Mixed Finite Element Methods for the Extended Fisher-Kolmogorov (EFK) Equation

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Abstract

We consider the EFK equation :

$$u_t + \gamma\Delta^2 u - \Delta u + f(u) = 0, \quad (x,t) \in \Omega \times (0,T],$$

where $\gamma > 0$, $f(u) = u^3 - u$, $0 < T < \infty$, and Ω is a bounded domain in $\Re^d$, $d \leq 2$ with boundary $\partial\Omega$. When $\gamma = 0$, we obtain the standard Fisher-Kolmogorov equation. However, by adding a stabilizing fourth order derivative term to the Fisher-Kolmogorov equation and describe the model as the extended Fisher-Kolmogorov (EFK) equation.

The EFK equation occurs in a variety of applications such as pattern formation in bi-stable systems, propagation of domain walls in liquid crystals, travelling waves in reaction diffusion equations.

As far as computational studies are concerned, there are some numerical experiments without any convergence analysis. Therefore, an attemt has been made to derive numerical methods for EFK equation. The different conforming finite element techniques which are used to approximate the solution needs higher degree polynomials and an imposition of the inter-element C^1-continuity conditions on the approximating spaces become computationally very expensive.

We have proposed methods with convergence analysis which either avoid using higher degree polynomials or violate interelement C^1-continuity requirement. To violate the interelement C^1 continuity condition, we have used mixed finite element methods which violates the inter-element C^1-continuity requirement on the approximating spaces. First, we split the EFK equation into two second order equations by setting $-\Delta u = v$. Then, we apply C^0-piecewise linear elements for approximating both u, v and derive *a priori* error estimates. Finally, we discuss the computational experiments.