## Gross proof time

Here is a computer generated proof of a bound on the length of the commutator ab $\overline{ab}$  for a linear norm on the free group with the lengths of the generators bounded above by 1.

- 1.  $|\bar{a}| \le 1.0$
- $2. |\overline{bab}| \le 1.0 \text{ using } |\overline{a}| \le 1.0$
- $3. |\overline{b}| \leq 1.0$
- 4.  $|a\overline{b}\overline{a}| \le 1.0 \text{ using } |\overline{b}| \le 1.0$
- 5.  $|\overline{a}\overline{b}ab\overline{a}\overline{b}| \le 2.0$  using  $|\overline{a}\overline{b}a| \le 1.0$  and  $|b\overline{a}\overline{b}| \le 1.0$
- 6. |a| ≤ 1.0
- 7.  $|ba\overline{b}| \le 1.0 \text{ using } |a| \le 1.0$
- 8.  $|b| \le 1.0$
- 9.  $|\overline{a}ba| \le 1.0 \text{ using } |b| \le 1.0$
- 10.  $|ab\overline{a}\overline{b}ab| \le 2.0$  using  $|ab\overline{a}| \le 1.0$  and  $|\overline{b}ab| \le 1.0$
- 11.  $|\overline{a}ab\overline{a}\overline{b}aba| \le 2.0 \text{ using } |ab\overline{a}\overline{b}ab| \le 2.0$
- 12.  $|b\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}| \le 3.0 \text{ using } |b\overline{a}\overline{b}| \le 1.0 \text{ and } |aab\overline{a}\overline{b}ab\overline{a}| \le 2.0$
- 13.  $|\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}| \le 4.0$  using  $|\overline{a}\overline{b}a| \le 1.0$  and  $|b\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}| \le 3.0$
- 14.  $|\overline{b}\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}b| \le 4.0$  using  $|\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}| \le 4.0$
- 16.  $|\overline{b}ab\overline{a}\overline{b}a| \le 2.0$  using  $|\overline{b}ab| \le 1.0$  and  $|\overline{a}\overline{b}a| \le 1.0$
- 17.  $|a\overline{b}ab\overline{a}\overline{b}a\overline{a}| \le 2.0 \text{ using } |\overline{b}ab\overline{a}\overline{b}a| \le 2.0$
- 19.  $|\overline{ba}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}baab\overline{a}bab\overline{a}b|$  ≤ 8.0 using  $|\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}$
- $20. \ |b\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{$
- 21.  $|\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}| \le 12.0$  using  $|\overline{a}\overline{b}ab\overline{a}\overline{b}aa| \le 2.0$  and  $|b\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}$

- 24. |สิโอฝลชิโอลปสิโอ

- 27. |สิโอฝลิโอลปสิโอล

- 30. |สิโอฝลชิอลฝลชิอฝลชิอลป
- 31. |ចិត្តចិត្តមានចិត្តម
- $33. \ | \overline{a}\overline{b}ab\overline{a}\overline{b}aab\overline{$

- - $|b a \overline{b}a ba b a \overline{b}a b a \overline{b}a b a \overline{b}a b a \overline{b}a b \overline{a} \overline{b}a \overline{b}$
- |สิโวลbสิโวลอสิโลลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโลลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิโวลอสิ

- - |តិសិងសិធិbaabតិសិងសិធិbaabតិសិងសិធិbaabតិសិងសិធិbaabតិសិងសិធិbaabតិសិងសិធិbabតិសិងសិធិbabតិសិងសិធិbaabតិសិធិbaabតិសិងសិធិbaabតិសិធិ

- 43. |abāb̄abāb̄a| ≤ 2.111111111111111 using

- 45.  $|aa| \le 2.0$  using  $|a| \le 1.0$  and  $|a| \le 1.0$
- 46.  $|baa\overline{b}| \le 2.0 \text{ using } |aa| \le 2.0$
- 47.  $|ab\overline{a}\overline{b}aab| \le 3.0$  using  $|ab\overline{a}| \le 1.0$  and  $|\overline{b}aab| \le 2.0$
- 48.  $|\overline{a}ab\overline{a}\overline{b}aaba|$  ≤ 3.0 using  $|ab\overline{a}\overline{b}aab|$  ≤ 3.0
- 49.  $|aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab| \le 5.0$  using  $|aab\overline{a}\overline{b}aab\overline{a}| \le 3.0$  and  $|\overline{b}aab| \le 2.0$
- 50.  $|\overline{a}\overline{b}aab\overline{a}| \le 2.0$  using  $|\overline{a}\overline{b}a| \le 1.0$  and  $|ab\overline{a}| \le 1.0$
- 51.  $|\overline{b}\overline{a}\overline{b}aab\overline{a}b| \le 2.0 \text{ using } |\overline{a}\overline{b}aab\overline{a}| \le 2.0$
- 52.  $|b\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab| \le 7.0$  using  $|b\overline{a}\overline{b}aab\overline{a}\overline{b}| \le 2.0$  and  $|aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab| \le 5.0$
- 53.  $|\overline{a}b\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}$
- 54.  $|\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}|$  ≤ 8.0 using  $|\overline{a}\overline{b}a|$  ≤ 1.0 and  $|\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}|$  ≤ 7.0
- 55.  $|\overline{b}\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}\overline{b}aab\overline{a}$
- 56.  $|\overline{b}aab\overline{a}\overline{b}a| \le 3.0 \text{ using } |\overline{b}aab| \le 2.0 \text{ and } |\overline{a}\overline{b}a| \le 1.0$
- 57.  $|a\overline{b}aab\overline{a}\overline{b}a\overline{a}| \le 3.0 \text{ using } |\overline{b}aab\overline{a}\overline{b}a| \le 3.0$
- 58. |สิโภลอสสิโภลอสิ
- 59. | $\overline{b}$ aab $\overline{a}\overline{b}$ aab
- $60. \ |a\overline{b}aab\overline{a}\overline$
- $61. \ |\overline{b}aab\overline{a}\overline{$
- $62. \ |a\overline{b}aab\overline{a}\overline$
- 63.  $|ar{b}$ aabā $ar{b}$ aabā $ar{b}$
- 64. |aab $\overline{a}$ D̄aab $\overline{a}$ D̄a
- 65. |baabāt̄ba
- 66. |aabatDa

- 72.  $|ab\overline{a}\overline{b}aab\overline{$

- 78. |abāb̄abāb̄abāb̄abāb̄ab| ≤ 3.11111111111111111111 using |abā| ≤ 1.0 and |b̄abāb̄abāb̄ab| ≤ 2.111111111111111111

- 81. |bสิโจปลชิโจ

- 86. | $ba\bar{b}aba\bar{b}ab\bar{a}\bar{b}aba\bar{b}ab\bar{a}\bar{b}aba\bar{b}ab\bar{a}\bar{b}aba\bar{b}ab\bar{a}\bar{b}ab\bar{$

- 90.  $|\overline{D}aba\overline$
- 92.  $|\overline{a}\overline{b}aba\overline{b}a$
- 93.  $|ar{b}$  aba $ar{b}$  ab
- 94.  $|\overline{a}\overline{b}ab\overline{a}\overline{b}a$ 
  - $|bar{a}babaar{a}bab{a}babar{a}bab{b}abbabar{a}bab{b}abbabar{a}bab{a}bababar{a}bab{a}bab{b}abbabar{a}bab{a}bab{b}abbab{b}a$

- 23.6666666666664 using

- 97. |D̄abātD̄abātD̄ab = ≤ 2.55555555555555555554 using |D̄abātD̄ab| ≤ 1.555555555555556 and |ātD̄a| ≤ 1.0
- 98. |abababababaa| ≤ 2.55555555555555554 using |bababababa| ≤ 2.55555555555555555
- 99. | $\overline{a}$ Dab $\overline{a}$
- 101. |aabābābābābābābābābābābā|≤ 5.111111111111111111 using |aabābābābā| ≤ 2.0 and |b̄abābabābabābabāba| ≤ 3.11111111111111111
- 102. |baabābabābabābabābabābabābabā| ≤ 5.111111111111111111 using |aabābabābabābabābabābabābabābab | ≤ 5.11111111111111111
- 103. |БааЬāБаЬāБаЬāБаЬāБаЬāБаЬāБаЬāБа| ≤ 6.11111111111111111111 using |БааЬāБаЬāБаЬāБаЬāБаЬāБаЬāБаЬаБаЬаЬ ≤ 5.11111111111111111 and |āБа| ≤ 1.0
- 104. |aБaabāБabāБabāБabāБabāБabāБabāБabāБaā| ≤ 6.111111111111111111 using |БaabāБabāБabāБabāБabāБabāБabāБabāБab
- 6.1111111111111111
- 10.777777777777777
- 108. | $\bar{D}$ aabā $\bar{D}$ abā $\bar$  $|\overline{\mathrm{D}}$ aabā $\overline{\mathrm{D}}$ aba $\overline{\mathrm{D}}$ abā $\overline{$
- 24.222222222222222 using |ba\(\text{ba}\)ba\(\text{a}\)ba\(\text{ba}\)ba\(\tex
- 110. |ब्राठ्यकि।अस्क्रिकोञ्चक्रिकेञ्चक्रिकोञ्चक्रिकोञ्चक्रिकोञ्चक्रिकेञ्चक् 24.22222222222 using
  - 24.222222222222
- $\leq$  47.88888888888886 using
  - 23.6666666666664 and
  - |аbatibabat 24.222222222222
- 112. | ঢিਕ্ਰটিaba্টিaba  $\leq$  47.88888888888886 using
  - ≤47.88888888888888
- ≤52.555555555555 using
  - |bឨ៑្រងច្ឆ្លាំ៦៦ឨ្យា១២ឨ្យា១២
- ≤52.555555555555 using
  - ≤ 52.555555555555

- ≤ 69.0 using
- ≤69.0
- $\leq$  92.666666666666 using 23.66666666666664
- $\leq$  93.666666666666666 using labal  $\leq$  1.0 and
- 119. |abābabābabābabābabābabābabāba| ≤ 4.9298245614035086 using
  - $\leq$  93.666666666666666 by taking 19th power
- 120. |babābabābabābabābabābabābabababab| ≤ 4.9298245614035086 using |abābabābabābabābabābabābabāba| ≤ 4.9298245614035086

- 123. |ब्रिक्किक्रिकिक्
- 4.9298245614035086 and  $|\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}ab\overline{a}\overline{b}| \le 7.9298245614035086$