Computer Vision

Image Filtering in Frequency Domain

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Introduction

- Fourier Series

  Any periodic function can be expressed as the sum of sines and /or cosines of different frequencies, each multiplied by a different coefficients.

- Fourier Transform

  Any function that is not periodic can be expressed as the integral of sines and /or cosines multiplied by a weighing function.

Jean Baptiste Joseph Fourier, French mathematician and physicist (03/21/1768-05/16/1830)

Example of Fourier Series

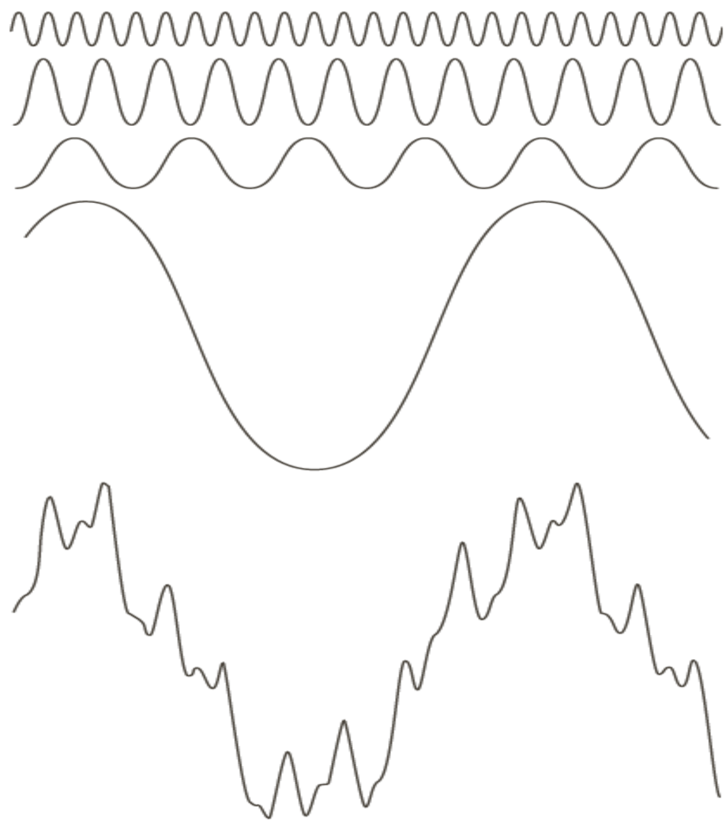


FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

Preliminary Concepts

$$j = \sqrt{-1}, \text{ a complex number}$$

$$C = R + jI$$

the conjugate

$$C^* = R - jI$$

$$|C| = \sqrt{R^2 + I^2} \text{ and } \theta = \arctan(I / R)$$

$$C = |C|(\cos\theta + j\sin\theta)$$

Using Euler's formula,

$$C = |C| e^{j\theta}$$

Fourier Series

A function $f(t)$ of a continuous variable t that is periodic with period, T, can be expressed as the sum of sines and cosines multiplied by appropriate coefficients

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi n}{T}t}$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-j\frac{2\pi n}{T}t} dt \qquad \text{for } n = 0, \pm 1, \pm 2, \ldots$$

1-D Fourier Transform: Continuous Variable

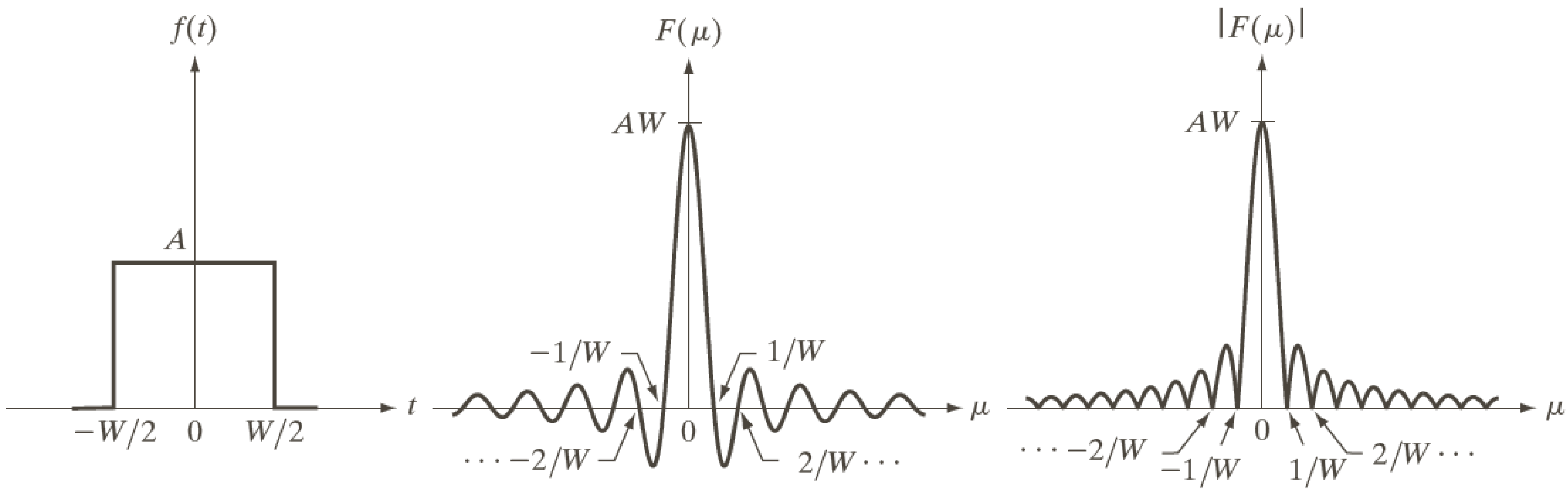
The *Fourier Transform* of a continous function $f(t)$

$$F(\mu) = \Im\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

The *Inverse Fourier Transform* of $F(\mu)$

$$f(t) = \Im^{-1}\{F(\mu)\} = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

1-D Fourier Transform: Continuous Variable



a b c

FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

1-D Discrete Fourier Transform

$$F(\mu) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi\mu x/M}, \qquad \mu = 0, 1, \ldots, M-1$$

$$f(x) = \frac{1}{M} \sum_{\mu=0}^{M-1} F(\mu) e^{j2\pi\mu x/M}, \qquad x = 0, 1, 2, \ldots, M-1$$

1. The domain (values of μ) over which the value of $F(\mu)$ range is called the Frequency Domain.
2. Each of the *M* terms of $F(\mu)$ is called a Frequency Component of the transform.

2-D Fourier Transform: Continuous Variable

$$F(\mu,\nu)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)e^{-j2\pi(\mu t+\nu z)}dtdz$$

and

$$f(t,z)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mu,\nu)e^{j2\pi(\mu t+\nu z)}d\mu d\nu$$

2-D Discrete Fourier Transform and Its Inverse

DFT:

$$F(\mu,\nu)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(\mu x/M+\nu y/N)}$$

$$\mu=0,1,2,\ldots,M-1;\nu=0,1,2,\ldots,N-1;$$

$f(x,y)$ is a digital image of size $\mathrm{M}\times\mathrm{N}$.

IDFT:

$$f(x,y)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(\mu,\nu)e^{j2\pi(\mu x/M+\nu y/N)}$$

Properties of 2-D DFT

Relationships between Samples in the Frequency and Spatial Domains

Let ΔT and ΔZ denote the separations between samples, then the seperations between the corresponding discrete, frequency domain variables are given by

$$\Delta\mu = \frac{1}{M\Delta T}$$

and $$\Delta\nu = \frac{1}{N\Delta Z}$$

Properties of 2-D DFT

Translation and Rotation

$$f(x,y)e^{j2\pi(\mu_0 x/M+\nu_0 y/N)} \Leftrightarrow F(\mu-\mu_0,\nu-\nu_0)$$

and

$$f(x-x_0,y-y_0) \Leftrightarrow F(\mu,\nu)e^{-j2\pi(\mu x_0/M+\nu y_0/N)}$$

Using the polar coordinates

$x=r\cos\theta \quad y=r\sin\theta \quad \mu=\omega\cos\varphi \quad \nu=\omega\sin\varphi$

results in the following transform pair:

$$f(r,\theta+\theta_0) \Leftrightarrow F(\omega,\varphi+\theta_0)$$

Properties of 2-D DFT

Periodicity

$2-D$ Fourier transform and its inverse are infinitely periodic

$$F(\mu,\nu)=F(\mu+k_1M,\nu)=F(\mu,\nu+k_2N)=F(\mu+k_1M,\nu+k_2N)$$

$$f(x,y)=f(x+k_1M,y)=f(x,y+k_2N)=f(x+k_1M,y+k_2N)$$

$$f(x)e^{j2\pi(\mu_0 x/M)}\Leftrightarrow F(\mu-\mu_0)$$

$$\mu_0=M/2,\quad f(x)(-1)^x\Leftrightarrow F(\mu-M/2)$$

$$f(x,y)(-1)^{x+y}\Leftrightarrow F(\mu-M/2,\nu-N/2)$$

Properties of 2-D DFT

Symmetry

	Spatial Domain[†]		Frequency Domain[†]
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

[†]Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

Properties of 2-D DFT

Fourier Spectrum and Phase Angle

2-D DFT in polar form

$$F(u,v) = |F(u,v)|\, e^{j\phi(u,v)}$$

Fourier spectrum

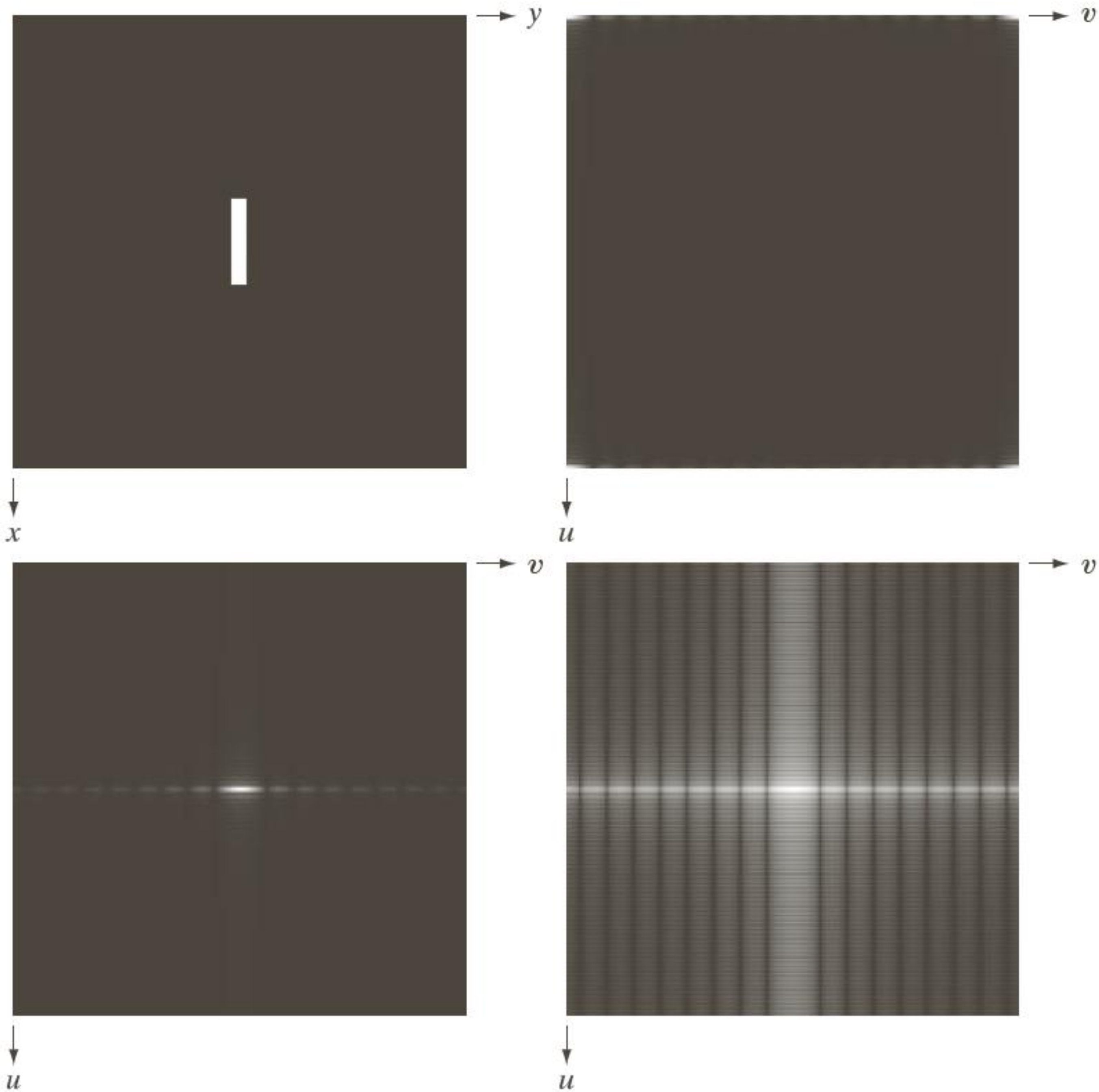
$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v) \right]^{1/2}$$

Power spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

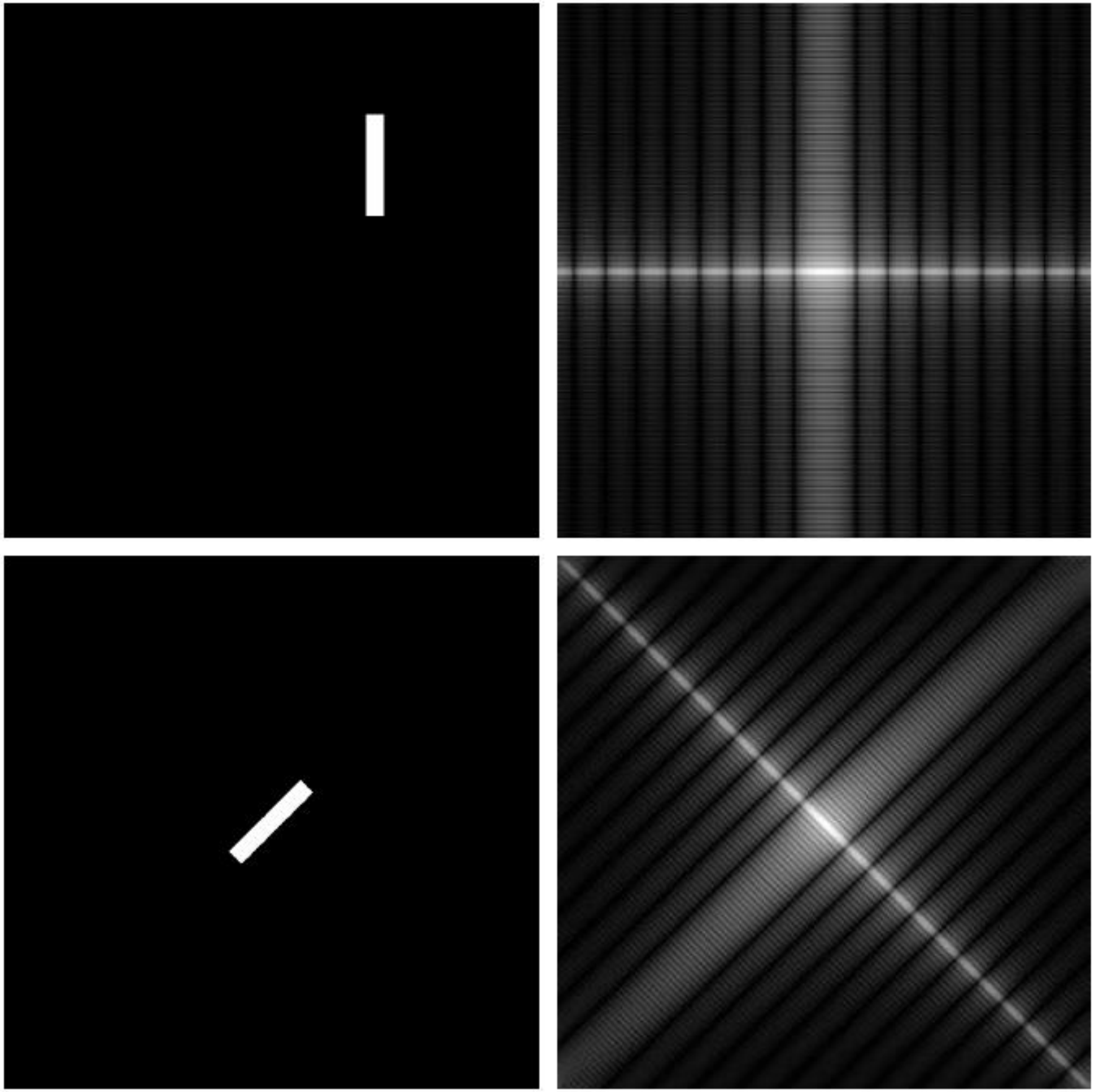
Phase angle

$$\phi(\mathrm{u,v}) = \arctan\left[\frac{I(u,v)}{R(u,v)} \right]$$

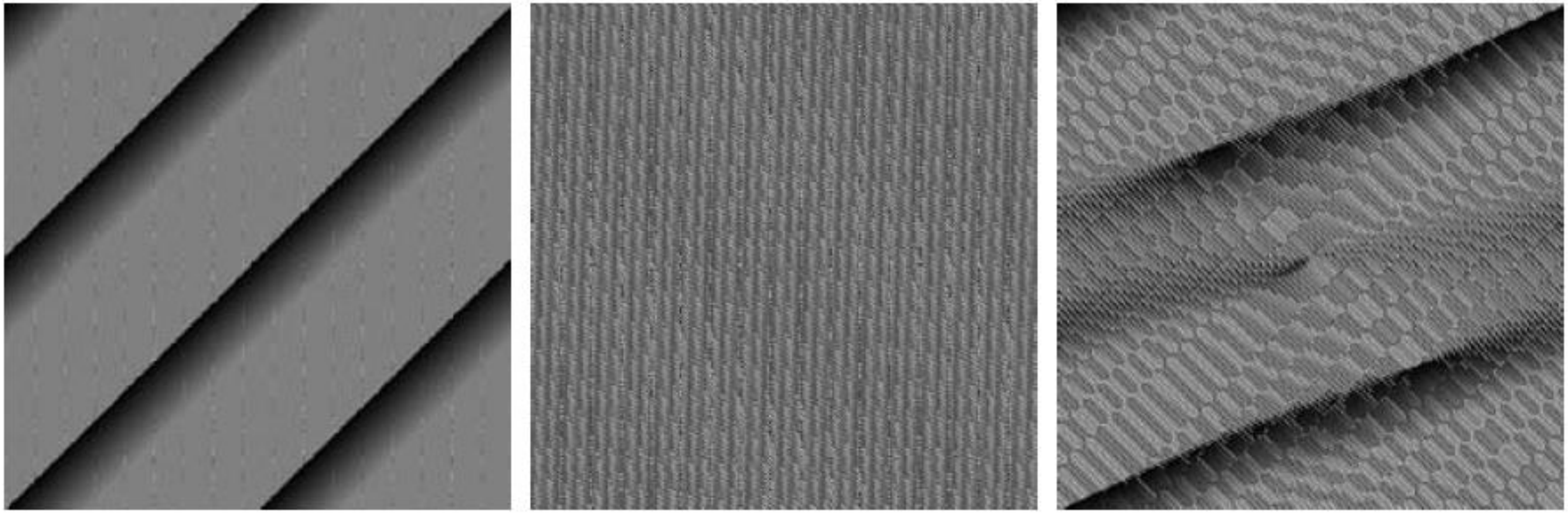


a b
c d

FIGURE 4.24
(a) Image. (b) Spectrum showing bright spots in the four corners. (c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.



Phase Angle: Example



a b c

FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

Phase Angle and The Reconstructed: Example

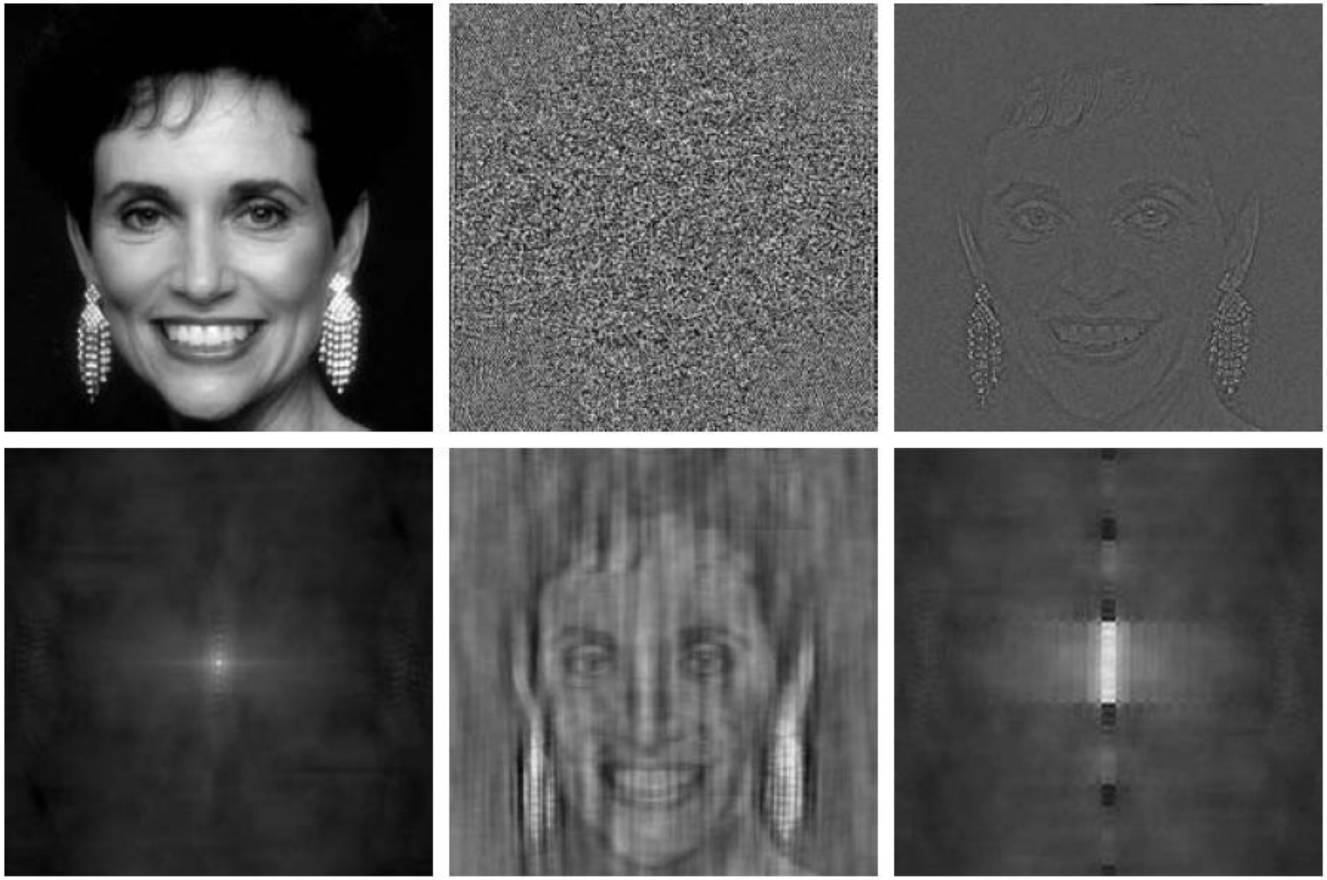


FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

Properties of 2-D DFT: Fourier Spectrum and Phase Angle

2-D DFT in polar form

$$F(u,v) = |F(u,v)| e^{j\phi(u,v)}$$

Fourier spectrum

$$|F(u,v)| = \left[R^2(u,v) + I^2(u,v) \right]^{1/2}$$

Power spectrum

$$P(u,v) = |F(u,v)|^2 = R^2(u,v) + I^2(u,v)$$

Phase angle

$$\phi(\mathrm{u,v}) = \arctan\left[\frac{I(u,v)}{R(u,v)} \right]$$

Filtering in Frequency Domain: Steps

1. Multiply the input image by $-1^{(x+y)}$ to center the transform
2. Compute $F(u, v)$, the DFT of the image
3. Multiply $F(u, v)$ by a filter function $\mathrm{H}(u, v)$
4. Compute the inverse DFT of the result in (3)
5. Obtain the real part of the result in (4)
6. Multiply the result in (5) by $-1^{(x+y)}$

Filtering in Frequency Domain

- Let $f(x,y)$ is the input image and $F(u,v)$ its Fourier transform. Then the Fourier transform the output image is given by

$$G(u,v) = H(u,v)F(u,v)$$

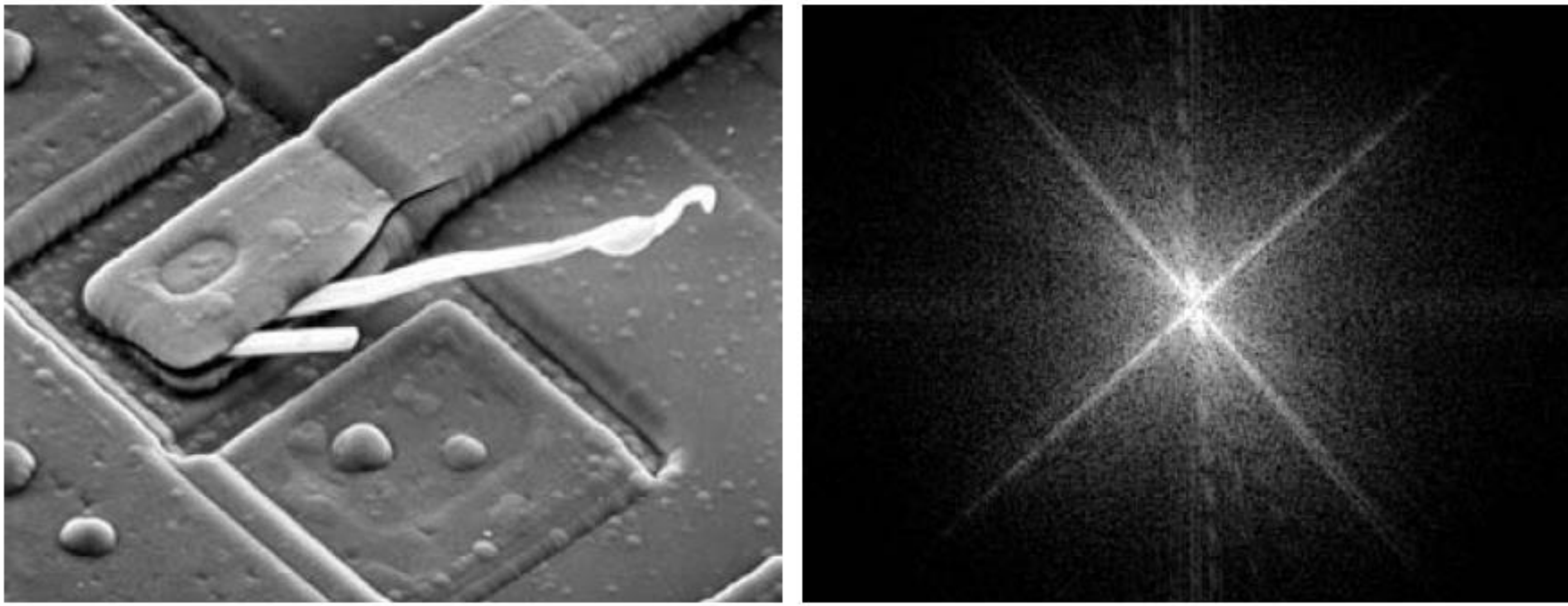
- Computing the inverse transform to obtain the processed result

$$g(x,y) = \mathfrak{I}^{-1}\{H(u,v)F(u,v)\}$$

$F(u,v)$ is the DFT of the input image

$H(u,v)$ is a filter function.

Filtering in Frequency Domain

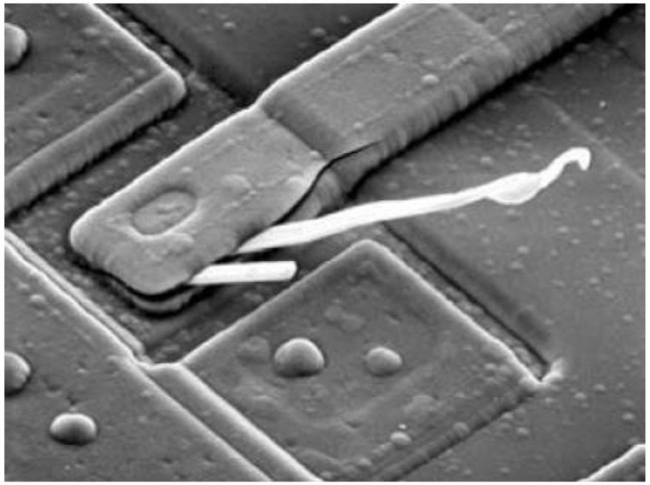


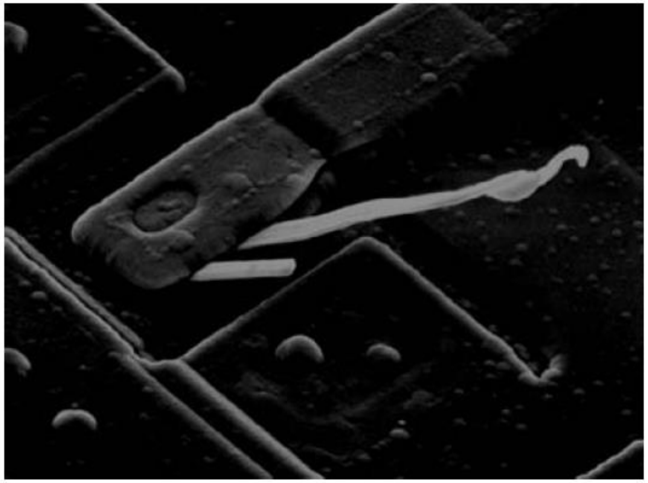
a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

Filtering in Frequency Domain: Notch Filter

In a filter H(u,v) that is 0 at the center of the transform and 1 elsewhere, what's the output image?





Filtering in Frequency Domain: Properties

- Low frequencies in the Fourier transform are responsible for the general gray-level appearance of an image over smooth regions.
- High frequencies are responsible for detail, such as edges and noise.
- A filter that attenuates high frequencies while "passing" the low frequencies is called *lowpass filter.*
- A filter that attenuates low frequencies while "passing" the high frequencies is called *highpass filter.*

Filtering in Frequency Domain

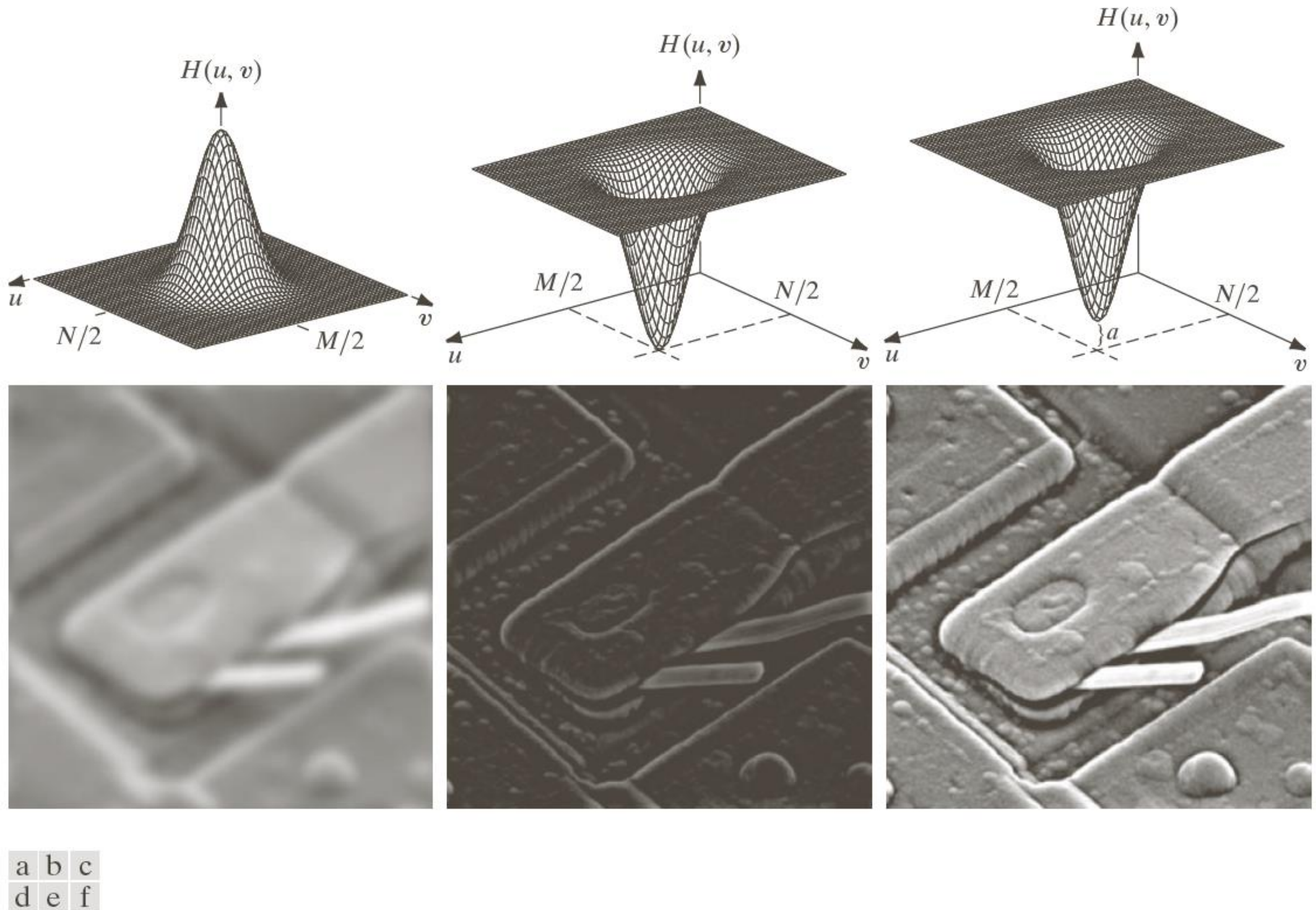


FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

2-D Convolution Theorem

1-D convolution

$$f(x)\star h(x) = \sum_{m=0}^{M-1} f(m)h(x-m)$$

2-D convolution

$$f(x,y)\star h(x,y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)h(x-m,y-n)$$

$$x = 0,1,2,\ldots,M-1;\ y = 0,1,2,\ldots,N-1.$$

$$f(x,y)\star h(x,y) \Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y) \Leftrightarrow F(u,v)\star H(u,v)$$

1-D Impulses and the Shifting Property: Continuous

A *unit impulse* of a continuous variable t located at t=0, denoted $\delta(t)$, defined as

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\int_{-\infty}^{\infty} \delta(t)dt = 1$$

The *sifting property* $\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)$

$$\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$$

1- D Impulses and the Shifting Property: Discrete

A *unit impulse* of a discrete variable x located at x=0, denoted $\delta(x)$, defined as

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\sum_{x=-\infty}^{\infty} \delta(x) = 1$$

The *sifting property*

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x - x_0) = f(x_0)$$

$$\sum_{x=-\infty}^{\infty} f(x)\delta(x) = f(0)$$

2-D Impulses and the Shifting Property: Continuous

The impulse $\delta(t,z)$, $\qquad \delta(t,z) = \begin{cases} \infty & \text{if } t = z = 0 \\ 0 & \text{otherwise} \end{cases}$

and $\qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(t,z)\,dt\,dz = 1$

The sifting property

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)\delta(t,z)\,dt\,dz = f(0,0)$$

and

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,z)\delta(t-t_0, z-z_0)\,dt\,dz = f(t_0, z_0)$$

2-D Impulses and the Shifting Property: Discrete

The impulse $\delta(x, y)$, $\qquad \delta(x, y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$

The sifting property

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y)\delta(x, y) = f(0, 0)$$

and

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y)\delta(x - x_0, y - y_0) = f(x_0, y_0)$$

2-D Impulses and the Shifting Property: Discrete

For a unit impulse located at origin (0,0),

$$\sum_{x=0}^{m-1}\sum_{y=0}^{n-1} f(x,y)\delta(x,y) = f(0,0)$$

Fourier transform of a unit impulse located at origin

$$F(u,v) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \delta(x,y)e^{-j2\pi(\frac{ux}{M}+\frac{vy}{N})}$$

$$= \frac{1}{MN}$$

Relation between Spatial and Fourier domain

Let $f(x,y) = \delta(x,y)$.

$$f(x,y) * h(x,y) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta(m,n)h(x-m,y-n)$$

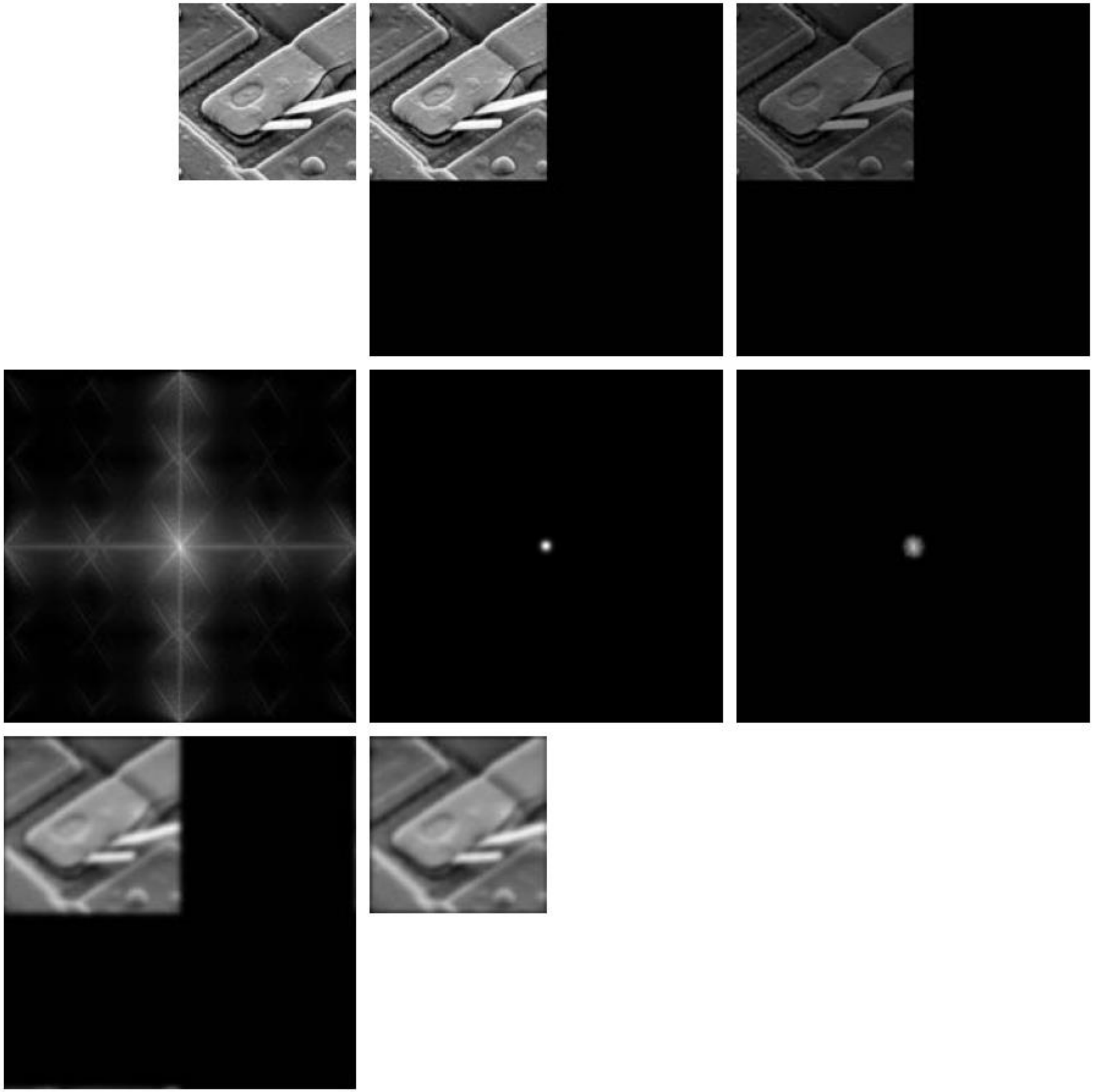
$$= \frac{1}{MN}h(x,y)$$

$$f(x,y) * h(x,y) \Longleftrightarrow F(u,v)H(u,v)$$

$$\delta(x,y) * h(x,y) \Longleftrightarrow \Im[\delta(x,y)]H(u,v)$$

$$h(x,y) \Longleftrightarrow H(u,v)$$

Given a filter in the frequency domain, we can obtain the corresponding filter in the spatial domain by taking the inverse Fourier transform of the former. The reverse is also true.



a	b	c
d	e	f
g	h	

FIGURE 4.36 (a) An $M \times N$ image, f. (b) Padded image, f_p of size $P \times Q$. (c) Result of multiplying f_p by $(-1)^{x+y}$. (d) Spectrum of F_p. (e) Centered Gaussian lowpass filter, H, of size $P \times Q$. (f) Spectrum of the product HF_p. (g) g_p, the product of $(-1)^{x+y}$ and the real part of the IDFT of HF_p. (h) Final result, g, obtained by cropping the first M rows and N columns of g_p.

Spatial Domain vs. Frequency Domain Filtering

Let H(u) denote the 1-D frequency domain Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma^2}$$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$$

1. Both components are Gaussian and real
2. The functions behave reciprocally

Spatial Domain vs. Frequency Domain Filtering

Let $H(u)$ denote the difference of Gaussian filter

$$H(u) = Ae^{-u^2/2\sigma_1^2} - Be^{-u^2/2\sigma_2^2}$$

with $A \geq B$ and $\sigma_1 \geq \sigma_2$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\sigma_2 Ae^{-2\pi^2\sigma_2^2 x^2}$$

High-pass filter or low-pass filter ?

Spatial Domain vs. Frequency Domain Filtering

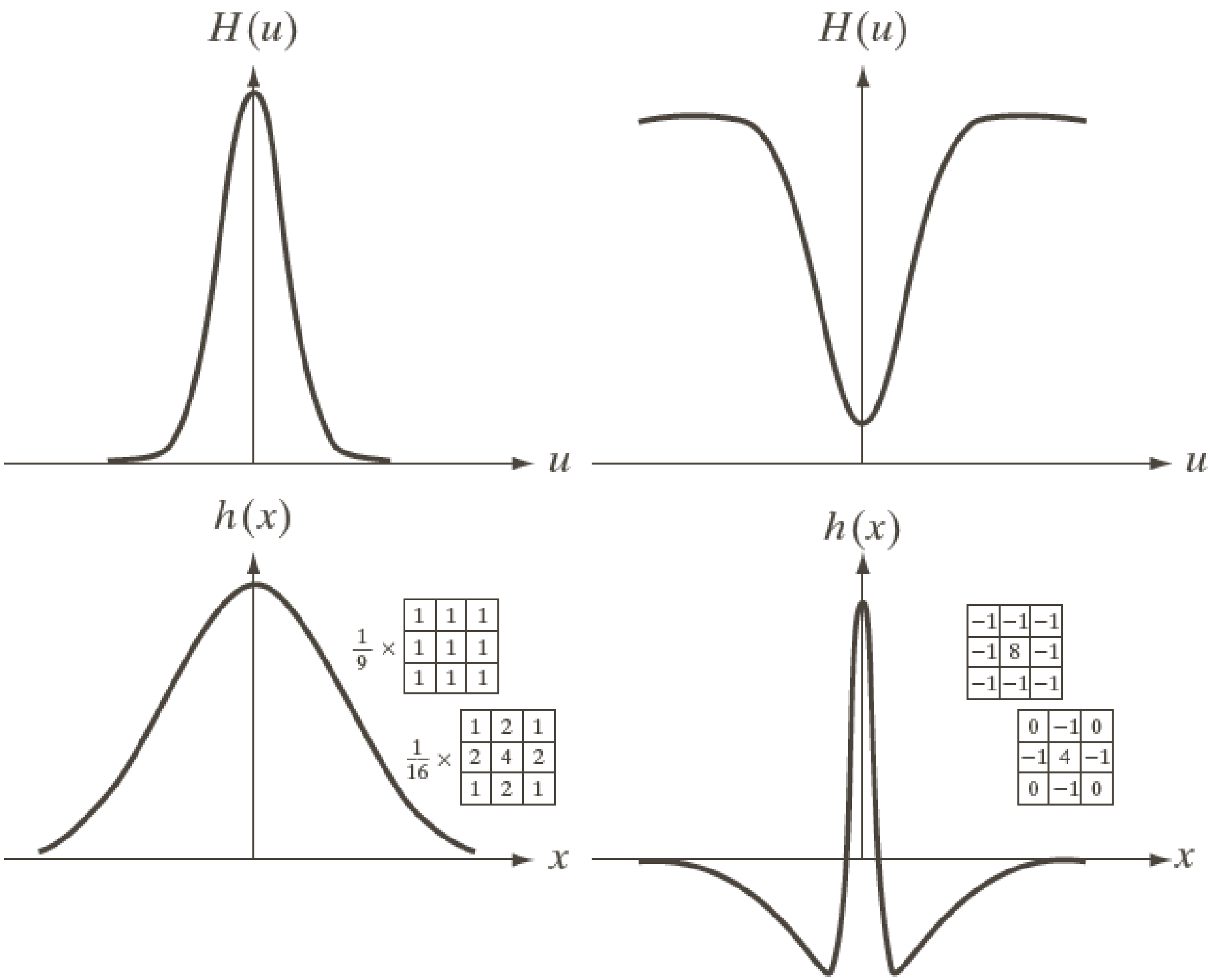
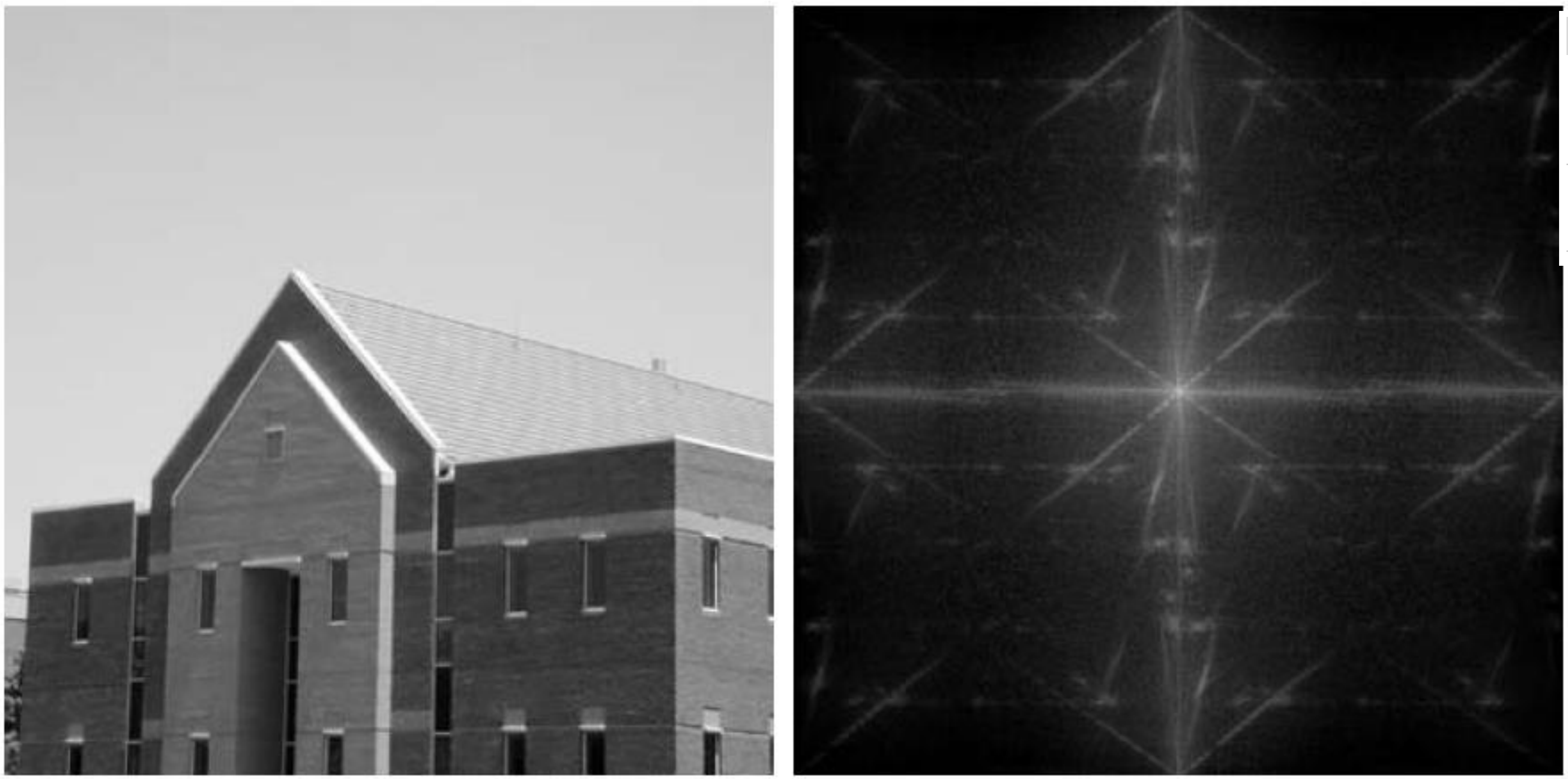


FIGURE 4.37
(a) A 1-D Gaussian lowpass filter in the frequency domain. (b) Spatial lowpass filter corresponding to (a). (c) Gaussian highpass filter in the frequency domain. (d) Spatial highpass filter corresponding to (c). The small 2-D masks shown are spatial filters we used in Chapter 3.

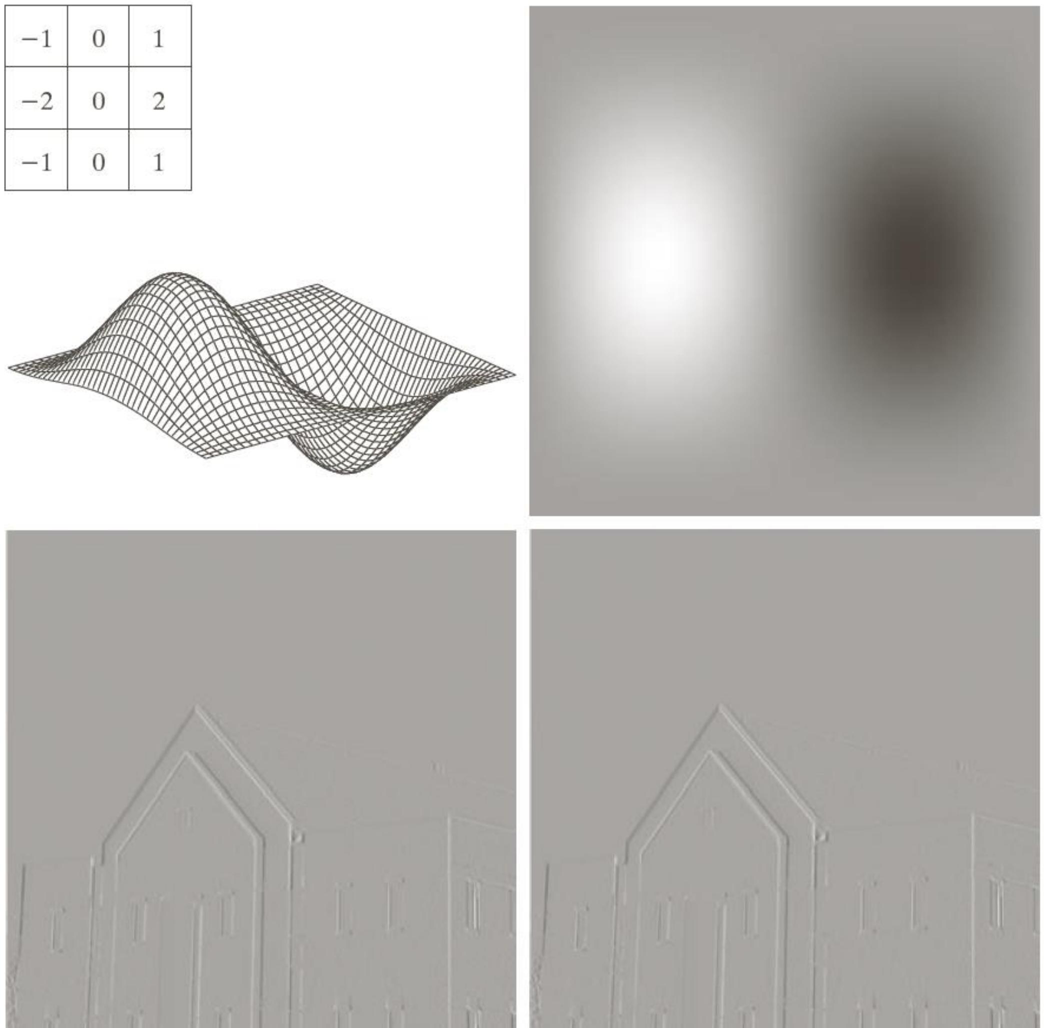
Spatial Domain vs. Frequency Domain Filtering



a b

FIGURE 4.38 (a) Image of a building, and (b) its spectrum.

Spatial Domain vs. Frequency Domain Filtering



−1	0	1
−2	0	2
−1	0	1

a	b
c	d

FIGURE 4.39 (a) A spatial mask and perspective plot of its corresponding frequency domain filter. (b) Filter shown as an image. (c) Result of filtering Fig. 4.38(a) in the frequency domain with the filter in (b). (d) Result of filtering the same image with the spatial filter in (a). The results are identical.

Smoothing Filters

- Edges and other sharp transitions (such as noise) contribute significantly to the high frequency content of Fourier transform.
- Smoothing (blurring) is achieved in frequency domain by attenuating a specified range of high frequency components.
- Three types of lowpass filters: Ideal, Butterworth and Gaussian filters
- These filters cover the range from very sharp (ideal) to very smooth (Gaussian) filter functions.

Smoothing Filters: Ideal Lowpass Filters (ILPF)

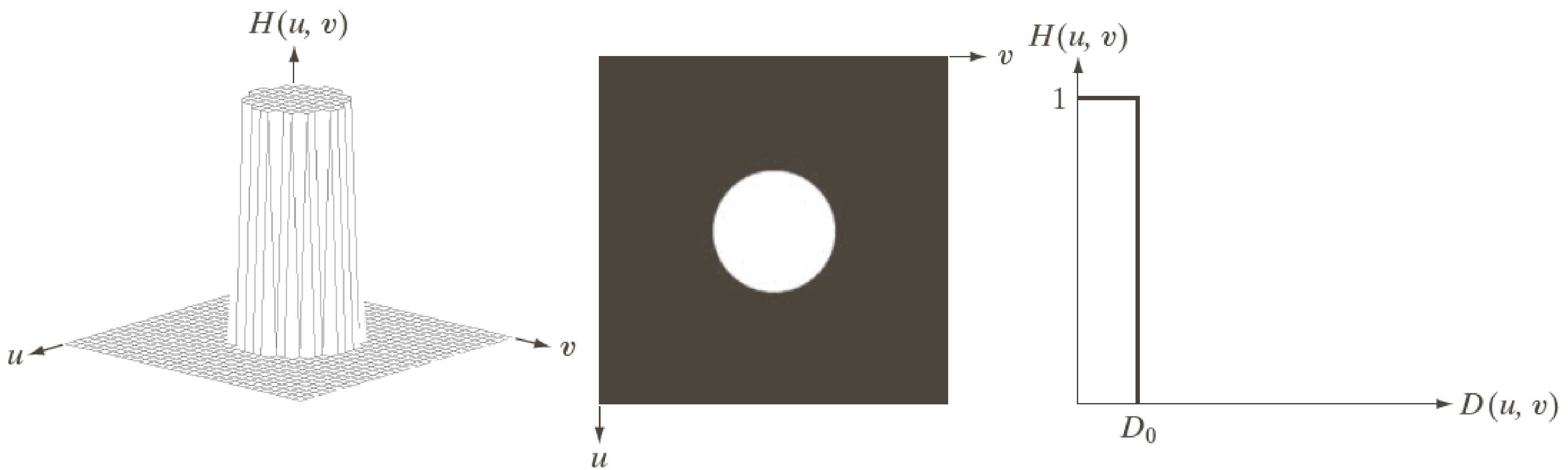
Ideal Lowpass Filters (ILPF)

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) \le D_0 \\ 0 & \text{if } D(u,v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u,v)$ is the distance between a point (u,v) in the frequency domain and the center of the frequency rectangle

$$D(u,v) = \left[(u - P/2)^2 + (v - Q/2)^2 \right]^{1/2}$$

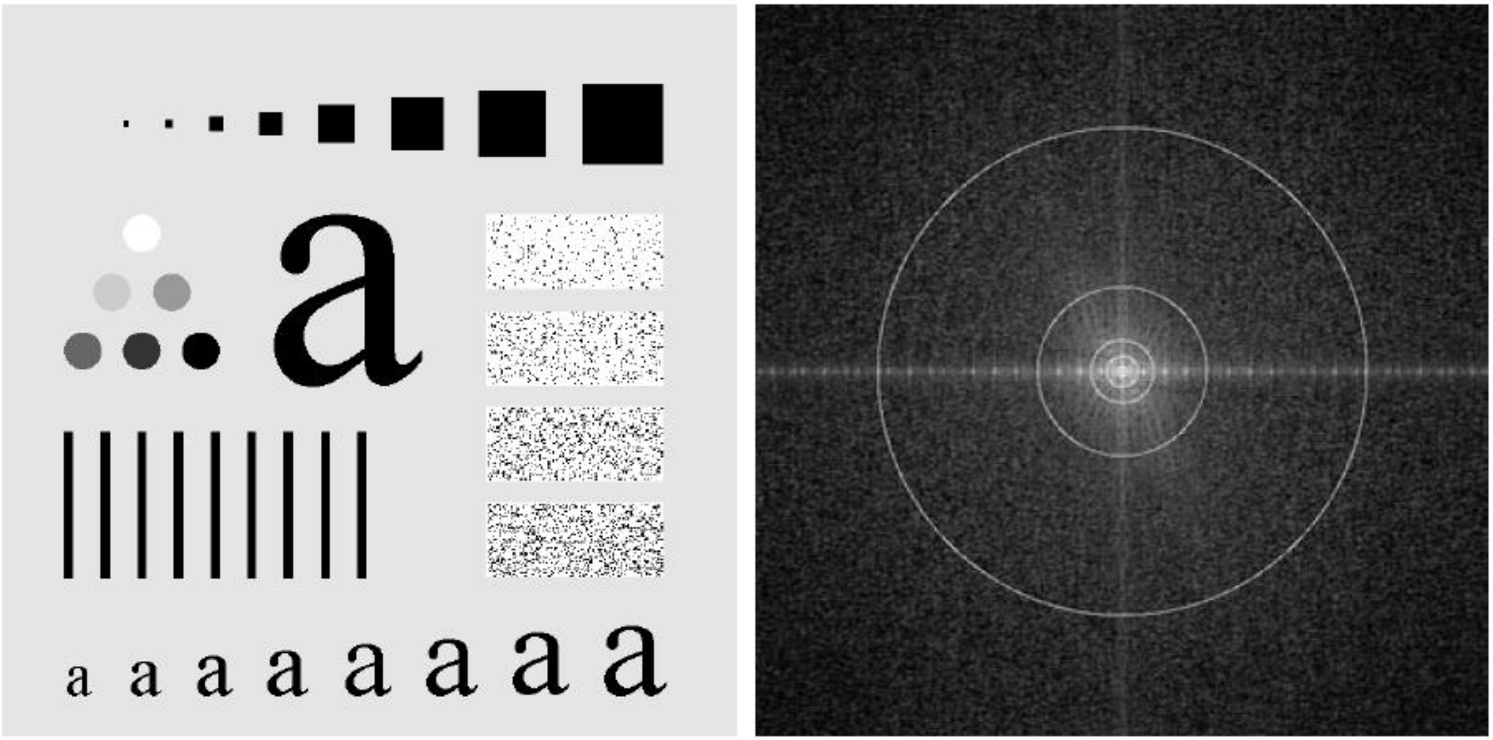
Smoothing Filters: ILPF



a b c

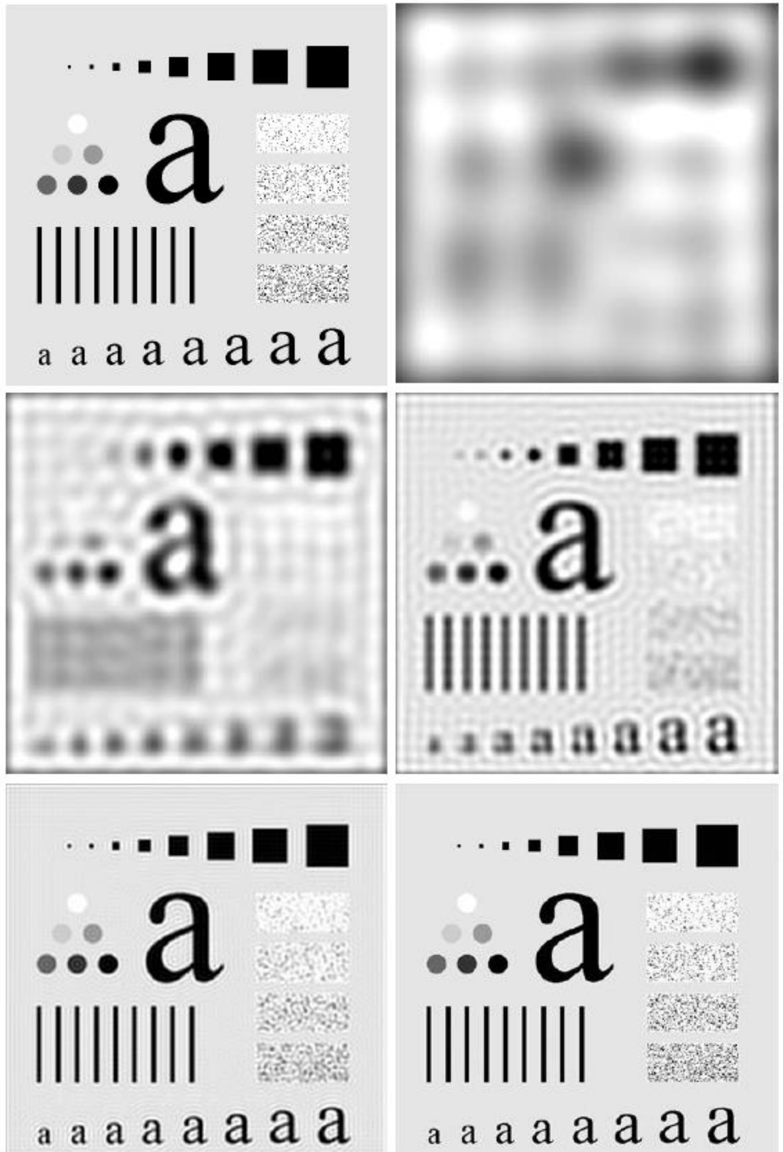
FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Smoothing Filters: ILPF



a b

FIGURE 4.41 (a) Test pattern of size 688 × 688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

Smoothing Filters: Spatial Representation of ILPF

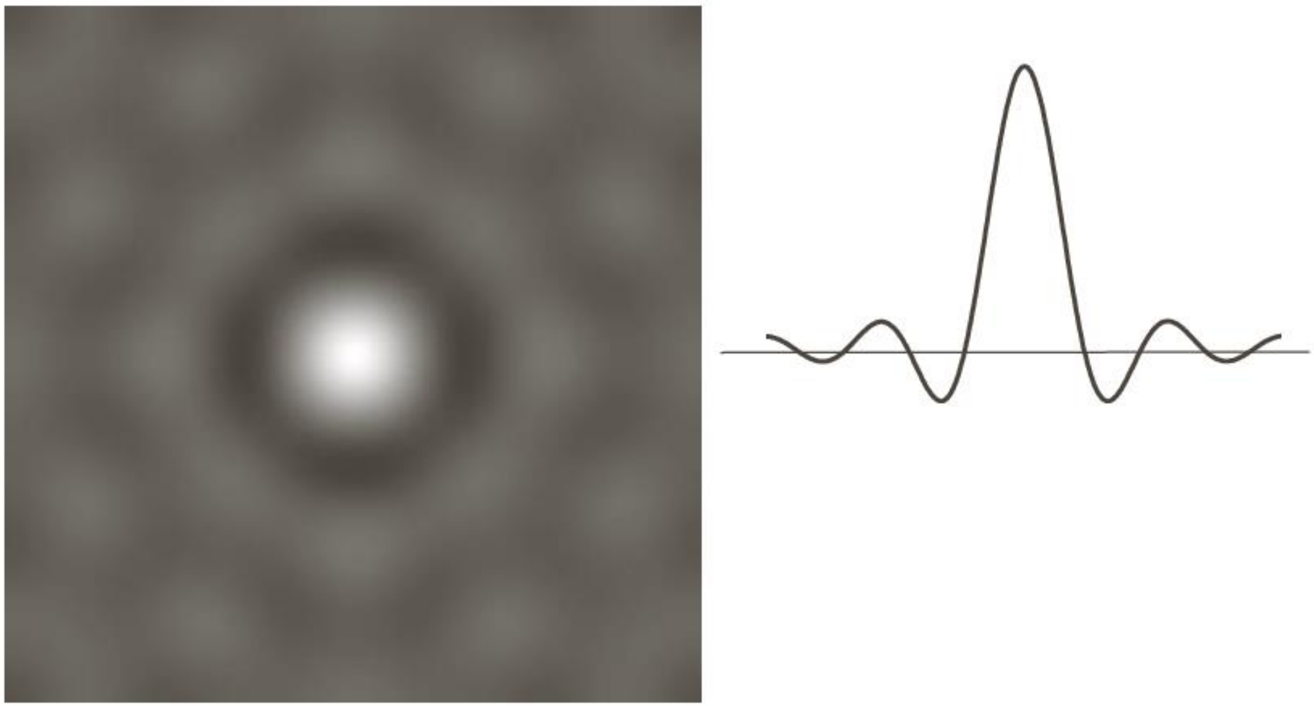


FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000×1000. (b) Intensity profile of a horizontal line passing through the center of the image.

Smoothing Filters: Butterworth Lowpass Filters (BLPF)

Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u,v) = \frac{1}{1+\left[D(u,v)/D_0\right]^{2n}}$$

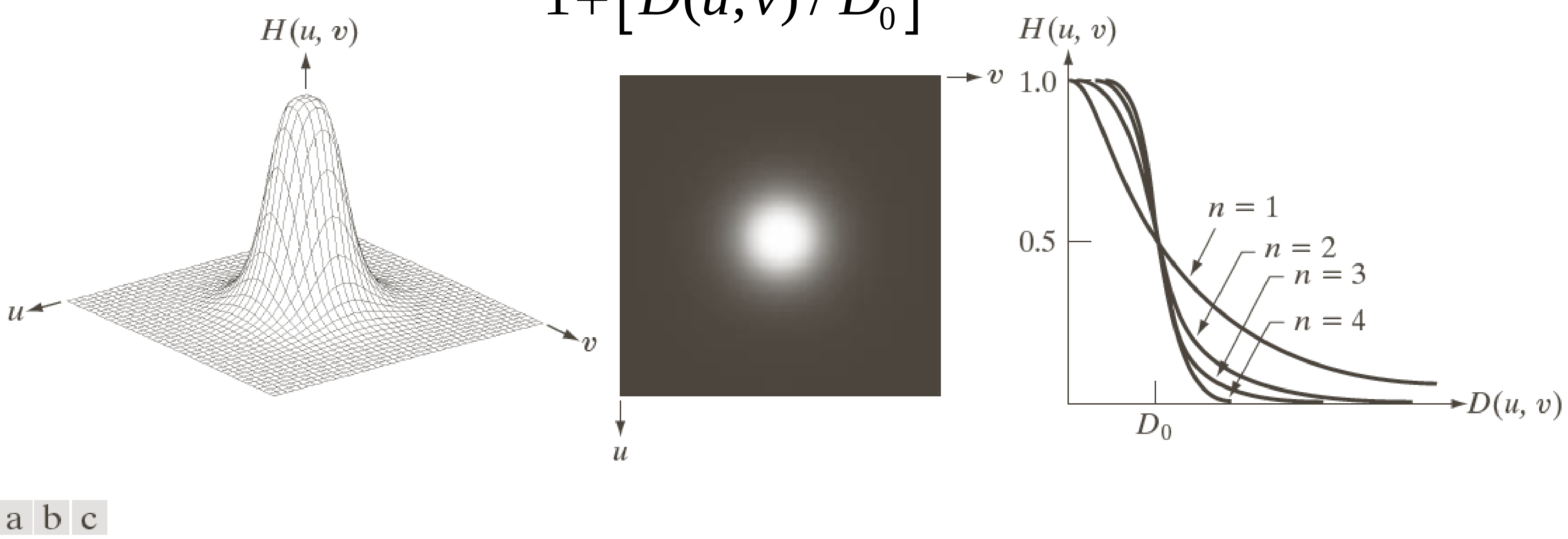
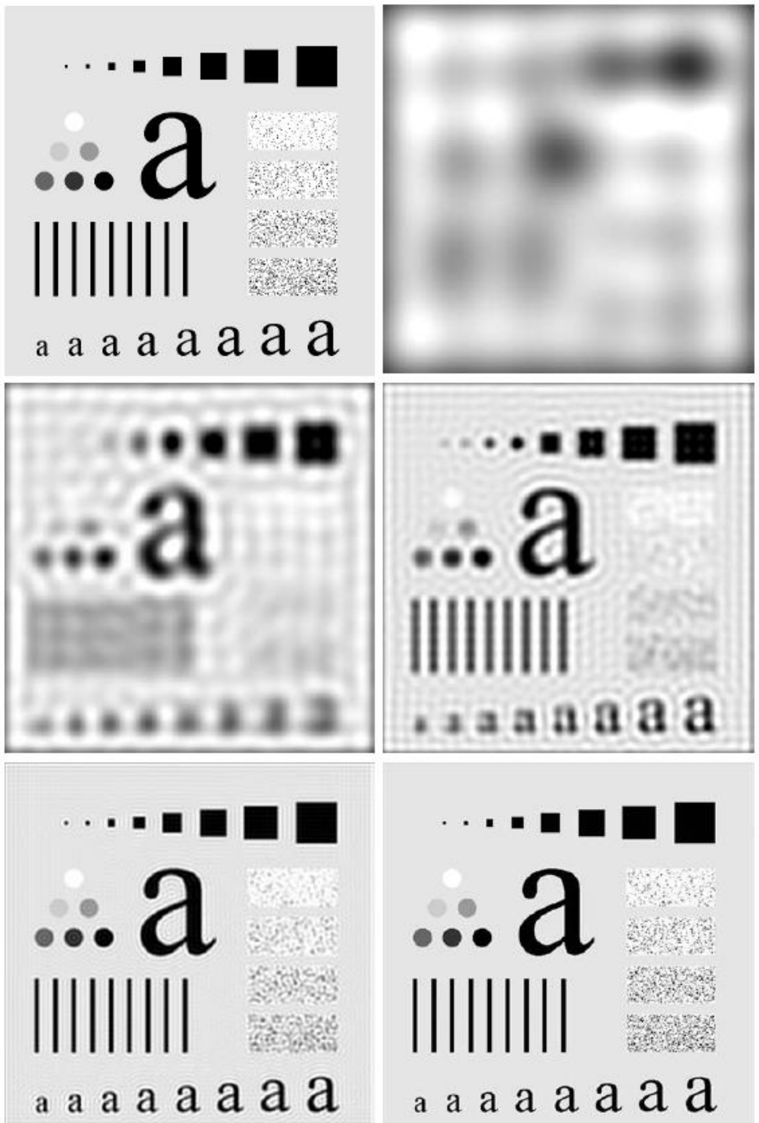
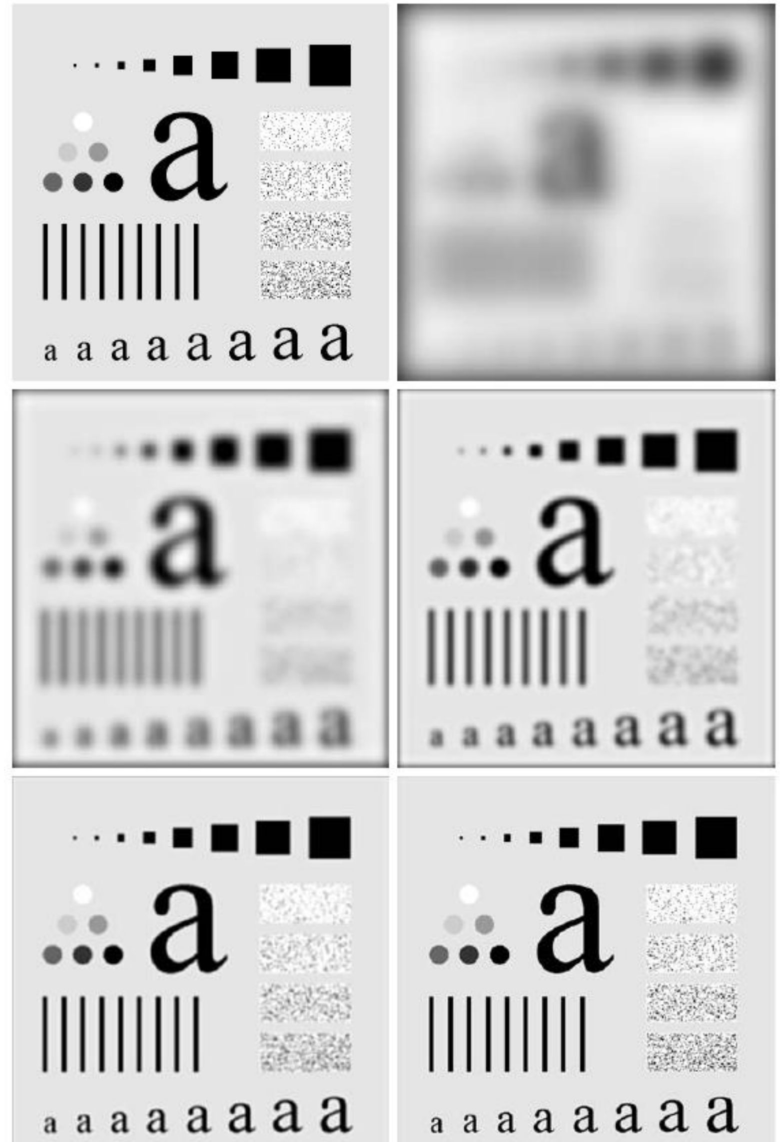


FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Smoothing Filters: Spatial Representation of BLPF

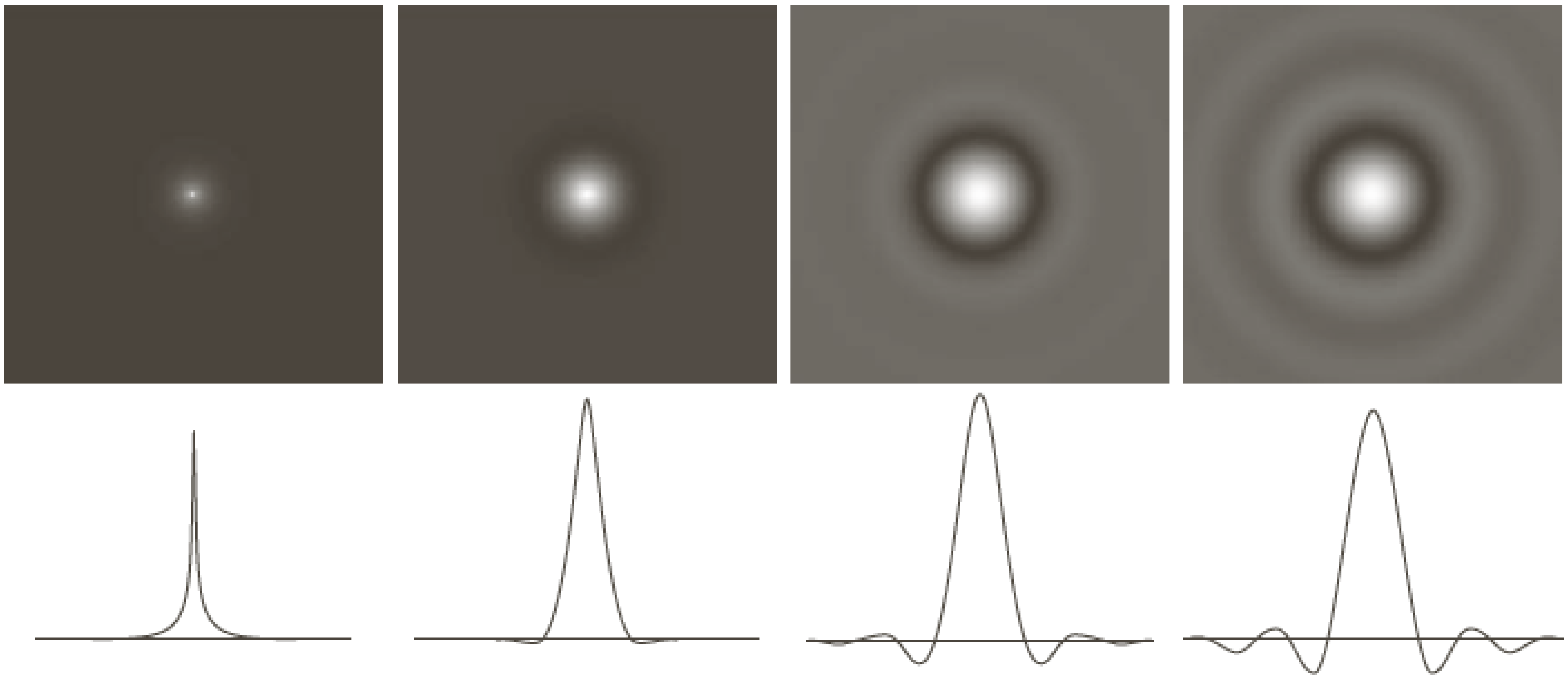


FIGURE 4.46 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding intensity profiles through the center of the filters (the size in all cases is 1000 × 1000 and the cutoff frequency is 5). Observe how ringing increases as a function of filter order.

Smoothing Filters: Gaussian Lowpass Filters (GLPF)

Gaussian Lowpass Filters (GLPF) in two dimensions is given

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

Smoothing Filters: GLPF

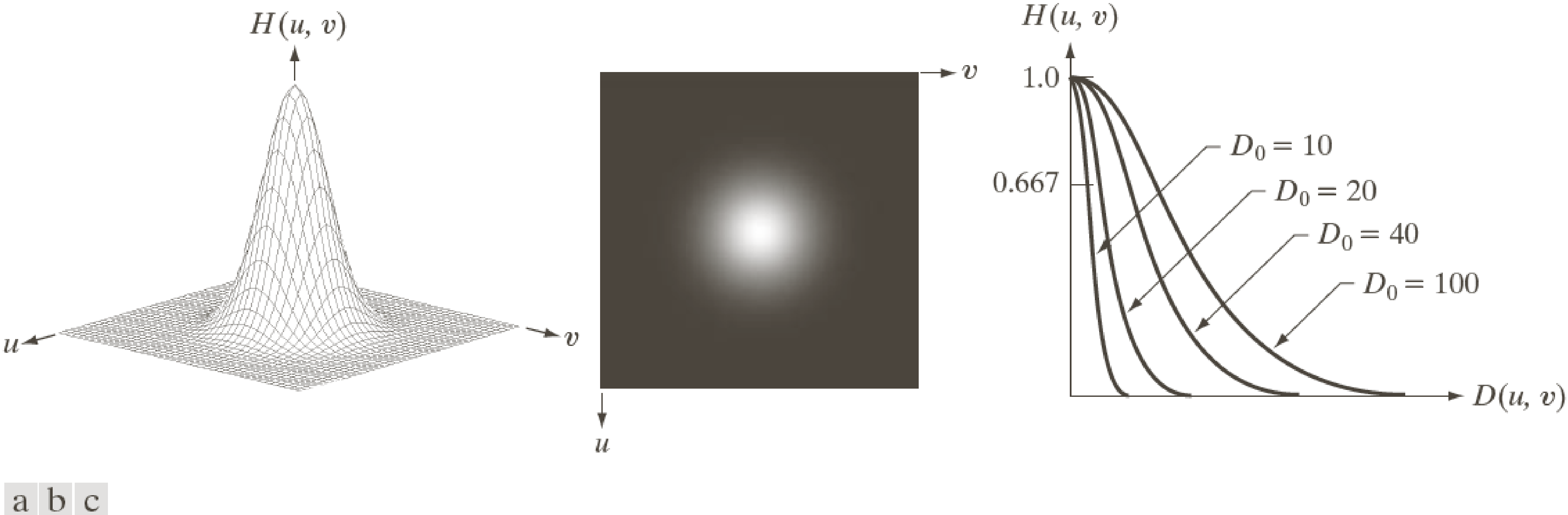
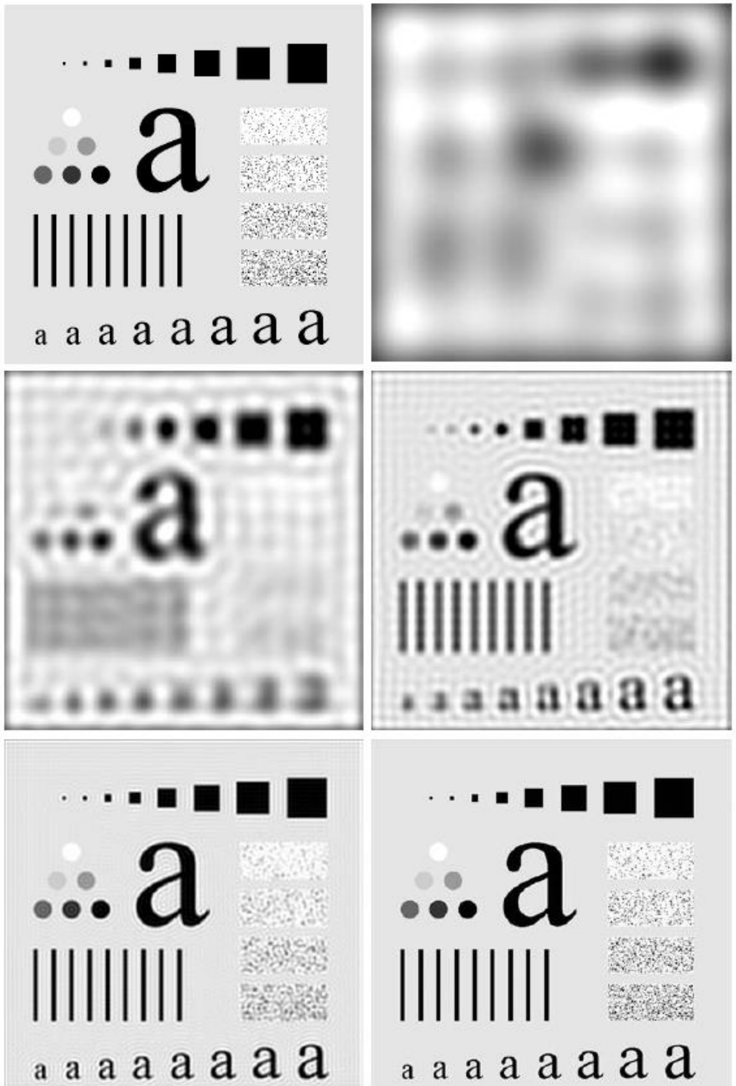
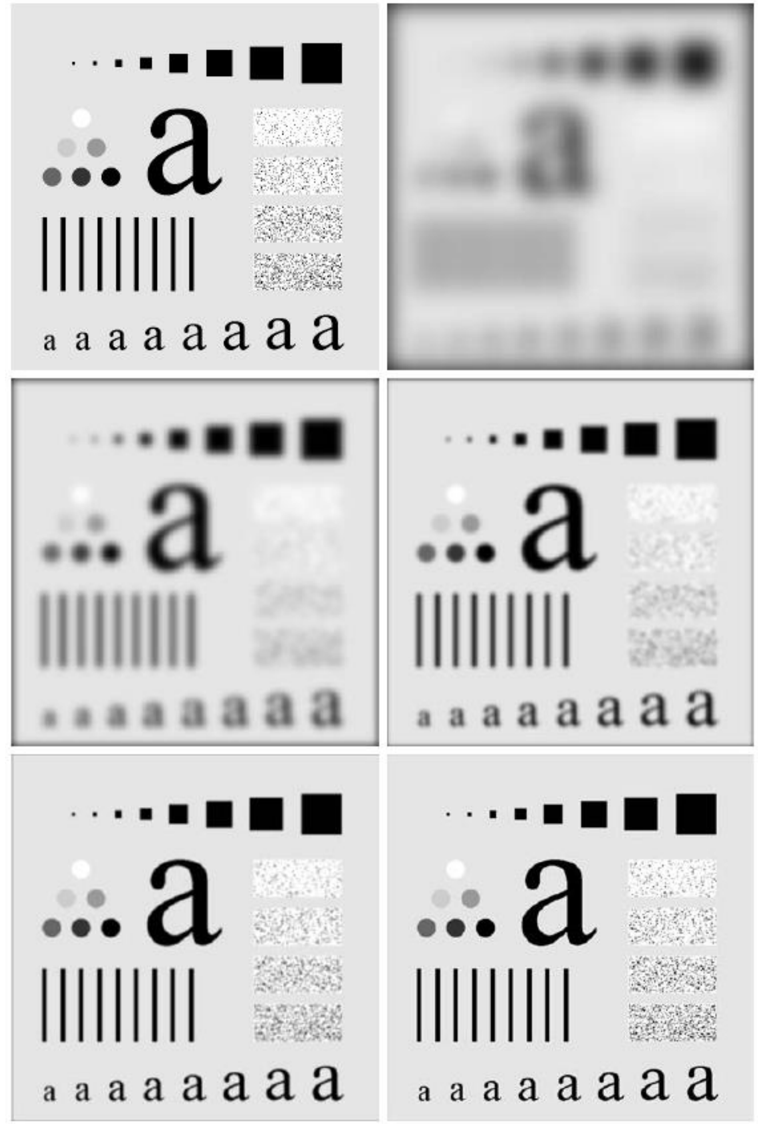


FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.



a b
c d
e f

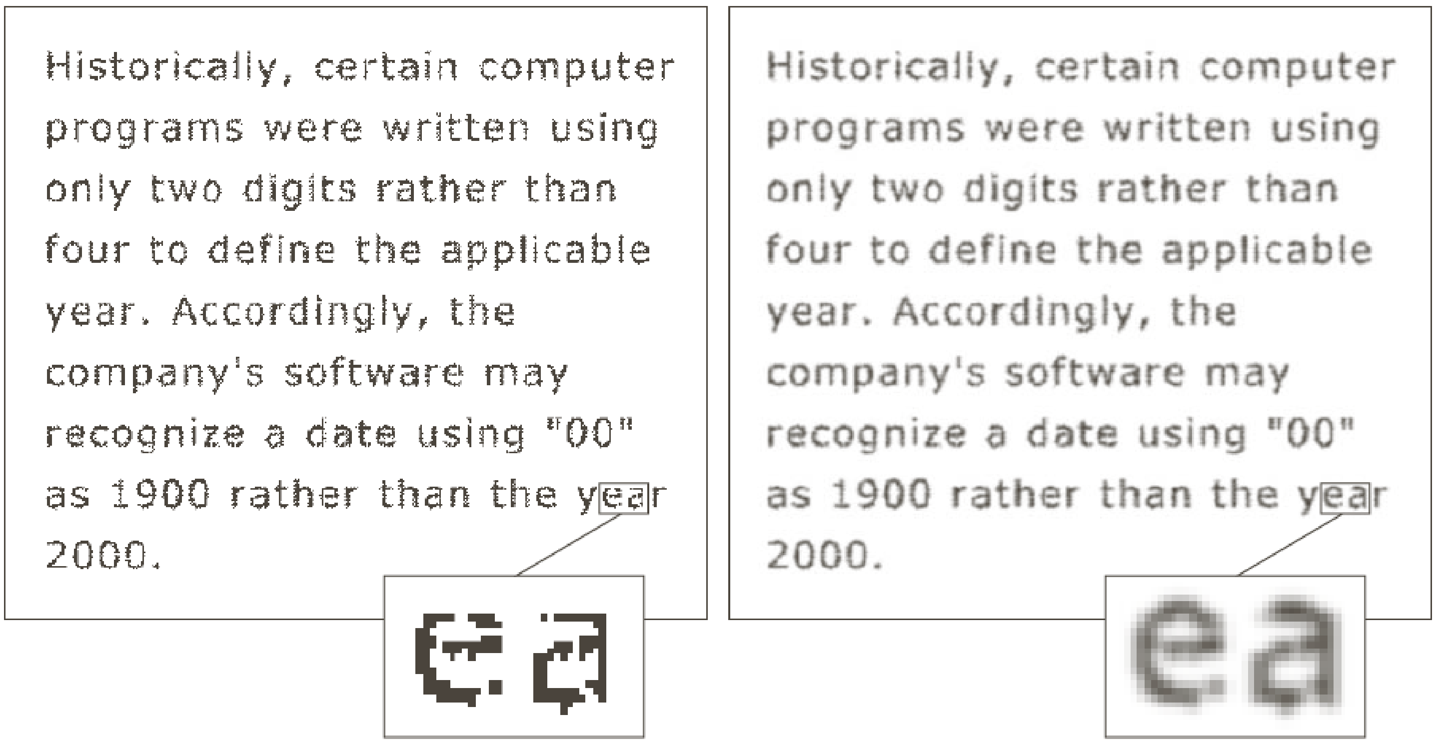
FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.



a b
c d
e f

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Smoothing Filters: Example of smoothing by GLPF



a b

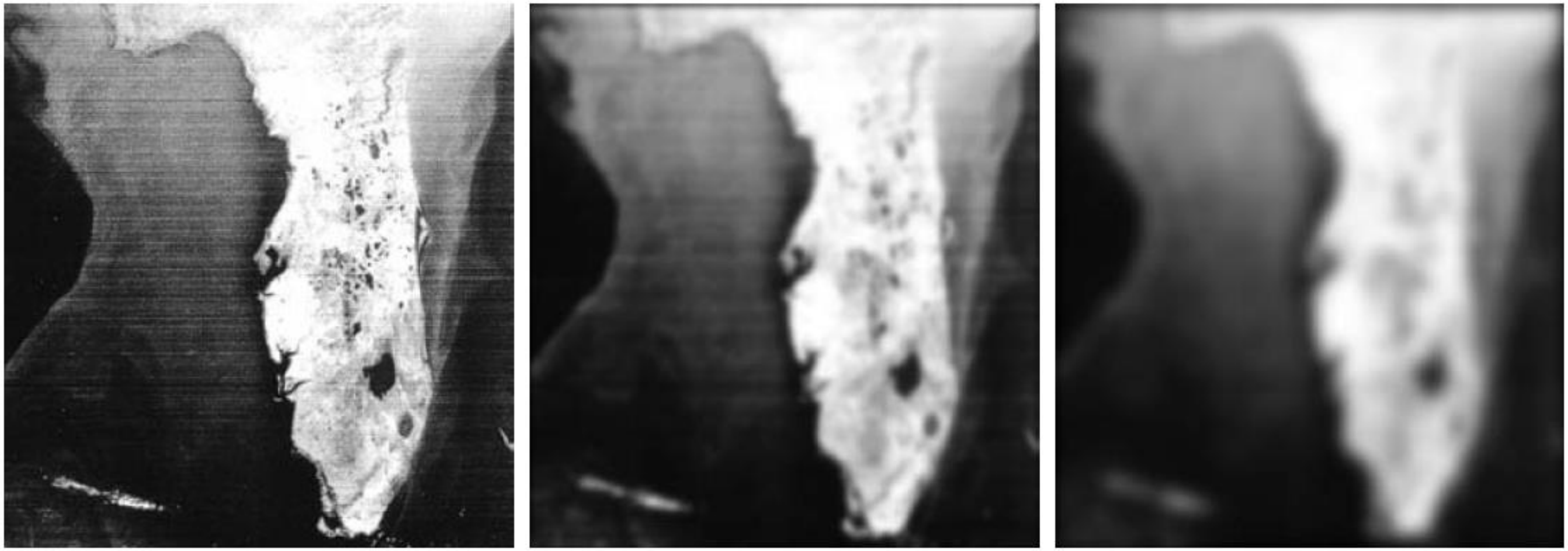
FIGURE 4.49
(a) Sample text of low resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).

Smoothing Filters: Example of smoothing by GLPF

a b c

FIGURE 4.50 (a) Original image (784 × 732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Smoothing Filters: Example of smoothing by GLPF



a b c

FIGURE 4.51 (a) Image showing prominent horizontal scan lines. (b) Result of filtering using a GLPF with $D_0 = 50$. (c) Result of using a GLPF with $D_0 = 20$. (Original image courtesy of NOAA.)

Sharpening Filters

- Image sharpening can be achieved in the frequency domain by highpass filtering.
- It attenuates low-frequency components without disturbing high-frequency information in the Fourier transform.

Sharpening Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Sharpening Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u,v) = \frac{1}{1+\left[D_0 / D(u,v)\right]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

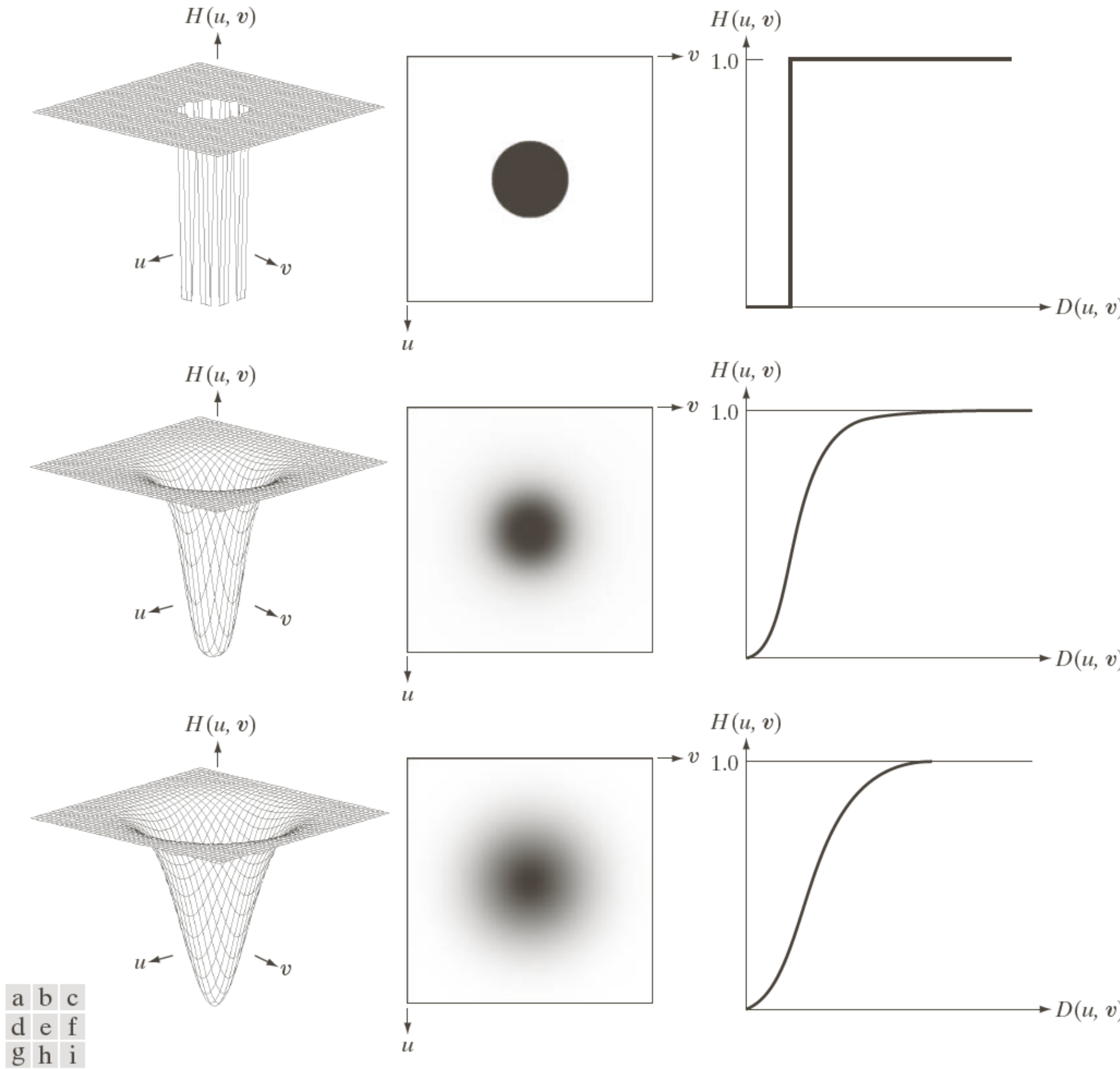


FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Sharpening Filters: Spatial Representation

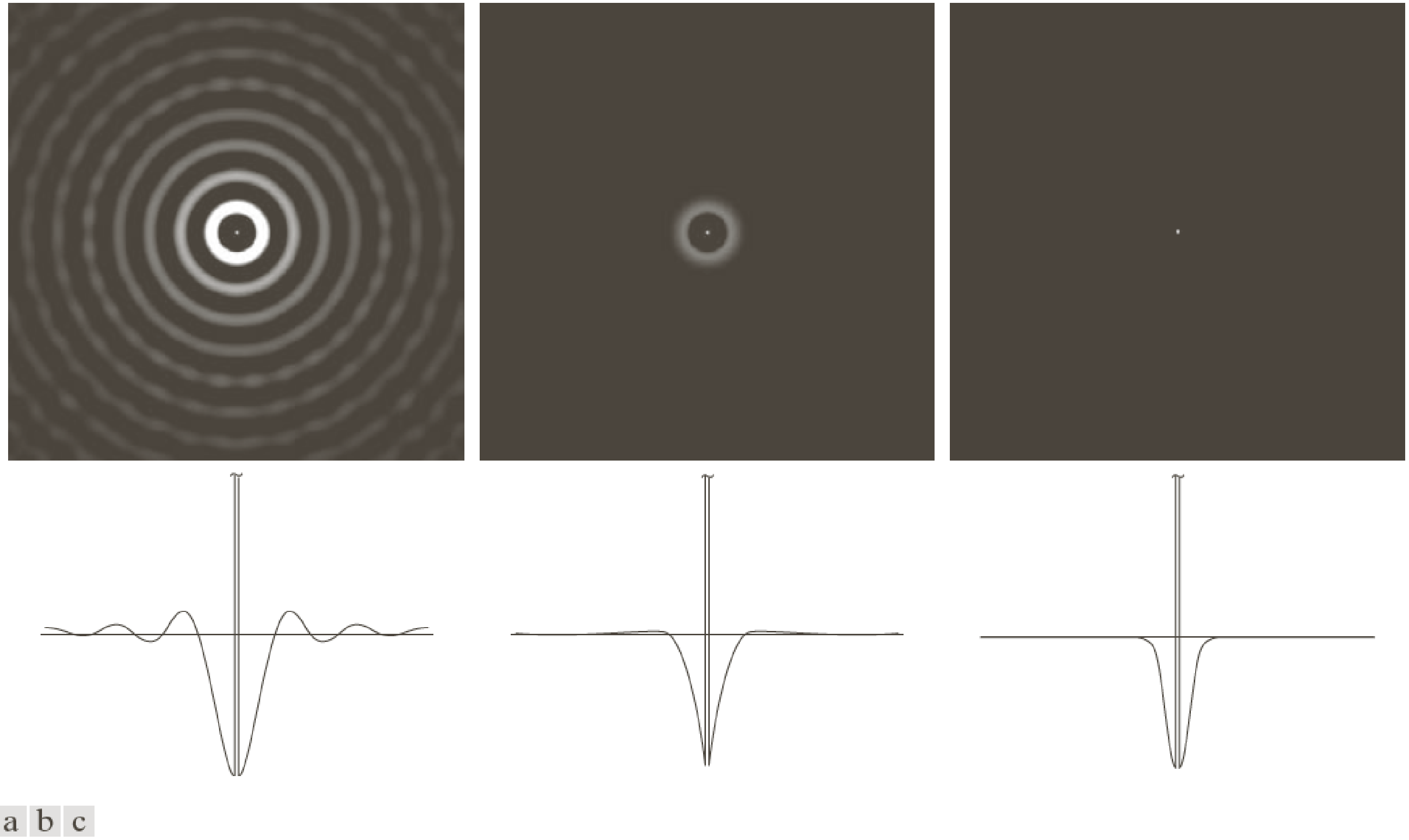


FIGURE 4.53 Spatial representation of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding intensity profiles through their centers.

Sharpening Filters: Result of IHPF

a b c

FIGURE 4.54 Results of highpass filtering the image in Fig. 4.41(a) using an IHPF with $D_0 = 30, 60$, and 160.

Sharpening Filters: Result of BHPF

a b c

FIGURE 4.55 Results of highpass filtering the image in Fig. 4.41(a) using a BHPF of order 2 with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). These results are much smoother than those obtained with an IHPF.

Sharpening Filters: Result of GHPF

a b c

FIGURE 4.56 Results of highpass filtering the image in Fig. 4.41(a) using a GHPF with $D_0 = 30, 60,$ and 160, corresponding to the circles in Fig. 4.41(b). Compare with Figs. 4.54 and 4.55.

Laplacian in Frequency Domain

$$H(u,v) = -4\pi^2(u^2 + v^2)$$

$$H(u,v) = -4\pi^2\left[(u - P/2)^2 + (v - Q/2)^2)\right]$$

$$= -4\pi^2 D^2(u,v)$$

The Laplacian image

$$\nabla^2 f(x,y) = \Im^{-1}\left\{H(u,v)F(u,v)\right\}$$

Enhancement is obtained

$$g(x,y) = f(x,y) + c\nabla^2 f(x,y) \qquad c = -1$$

Laplacian in Frequency Domain

The enhanced image

$$
\begin{aligned}
g(x,y) &= \Im^{-1}\left\{F(u,v) - H(u,v)F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 - H(u,v)\right]F(u,v)\right\} \\
&= \Im^{-1}\left\{\left[1 + 4\pi^2 D^2(u,v)\right]F(u,v)\right\}
\end{aligned}
$$

Laplacian in Frequency Domain

a b

FIGURE 4.58
(a) Original, blurry image. (b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Thank You

Questions?