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# Seismic data analysis using synchrosqueezing short time Fourier transform

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## Abstract

The synchrosqueezing wavelet transform (SWT) reallocates the wavelet transform values to different points, hence produces a sharp spectral decomposition for the input signal. The SWT method was widely used for de-noising, spectral decomposition, etc. In this paper, a new synchrosqueezing method was proposed based on short time Fourier transform. The proposed method reassigns the short time Fourier transform values to different points, thus produces a concentrated time–frequency map. Furthermore, the proposed method has an inverse formula, which allows the reconstruction of the input signal from its spectral decomposition. Examples showed that the proposed method is effective for revealing the time–frequency characterizations of non-stationary signals.

Keywords: time–frequency analysis, synchrosqueezing method, short time Fourier transform

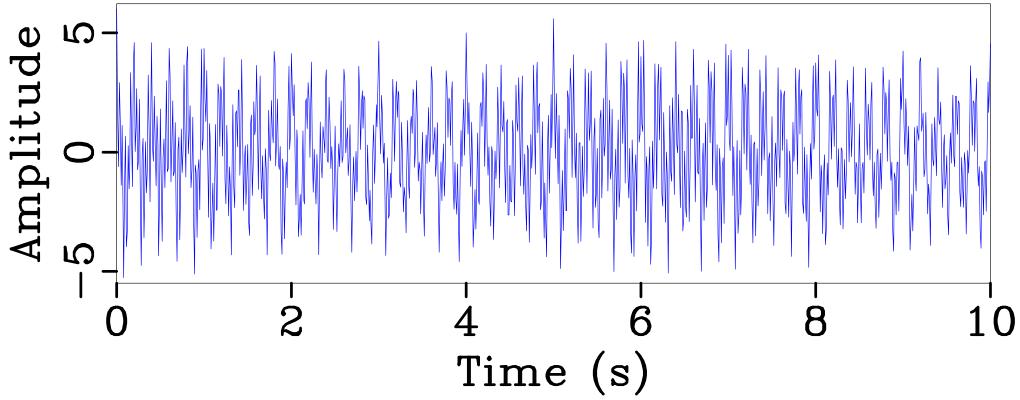
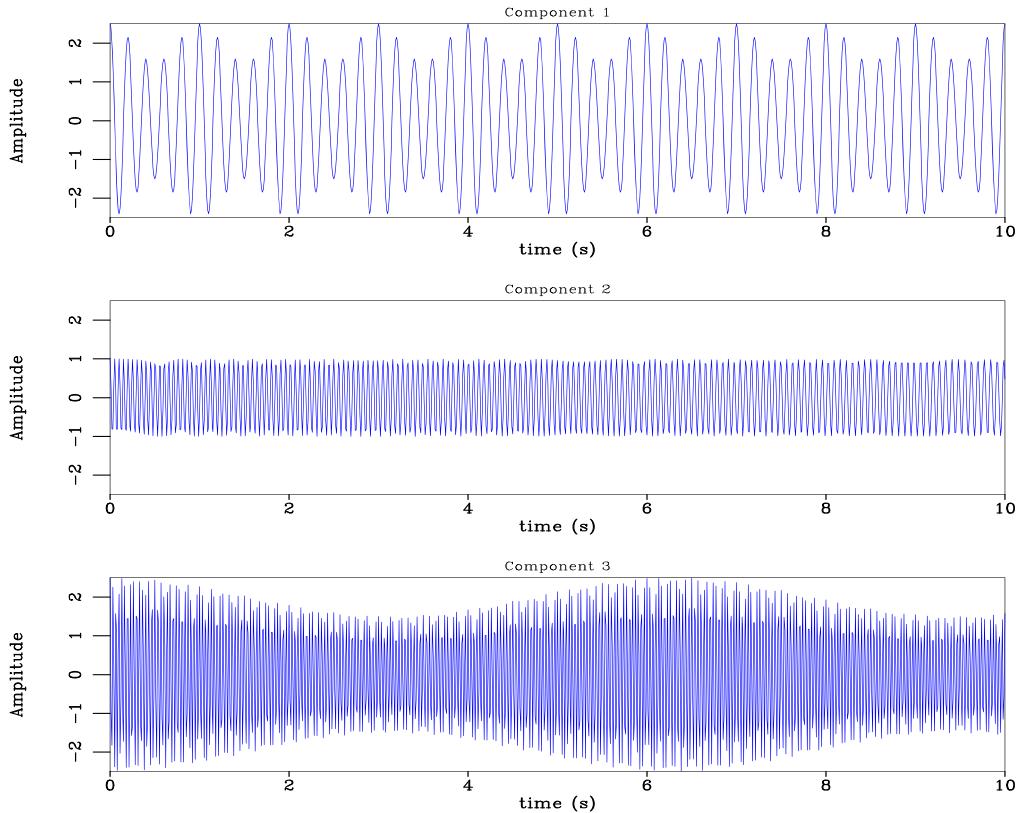
(Some figures may appear in colour only in the online journal)

## Introduction

Time–frequency distribution is a powerful tool for non-stationary signal analysis, which was widely used in seismic data interpretation (Castagna *et al* 2003, Reine *et al* 2009, Chen *et al* 2014, Liu *et al* 2016). Conventional time–frequency methods are either ‘linear’ or ‘quadratic’. For example, short time Fourier transform (STFT) (Cohen 1989), wavelet transform (WT) (Mallat 1989), and S-transform (ST) (Stockwell *et al* 1996), are linear. These linear methods pick up sections of the input signal with windows moving along the time axis. The linear methods were widely used in wide range of applications, however, the Fourier transform of the windowed section usually generates spurious frequencies, and these frequencies make the true time–frequency map unclear (Tary *et al* 2014). The Wigner–Ville transform and its variants, such as the Cohen class are quadratic. Quadratic methods produce interference terms and make the time–frequency densities negative, thus generate misleading results. Furthermore, the inversions for the quadratic methods are less straightforward (Daubechies *et al* 2011, Huang *et al* 2015).

The empirical mode decomposition (EMD) (Huang *et al* 1998), was introduced in the late 1990s, aims to extract symmetric, narrow-band waveforms called intrinsic mode functions (IMF) in a data-driven manner. The EMD method was widely used in signal processing. However, the EMD suffers from mode mixing and splitting problems. In order to solve the above problems, alternative methods were proposed based on EMD such as ensemble EMD (EEMD) (Wu and Huang 2009), and complete ensemble empirical decomposition (CEEMD) (Torres *et al* 2011). In spite of its usefulness in a wide range of applications, the EMD method and its variants lack firm mathematical foundations. The synchrosqueezing wavelet transform (SWT) was recently proposed (Daubechies *et al* 2011), with a rigorous theoretical foundation, this method captures the philosophy of the EMD method but uses a different method to construct its IMF. Auger *et al* (2013) showed the SWT can be viewed as a reassignment method, this method sharpens the time–frequency map of the wavelet transform by reallocating its values to different points. Herrera *et al* (2014), Chen *et al* (2014) applied the SWT method to seismic data analysis. Synchrosqueezing method can be used by many classical time–frequency methods in order to get concentrated time–frequency

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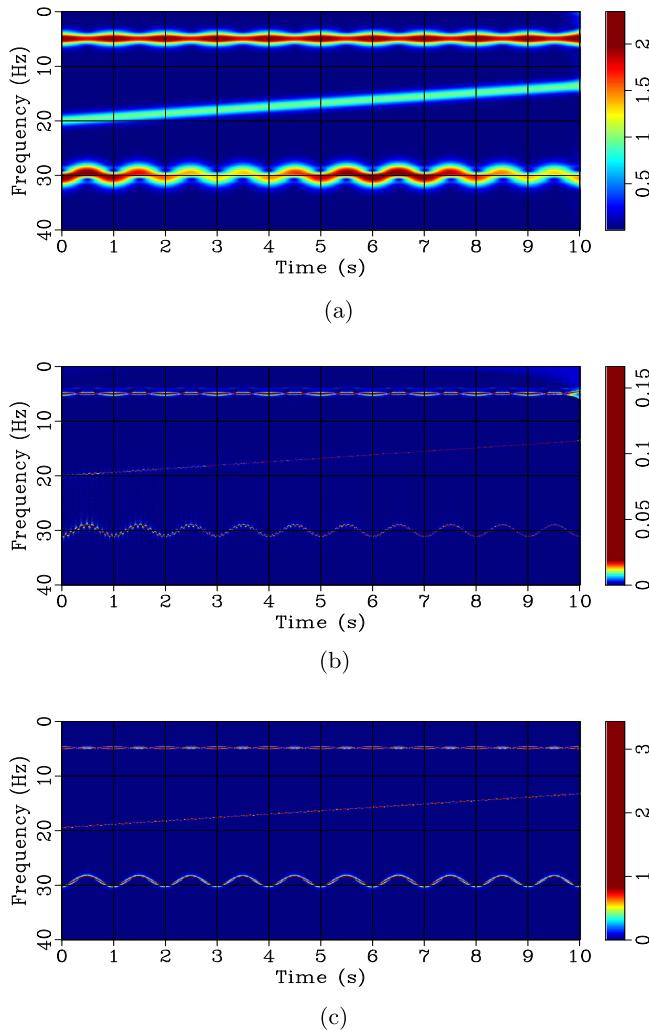
**Figure 1.** Synthetic signal.**Figure 2.** Components of the synthetic signal of figure 1.

representations. For example, the synchrosqueezing S-transform (Huang *et al* 2016, 2017), the matching synchrosqueezing wavelet transform (Wang *et al* 2016), the nonlinear squeezing time–frequency transform (Wang *et al* 2015), and the second-order synchrosqueezing transform (Oberlin *et al* 2015), synchrosqueezing method also be used as filter banks, subsampling and processing (Holighaus *et al* 2016).

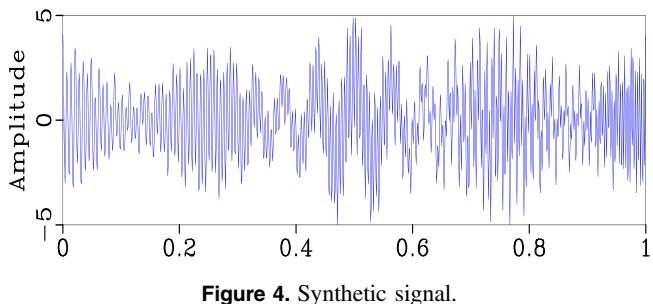
The spectrogram can be viewed as a variation of the Wigner–Ville distribution (Auger *et al* 2013). The energies spread over the instantaneous frequencies due to the windowing process. In order to get a concentrated time–frequency map, Auger and Flandrin (1995) proposed the reassignment method, which reallocates its values to different points. Auger *et al* (2012) also presented a new Levenberg–

Marquardt method, which makes the reassignment method adjustable. The synchrosqueezing method with the advantage of inversion has received new attention (Oberlin *et al* 2014).

In this paper, a new time–frequency method named synchrosqueezing short time Fourier transform (SSTFT) was proposed. The proposed method is a combination of the STFT and the synchrosqueezing method. The synchrosqueezing process squeezes the energies of STFT to the instantaneous frequencies and therefore generates a concentrated time–frequency map (Mallat 2009). Firstly, the forward and inverse transforms were given. The proofs of the forward and inverse transforms are different from those of Oberlin *et al* (2014), but similar to the proofs of Daubechies *et al* (2011) and Huang *et al* (2016). Numerical results showed that the proposed method is effective



**Figure 3.** Time–frequency map for the synthetic signal of figure 1: (a) time–frequency map using local attribute. (b) Time–frequency map using SWT, with a 64-point length Morlet wavelet. (c) Time–frequency map using SSTFT, with a 64-point length Gaussian window.



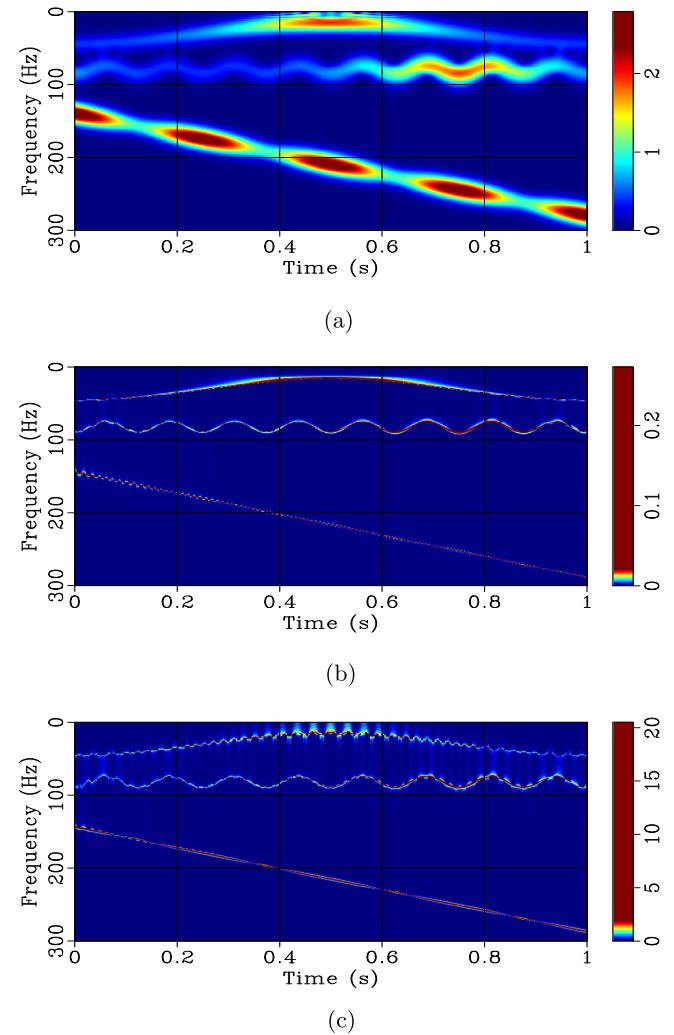
**Figure 4.** Synthetic signal.

for revealing the characterizations of non-stationary signals. Finally, the proposed method was used for seismic data interpretation.

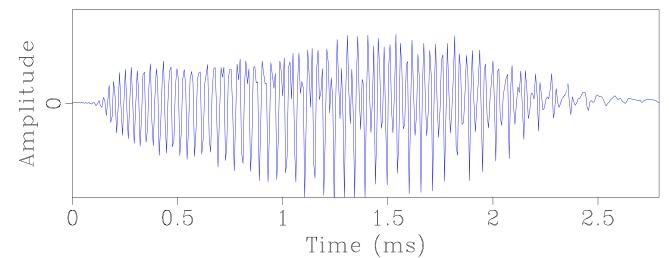
## Theorem

### Synchrosqueezing short time Fourier transform (SSTFT)

The SWT was used to analyze a wide variety of signals. The appendix gives a short introduction to the SWT algorithm.

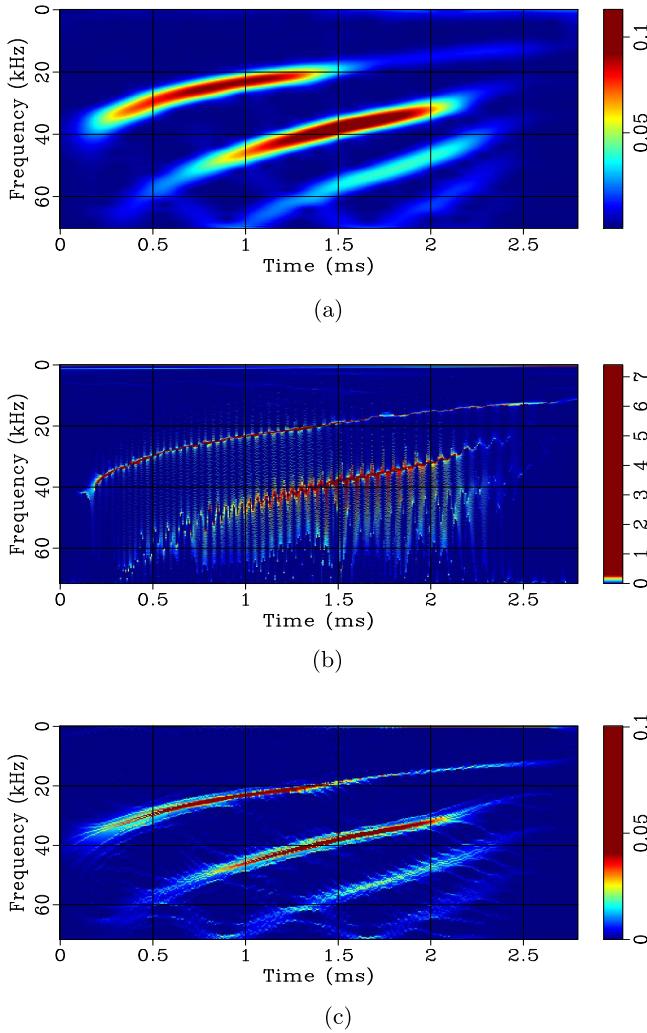


**Figure 5.** Time–frequency map for the synthetic signal of figure 4: (a) time–frequency map using local attribute. (b) Time–frequency map using SWT, with a 64-point length Morlet wavelet. (c) Time–frequency map using SSTFT, with a 64-point length Gaussian window.

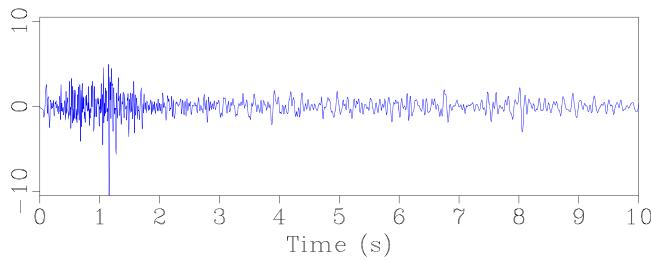


**Figure 6.** Bat signal.

For its concentration property, the synchrosqueezing method was used by other transforms to generate concentrated time–frequency representations. For example the synchrosqueezing S-transform (Huang *et al* 2016, 2017). In the following, the synchrosqueezing method is used to sharpen the STFT map, and therefore, generates a concentrated time–frequency map named SSTFT.



**Figure 7.** Time–frequency map for the bat signal of figure 6: (a) time–frequency map using short time Fourier transform, with a 64-point length Gaussian window. (b) Time–frequency map using Oberlin’s method, with a 64-point length Gaussian window. (c) Time–frequency map using SSTFT, with a 64-point length Gaussian window.



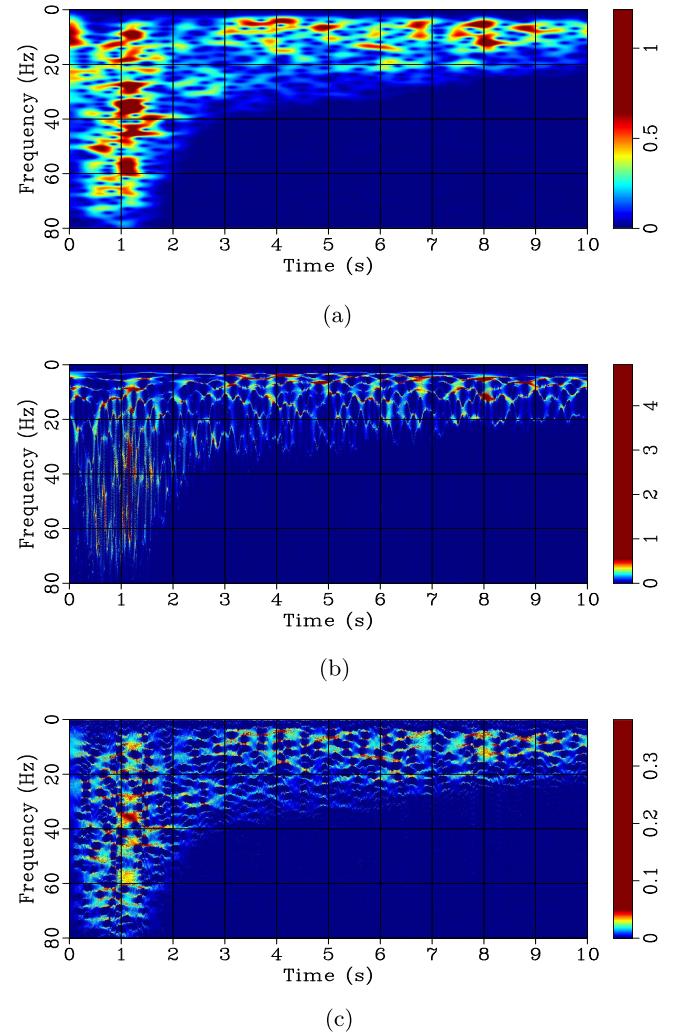
**Figure 8.** A trace from a marine survey.

#### The forward SSTFT

The STFT of a signal  $x(t)$  is (Auger *et al* 2013)

$$W_x(\omega, \tau) = e^{-i\omega\tau} \int_{-\infty}^{\infty} x(t) \overline{g(t-\tau)} e^{-i\omega(t-\tau)} dt, \quad (1)$$

where  $\overline{g(t-\tau)}$  is the complex conjugate of the window function  $g(t-\tau)$ ,  $\omega$  is the angular frequency,  $t$  is the time and  $\tau$  is the time translation. Let  $G(t, \omega) = g(t)e^{i\omega t}$ , then



**Figure 9.** Time–frequency map for the trace of figure 8: (a) time–frequency map using local attribute. (b) Time–frequency map using SWT, with a 64-point Morlet Wavelet. (c) Time–frequency map using SSTFT, with a 64-point Gaussian window.

equation (1) can be rewritten as

$$W_x(\omega, \tau) = e^{-i\omega\tau} \int_{-\infty}^{\infty} x(t) \overline{G(t-\tau, \omega)} dt. \quad (2)$$

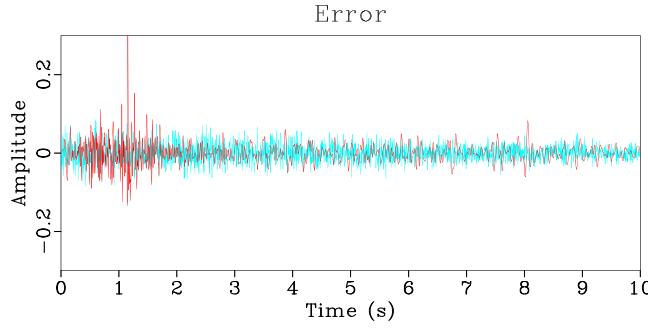
The spectrogram is  $\|W_x(\omega, \tau)\|^2$ , which can be viewed as the 2D smoothing of the Wigner–Ville distribution of the analyzed signal by the Wigner–Ville distribution of the analyzing window (Auger *et al* 2013). The smoothing process will decrease the resolution of STFT. For example a sinusoidal  $f(t) = e^{i\xi_0 t}$ , the Fourier transform of which is the Dirac  $\hat{f}(\omega) = 2\pi\delta(\omega - \xi_0)$ , has a STFT (Mallat 2009):

$$W_f(\omega, \tau) = \hat{g}(\omega - \xi_0) e^{-i\tau(\omega - \xi_0)}. \quad (3)$$

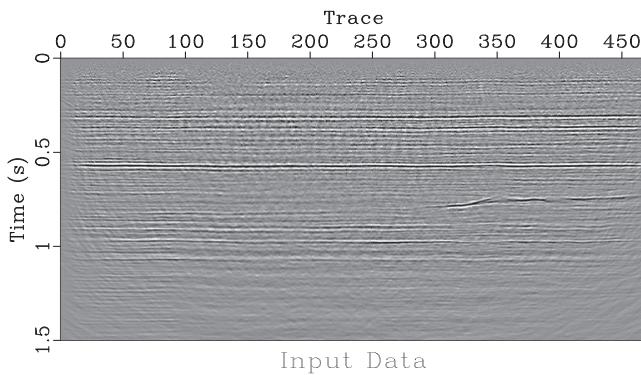
For the STFT of the sinusoidal function, its energies are spread over the interval  $[\xi_0 - \sigma_{\omega}, \xi_0 + \sigma_{\omega}]$ , where  $\sigma_{\omega}$  is the standard deviation of the function  $\hat{g}(\omega)$ . Whereas, the STFT of a Dirac function  $f(t) = \delta(t - u_0)$  is

$$W_f(\omega, \tau) = g(u_0 - \tau) e^{-i\omega u_0}. \quad (4)$$

For the Dirac function mentioned above, its energies are spread over in the time interval  $[u_0 - \sigma_t, u_0 + \sigma_t]$ , where  $\sigma_t$  is the standard deviation of the function  $g(u)$ .



**Figure 10.** Reconstruction errors of SWT (the blue line) and SSTFT (the red line) for the trace of figure 8.



**Figure 11.** Real seismic data.

In order to improve the time–frequency resolution, the synchrosqueezing method was used with the STFT to generate a concentrated time–frequency representation. Similar to the SWT, for any  $W_x(\omega, \tau) \neq 0$ , a candidate instantaneous frequency for the signal  $x$  is following. According to Plancherel’s theorem, equation (2) can be written as

$$W_x(\omega, \tau) = \frac{1}{2\pi} e^{-i\omega\tau} \int_{-\infty}^{\infty} \hat{x}(\xi) \overline{\hat{g}(\xi - \omega)} e^{i\xi\tau} d\xi, \quad (5)$$

where  $\xi$  is the angular frequency,  $\hat{x}(\xi)$  is the Fourier transform of  $x(t)$ ,  $\hat{g}(\xi)$  is the Fourier transform of  $g$ . To motivate the idea, let  $x(t) = A \cos(\omega_0 t)$ , its Fourier transform is

$$\hat{x}(\xi) = A\pi[\delta(\xi - \omega_0) + \delta(\xi + \omega_0)]. \quad (6)$$

Take a window  $g$  that is concentrated on the positive frequency axis:  $\widehat{g}(\xi) = 0$  for  $\xi < 0$ . Substituting equation (6) into (5) yields

$$W_x(\omega, \tau) = \frac{A}{2} \overline{\widehat{g}(\omega_0 - \omega)} e^{-i(\omega - \omega_0)\tau}. \quad (7)$$

For any  $(\omega, \tau)$  with  $W_x(\omega, \tau) \neq 0$ , a candidate instantaneous frequency  $\tilde{\omega}_x(\omega, \tau)$  for the signal  $x$  can be computed by

$$\tilde{\omega}_x(\omega, \tau) = -i(W_x(\omega, \tau))^{-1} \frac{\partial}{\partial \tau} (W_x(\omega, \tau)) + \omega. \quad (8)$$

If the frequency variables  $\omega, \tilde{\omega}$  are discretized i.e.  $W_x(\omega, \tau)$  was computed only at discrete value  $\omega_k$ , where

$\omega_k - \omega_{k-1} = (\Delta \omega)_k$  The SSTFT is given by

$$SW_x(\tilde{\omega}_l, \tau) = (\Delta \tilde{\omega})^{-1} \sum_{\omega_k: |\tilde{\omega}(\omega_k, \tau) - \tilde{\omega}_l| \leq \Delta \tilde{\omega}/2} W_x(\omega_k, \tau) e^{i\omega_k \tau} (\Delta \omega)_k. \quad (9)$$

This is the forward transform. The energies of the STFT be squeezed to the instantaneous frequencies locations according to the equation (9) in order to get a concentrated time–frequency representation. The SSTFT is a combination of the STFT and the synchrosqueezing method.

#### The inverse SSTFT

The following argument shows that the signal can be reconstructed.

$$\begin{aligned} & \int_{-\infty}^{+\infty} W_x(\omega, \tau) e^{i\omega\tau} d\omega \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(t) \overline{g(t - \tau)} e^{-i\omega t} e^{i\omega\tau} d\omega dt \\ &= \int_{-\infty}^{+\infty} \left( \int_{-\infty}^{+\infty} x(t) \overline{g(t - \tau)} e^{i\omega t} dt \right) e^{i\omega\tau} d\omega \\ &= \int_{-\infty}^{+\infty} \left( \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{x}(\xi) \overline{\hat{g}(\xi - \omega)} e^{-i(\xi - \omega)\tau} d\xi \right) e^{i\omega\tau} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{x}(\xi) e^{i\xi\tau} d\xi \int_{-\infty}^{+\infty} \overline{\hat{g}(\xi - \omega)} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{x}(\xi) e^{i\xi\tau} d\xi \int_{-\infty}^{+\infty} \overline{\hat{g}(\omega)} d\omega \\ &= x(\tau) \int_{-\infty}^{+\infty} \overline{\hat{g}(\omega)} d\omega. \end{aligned} \quad (10)$$

Suppose the window function is real, we derive that  $\widehat{g}(-\omega) = \overline{\widehat{g}(\omega)}$ . Equation (10) can be written as

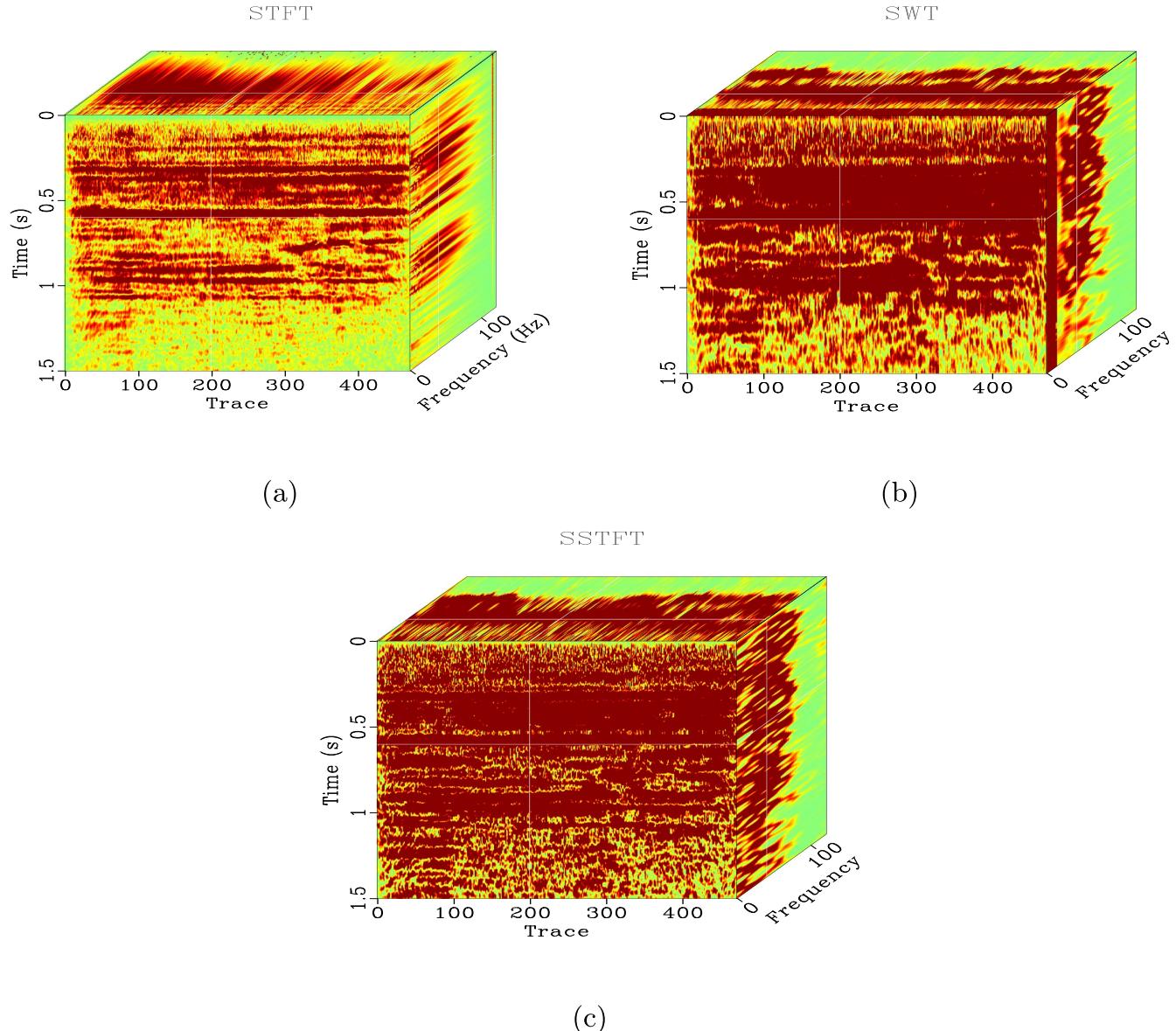
$$\begin{aligned} & \int_{-\infty}^{+\infty} W_x(\omega, \tau) e^{i\omega\tau} d\omega \\ &= x(\tau) \int_{-\infty}^{+\infty} \overline{\widehat{g}(\omega)} d\omega \\ &= x(\tau) \int_0^{+\infty} \widehat{g}(\omega) + \overline{\widehat{g}(\omega)} d\omega \\ &= x(\tau) \int_0^{+\infty} 2\Re(\widehat{g}(\omega)) d\omega. \end{aligned} \quad (11)$$

If we let  $E_g = \int_{-\infty}^{+\infty} \overline{\widehat{g}(\omega)} d\omega$ , the signal can be reconstructed by

$$x(\tau) = (E_g)^{-1} \int_{-\infty}^{\infty} W_x(\omega, \tau) e^{i\omega\tau} d\omega. \quad (12)$$

The following is the discrete reconstruction formula. From equations (9) and (12) we have

$$\begin{aligned} x(\tau) &\approx (E_g)^{-1} \sum_k W_x(\omega_k, \tau) e^{i\omega_k \tau} (\Delta \omega)_k \\ &= (E_g)^{-1} \sum_l SW_x(\tilde{\omega}_l, \tau) \Delta \tilde{\omega}. \end{aligned} \quad (13)$$



**Figure 12.** Time–frequency cubes for the real data of figure 11: (a) time–frequency using local attribute. (b) Time–frequency using SWT, with a 64-point length Morlet wavelet. (c) Time–frequency using SSTFT, with a 64-point length Gaussian window.

### Relations with Oberlin’s Fourier based synchrosqueezing transform

We underline the difference between the proposed method and Oberlin’s Fourier based synchrosqueezing transform (Oberlin *et al* 2014).

The Oberlin’s method has a threshold  $\gamma$ , the values of the STFT greater than the threshold can pass. The proposed method uses all the values of the STFT.

The compressing windows of the Oberlin’s method are  $\delta$  dependent, which usually are the Gaussian windows. The proposed method uses rectangle windows, which are constant.

The inversion of the Oberlin’s method is incomplete, some data may have been removed by the threshold. The inversion of the proposed method is complete since it uses all the data.

### Examples

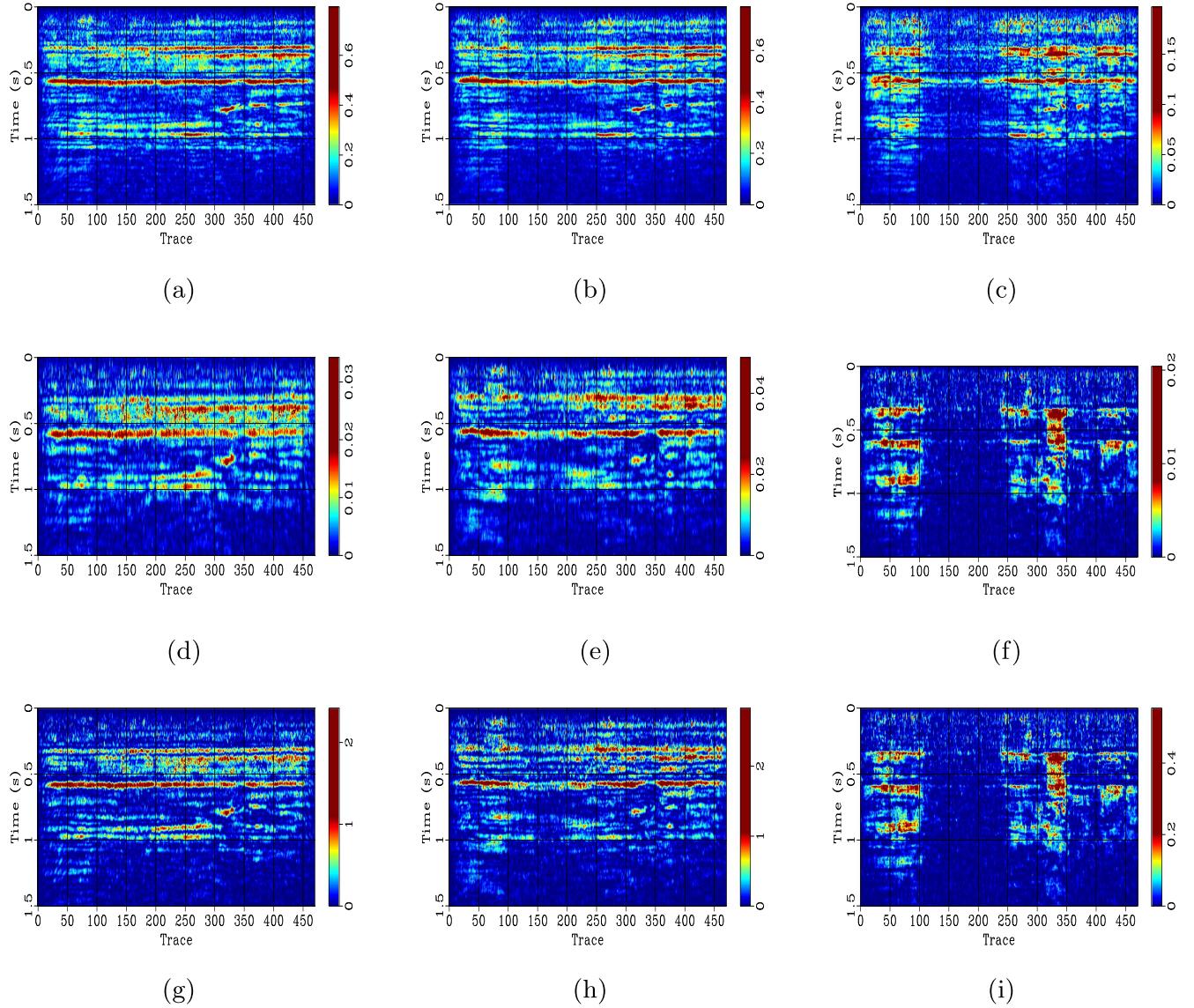
Synthetic signals and real field data are used to test the proposed method.

#### Benchmark examples

Firstly, a simple synthetic signal is used to test the proposed method. Figure 1 is a synthetic signal  $s(t)$ , which consists of three components  $s_1(t)$ ,  $s_2(t)$ ,  $s_3(t)$  as displayed in figure 2.

$$\begin{aligned} s(t) &= s_1(t) + s_2(t) + s_3(t), \\ s_1(t) &= (2 + 0.5 \cos(2\pi t)) \times \cos(10\pi t) \\ s_2(t) &= e^{-0.001t} \times \cos(40\pi t - 2.0t^2) \\ s_3(t) &= (2 + 0.5 \cos(t)) \times (60\pi t + \sin(2\pi t)). \end{aligned} \quad (14)$$

The SWT (Daubechies *et al* 2011), and the local attribute method (Liu *et al* 2011), are used for comparative analyses. The



**Figure 13.** Constant slices for the real data of figure 11. Local attribute method: (a) 20 Hz. (b) 30 Hz. (c) 50 Hz. SWT method: (d) 20 Hz. (e) 30 Hz. (f) 50 Hz. SSTFT method: (g) 20 Hz. (h) 30 Hz. (i) 50 Hz.

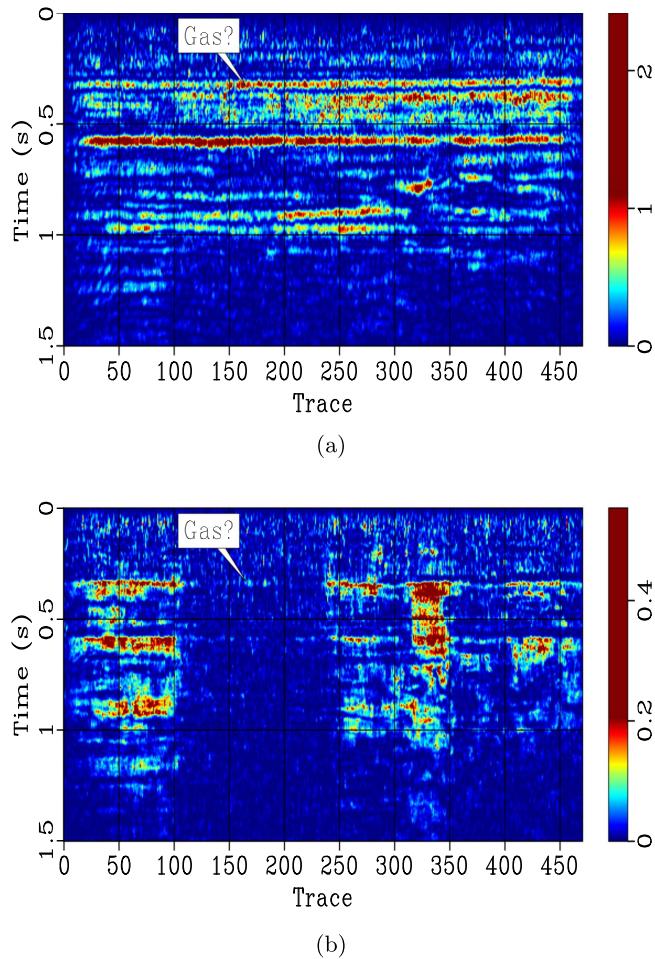
local attribute method is a non-stationary time–frequency method. Figure 3(a) is the time–frequency map using local attribute method for the synthetic signal of figure 1. The time–frequency representations of SWT and SSTFT for the synthetic signal are displayed respectively in figures 3(b) and (c). For the SWT method, a 64-point length Morlet wavelet was used. For the SSTFT method, a 64-point length Gaussian window was used. From the figures, we see that the energies spread over the instantaneous frequencies locations for the local attribute method. The SSTFT and the SWT methods squeeze the energies to the instantaneous frequencies locations.

Figure 4 is another synthetic signal used by (Hou and Shi 2013). The signal consists of three components with variable frequencies and amplitudes. Figure 5(a) is the time–frequency representation using local attribute method. Figures 5(b) and (c) are time–frequency representations using respectively the SWT and the proposed method. All three methods correctly identify the three components. The proposed SSTFT method squeezes the time–frequency

energies to the instantaneous frequencies locations, which make a concentrated time–frequency map.

Figure 6 is a 400 samples long recording of a bat chirp sampled with a sampling period  $7\ \mu\text{s}$ . This gives a sampling rate of 143 kHz. Figures 7(a)–(c) are the time–frequency representations using STFT, with a 64-point length Gaussian window, the Oberlin’s synchrosqueezing STFT method (Oberlin *et al* 2014), with a 64-point Gaussian window and the proposed method with a 64-point Gaussian window. From the figures, we see that the bat chirp signal consists of three separated components. The Oberlin’s synchrosqueezing method cannot reveal the high-frequency component.

The last example is a single seismic trace from a marine survey as displayed in figure 8. Figure 9(a) is the time–frequency map using local attribute method. The time–frequency representations of SSTFT and SWT for the trace are displayed respectively in figures 9(b) and (c). From the figures we see that the SSTFT and SWT methods squeeze the energies to the instantaneous frequencies locations. From the above figures,



**Figure 14.** Constant slices for the real data of figure 11: (a) 20 Hz slice of SSTFT. (b) 50 Hz slice of SSTFT.

we can see that the energies for the SWT and SSTFT methods are not smoothly distributed due to the synchrosqueezing process. A local Gaussian smooth operator could be implemented to smooth the roughness.

Figure 10 shows the errors of reconstruction. The blue line is the reconstruction error for SWT method, and the red line is the error for SSTFT method. Both lines prove that the two methods are capable of reconstructing the original signal while keeping a small error.

#### Real data

Figure 11 is a 2D section from a land survey previously analyzed by Fomel (2007) and Liu *et al* (2011). Figures 12(a)–(c) are the time–frequency cubes using respectively local attribute method, SWT method, and SSTFT method. The front panels for the three cubes are the 40 Hz constant slices, the right panels are the 200th trace time–frequency maps, and the top panels are 0.6 s time depth time–frequency maps. All the three methods reveal the time-dependent frequency response of the seismic data. For the local attribute method (Liu *et al* 2011), the deep layers have weak signals, whereas, for the SWT and SSTFT methods have relatively strong signals for the deep layers. The time–frequency response of

local attribute method is mainly concentrated on the strong reflection layers, whereas, the time–frequency response are blended together for the SWT and SSTFT methods. We then extract the 20, 30 and 50 Hz constant slices for the three different methods mentioned above. Figures 13(a)–(c) are the 20, 30 and 50 Hz slices for local attribute method, figures 13(d)–(f) are the 20, 30 and 50 Hz slices for SWT method, and figures 13(g)–(i) are the 20, 30 and 50 slices for SSTFT method. From these 20 and 30 Hz constant slices, we see the energies are more concentrated for SSTFT and local attribute methods than the SWT method. However, for the 50 Hz constant slices, the energies are more concentrated for the local attribute method than those of the SWT and SSTFT methods.

Low-frequency anomalies can be used as hydrocarbon indicators, which may be attributed to the abnormal high-frequency attenuation in the gas filled reservoirs (Castagna *et al* 2003). The mechanisms for low-frequency anomalies of hydrocarbon reservoirs are still not clearly understood (Ebrom 2004, Kazemeini *et al* 2009). Figures 14(a) and (b) are the 20 and 50 Hz constant slices of the SSTFT method. Comparing the slices, a low-frequency anomaly in the top-left part of the section is apparent indicated by the text box, which might be viewed as an indicator of the gas presentation (Castagna *et al* 2003).

## Conclusions

SSTFT is a concentrated version of the STFT, which improves the time–frequency resolution of the STFT by synchrosqueezing method. Since seismic signals are non-stationary, the proposed method can be used as a tool for spectral anomalies detection, and thus improves the prediction of oil and gas reservoirs. Further applications include image processing, de-noising etc.

## Acknowledgments

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## Appendix. Synchrosqueezing wavelet transform

Synchrosqueezing wavelet transform was a special case of reassignment method (Chassande-Mottin *et al* 1997, Daubechies *et al* 2011, Auger *et al* 2013), which based on the wavelet transform with the aim of sharpening the time–frequency map by reallocating the point of computation to the instantaneous

frequency of the input signal. The continuous wavelet transform of a signal  $x(t)$  is defined by (Mallat 2009)

$$W_x(a, b) = \int_{-\infty}^{\infty} x(t) a^{\frac{1}{2}} \overline{\psi\left(\frac{t-b}{a}\right)} dt, \quad (\text{A.1})$$

where  $\overline{\psi}$  is the complex conjugate of a wavelet  $\psi$ ,  $a$  is the scale variable, and  $b$  is the time location. Consider the purely harmonic signal  $x(t) = A \cos(\omega t)$ , whose Fourier transform is

$$\hat{x}(\xi) = A\pi[\delta(\xi - \omega) + \delta(\xi + \omega)]. \quad (\text{A.2})$$

By Parseval's theorem (Mallat 2009), we can rewrite equation (A.1) as

$$W_x(a, b) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{x}(\xi) a^{\frac{1}{2}} \overline{\widehat{\psi}(a\xi)} e^{ib\xi} d\xi, \quad (\text{A.3})$$

where  $\xi$  is the angular frequency. Suppose  $\widehat{\psi}$  is an analytical signal, its frequency contents are positive. Substituting equation (A.2) into (A.3) yields

$$W_x(a, b) = \frac{A}{2} a^{\frac{1}{2}} \overline{\widehat{\psi}(a\omega)} e^{ib\omega}. \quad (\text{A.4})$$

Daubechies *et al* (2011) pointed out that the instantaneous frequency can be computed for  $\omega(a, b)$  with any  $(a, b)$  for which  $W_x(a, b) \neq 0$  by

$$\omega(a, b) = -i W_x(a, b)^{-1} \frac{\partial W_x(a, b)}{\partial b}, \quad (\text{A.5})$$

the time-scale plane is translated to the time-frequency plane according to the map  $(a, b) \rightarrow (a, \omega(a, b))$ . If the discrete points for the continuous variables  $a$  and  $\omega$  are taken  $a_k$ ,  $(\Delta a)_k = a_k - a_{k-1}$  and  $\omega_l$ ,  $(\Delta\omega)_l = \omega_l - \omega_{l-1}$ . We can get the synchrosqueezing wavelet transform based on the above assumption as (Daubechies *et al* 2011, Thakur *et al* 2013),

$$T_x(\omega_l, b) = (\Delta\omega)^{-1} \sum_{a_k : |\omega(a_k, b) - \omega_l| \leq \Delta\omega/2} W_x(a_k, b) a_k^{-\frac{3}{2}} (\Delta a)_k. \quad (\text{A.6})$$

We then get the squeezed time-frequency representation by summing different contributions to the center frequency, which sharpen the time-frequency map. The following shows that the signal can be reconstructed after the synchrosqueezing. Assuming  $C_\psi = \frac{1}{2} \int_0^\infty \widehat{\psi}(\xi) \frac{d\xi}{\xi}$ , the reconstruction equation is (Daubechies *et al* 2011),

$$x(b) = \Re \left[ C_\psi^{-1} \int_0^\infty W_x(a, b) a^{\frac{-3}{2}} da \right], \quad (\text{A.7})$$

where  $\Re$  takes the real part of a complex number. The discrete reconstruction formula is (Daubechies *et al* 2011),

$$\begin{aligned} x(b) &\approx \Re \left[ C_\psi^{-1} \sum_k W_x(a_k, b) a_k^{-\frac{3}{2}} (\Delta a)_k \right] \\ &= \Re \left[ C_\psi^{-1} \sum_l T_x(\omega_l, b) (\Delta\omega)_l \right]. \end{aligned} \quad (\text{A.8})$$

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