

rein's original proof ([26], [27]; see also [40]) uses properties of perturbation determinants and the integral representation of holomorphic functions on the upper half-plane with a bounded positive imaginary part. In 1995, Voiculescu [47] approached the trace formula from a different direction and gave an alternative proof without using functional theory for the case of bounded self-adjoint operators. Later, Nha and Mohapatra [40] extended these ideas to the unbounded self-adjoint and unitary cases [41]. There is also the interesting approach of Birman and Solomyak [4], [5], [6] based on the theory of double operator integrals. More recently there has been work by Potapov, Sukochev and Zainlin [34] giving yet another proof of Krein's formula.

Let us begin by stating the theorem and its corollary in a finite-dimensional Hilbert space, the proof of which uses the minimax principles for eigenvalues.

## INSA AWARD LECTURE SRINIVASA RAMANUJAN MEDAL (FOR MATHEMATICAL SCIENCES)

**Recipient: Professor Kalyan B Sinha, JNCASR**

Theorem: Let  $H$  and  $H_0$  be two self-adjoint operators in a finite-dimensional Hilbert space  $\mathcal{H}$ , in which  $E_H(\lambda)$  and  $E_{H_0}(\lambda)$  are the spectral families of  $H$  and  $H_0$  respectively. Then there exists a unique real-valued bounded function  $\xi$  such that

- (i)  $\xi(\lambda) = \text{Tr}[E_{H_0}(\lambda)(H - H_0)]$ ,  $\lambda \in \mathbb{R}$ ;
- (ii)  $\int_{\mathbb{R}} \xi(\lambda) d\lambda = \text{Tr}(H - H_0)$ ;
- (iii) for  $\phi \in C^1(\mathbb{R})$  (set of all once continuously differentiable functions on  $\mathbb{R}$ ),

Date: 27 June 2019 (Thursday)  $\text{Tr}[\phi(H) - \phi(H_0)] = \int_{\mathbb{R}} \phi'(\lambda) \xi(\lambda) d\lambda.$  (2.1)

Time: 4 pm

Furthermore,  $\xi$  has support in  $[a, b]$ , where  $a = \min\{\inf \sigma(H), \inf \sigma(H_0)\}$

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- v) If  $H - H_0 = \tau|g\rangle\langle g|$  with  $\tau > 0$ ,  $\|g\| = 1$  (we have up to a constant factor rank one perturbations), then  $\xi$  is a  $\{0, 1\}$ -valued function.

**Trace is a kind of 'special non-commutative**

**integration'** and trace formulae attempt to

**relate traces of certain expressions, involving**

**operators with associated geometric/**

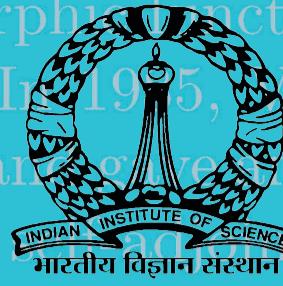
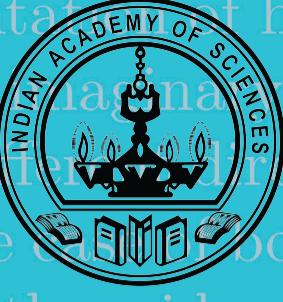
**topological quantities. A simple example,**

In an infinite-dimensional Hilbert space, the relation

$\text{Tr}[E_H(\lambda) - E_{H_0}(\lambda)]$  will not make sense in general because  $E_{H_0}(\lambda)$  -

Hilbert space, will be discussed, in which

some of these features appear naturally.



**Tea/ coffee and refreshments will be served after the talk.**

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**or Renee M. Borges, Convenor, INSA Bangalore Chapter.**