PVS Assignment 1

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- **Q3**

```
data BAG:= P Nat Nat  % The Prouct type. The first coordinate indicates the number of
                      % black balls, and the second indicates the number of whit balls.

f::BAG -> BAG
f (P m n)
| m+n<=1 = P m n
| otherwise = case () of
  () | a<m && b<m = f(P (m-2) (n+1))                  % both black
     | (a<m && b>=m) || (a>=m && b<m) = f(P m (n-1)) % different colours
     | otherwise = f(P m (n-1))                       % both white

       where (a,b) = randomPair(m+n)                  % <m is black, else white
```

What is the colour of the last ball? -

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first(f (P m n))
```

1 indicates a black ball and 0 indicates a white ball.

Inductive proof:

Proof: Let us induct on $m + n$. Base cases: $m + n = 0$ or 1. Trivial since

```
f (P m n) = P m n
```

and there are no draws from the bag.

Induction step: Assume the hypothesis for $m + n = k$.

Let $m + n = k + 1$, $randomPair$ chooses two (distinct) balls. The $case()$ condition appropriately adds an extra ball whose colour depends on a and b. The new arguments m' and n' have the property that $m' + n' = k$ in all the three cases of $case()$. Thus f satisfies the specification by the induction hypothesis. □

- **Q4**

```
findMajority :: [Char] -> Char
findMajority L = aux L 'F' 0

aux :: [Char] -> Char -> Int -> Char
aux [] x i = x
aux v:vs x i
| i=0 = aux vs v 1
| x == v = aux vs v (i+1)
| otherwise = aux vs x (i-1)
```

Proof: Induct on size of the list of votes (L). Base case: $|L| = 1$.

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findmajority [x] = aux L 'F' 0 = x
```

which is the majority.

Induction step: Assume correctness for $|L| = k$.
Let $|L| = k + 1$. Assume that the majority element for the first k elements is x. Then, $i > 0$ and the number of elements $\neq x$ are at most $k/2 - i$, and regardless of the last vote, the majority remains with x (since we are guaranteed a majority).
If $i = 0$, then x is not the majority element. Since we are guaranteed a majority at the end of the list, and we have no majority for the first k elements, the majority is induced by the last element of L, which is exactly what *aux* assigns as the majority element (first guarded line). □