Spatial Wideband Effect and Beam Squint in IRS Systems

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Overview

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Introduction

- Large number of antennas can simultaneously serve many users in the same time-frequency band, improve spectrum and energy efficiency, spatial resolution, and network coverage.
- When there are a large number of antennas, the transmitted wideband signal will be sensitive to the physical propagation delay across the large array aperture, which is called the spatial-wideband effect.
- Most works ignore the non-negligible time delay across the array aperture by directly extending the standard MIMO chanel model and only consider the frequency selective effect induced by multipath delay spread.

Authors and Journal Published

- **Spatial and Frequency Wideband Effects in Millimeter-Wave Massive MIMO Systems**
- Bolei Wang, Feifei Gao, Shi Jin, Hai Lin, Geoffrey Ye Li
- IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 66, NO. 13, JULY 1, 2018

Contribution of Paper 1

- Identified spatial-wideband effect in mMIMO systems and mathematically modeled it from an array-signal processing POV.
- Described the mmWave channel model as a function of channel gains, DoA/DoD, time delays.
- Gave algorithms for uplink and downlink channel estimation while taking spatial-wideband effect into account with very less amount of training overhead.

Spatial Wideband Effect

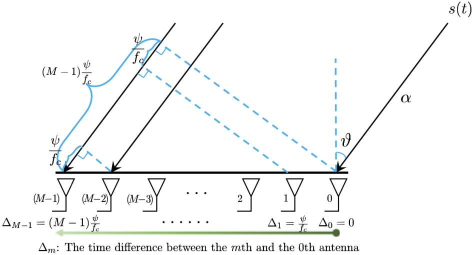


Figure: Illustration of Spatial Wideband Effect

- $$(\text{normalised angle})\psi = \frac{d\sin(\nu)}{\lambda_c}$$

- $$(\text{time delay between 2 adjacent antennas})\Delta t_{adjacent} = \frac{d\sin(\nu)}{c} = \frac{\psi}{f_c}$$

Spatial Wideband Effect

- $s(t)$ is transmitted with $s[i]$ as transmit symbols and $g(t)$ to shape the pulse with T_s as symbol period.
- $$s(t) = \sum_{i=-\infty}^{\infty} s[i]g(t - iT_s)$$
- Modulate with carrier and 1st antenna of ULA receives
$$\Re(\alpha s(t)e^{j2\pi f_c t})$$
- Other antennas will receive delayed versions of the above passband signal
- Baseband signal at m'th antenna is :
$$\alpha s(t - \frac{m\psi}{f_c})e^{-j2\pi m\psi}$$

Spatial Wideband Effect

- When M is small or transmission bandwidth of $s(t)$,

$$\frac{m\psi}{f_c} << T_s \text{ and } s(t - \frac{m\psi}{f_c}) \approx s(t)$$

  hold.

- So we can neglect the delay across the array aperture and the recieved signal is

$$\mathbf{y}(t) = \alpha s(t)\mathbf{a}(\psi)$$

  where $\mathbf{a}(\psi)$ is the spatial steering vector.

- Therefore the channel can be modeled as

$$\mathbf{h} = \sum_{i=1}^{L} \alpha_l \mathbf{a}(\psi_l)\delta(t - \tau_l)$$

Spatial Wideband Effect

- For typical mMIMO systems, $\frac{m\psi}{f_c}$ is comparable to T_s. Therefore it is wrong to neglect it's effect!
- The algorithms which don't consider the spatial-wideband effect suffer from performance loss when either the number of BS antennas or the transmission bandwidth becomes large

Channel Model with Dual-Wideband Effect

- 1 BS, M antennas
- N OFDM subcarriers
- P users, L_p paths for p'th user
- $\tau_{p,l,m}$: delay of the l'th path from p'th user to m'th antenna.
- Therefore, the received baseband signal at the m'th antenna from the p'th user is

$$y_{p,m}(t) = \sum_{l=0}^{L_p-1} \overline{\alpha}_{p,l} x_p(t - \tau_{p,l,m}) e^{-j2\pi f_c \tau_{p,l,m}}$$

- $$\tau_{p,l,m} = \tau_{p,l} + \frac{m\psi}{f_c}$$

$$\implies \boxed{y_{p,m}(t) = \sum_{l=0}^{L_p-1} \alpha_{p,l} x_p(t - \tau_{p,l} - \frac{m\psi}{f_c}) e^{-j2\pi m\psi_{p,l}}}$$

Channel Model with Dual-Wideband Effect

- Therefore the spacial-time uplink channel is given by:

$$[\mathbf{h}_{ST,p}(t)]_m = \sum_{l=0}^{L_p-1} \alpha_{p,l}[\mathbf{a}(\psi_{p,l})]_m \delta\left(t - \tau_{p,l} - m\frac{\psi}{f_c}\right)$$

- Taking the fourier transform, the uplink spatial-frequency response of the p'th user at the m'th antenna:

$$[\mathbf{h}_{SF,p}(f)]_m = \sum_{l=0}^{L_p-1} \alpha_{p,l}[\mathbf{a}(\psi_{p,l})]_m e^{-j2\pi f \tau_{p,l,m}}$$

- If the OFDM subcarrier spacing is $\eta = \frac{f_s}{N}$, then overall spatial-frequency channel matrix from all M antennas can then be formulated as:

$$\mathbf{H}_p = [\mathbf{h}_{SF,p}(0), \mathbf{h}_{SF,p}(\eta), \ldots, \mathbf{h}_{SF,p}((N-1)\eta)]$$

Channel Model with Dual-Wideband Effect

- If the OFDM subcarrier spacing is $\eta = \frac{f_s}{N}$, then overall spatial-frequency channel matrix from all M antennas can then be formulated as (SFW channel):

$$\mathbf{H}_p = [\mathbf{h}_{SF,p}(0), \mathbf{h}_{SF,p}(\eta), \ldots, \mathbf{h}_{SF,p}((N-1)\eta)]$$

$$= \sum_{l=0}^{L_p-1} \alpha_{p,l}(\mathbf{a}(\psi_{p,l})\mathbf{b}^T(\tau_{p,l})) \circ \mathbf{\Theta}(\psi_{p,l})$$

- $\mathbf{b}(\tau_{p,l})$ can be viewed as the "frequency-domain steering vector".
- $\mathbf{\Theta}(\psi_{p,l})$ is the "phase-shift matrix" with

$$[\mathbf{\Theta}(\psi_{p,l})]_{m,n} = \exp\left(-j2\pi mn\eta\frac{\psi_{p,l}}{f_c}\right)$$

- Phase-shift matrix is all ones and has no effect when spatial wideband effect is neglected

Duality

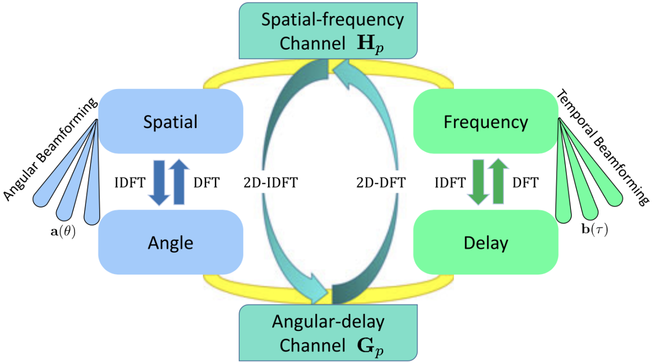


Figure: Angle-Delay duality, Spatial-frequency duality

Duality

- Angular beamforming can be achieved by weighting the antennas in spatial domain.
- Temporal beamforming can be achieved by weighting the subcarriers in frequency domain.
- There are 2 consequences of Spatial-Wideband effect:
  - Coupling of spatial steering vectors with frequency steering vectors due to non-negligible delay to the array aperture.
  - A phase shift due to extra distance travelled by the waves which causes frequency dependence of the radiation pattern, a.k.a "beam-squint" effect.
- These effects are negligible in small-scale MIMO systems.

Minimum CP Length for OFDM

- Implicitly assume that CP length is sufficient to overcome channel delay at each individual antenna
- $$N_{CP} = \left\lceil \frac{M-1}{2}\frac{f_s}{f_c} + \frac{1}{T_s} \max_{p \in \mathcal{I}(\mathcal{P})} \max_{l \in \mathcal{I}(\mathcal{L})} \tau_{p,l} \right\rceil$$
- 2nd term is due to multipath delay spread from user to BS.
- 1st term is due to delay spread by spatial-wideband effect.

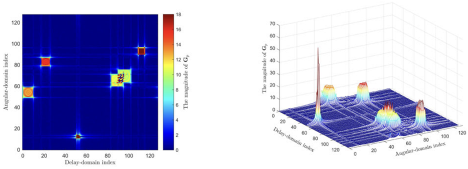
Asymptotic Characteristics of SFW Channels

Theorem 1

When $M \to \infty, N \to \infty$, the angular-delay channel($\mathbf{G}_p$) which is obtained by taking the 2D-IDFT of the spatial-frequency channel ($\mathbf{H}_p$),

$$\mathbf{G}_p = \mathbf{F}_M^H \mathbf{H}_p \mathbf{F}_N^*$$

is a sparse matrix and only contains L_p non-zero square regions, each corresponding to one of the channel paths.



(a) Grayscale of angular-delay domain

(b) Surface plot of angular-delay domain

Figure: 6-path SFW channel

Angular-Delay Orthogonality

- Let $\mathbf{h}_p = vec(\mathbf{H}_p)$

$$\implies \mathbf{h}_p = \sum_{l=1}^{L_p} \alpha_{p,l}[vec(\boldsymbol{\Theta}(\psi_{p,l}) \circ vec(\mathbf{a}(\psi_{p,l})\mathbf{b}^T(\tau_{p,l}))]$$

$$\implies \mathbf{h}_p = \sum_{l=1}^{L_p} \alpha_{p,l} diag(vec(\boldsymbol{\Theta}(\psi_{p,l})))(\mathbf{b}^T(\tau_{p,l}) \otimes \mathbf{a}(\psi_{p,l}))$$

$$\implies \boxed{\mathbf{h}_p = \sum_{l=1}^{L_p} \alpha_{p,l}\mathbf{p}(\psi_{p,l}, \tau_{p,l})}$$

- $\mathbf{p}(\psi_{p,l}, \tau_{p,l})$ form a set of basis vectors for $l \in \mathcal{I}(\mathcal{L})$ which span $\mathbf{h}_p$

Angular-Delay Orthogonality

Theorem 2

When $M \to \infty, N \to \infty$,

$$\lim_{M,N\to\infty} \frac{1}{MN}\mathbf{p}(\psi_1,\tau_1)^H\mathbf{p}(\psi_2,\tau_2) = \begin{cases} 1 \text{ if } \psi_1 = \psi_2, \tau_1 = \tau_2 \\ 0 \text{ otherwise} \end{cases}$$

- Asymptotically, these basis vectors are orthogonal.
- i.e if 2 users don't share the same path (same DoA and same path delay), then their vectorized SFW channels are orthogonal.
- We can exploit this orthogonality by simultaneously scheduling orthogonal users without any pilot contamination.

Preamble for Uplink Channel Estimation

- Apply conventional MIMO-OFDM channel estimation techniques for each antenna at BS and obtain $\hat{\mathbf{H}}_p$
- We do this to obtain initial DoA and delay estimates to facilitate further uplink and downlink channel estimations with small number of pilot resources.

Extracting Angular-Delay Signature

- We find $\hat{\mathbf{G}}_p = \mathbf{F}_M^H \hat{\mathbf{H}}_p \mathbf{F}_N^*$
- Since M, N are usually finite, the non-zero squares will be spread out due to power-leakage effect. but nevertheless, we can estimate L_p.
- We can also get a coarse estimate of $(\psi_{p,l}, \tau_{p,l})$ by looking at the index of the square grid of $\hat{\mathbf{G}}_p$.
- We improve these estimates by following the procedure described in the paper and obtain the angular-delay signature of each user:

$$\mathcal{B}_p = \{(\psi_{p,l}, \tau_{p,l}) | l \in \mathcal{I}(\mathcal{L})\}$$

i.e a user is characterized by it's DoAs and time delays.

Soft Grouping Strategy for Uplink Channel Estimation

- Delay signatures are assumed to be constant over channel coherence times. Thus, only gains are left to be estimated.
- We now change the delays of each user to make their delay signatures orthogonal but not necessarily in completely non-overlapping intervals for them to train by the same pilot sequence.
- $$\tilde{\mathcal{B}}_p = (\psi_{p,l}, \tilde{\tau}_{p,l})$$
- To combat finite M,N we ensure that Angular-Delay signatures of different users are well separated

$$dist(\tilde{\mathcal{B}}_p, \tilde{\mathcal{B}}_r) \geq \Omega$$

Soft Grouping Strategy for Uplink Channel Estimation

- Now all user send pilot signal "1" over all their subcarriers in the training block.
- The received signal from the entire OFDM block is:

$$\mathbf{Y}_U = \sum_{p \in \mathcal{I}(\mathcal{P})} \sqrt{E_p}\mathbf{H}_p + \mathbf{W}_U$$

- From angular-delay orthogonality, the ML estimate of channel gain is :

$$\hat{\alpha}_{p,l} = \frac{1}{MN\sqrt{E_p}}\mathbf{p}(\psi_{p,l}, \tau_{p,l})^H vec(\mathbf{Y}_U)$$

- We now have gains, delays and DoAs and hence can completely estimate the uplink channel!

Downlink Channel Estimation

- DoAs and path delays are approximately same for uplink and downlink channels. (angle-delay reciprocity)
- $$\psi_{p,l}^{D} = \frac{f_c^{D}}{f_c}\psi_{p,l}$$
- Again use soft grouping strategy and appropriately modify the delays to get downlink angle-delay signatures of all users and model the channel as:

$$\mathbf{H}_p^{D} = \sum_{l=0}^{L_p-1} \beta_{p,l}(\mathbf{a}(\psi_{p,l}^{D})\mathbf{b}^{T}(\tilde{\tau}_{p,l}^{D})) \circ \mathbf{\Theta}(\psi_{p,l}^{D})$$

- Only left to estimate gains of downlink channel.

Downlink Channel Estimation

- Assume all users have L_M paths.
- Vectorize the downlink channel:

$$\mathbf{h}_p^D = vec(\mathbf{H}_p^D)$$

$$(\mathbf{h}_p^D)^H = \sum_{l=0}^{L_p-1} \beta_{p,l}^* \mathbf{p}^H(\psi_{p,l}^D, \tilde{\tau}_{p,l}^D) = \beta_p^H \mathbf{P}_p^H$$

- Choose beamforming matrix $\mathbf{B}_p = \frac{1}{MN}\mathbf{P}_p$, then overall beamforming matrix is

$$\mathbf{B}^D = \sum_{p \in \mathcal{I}(\mathcal{P})} \mathbf{B}_p$$

- $\mathbf{S}$ is an $L_M \times L_M$ training matrix.

Downlink Channel Estimation

- The m'th antenna sends a training symbol $[\mathbf{B}^D[\mathbf{S}]_{:,q}]_{nM+m}$ on the n'th subcarrier of the q'th training block.
- The p'th user sums up the signals from all N subcarriers from the q'th training block and receives

$$\mathbf{y}_{p,q} = ((\overline{\mathbf{H})_p^D})^H \mathbf{B}^D [\mathbf{S}]_{:,q} + \mathbf{w}_{p,q}$$

- The user then collects signals from all L_M training block to receive

$$\mathbf{y}_p^H = (\mathbf{h}_p^D)^H \mathbf{B}^D \mathbf{S}^H + \overline{\mathbf{w}}_p^H$$

$$\implies \mathbf{y}_p^H = \beta_p \mathbf{P}_p^H \mathbf{B}_p \mathbf{S}^H + \sum_{r \neq p} \beta_p \mathbf{P}_p^H \mathbf{B}_r \mathbf{S}^H + \overline{\mathbf{w}}_p^H$$

Downlink Channel Estimation

- The summation term will be zero asymptotically (Theorem 2)!
- Now estimate the channel gain

$$\hat{\beta}_p^H = \frac{1}{L_M}\mathbf{y}_p^H\mathbf{S}$$

- The vectorized channel found from the estimate is :

$$\boxed{(\hat{\mathbf{h}}_p^D)^H = \hat{\beta}_p^H \mathbf{P}_p^H}$$

Authors and Journal Published

- **Wideband Channel Estimation for IRS-Aided Systems in the Face of Beam Squint**
- Siqi Ma , Wenqian Shen , Jianping An , Lajos Hanzo
- IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 20, NO. 10, OCTOBER 2021

Contribution of Paper 2

- Apply concepts of previous paper to an IRS aided Communication system. Consider Spatial wideband effect and beam squint at the IRS instead of at BS.
- Formulate the estimation problem of the cascaded BS-IRS-User channel.
- Show the effect of beam-squint on estimating effective cascaded DoA+DoD (has a frequency-dependent spurious peak which can harm the performance of the algorithm)
- Propose a Twin-Stage OMP algorithm robust to the above effect.
- Propose pilot-design procedure which takes into consideration both stage of the TS-OMP algorithm.
  - Reduce effect of false angles in 1st stage
  - High grade of orthogonality in measurement matrix for ease of estimation of channel gains in 2nd stage

Channel Model

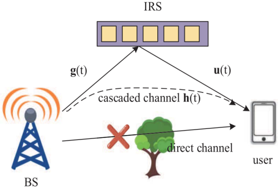


Figure: IRS Aided Communication System

- Neglect direct path. Otherwise estimate with traditional methods by turning off the IRS.
- $\mathbf{g}(t)$: CIR of BS-IRS channel, L_1 paths
- $\mathbf{u}(t)$: CIR of IRS-User channel, L_2 paths
- M IRS elements, N_p OFDM subcarriers, W bandwidth of transmission, sufficient CP length to combat all multipath delays

Channel Model

- $$g_m(t) = \sum_{l_1=1}^{L_1} \overline{\alpha}_{l_1} e^{-j2\pi f_c \tau_{l_1,m}^{TR}} \delta(t - \tau_{l1,m}^{TR})$$
- $$\mathbf{g}(t) = [g_1(t), g_2(t), \ldots, g_M(t)]^T$$
- $$u_m(t) = \sum_{l_2=1}^{L_2} \overline{\beta}_{l_2} e^{-j2\pi f_c \tau_{l_2,m}^{RR}} \delta(t - \tau_{l_2,m}^{RR})$$
- $$\mathbf{u}(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T$$
- $\theta = [\theta_1, \theta_2, \ldots, \theta_M]^T$ are the reflection coefficients of the IRS. Set magnitude response to 1 for maximum reflected signal power.

Channel Model

- The reflected signal by m'th IRS element :

$$r_m(t) = \theta_m(g_m(t) * s(t))$$

- The signal recieved by user from m'th IRS element :

$$y_m(t) = u_m(t) * r_m(t) + n_m(t) = \theta_m h_m(t) * s(t) + n_m(t)$$

$$\implies h_m(t) = g_m(t) * r_m(t)$$

- Let

$$\tau_{l_1,m}^{TR} = \tau_{l_1}^{TR} + (m-1)\frac{d\sin(\chi_{l_1}^{TR})}{f_c}$$

$$\tau_{l_2,m}^{RR} = \tau_{l_2}^{RR} - (m-1)\frac{d\sin(\nu_{l_2}^{RR})}{f_c}$$

(DoA and DoD are measured in opposite directions)

Channel Model

- Therefore

$$h_m(f) = \sum_{l_3=1}^{L_1 L_2} c_{l_3}^C e^{-j2\pi(m-1)\varphi_{l_3}^C(1+\frac{f}{f_c})} e^{-j2\pi f \tau_{l_3}^C}$$

-

$$h_m(f) = \sum_{l_3=1}^{L_1 L_2} c_{l_3}^C \mathbf{a}\left(\left(1 + \frac{f}{f_c}\right)\varphi_{l_3}^C\right) e^{-j2\pi f \tau_{l_3}^C}$$

The cascaded channel is characterised by effective gains, effective delays, and effective angles over all L_1 paths from BS-IRS and L_2 paths from IRS-User

$$\boxed{\tau_{l_3}^C = \tau_{l_1}^{TR} + \tau_{l2}^{RR}}$$

$$\boxed{\varphi_{l_3}^C = \varphi_{l_1}^{TR} - \varphi_{l_2}^{RR}}$$

$$\boxed{c_{l_3}^C = \alpha_{l_1}\beta_{l_2}}$$

Channel Model

- From principle of orthogonality, we know that :

$$\lim_{M\to\infty} \mathbf{a}^H\left(\left(1+\frac{f}{f_c}\right)\varphi_1\right)\mathbf{a}\left(\left(1+\frac{f}{f_c}\right)\varphi_2\right) = \delta(\varphi_1-\varphi_2)$$

- The above results can be extended to planar arrays as well.

Effect of Beam Squint

- In the above channel model, the steering vectors are dependent of the frequency at which we are observing,
  $\implies$ the angular spread of the effective spatial angle $(1 + \frac{f}{f_c})\varphi^C_{l_3}$ is different for different subcarriers!
- Due to the cascading of channels, the effective range of $\varphi^C_{l_3}$ is $(-1, 1)$
- To find the effective angle of the cascaded channel, we can try to find the peak of the correlation between the steering vectors and the cascaded channel and utilise the principle of orthogonality

$$\Gamma^f(x) = \mathbf{a}^H\left(\left(1 + \frac{f}{f_c}\right)x\right)\mathbf{h}(f)$$

- In the absence of beam squint, The false angle obtained are frequency independent, so we can narrow down the search range of x to $(-1/2, 1/2)$ and find the true effective cascaded angle

Effect of Beam Squint

- But when we consider the effect of beam squint, the peaks of the correlation function will be obtained at $x = \varphi^C_{l_3}, \varphi^C_{l_3} \pm \frac{f_c}{f+f_c}$ which is dependent on the frequncy
- Thus the squint over all subcarriers is :

$$\frac{f_c}{0+f_c} - \frac{f_c}{W+f_c}$$

- The peak of the false angle is very similar to the peak of the actual angle. This will cause high interference in estimating the actual angle!

Effect of Beam Squint

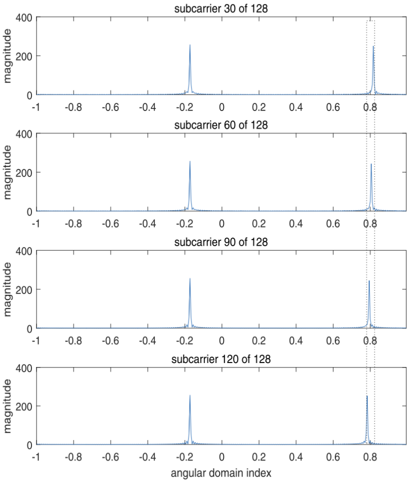


Figure: Demonstration of presence of false angles

Overview

- We can see that the false angle peak is dependent on the subcarrier frequncy and the peak occurs at different angles for different subcarriers as illustrated by the figure.
- We accumulate correlation function across different subcarriers for the same training symbol. This will cause the peak at the actual angle to grow much faster than the peak at the false angle.
- OMP based algorithm is then used to obtain path delays and gains of the cascaded channel.

Equivalent Angle Estimation

- Not all angles in the cascaded system are unique as many physical paths may have same effective angle due to contribution from both DoA and DoD.
- Let the number of unique angles in the system be N_a
- Let J_i paths share the effective angle $\overline{\varphi}_i$

$$\sum_{i=1}^{N_a} J_i = L_1 L_2$$

- The effective channel can now be represented as (pilot signal has $s(f) = 1$)

$$y(f) = \sum_{i=1}^{N_a} \theta^T \mathbf{a}((1 + \frac{f}{f_c})\overline{\varphi}_i) \sum_{j_i=1}^{J_i} \overline{c}_{j_i} e^{-j2\pi f \overline{\tau}_{j_i}} + n(f)$$

Equivalent Angle Estimation

- $h(f)$ can be decoupled into a component which consists of N_d steering vectors (resolution $2/N_d$) and a component which consists of channel gains and delays.

$$h(f) = A(f)z(f)$$

- $z(f)$ is of size N_d (can store gains and delays for all steering vectors) but has only N_a non-zero elements (the number of uniques angles in the system)
- Therefore, for the n_p'th subcarrier

$$y(n_p) = \theta^T \mathbf{A}(n_p)\mathbf{z}(n_p) + n(n_p)$$

- We send N_s OFDM symbols for training and the IRS configuration changes for every OFDM symbol, Thus :

$$\mathbf{y}(n_p) = \mathbf{\Theta}\mathbf{A}(n_p)\mathbf{z}(n_p) + \mathbf{n}(n_p) = \mathbf{F}(n_p)\mathbf{z}(n_p) + \mathbf{n}(n_p)$$

Equivalent Angle Estimation

- We now accumulate information across N_{P1} subcarriers to reduce the effect of false angles

$$\bar{\mathbf{y}} = \bar{\mathbf{F}}\bar{\mathbf{z}} + \bar{\mathbf{n}}$$

- This is a sparse-recovery problem on $\bar{\mathbf{z}}$ and we can get the indices of the non-zero elements using a variation of the OMP algorithm and can find the equivalent angles from the returned index set $\mathcal{I}^a$.
- We can also estimate the non-zero elements of $\mathbf{z}(n_p)$ from the Least-Squares estimate :

$$[\mathbf{z}(n_p)]_{\mathcal{I}^a} = \left([\mathbf{F}(n_p)]_{:,\mathcal{I}^a} \right)^{\dagger} \mathbf{y}(n_p)$$

Delay and Gain Estimation

- Once we have the index set of the non-zero elements of $\bar{\mathbf{z}}$, one block of it corresponds to $\mathbf{z}(n_p)$
- The i'th non-zero element of $\mathbf{z}(n_p)$ can be expressed as:

$$[\mathbf{z}(n_p)]_{\mathcal{I}_i^a} = \sum_{j_i=1}^{J_i} \overline{c}_{j_i} e^{-j2\pi \frac{n_p W}{N_P} \overline{\tau}_{j_i}}$$

- Now collect over all N_p subcarriers

$$\mathbf{z}^{\mathcal{I}_i^a} = \sum_{j_i=1}^{J_i} \overline{c}_{j_i} \overline{\mathbf{b}}(\overline{\tau}_{j_i})$$

where $\overline{c}_{j_i}$ are the gains of the j_i'th path with angle $\overline{\varphi}_i$ and $\overline{\tau}_{j_i}$ are the delays for the same and $\overline{\mathbf{b}}$ can be viewed as the frequency steering vector.

Delay and Gain Estimation

- The above equation can be formulated as a sparse recovery problem as :

$$\mathbf{z}^{\mathcal{I}_i^a} = \mathbf{B}\overline{\mathbf{c}}^{\mathcal{I}_i^a}$$

where $\mathbf{B}$ contains N_τ steering vectors. for the sparse vector $\overline{\mathbf{c}}^{\mathcal{I}_i^a} \in \mathbb{C}^{N_\tau}$ which has J_i non-zero elements (one for each path with angle $\overline{\varphi}_i$.

- Now we utilise the OMP Algorithm to return the index set of the non-zero elements which correspond to the channel delays $\overline{\tau}_{j_i}$ and then use the LS estimator to find the channel gains $\overline{c}_{j_i}$.
- Now we can reconstruct the channel from the estimate values of $\{\overline{\varphi}_i, \overline{c}_{j_i}, \overline{\tau}_{j_i}\}$ as :

$$\boxed{\mathbf{h}(f) = \sum_{i=1}^{N_a} \mathbf{a}((1+\frac{f}{f_c})\overline{\varphi}_i) \sum_{j_i=1}^{J_i} \overline{c}_{j_i} e^{-j2\pi f \overline{\tau}_{j_i}}}$$

Pilot Design

- The Author's propose bespoke pilot design (which set of pilot subcarriers to choose) based on cross-entropy to improve the performance of the proposed TS-OMP Algorithm
- In the 1st stage of the TS-OMP Algorithm, we accumulate over many subcarrier frequencies. But, the resolution of the steering vectors in **A** is $2/N_d$.
- So the pilot subcarriers we accumulate over should have peaks which are far enough apart that they are resolvable by the steering vectors of the array
- Thus if the pilot subcarriers are denoted as $n_1, n_2, \ldots, n_{N_{P1}}$, We need atleast 2 of them to be resolvable :

$$\frac{f_c}{f_{n_1} + f_c} - \frac{f_c}{f_{n_{N_{P1}}} + f_c} \geq \frac{2}{N_d}$$

Pilot Design

- The 2nd stage of our problem is modeled as a sparse-recovery problem of $\bar{\mathbf{c}}^{\mathcal{I}_{\rangle}^{\dashv}}$ with measurement matrix $\mathbf{B}$.
- A high degree of orthogonality is preferred for the measurement matrix (which depends on the pilot subcarriers) to improve the estimates.
- Thus, we need $\mathbf{B}$ to be an Identity matrix.

$$\mathbf{B}^H\mathbf{B} \approx N_{P1}\mathbf{I}_{N_\tau}$$

- Therefore we solve the following optimization problem:

$$\text{min. } \mu(\mathbf{B}) = ||\mathbf{B}^H\mathbf{B} - N_{P1}\mathbf{I}_{N_\tau}||_2^2$$

$$\text{s.t } \frac{f_c}{f_{n_1} + f_c} - \frac{f_c}{f_{n_{N_{P1}}} + f_c} \geq \frac{2}{N_d}$$

Thank You