Topological properties:

E.g. · Cannot distinguish $[0,1] = X$ from $[0,2] = Z$ in terms of continuous functions.

· Can distinguish $[0,1] = X$ from $Y = [0,1] \cup [2,3]$. Namely

Property ☺ : X has ☺ if $\forall f : X \to \{0,1\}$ continuous map, f is constant.

Property ☆ : X has ☆ if $\forall x, y \in X$, $\exists f : [0,1] \to X$ continuous map s.t. $f(0) = x$ & $f(1) = y$.

Dangers with sets:

$$X = \{S \text{ a set} : S \notin S\} \qquad \left\{ \underline{\text{Not permitted}} \right\} \qquad \%$$

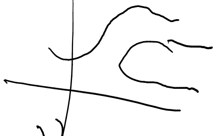
Question: Does $X \in X$?

- If $X \notin X$, by defn $X \in X$.
- If $X \in X$, by defn $X \notin X$.

Solution:
- Carefully define 'well-formed' expressions (which give sets)
- Axioms that give existence for sets. (in terms of other sets)

Carefully define arbitrary collections.

- Define $\{X_\alpha\}_{\alpha \in A}$, A 'index set' (e.g. $\mathbb{N}$). $\left\{\begin{array}{l}(a,b) \leftrightarrow \{\{a\},\{a,b\}\} \\ (c,d)\end{array}\right.$



- Try: function on A – to what?
  - What is a function anyway? $\{\{x\},\{x,f(x)\}\}$
  - A function $f: X \to Y$ is identified with its graph; what is this?

    $$\Gamma(f) = \{(x, f(x)) : x \in X\} \subset X \times Y$$

  - But not all subsets are graphs; have properties that characterize graphs
    - $\forall x \in X$, $\exists y \in Y$ s.t. $(x,y) \in \Gamma$
    - $\forall x \in X$, $\forall y_1, y_2 \in Y$ if $(x,y_1) \in \Gamma$ & $(x,y_2) \in \Gamma$ then $y_1 = y_2$

For collections, no <u>codomain</u>, instead have 'graph-like sets'

$\underline{\{X_\alpha\}_{\alpha\in A}}$: 'Graph-like set' corresponding to $\underset{\sim}{\Gamma} := \{(\alpha, X_\alpha) : \alpha\in A\}$

Formally, Γ. $\Gamma(X_\alpha)$ is a set

all collection of sets, not a set

· we do not have $\Gamma \subset X\times Y$, instead $\Gamma \subset X \times \mathrm{Set}$

· If $p\in\Gamma$, then $\exists$ S set s.t. $p = (\alpha, S)$, $\alpha\in A$.

· If $\alpha\in A$, $\exists$ S set s.t. $(\alpha, S)\in\Gamma$

· If $\alpha\in A$, S_1, S_2 sets s.t. $(\alpha, S_1)\in\Gamma$ & $(\alpha, S_2)\in\Gamma$ then $S_1 = S_2$

Then X_α is the unique set s.t. $(\alpha, X_\alpha)\in\Gamma$.