# **Computer Vision**

### **Introduction to Classifiers**

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Sri City, Chittoor



### **Previous Class**

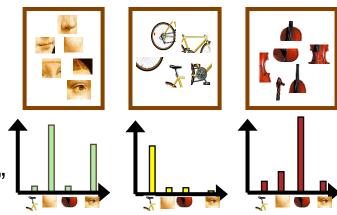
### Image features and categorization

Choosing right features
Object, Scene, Action, etc.

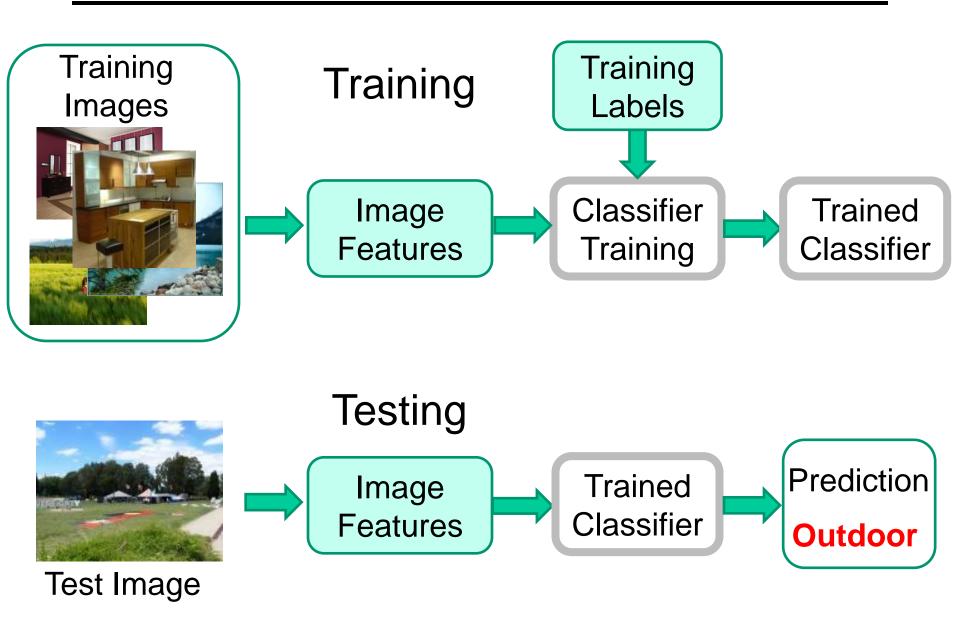


### **Bag-of-visual-words**

Extract local features
Learn "visual vocabulary"
Quantize features using visual vocabulary
Represent by frequencies of "visual words"



## Today's Class



## Today's Class

**Nearest Neighbor Classifier** 

**K - Nearest Neighbor Classifier** 

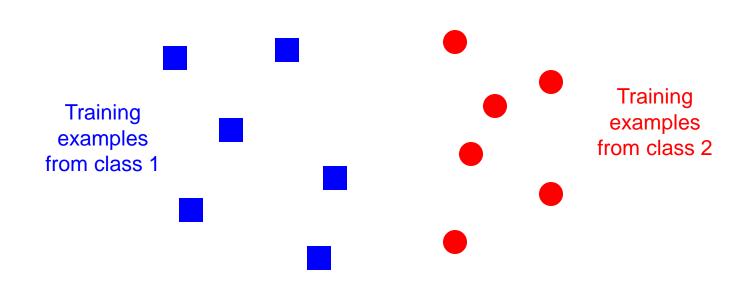
**Linear Classifier** 

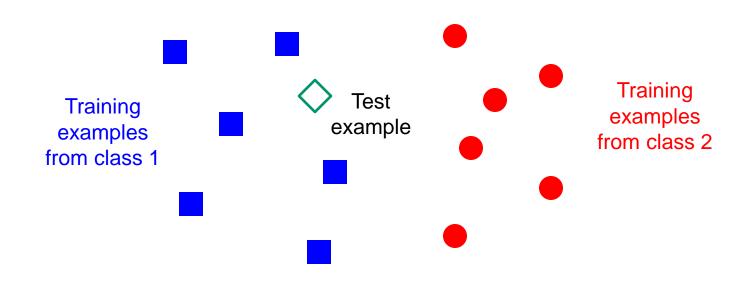
**Support Vector Machine** 

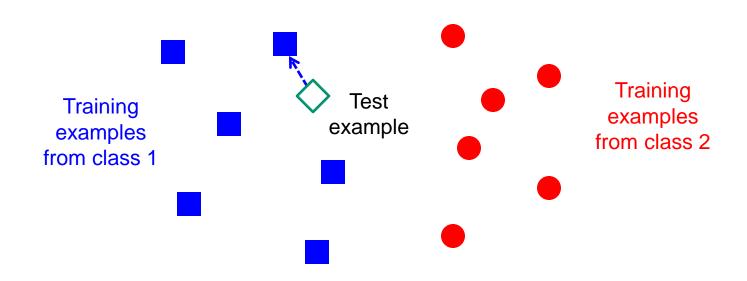
Non-linear SVM

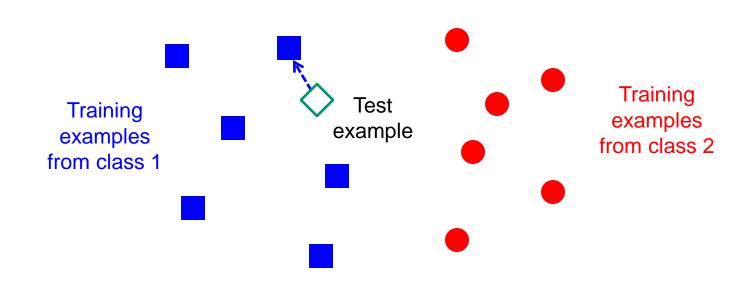
**Multi-class SVM** 

**Softmax Classifier** 







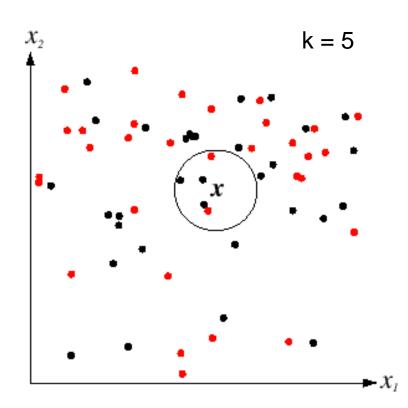


### $f(\mathbf{x})$ = label of the training example nearest to $\mathbf{x}$

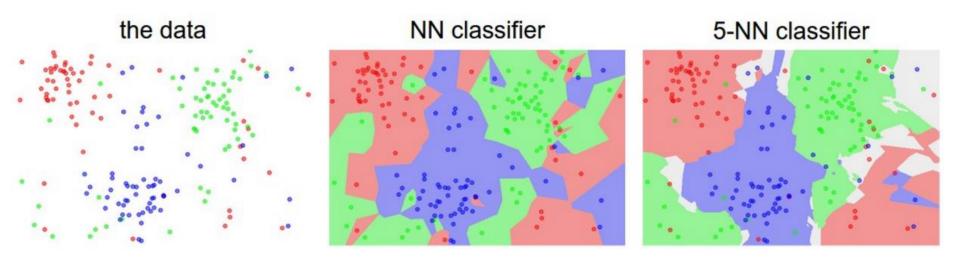
All we need is a distance function for our inputs No training required!

## K-nearest neighbor classifier

- For a new point, find the k closest points from training data
- Vote for class label with labels of the k points

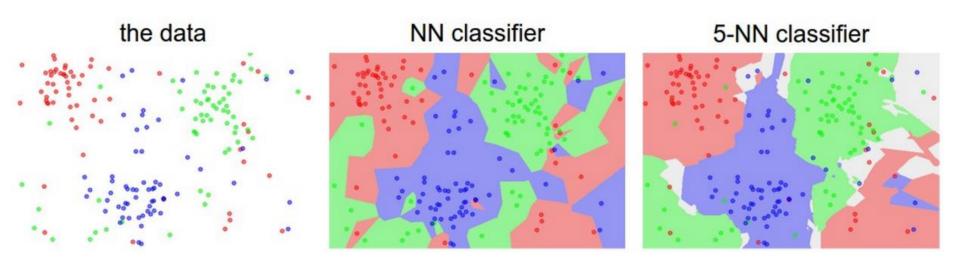


## K-nearest neighbor classifier



Credit: cs231n, <a href="http://cs231n.github.io/classification/">http://cs231n.github.io/classification/</a>

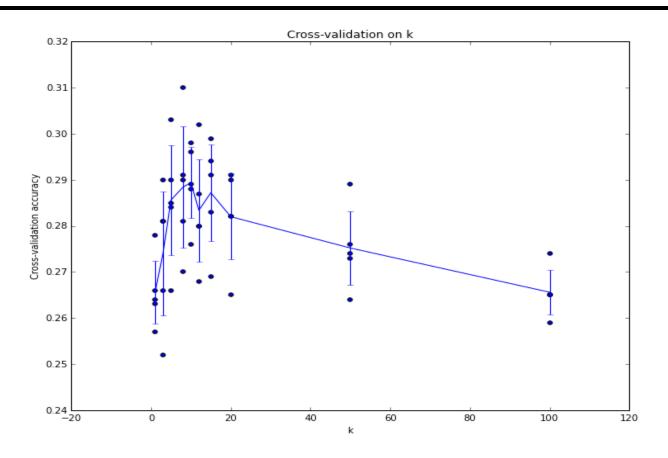
## K-nearest neighbor classifier



Which classifier is more robust to *outliers*?

Credit: cs231n, <a href="http://cs231n.github.io/classification/">http://cs231n.github.io/classification/</a>

### Choice of K in KNN classifier



Example of a 5-fold cross-validation run for the parameter  $\mathbf{k}$ . Note that in this particular case, the cross-validation suggests that a value of about  $\mathbf{k} = 7$  works best on this particular dataset (corresponding to the peak in the plot).

Credit: cs231n, <a href="http://cs231n.github.io/classification/">http://cs231n.github.io/classification/</a>

"Non-parametric" classifier: the entire training set is essentially the model parameters.

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#### **Pros:**

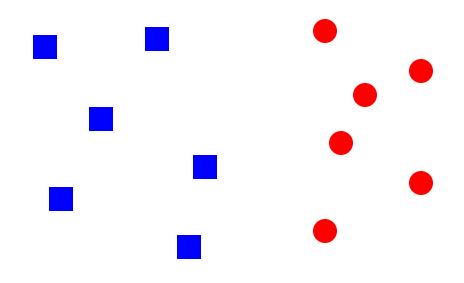
- Very fast at training time
- **Flexible**: all it requires is a way to compute similarity or distances between pairs of features. Applies to many different kinds of features.
- Works with any number of classes.
- Works well in practice for large datasets (but see cons)

"Non-parametric" classifier: the entire training set is essentially the model parameters.

#### Cons:

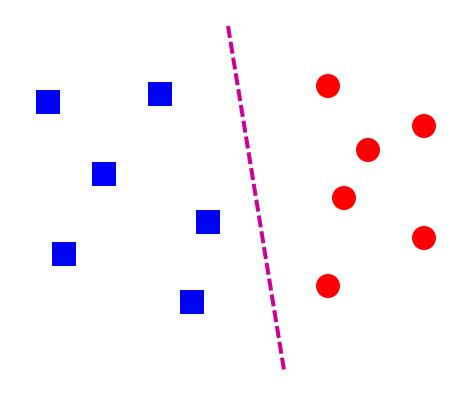
- The classifier must *remember* all of the training data and store it for future comparisons with the test data.
- This is **space inefficient** because datasets may easily be gigabytes in size.
- Slow at test time (need to compute distances between test example and every training example)
- Optimum value of **K** is not known.

## Linear classifiers – 2 class problem



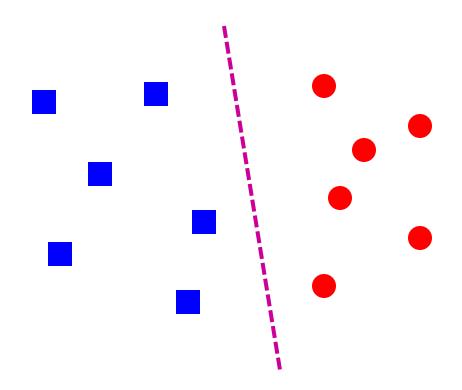
Find a *linear function* to separate the classes:

## Linear classifiers – 2 class problem



Find a *linear function* to separate the classes:

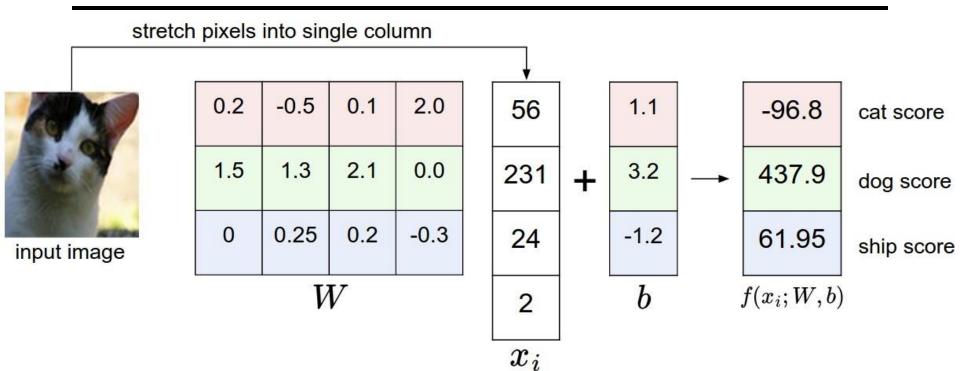
## Linear classifiers – 2 class problem



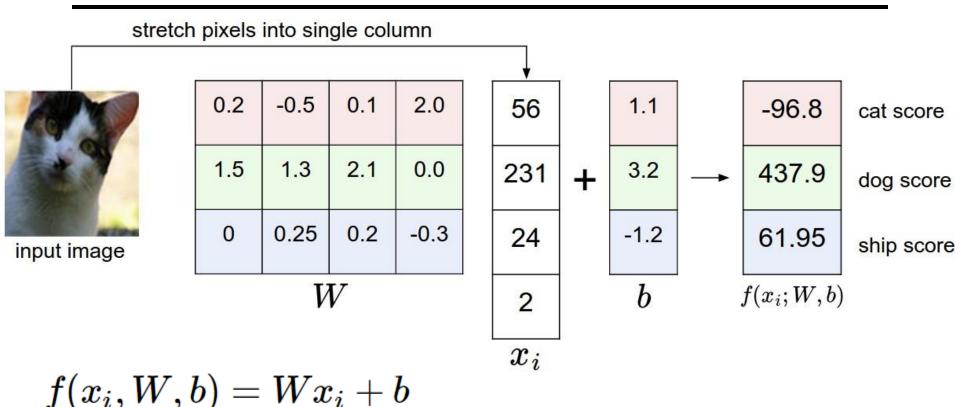
Find a *linear function* to separate the classes:

$$f(\mathbf{x}) = sign(\mathbf{w} \cdot \mathbf{x} + b)$$

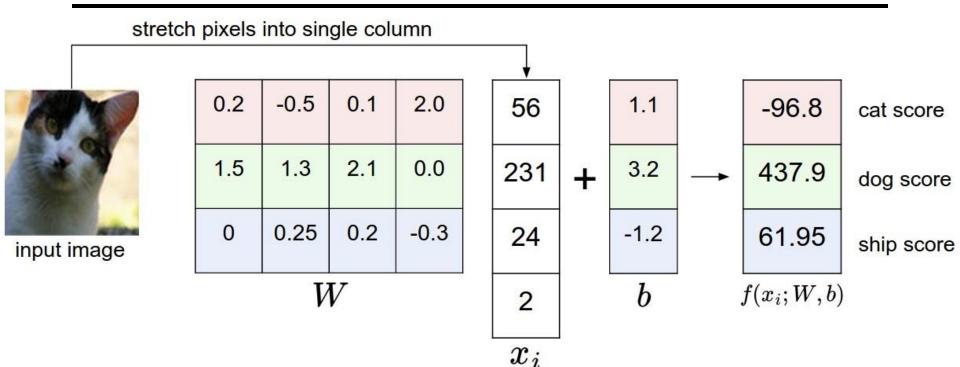
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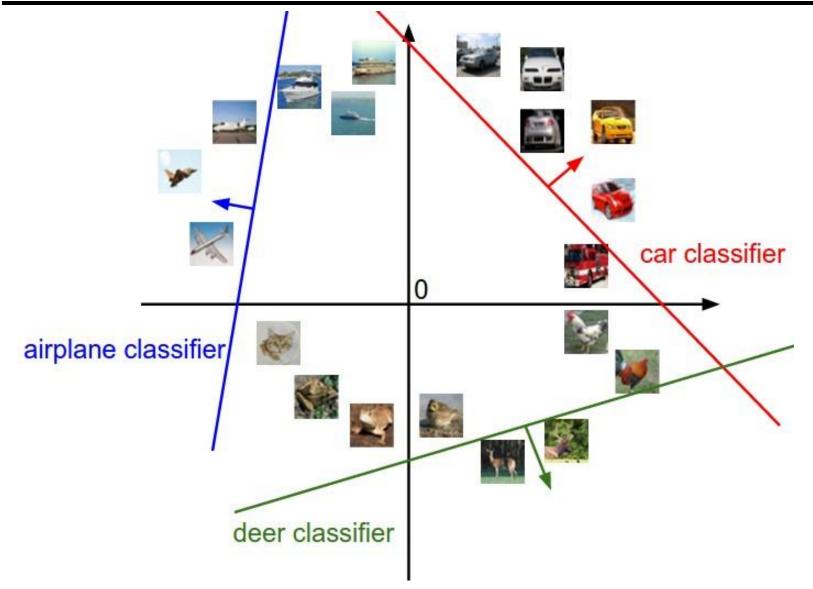
$$f(x_i,W,b)=Wx_i+b$$

Image  $x_i$  has all of its pixels flattened out to a single column vector of shape [D x 1].

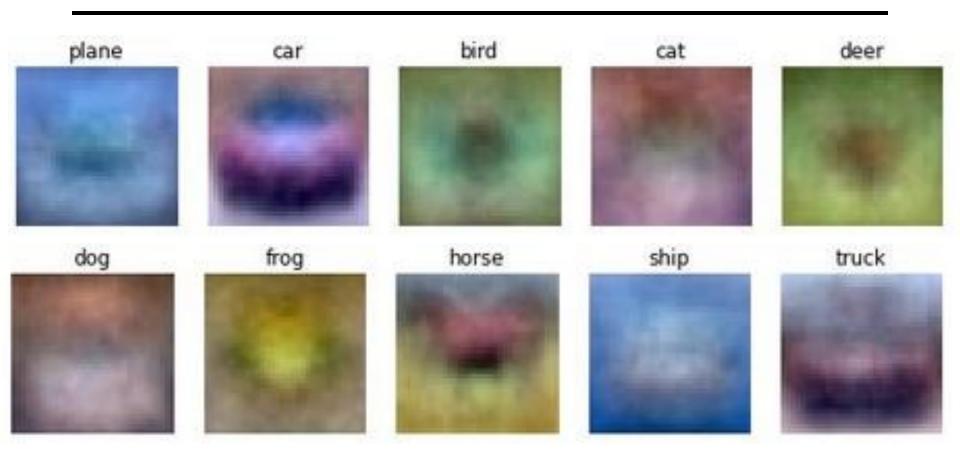
Matrix **W** (of size [K x D]), and vector **b** (of size [K x 1]) are the **parameters**.

**K** is the number of classes.

## Analogy of images as high-dimensional points

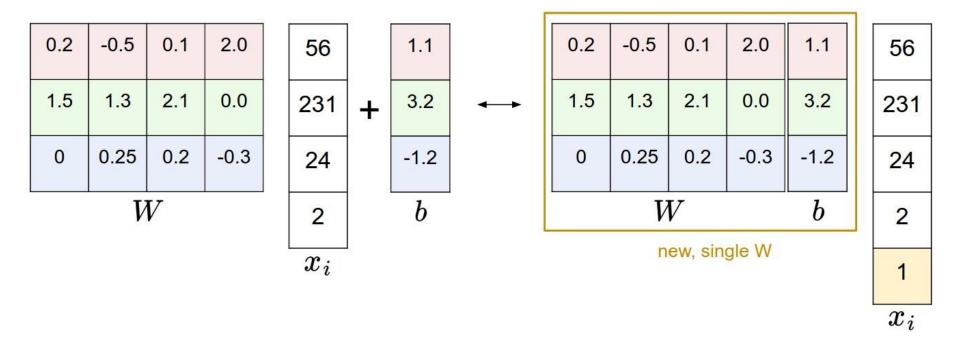


### Interpretation of linear classifiers as template matching



Example learned weights at the end of learning for CIFAR-10. Note that, for example, the ship template contains a lot of blue pixels as expected. This template will therefore give a high score once it is matched against images of ships on the ocean with an inner product.

### Bias Trick



### Linear classifiers

"Parametric" classifier: model defined by a small number of parameters (w, b)

#### Pros:

- Very fast at test time

#### Cons:

- Slow at training time: need to estimate the parameters
- Data may not be linearly separable

### NN pros:

- Simple to implement
- Decision boundaries not necessarily linear
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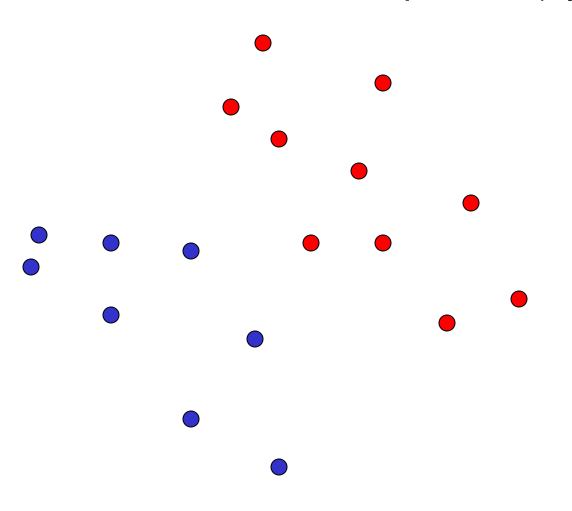
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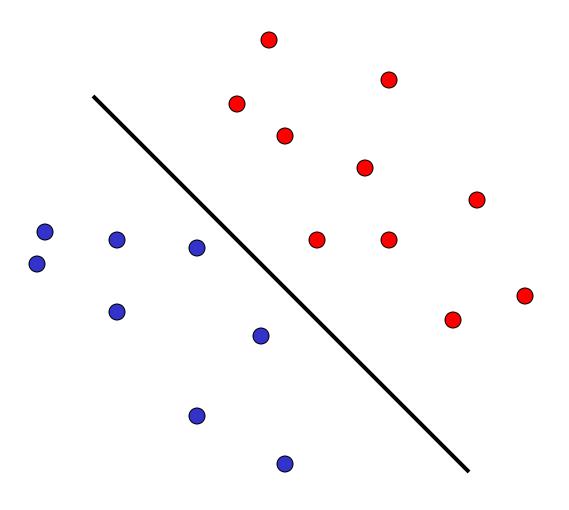
### Linear pros:

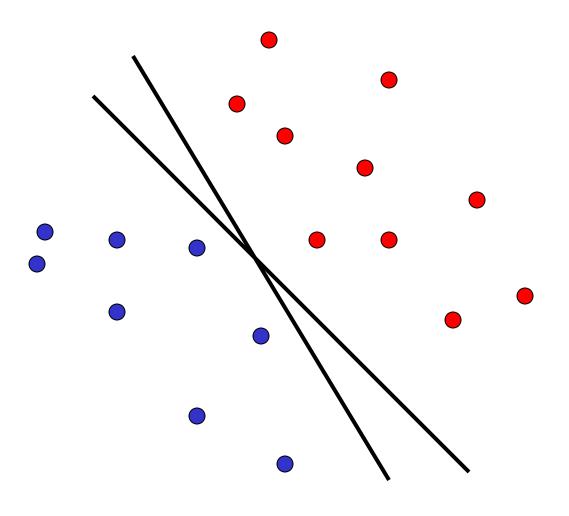
- Low-dimensional parametric representation
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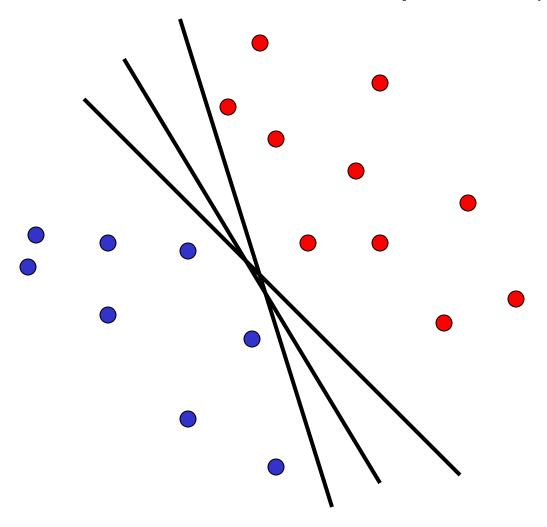
#### Linear cons:

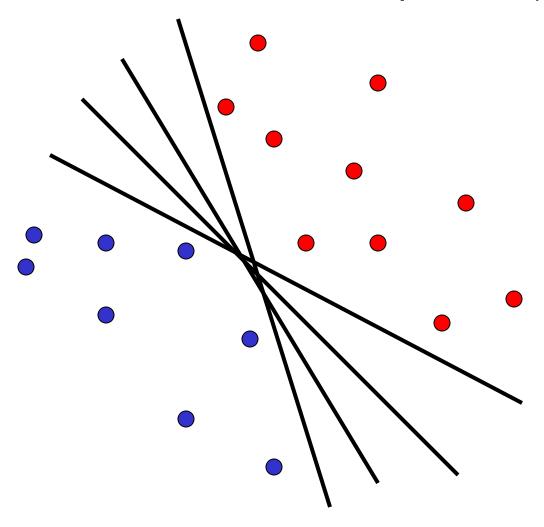
- How to train the linear function?
- What if data is not linearly separable?

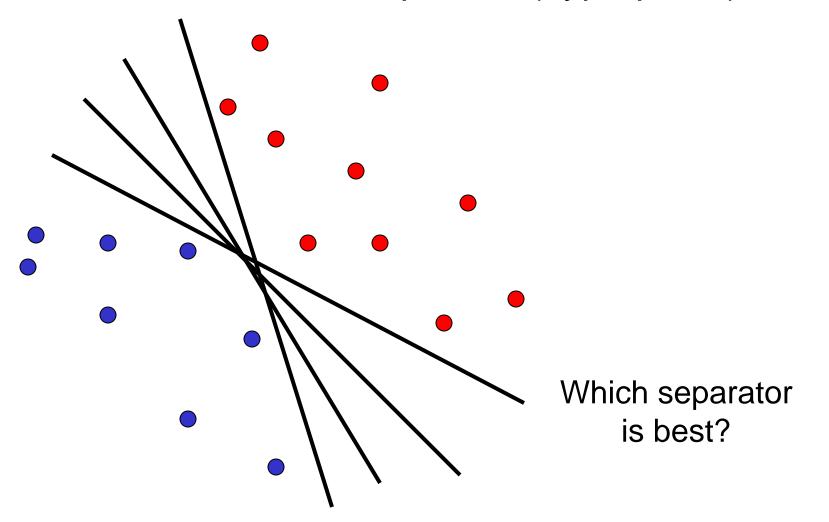




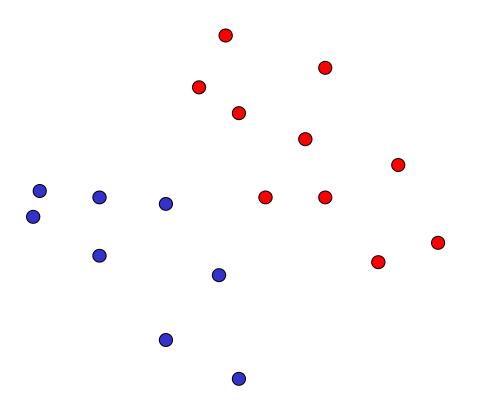




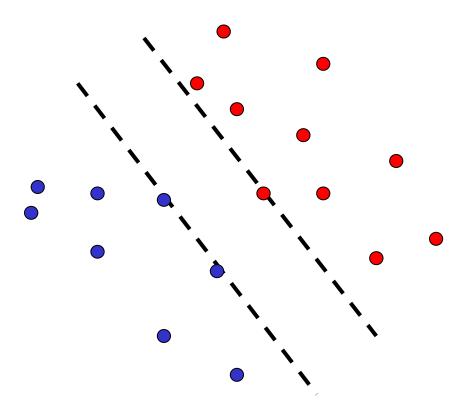




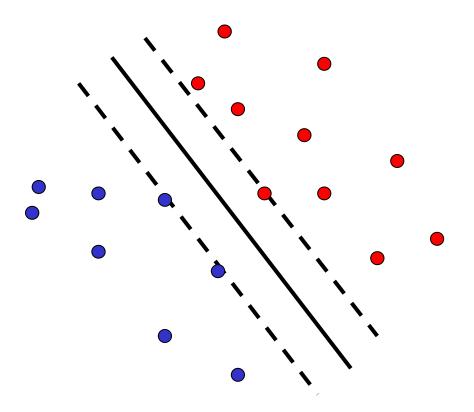
 Find hyperplane that maximizes the margin between the positive and negative examples



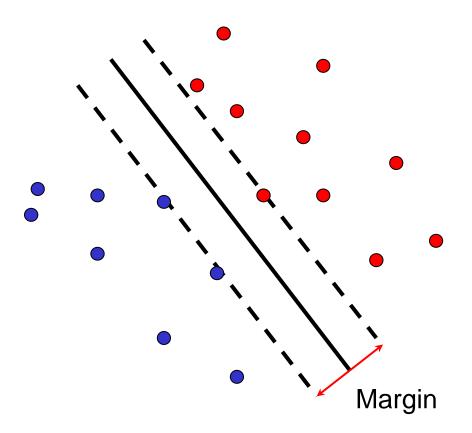
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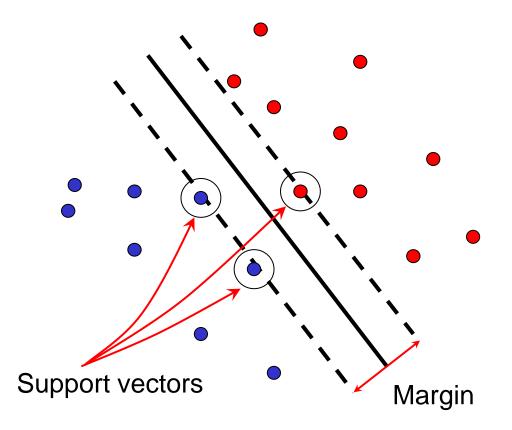
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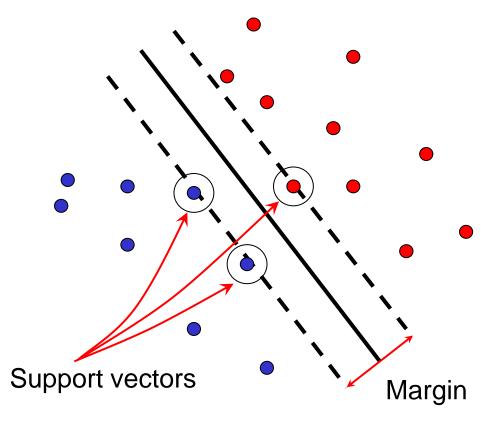
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 Find hyperplane that maximizes the margin between the positive and negative examples



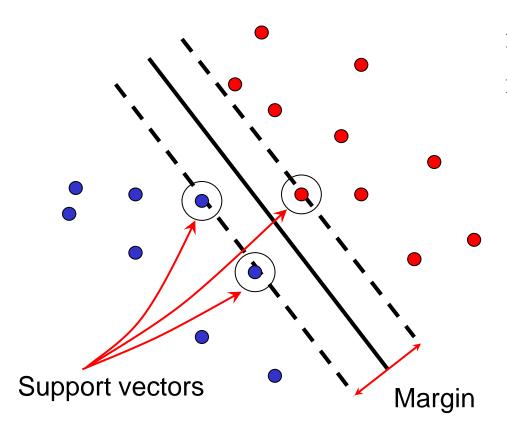
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$$\mathbf{x}_i$$
 positive  $(y_i = 1)$ :  $\mathbf{x}_i \cdot \mathbf{w} + b \ge 1$ 

$$\mathbf{x}_i$$
 negative  $(y_i = -1)$ :  $\mathbf{x}_i \cdot \mathbf{w} + b \le -1$ 

 Find hyperplane that maximizes the margin between the positive and negative examples

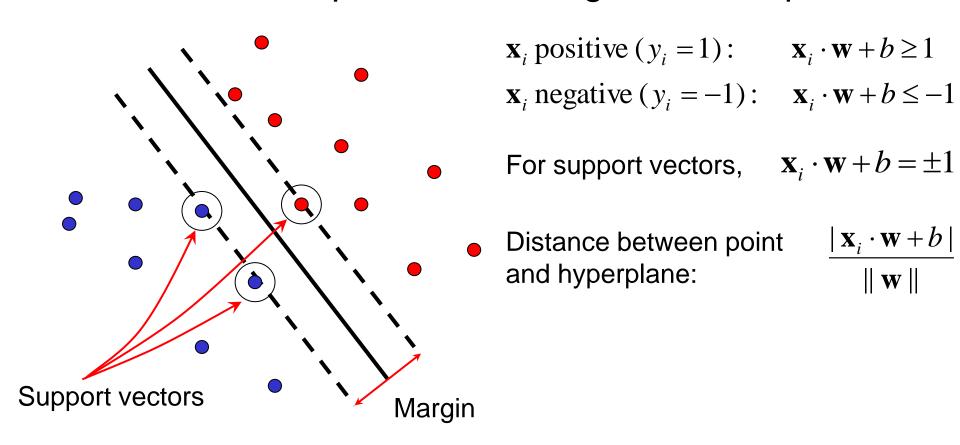


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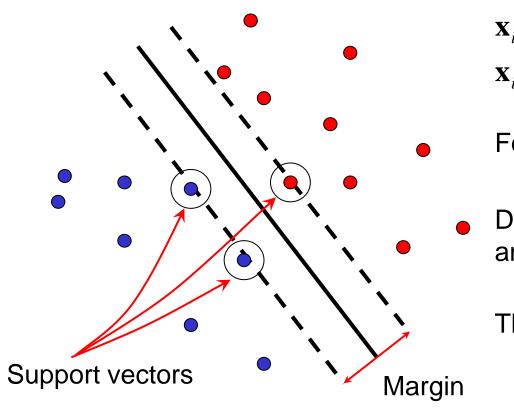
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For support vectors, 
$$\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$$

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For support vectors, 
$$\mathbf{x}_i \cdot \mathbf{w} + b = \pm 1$$

Distance between point 
$$|\mathbf{x}_i \cdot \mathbf{w} + b|$$
 and hyperplane:  $|\mathbf{w}|$ 

Therefore, the margin is  $2/||\mathbf{w}||$ 

1. Maximize margin 2 / ||w||

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- 2. Correctly classify all training data:

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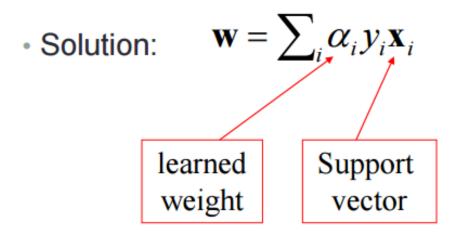
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Quadratic optimization problem:

$$\min_{\mathbf{w},b} \frac{1}{2} \|\mathbf{w}\|^2 \quad \text{subject to} \quad y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \ge 1$$



• The weights  $\alpha_i$  are non-zero only at support vectors.

• Solution: 
$$\mathbf{w} = \sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i}$$

$$b = y_{i} - \mathbf{w} \cdot \mathbf{x}_{i} \quad \text{(for any support vector)}$$

$$\mathbf{w} \cdot \mathbf{x} + b = \sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i} \cdot \mathbf{x} + b$$

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Classification function:

$$f(x) = \operatorname{sign}(\mathbf{w} \cdot \mathbf{x} + \mathbf{b}) \qquad \text{If } f(x) < 0, \text{ classify as negative,}$$

$$= \operatorname{sign}(\sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i} \cdot \mathbf{x} + \mathbf{b}) \quad \text{if } f(x) > 0, \text{ classify as positive}$$

$$\boxed{\text{Dot product only!}}$$

# SVM parameter learning

$$\min_{\mathbf{w},b} \frac{1}{2} \|\mathbf{w}\|^2$$

Separable data:  $\min_{\mathbf{w},b} \frac{1}{2} \|\mathbf{w}\|^2$  subject to  $y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \ge 1$ 

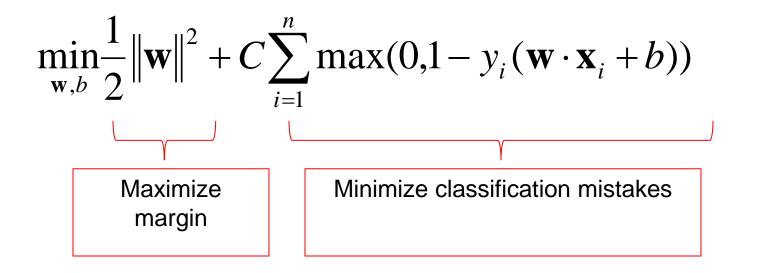
Maximize margin

Classify training data correctly

# SVM parameter learning

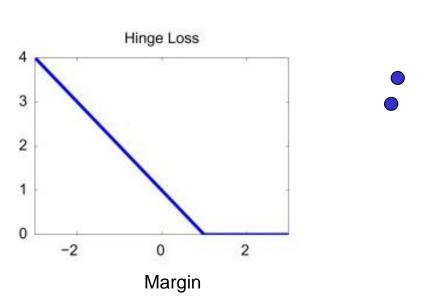
• Separable data:  $\min_{\mathbf{w},b} \frac{1}{2} \|\mathbf{w}\|^2$  subject to  $y_i(\mathbf{w} \cdot \mathbf{x}_i + b) \ge 1$ Maximize Max

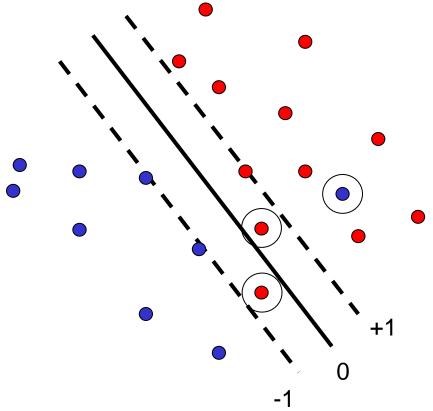
Non-separable data:



# SVM parameter learning

$$\min_{\mathbf{w},b} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \max(0,1-y_i(\mathbf{w} \cdot \mathbf{x}_i + b))$$

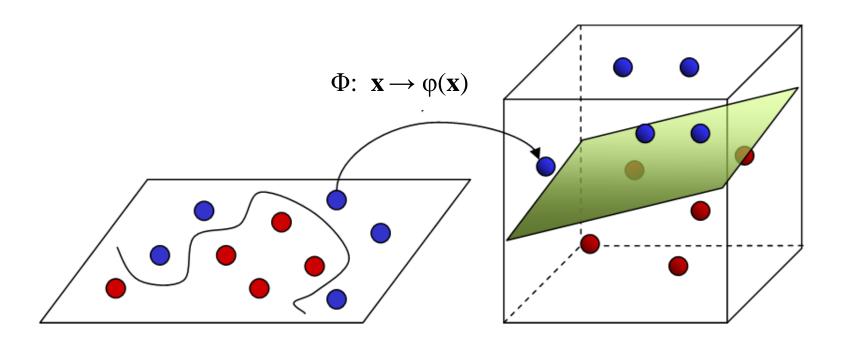




Demo: <a href="http://cs.stanford.edu/people/karpathy/svmjs/demo">http://cs.stanford.edu/people/karpathy/svmjs/demo</a>

#### Nonlinear SVMs

 General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable



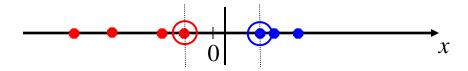
Input Space

**Feature Space** 

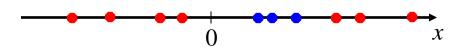
Image source

#### Nonlinear SVMs

Linearly separable dataset in 1D:

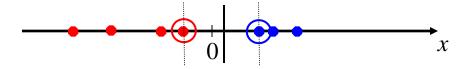


Non-linearly separable dataset in 1D:

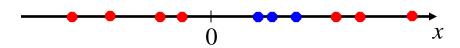


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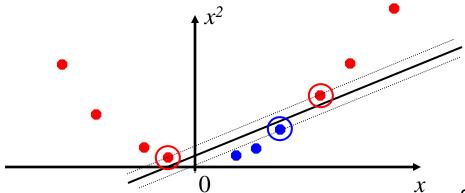
Linearly separable dataset in 1D:



Non-linearly separable dataset in 1D:



• We can map the data to a *higher-dimensional space*:



Slide credit: Andrew Moore

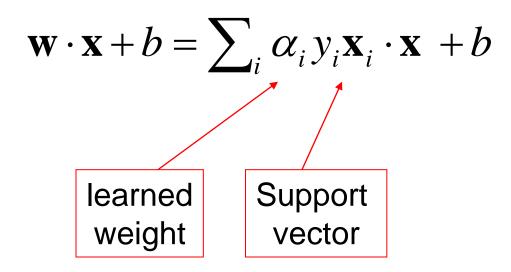
#### The kernel trick

- General idea: the original input space can always be mapped to some higher-dimensional feature space where the training set is separable
- The kernel trick: instead of explicitly computing the lifting transformation  $\varphi(\mathbf{x})$ , define a kernel function K such that

$$K(\mathbf{x}, \mathbf{y}) = \boldsymbol{\varphi}(\mathbf{x}) \cdot \boldsymbol{\varphi}(\mathbf{y})$$

#### The kernel trick

Linear SVM decision function:



#### The kernel trick

Linear SVM decision function:

$$\mathbf{w} \cdot \mathbf{x} + b = \sum_{i} \alpha_{i} y_{i} \mathbf{x}_{i} \cdot \mathbf{x} + b$$

Kernel SVM decision function:

$$\sum_{i} \alpha_{i} y_{i} \varphi(\mathbf{x}_{i}) \cdot \varphi(\mathbf{x}) + b = \sum_{i} \alpha_{i} y_{i} K(\mathbf{x}_{i}, \mathbf{x}) + b$$

This gives a nonlinear decision boundary in the original feature space

2-dimensional vectors  $\mathbf{x} = [x_1 \ x_2]$ ;

let 
$$K(x_i,x_j)=(1 + x_i^Tx_j)^2$$

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$$K(x_i,x_j) = (1 + x_i^T x_j)^2,$$
  
= 1+  $x_{il}^2 x_{jl}^2 + 2 x_{il} x_{jl} x_{i2} x_{j2} + x_{i2}^2 x_{j2}^2 + 2 x_{il} x_{jl} + 2 x_{i2} x_{j2}$ 

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$$= \varphi(\mathbf{x}_{i})^{T} \varphi(\mathbf{x}_{j}),$$
where  $\varphi(\mathbf{x}) = \begin{bmatrix} 1 & x_{1}^{2} & \sqrt{2} & x_{1}x_{2} & x_{2}^{2} & \sqrt{2}x_{1} & \sqrt{2}x_{2} \end{bmatrix}$ 

#### SVMs: Pros and cons

#### Pros

- Kernel-based framework is very powerful, flexible
- Training is convex optimization, globally optimal solution can be found
- SVMs work very well in practice, even with very small training sample sizes

#### Cons

- No "direct" multi-class SVM, must combine two-class SVMs (e.g., with one-vs-others)
- Computation, memory (esp. for nonlinear SVMs)

- $i^{th}$  example: image  $x_i$  and the label  $y_i$
- Score for the  $j^{th}$  class:  $s_j = f(x_i, W)_j$

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$$L_i = \sum_{j 
eq y_i} \max(0, s_j - s_{y_i} + \Delta)$$
 Hinge Loss Margin

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 Hinge Loss Margin

#### Problem: W is not necessarily unique

Regularization Penalty:

$$R(W) = \sum_{k} \sum_{l} W_{k,l}^2$$

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Hinge Loss:

$$L = \underbrace{\frac{1}{N} \sum_{i} L_{i}}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

Source: cs231n, http://cs231n.github.io/linear-classify/

Regularization Penalty:

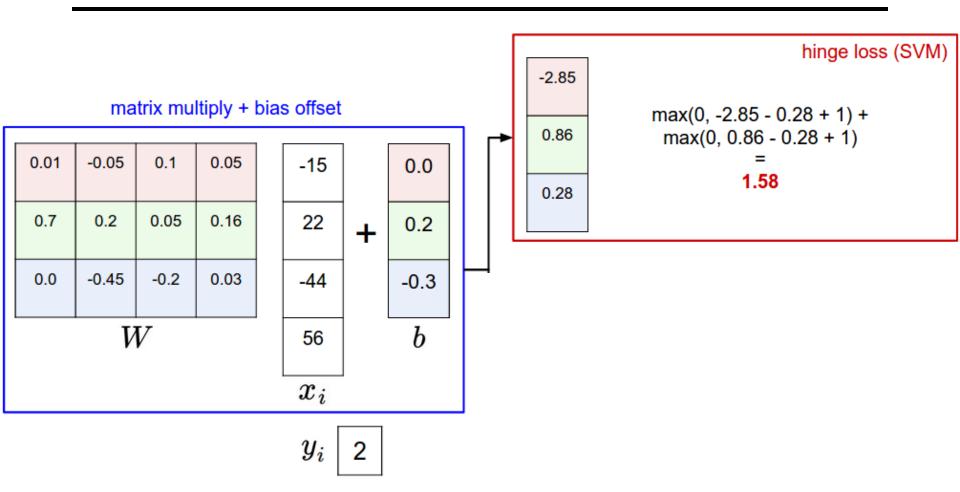
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Hinge Loss:

$$L = \underbrace{\frac{1}{N} \sum_{i} L_{i}}_{\text{data loss}} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

$$L = rac{1}{N} \sum_i \sum_{j 
eq y_i} \left[ \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + \Delta) 
ight] + \lambda \sum_k \sum_l W_{k,l}^2$$

# Hinge Loss



#### Softmax Classifier

 Interprets the class scores as the unnormalized log probabilities for each class and replace the *hinge loss* with a crossentropy loss that has the form:

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) = -s_{y_i} + \log\sum_j e^{s_j}$$

Source: cs231n, http://cs231n.github.io/linear-classify/

#### Softmax Classifier

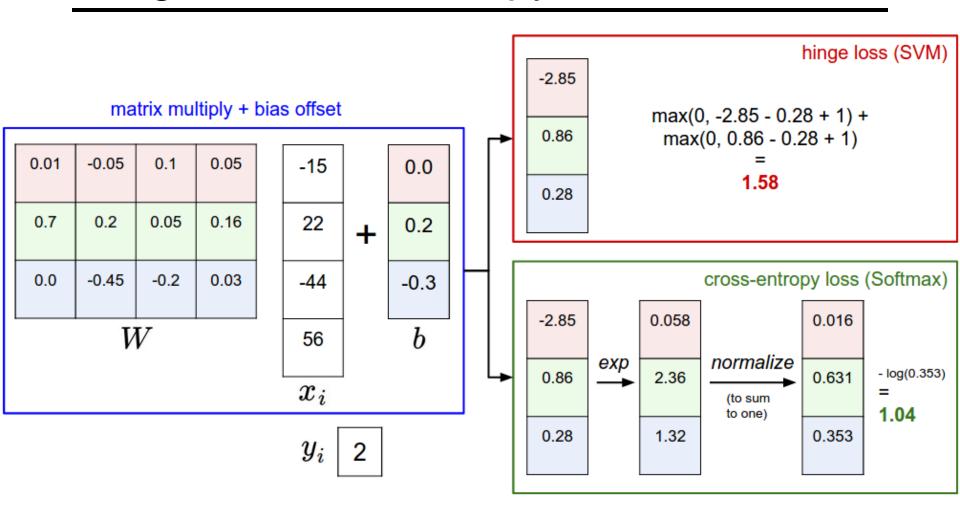
 Interprets the class scores as the unnormalized log probabilities for each class and replace the *hinge loss* with a crossentropy loss that has the form:

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) = -s_{y_i} + \log\sum_j e^{s_j}$$

Softmax Loss:

$$L = \frac{1}{N} \sum_{i} L_{i} + \underbrace{\lambda R(W)}_{\text{regularization loss}}$$

# Hinge vs Cross-entropy Loss



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#### **Next Lecture**

#### **Neural Networks**

