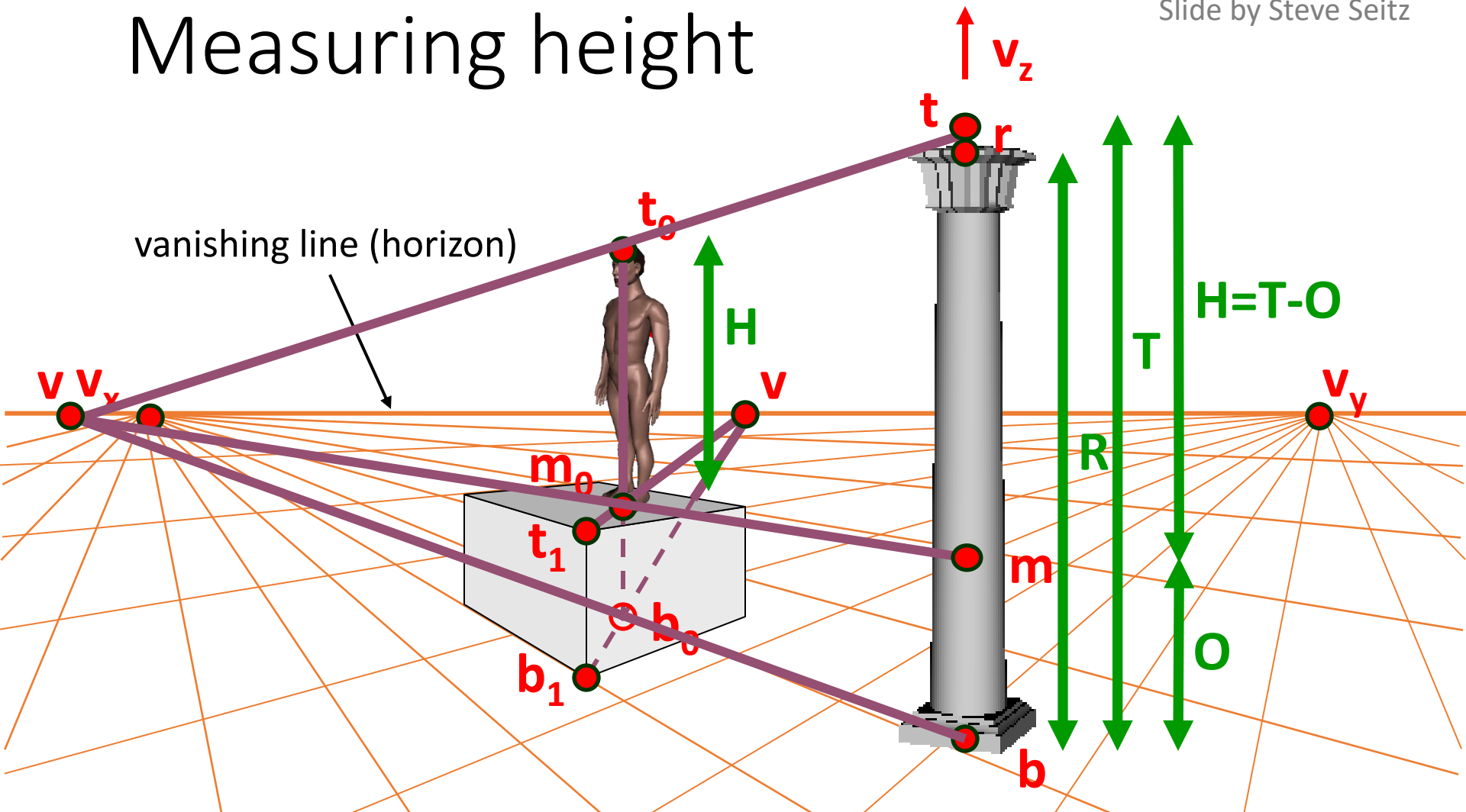
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Measuring height

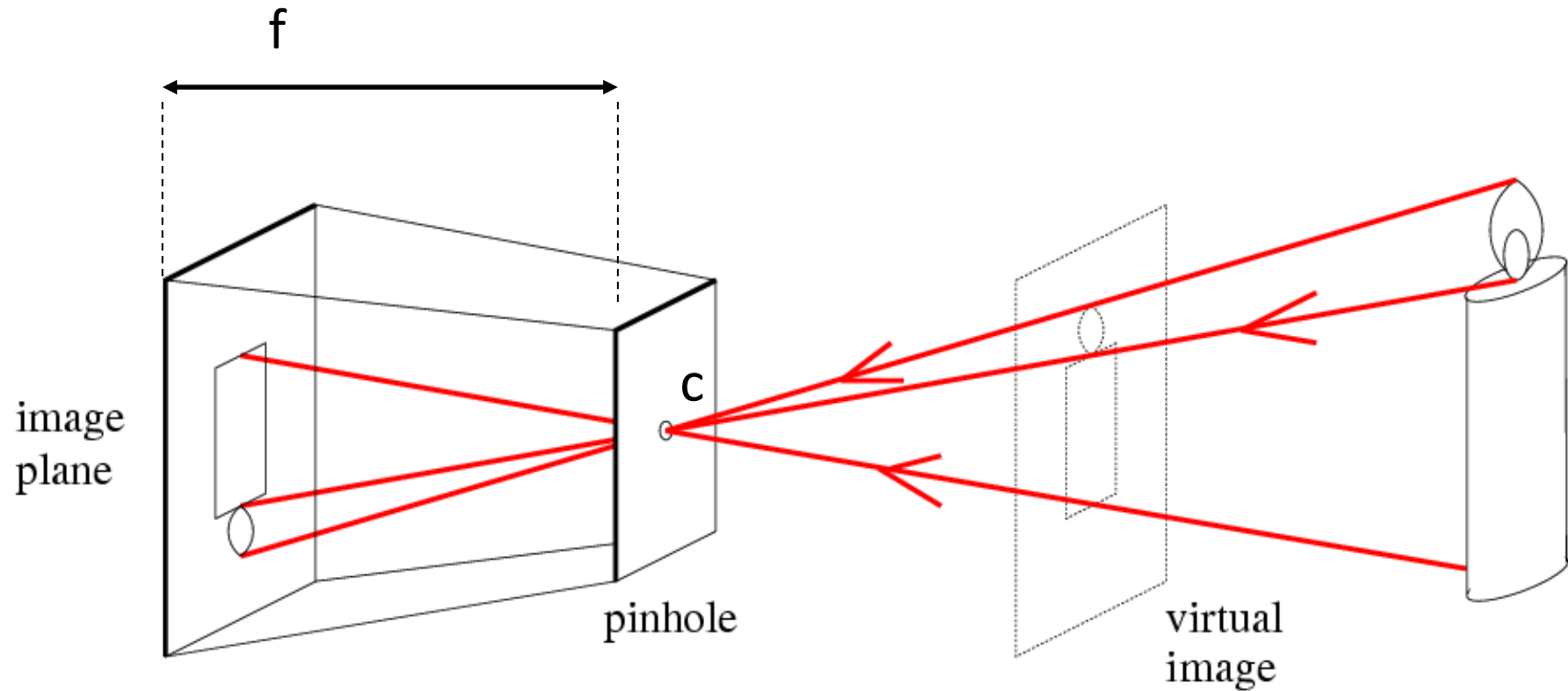


What if the point on the ground plane b_0 is not known?

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What about focus, aperture, DOF,
FOV, etc?

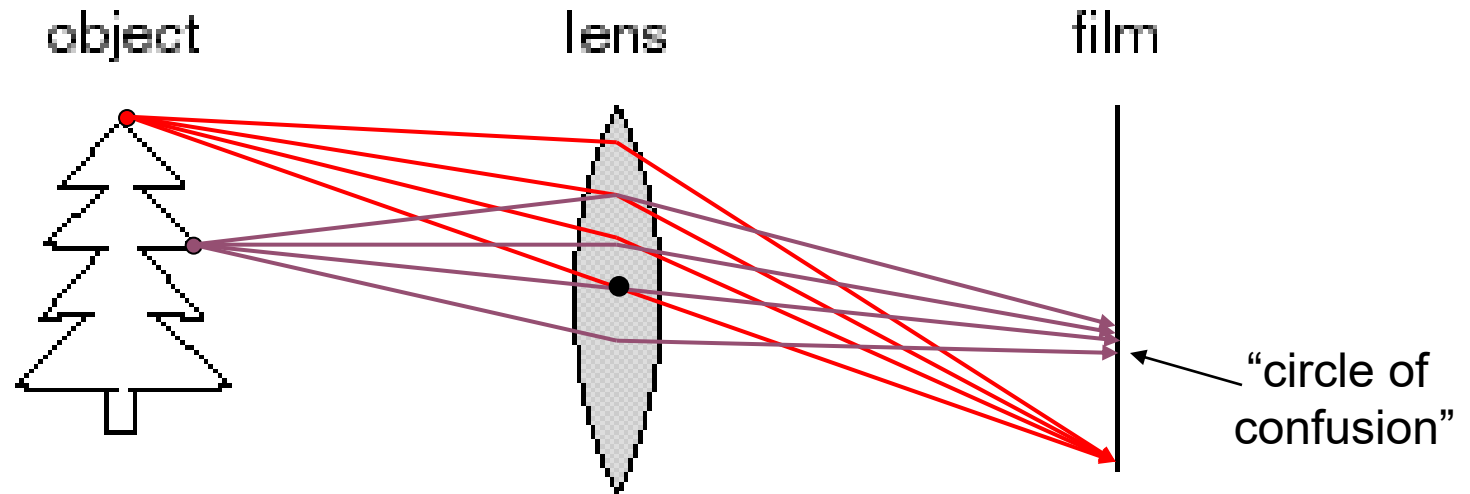
Previous: Pinhole camera



f = focal length

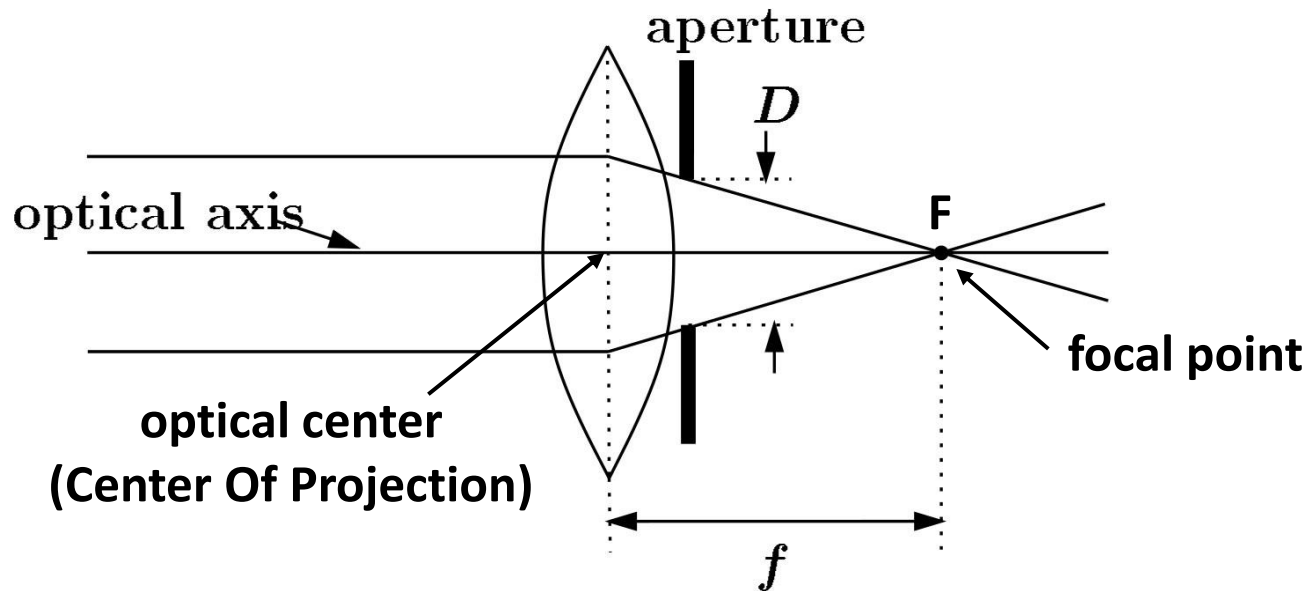
c = center of the camera

Adding a lens



- A lens focuses light onto the film
 - There is a specific distance at which objects are “in focus”
 - Other points project to a “circle of confusion” in the image
 - Changing the shape of the lens changes this distance

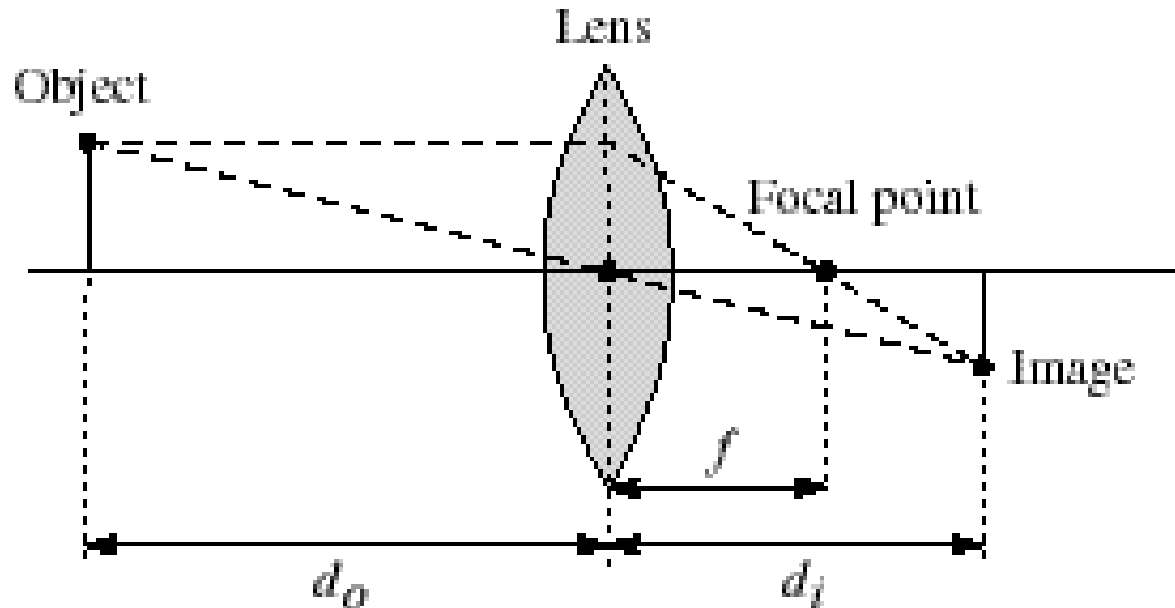
Lenses



A lens focuses parallel rays onto a single focal point

- Focal point at a distance f beyond the plane of the lens
 - f is a function of the shape and index of refraction of the lens
- Aperture of diameter D restricts the range of rays
- Real cameras use many lenses together (to correct for abnormalities)

Thin lenses



- Thin lens equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
- Any object point satisfying this equation is in focus

F-number in Camera

The f-number N or $f\#$ is given by:

$$N = \frac{f}{D}$$

where f is the focal length, and D is the diameter of the entrance pupil (*effective aperture*).

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It is also known as the **focal ratio**, **f-ratio**, or **f-stop**.

It is the **reciprocal** of the **relative aperture**.

The f-number is commonly indicated using the format f/N , where N is the f-number.

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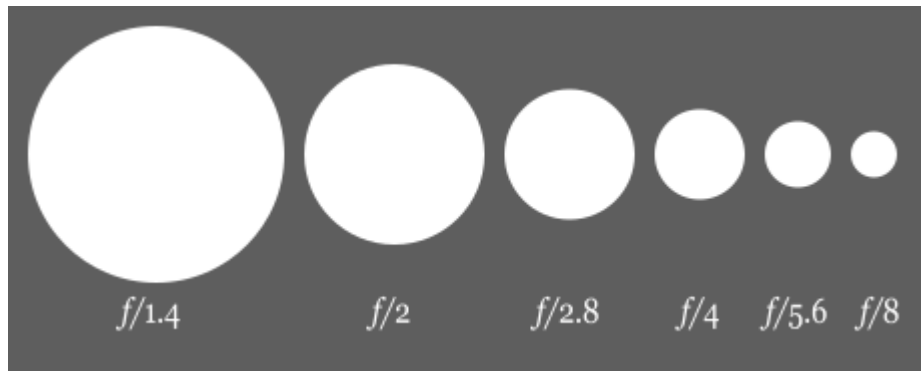
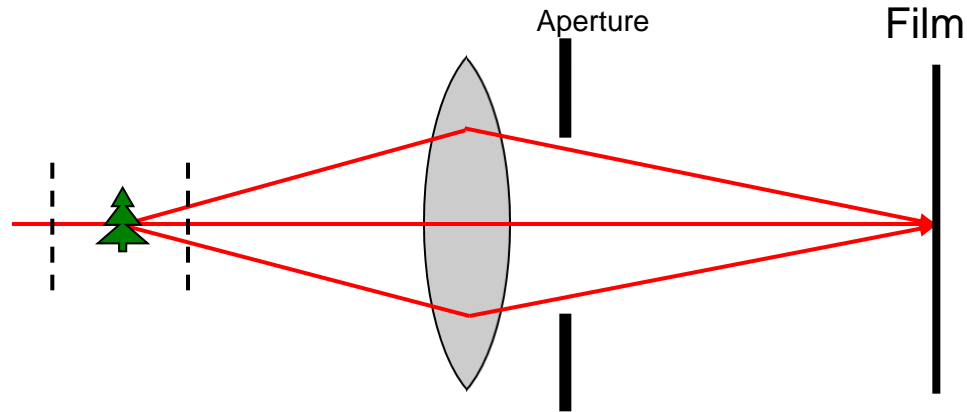


Diagram of decreasing apertures, i.e. increasing f-numbers.

Each aperture has half the light-gathering area of the previous one.

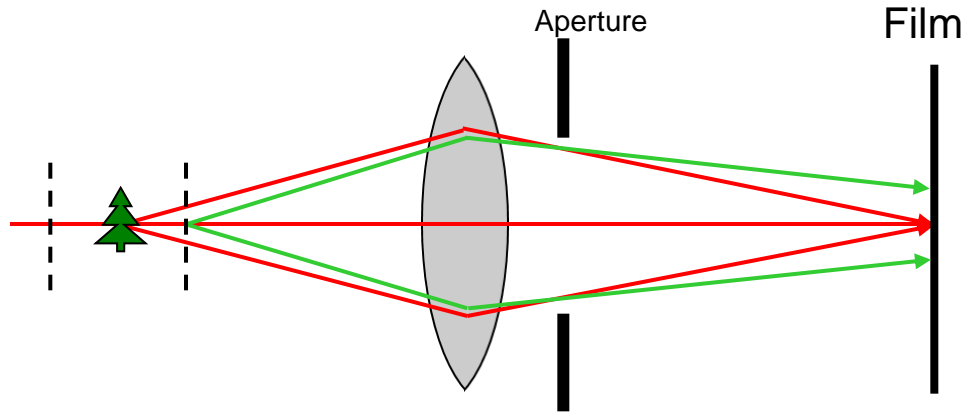
Depth of field



$f/5.6$

- Changing the aperture size affects depth of field
 - A smaller aperture increases the range in which the object is approximately in focus

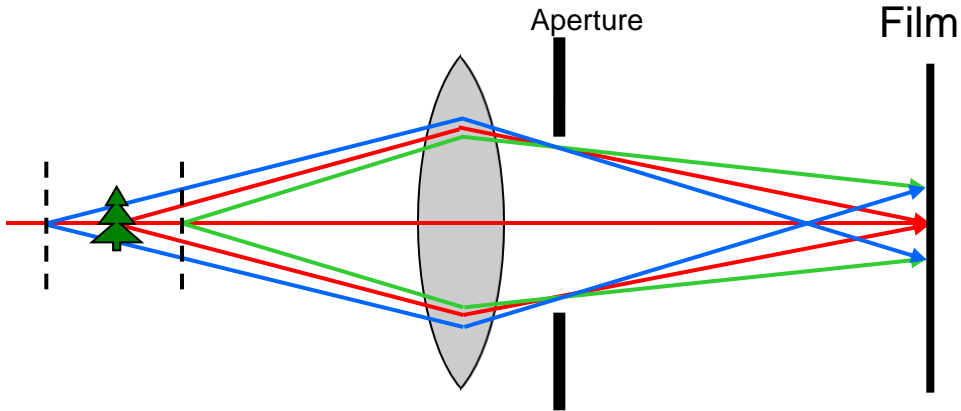
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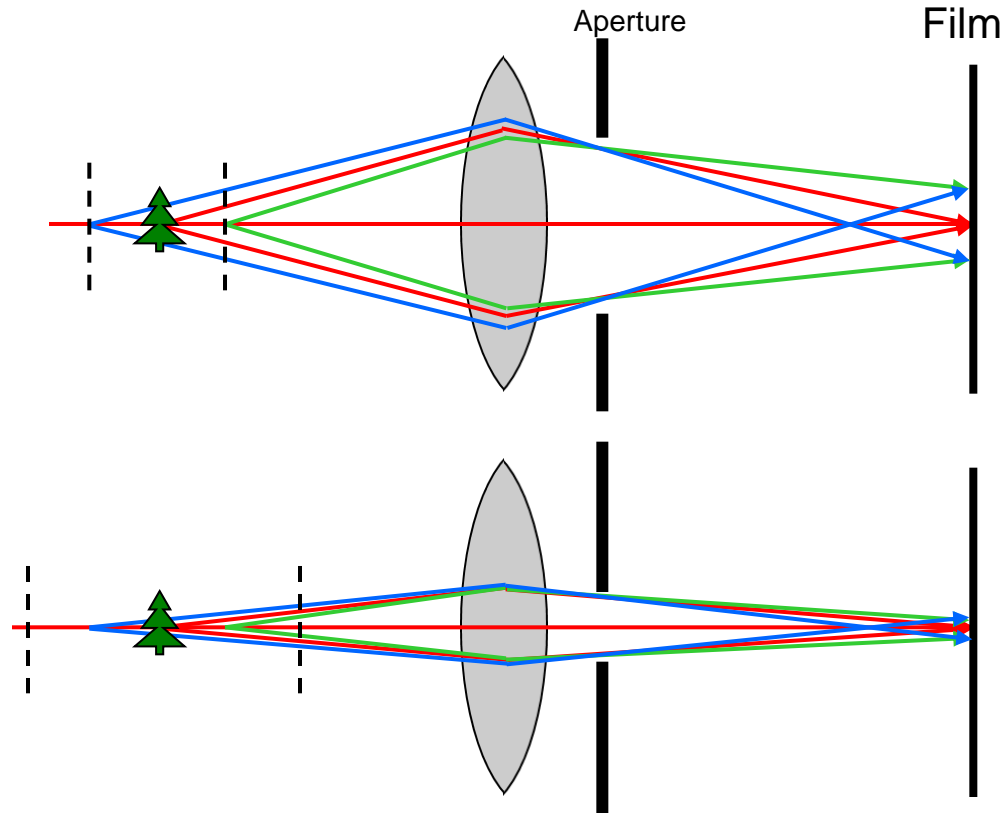
Depth of field



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Depth of field



$f/5.6$



$f/32$

- Changing the aperture size affects depth of field
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Varying the aperture

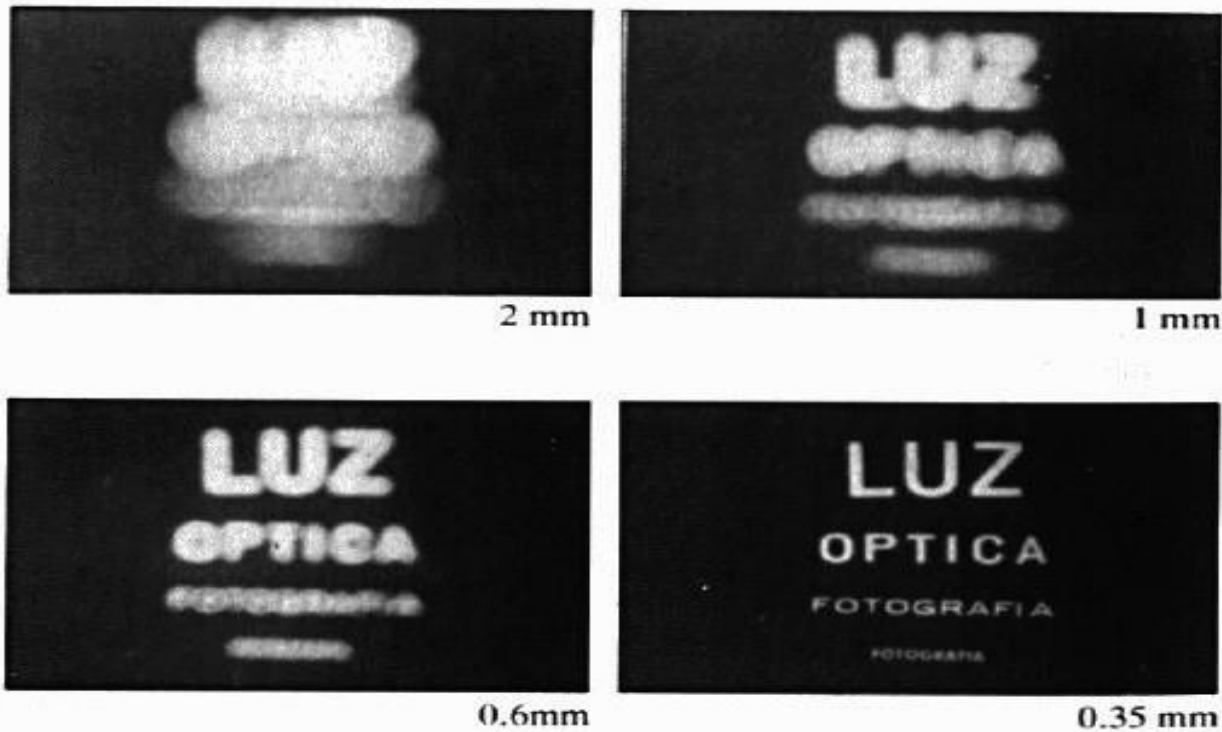


Large aperture = small DOF



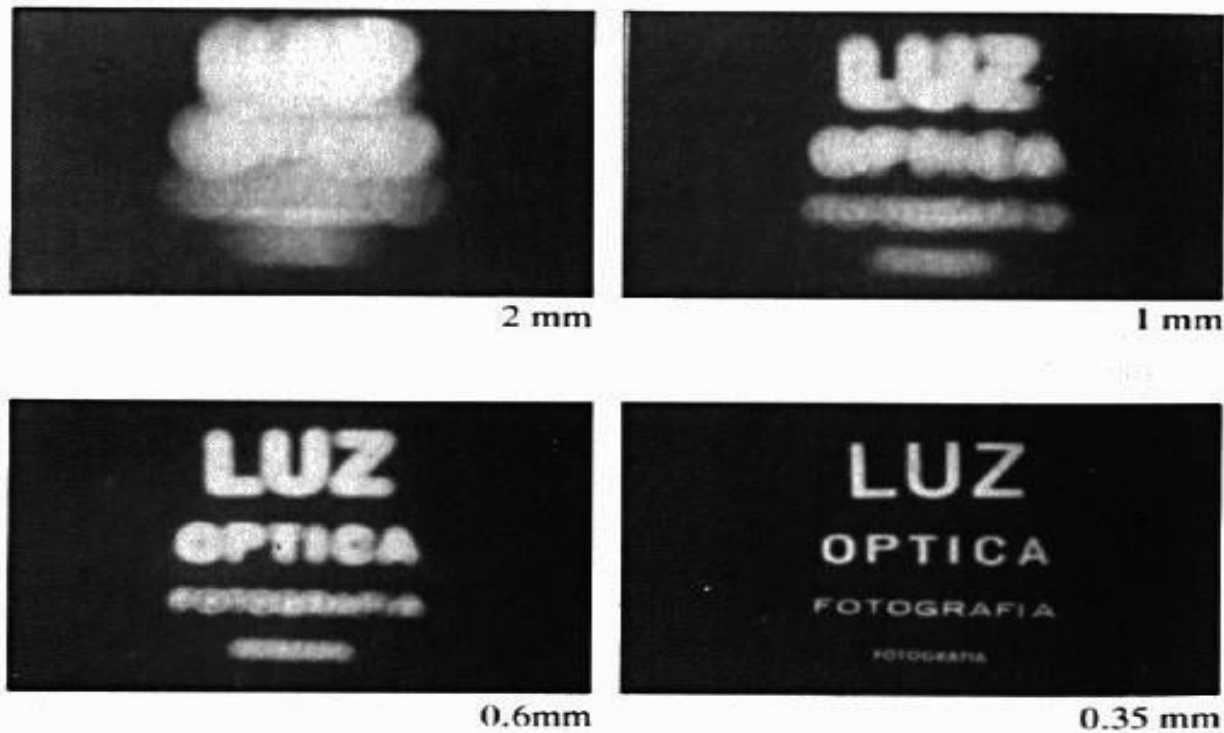
Small aperture = large DOF

Shrinking the aperture



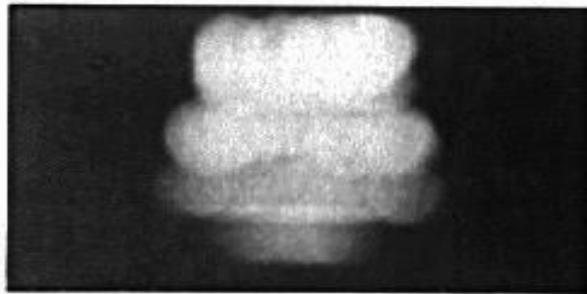
- Why not make the aperture as small as possible?

Shrinking the aperture

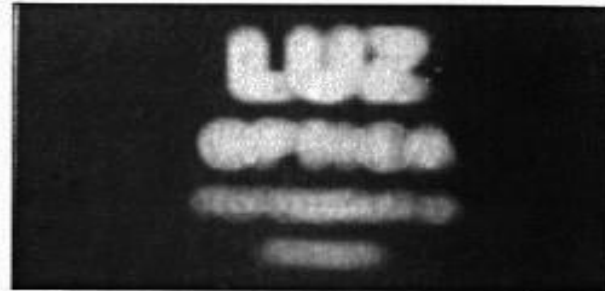


- Why not make the aperture as small as possible?
 - Less light gets through
 - Diffraction effects

Shrinking the aperture



2 mm



1 mm



0.6mm



0.35 mm



0.15 mm

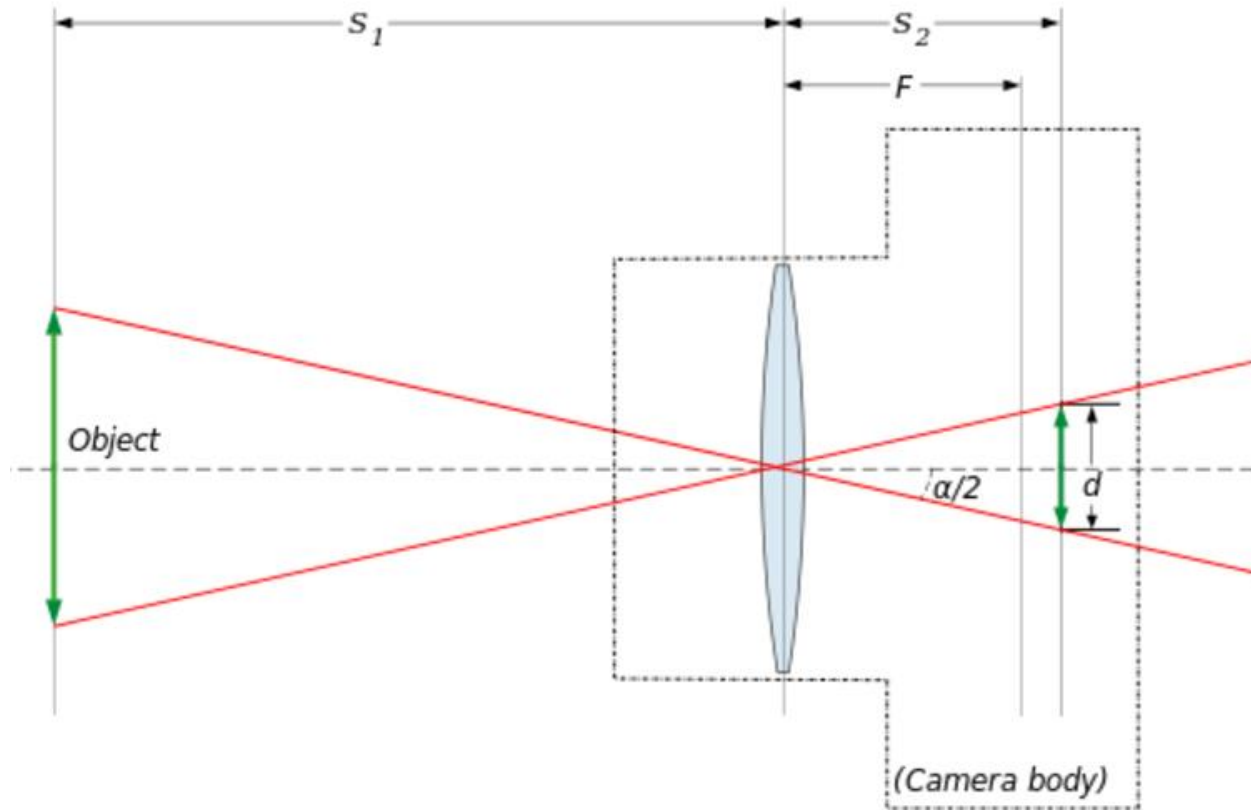


0.07 mm

Field of View

FOV depends on focal length and size of the camera retina

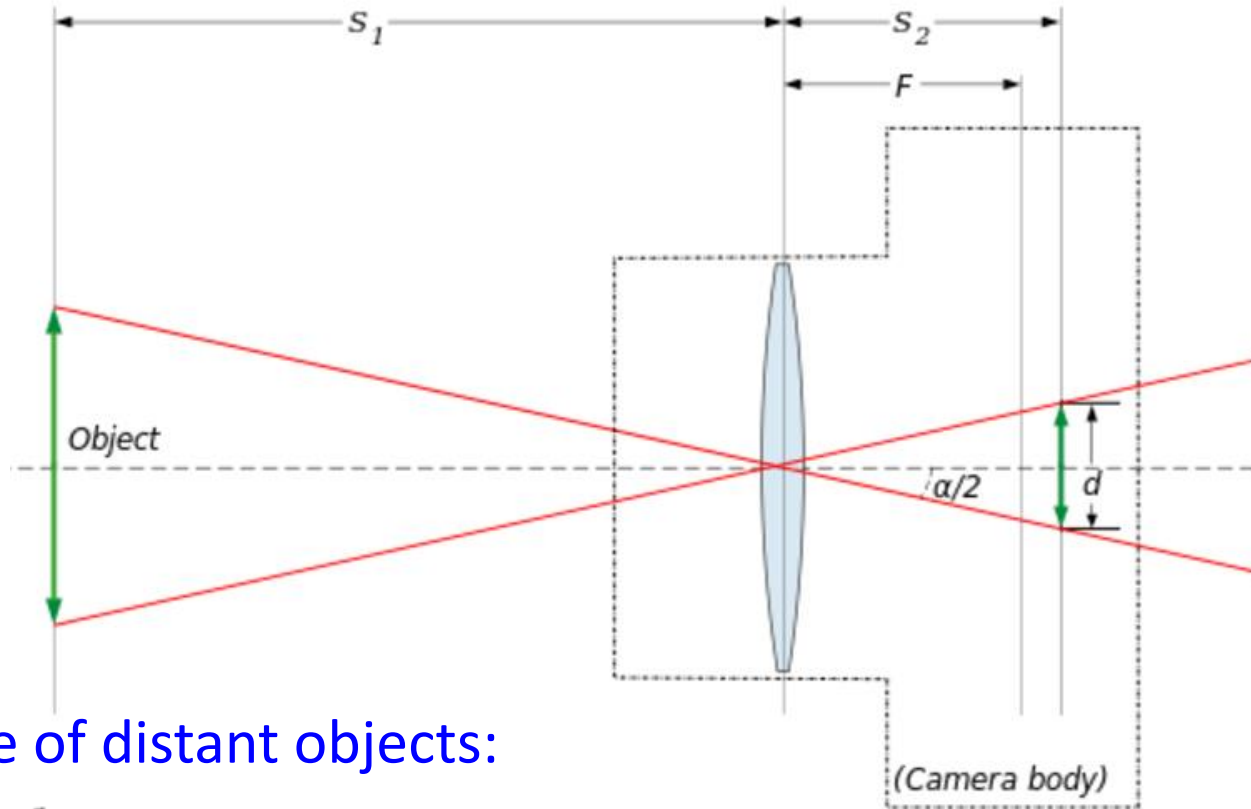
$$\alpha = 2 \arctan \frac{d}{2S_2}$$



Field of View

FOV depends on focal length and size of the camera retina

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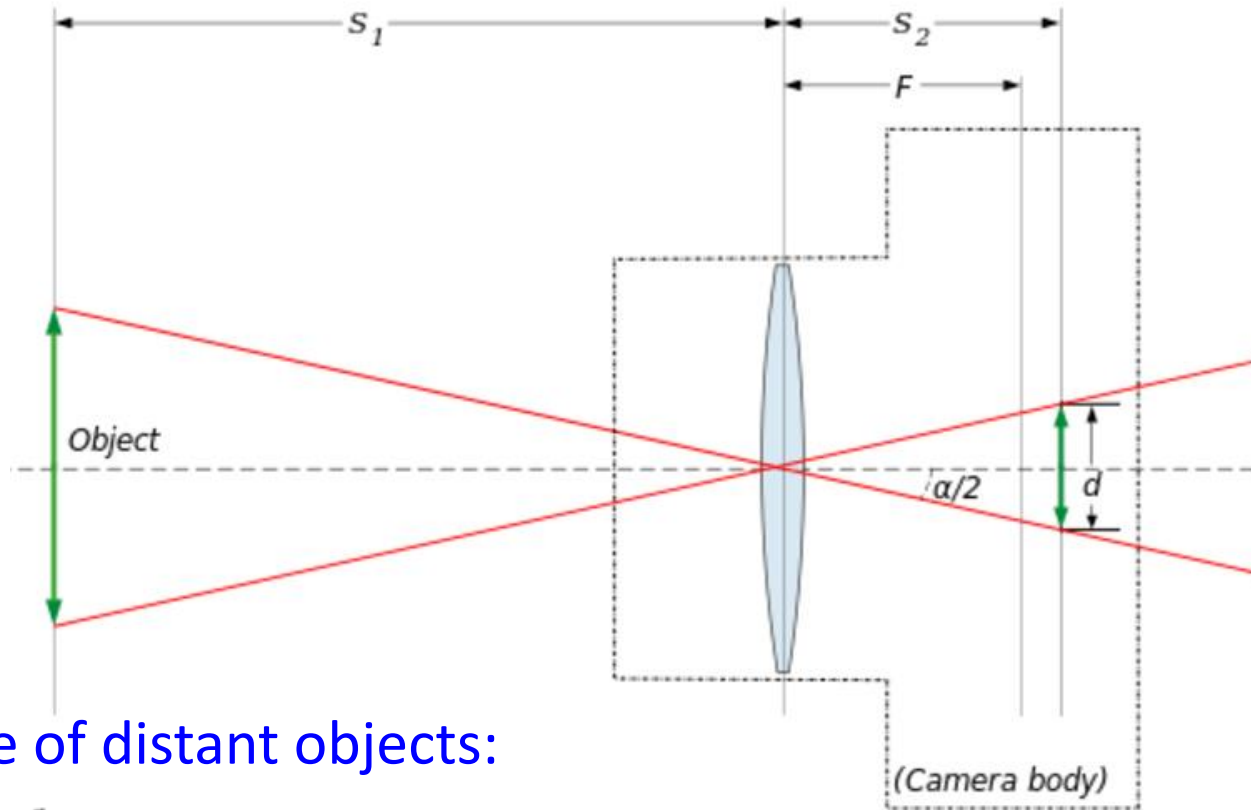
To project a sharp image of distant objects:

$$\alpha = 2 \arctan \frac{d}{2f} \text{ where } f = F$$

Field of View

FOV depends on focal length and size of the camera retina

$$\alpha = 2 \arctan \frac{d}{2S_2}$$

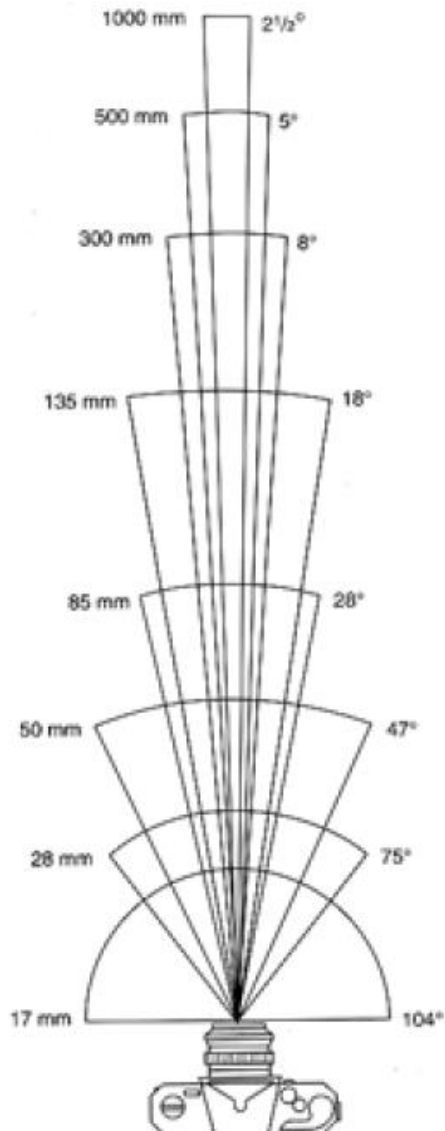


To project a sharp image of distant objects:

$$\alpha = 2 \arctan \frac{d}{2f} \text{ where } f = F$$

Larger focal length = smaller FOV

Field of View



17mm



28mm

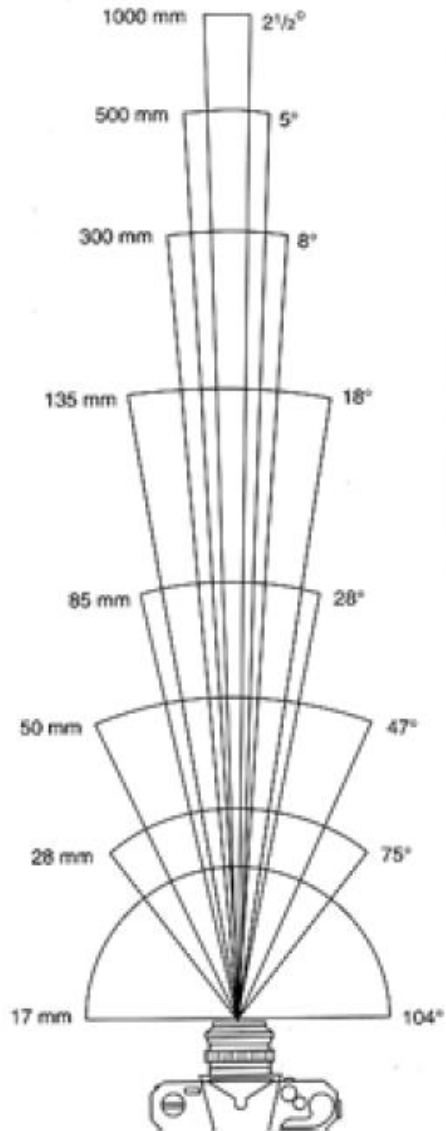


50mm

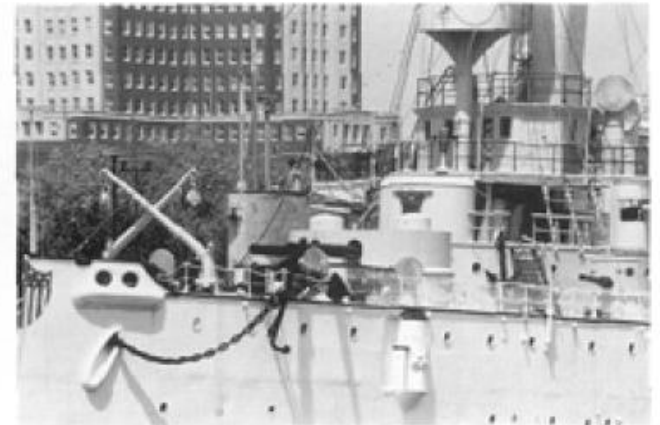


85mm

Field of View



135mm



300mm



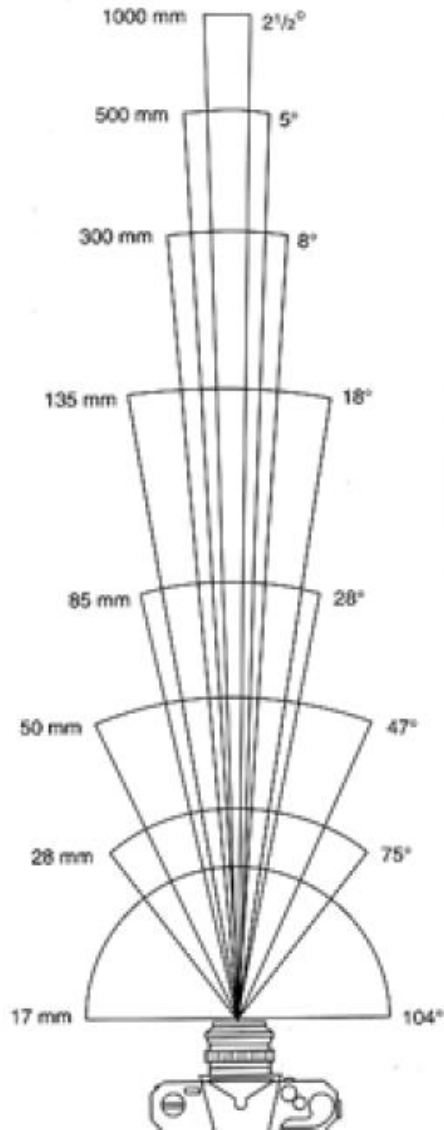
50mm



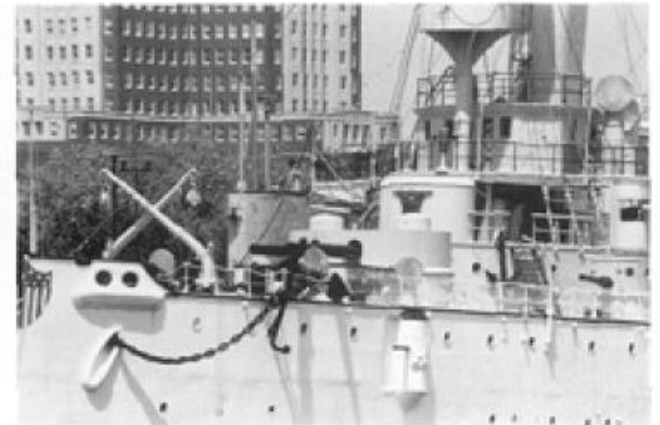
28mm

Field of View

Tripod is must to see



135mm



300mm

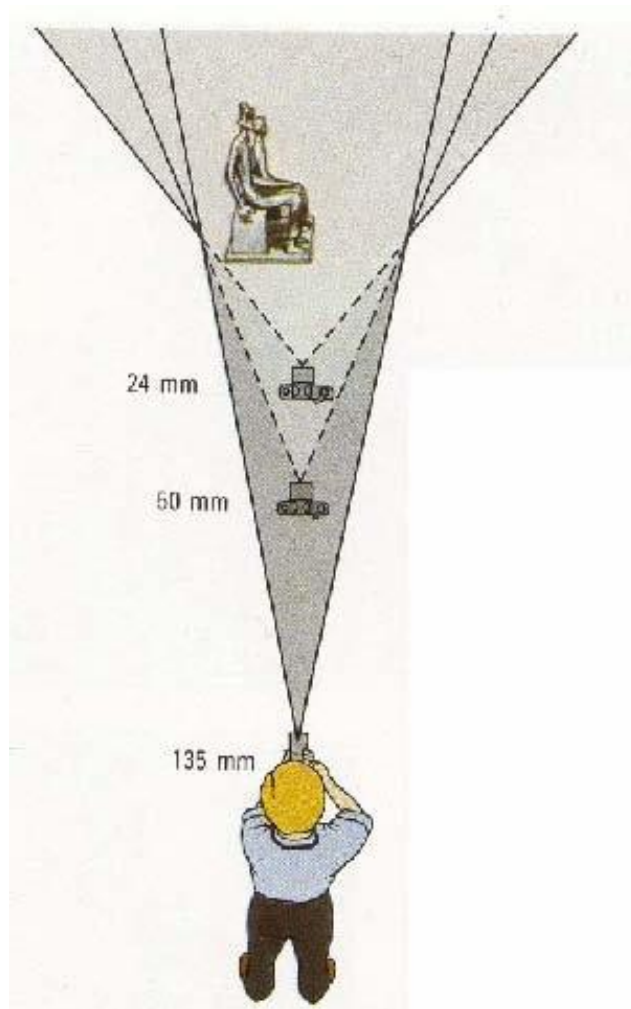


500mm



1000mm

Field of View / Focal Length



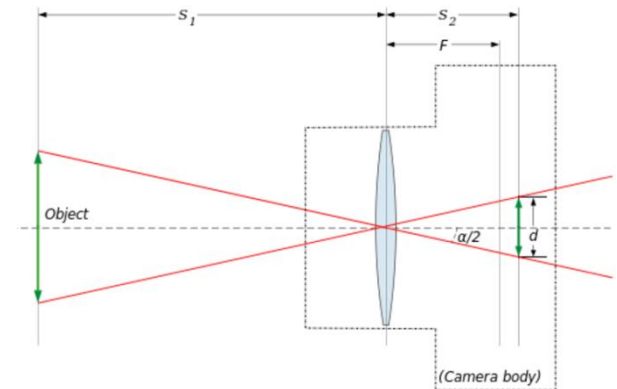
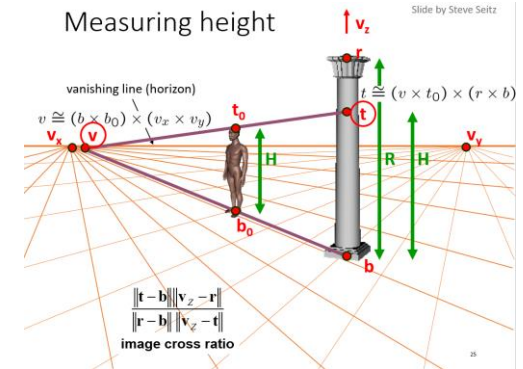
Large FOV, small f
Camera close to car



Small FOV, large f
Camera far from the car

Things to remember

- Measure the size of objects in the world from an image?
 - Use perspective cues
- Camera properties
 - focal length,
 - field of view,
 - depth of field,
 - aperture,
 - F-number



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Next class

- Image stitching

