# 5 - Orbits in 3D, Reference Frames and Coordinate Systems


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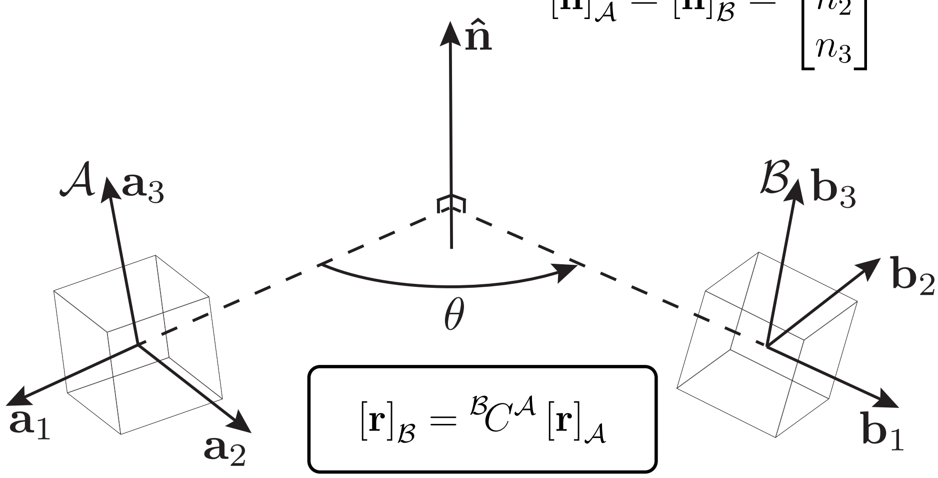
MAE 4060



## Direction Cosine Matrices

$$[\hat{\mathbf{n}}]_{\mathcal{A}} = [\hat{\mathbf{n}}]_{\mathcal{B}} = \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

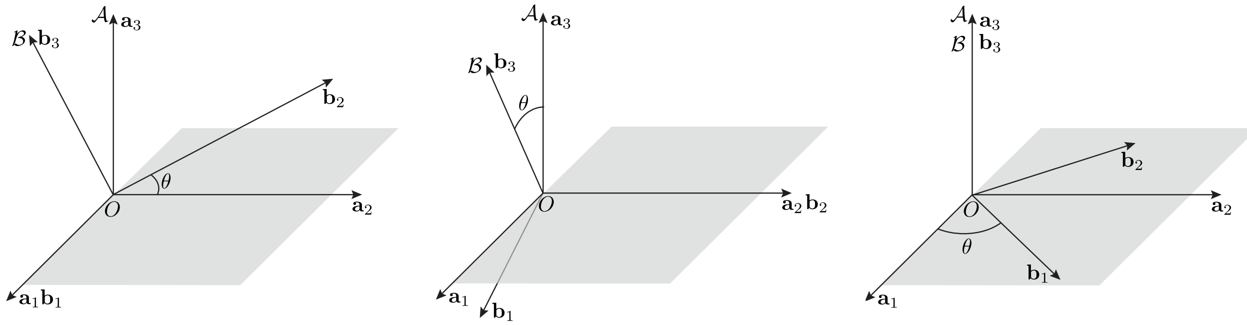



$$\boxed{[\mathbf{r}]_{\mathcal{B}} = {}^{\mathcal{B}}C^{\mathcal{A}}\,[\mathbf{r}]_{\mathcal{A}}}$$

$${}^{\mathcal{A}}C^{\mathcal{B}} = \left({}^{\mathcal{B}}C^{\mathcal{A}}\right)^{-1} = \left({}^{\mathcal{B}}C^{\mathcal{A}}\right)^{T}$$

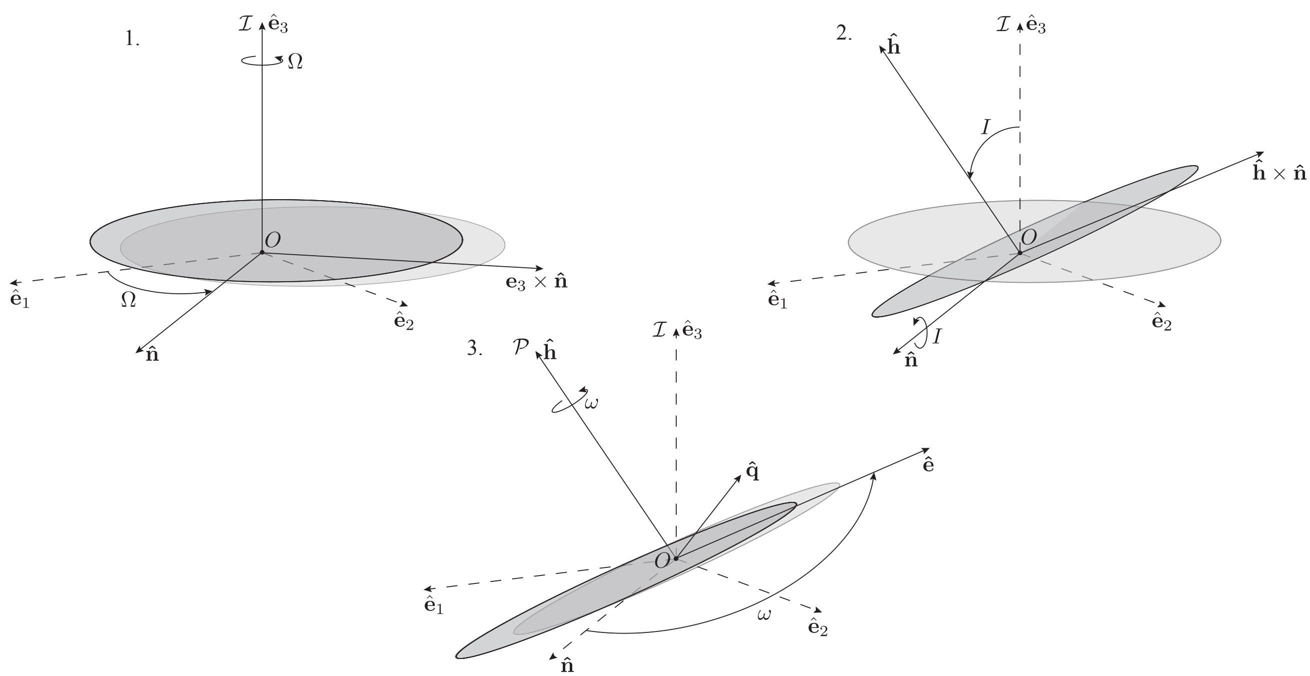
$${}^{\mathcal{I}}C^{\mathcal{F}_1}\,{}^{\mathcal{F}_1}C^{\mathcal{F}_2}\,{}^{\mathcal{F}_2}C^{\mathcal{F}_3}\ldots{}^{\mathcal{F}_{\mathcal{N}-1}}C^{\mathcal{F}_{\mathcal{N}}} = {}^{\mathcal{I}}C^{\mathcal{F}_{\mathcal{N}}}$$

# Simple Direction Cosine Matrices

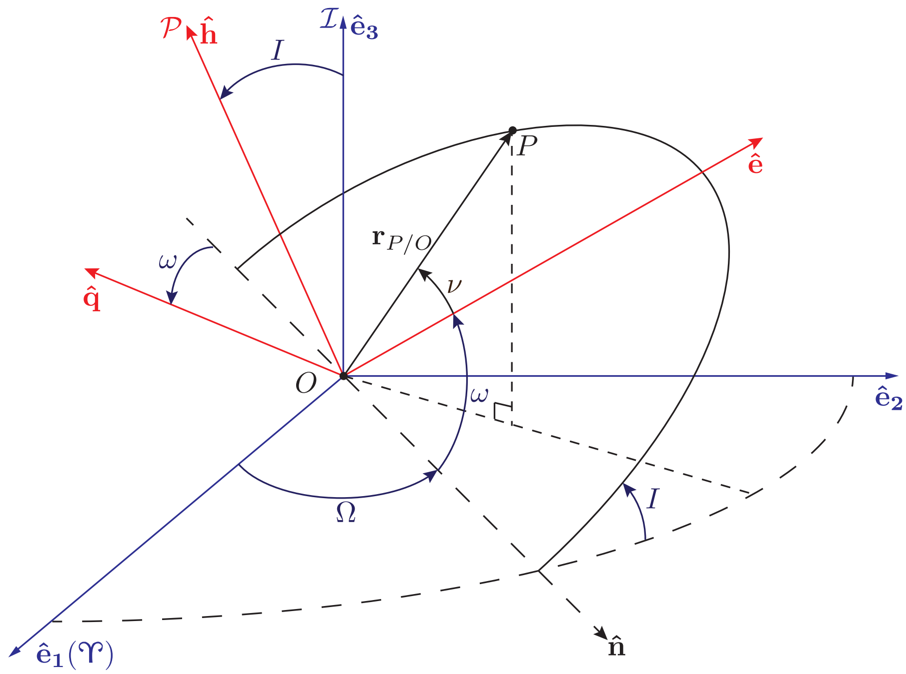



$$
{}^{\mathcal{B}}C^{\mathcal{A}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \qquad \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

# Inertial $\rightarrow$ Perifocal: 3-1-3 $(\Omega, I, \omega)$ Body-2 Rotation



# Orbits in 3D



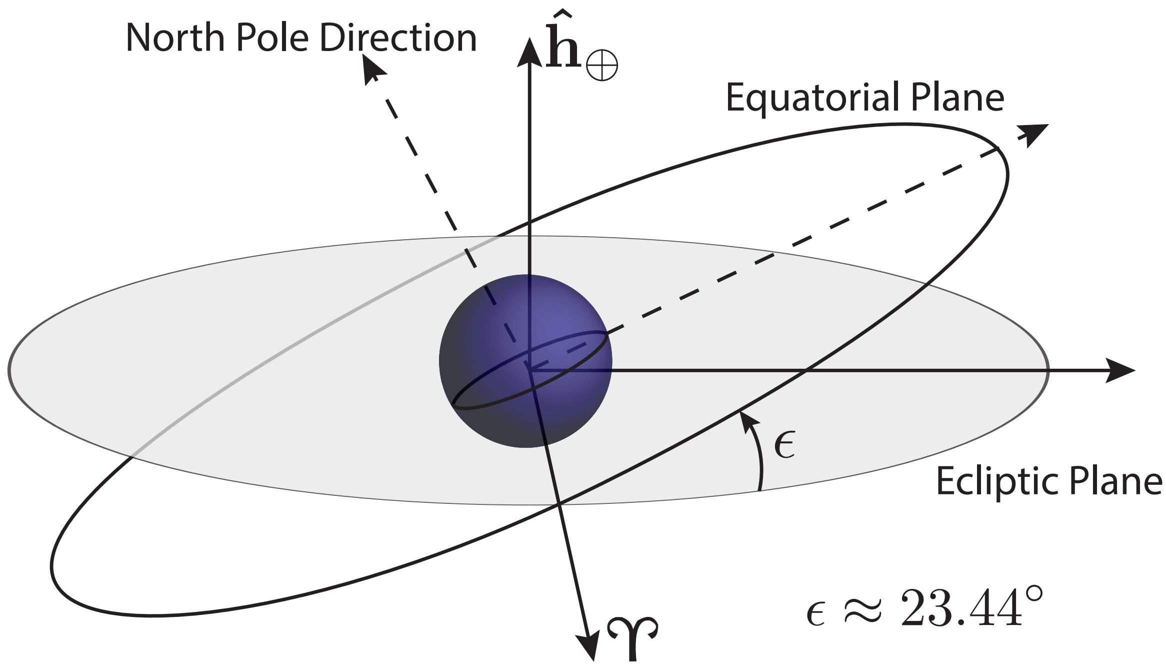

# Orbits in 3D (Math Version)

$$
{}^{\mathcal{P}}C^{\mathcal{I}} = \begin{bmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(I) & \sin(I) \\ 0 & -\sin(I) & \cos(I) \end{bmatrix} \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} =
$$

$$
\begin{bmatrix} -\sin(\Omega)\sin(\omega)\cos(I) + \cos(\Omega)\cos(\omega) & \sin(\Omega)\cos(\omega) + \sin(\omega)\cos(I)\cos(\Omega) & \sin(I)\sin(\omega) \\ -\sin(\Omega)\cos(I)\cos(\omega) - \sin(\omega)\cos(\Omega) & -\sin(\Omega)\sin(\omega) + \cos(I)\cos(\Omega)\cos(\omega) & \sin(I)\cos(\omega) \\ \sin(I)\sin(\Omega) & -\sin(I)\cos(\Omega) & \cos(I) \end{bmatrix}
$$

$$
\left[\mathbf{r}_{P/O}\right]_{\mathcal{I}} = {}^{\mathcal{I}}C^{\mathcal{P}} \begin{bmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{bmatrix}_{\mathcal{P}} = r \begin{bmatrix} \cos(\Omega)\cos(\nu+\omega) - \sin(\Omega)\sin(\nu+\omega)\cos(I) \\ \sin(\Omega)\cos(\nu+\omega) + \sin(\nu+\omega)\cos(I)\cos(\Omega) \\ \sin(I)\sin(\nu+\omega) \end{bmatrix}
$$

## Solar System Reference Planes




## Special Cases

- $I = 0$, Longitude of Periapsis:

$$\pi \equiv \varpi \triangleq \omega + \Omega$$

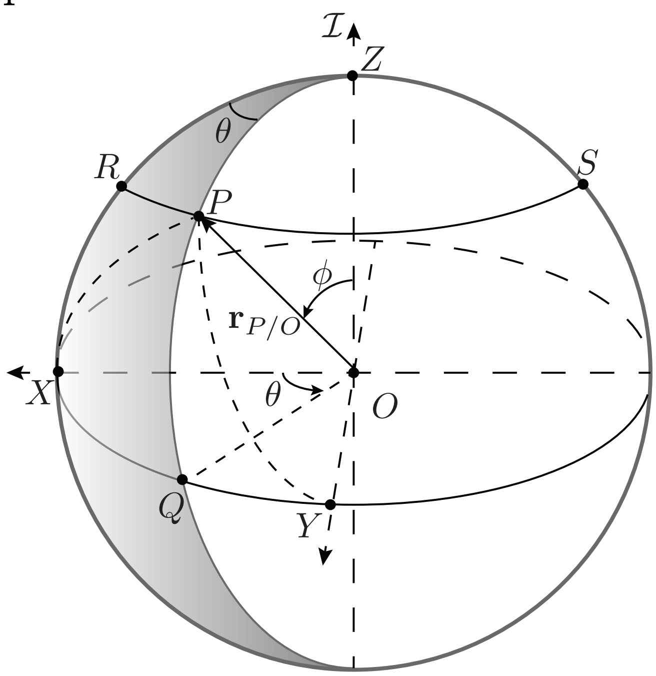
- $e = 0$, Argument of Lattitude:

$$u \equiv \theta \triangleq \nu + \omega$$

- $e = I = 0$, True Longitude:

$$l \triangleq \varpi + \nu = \Omega + \omega + \nu$$

# Spherical Coordinates



Adapted from Green (1985)

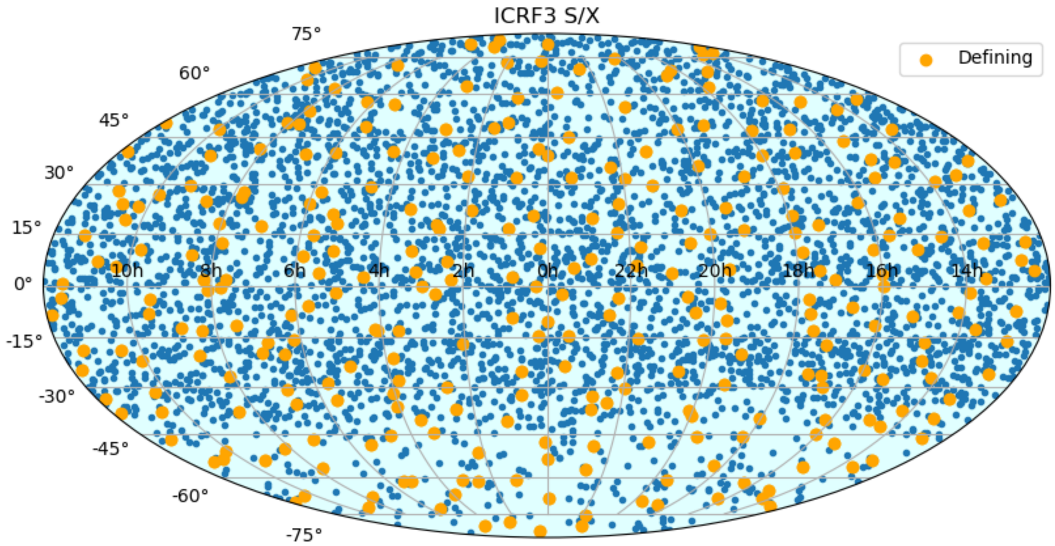
$$[\hat{\mathbf{r}}_{P/O}]_{\mathcal{I}} = \begin{bmatrix} \cos\theta\sin\phi \\ \sin\theta\sin\phi \\ \cos\phi \end{bmatrix}_{\mathcal{I}}$$

Adapted form Green (1985)

# Spherical Coordinate Systems

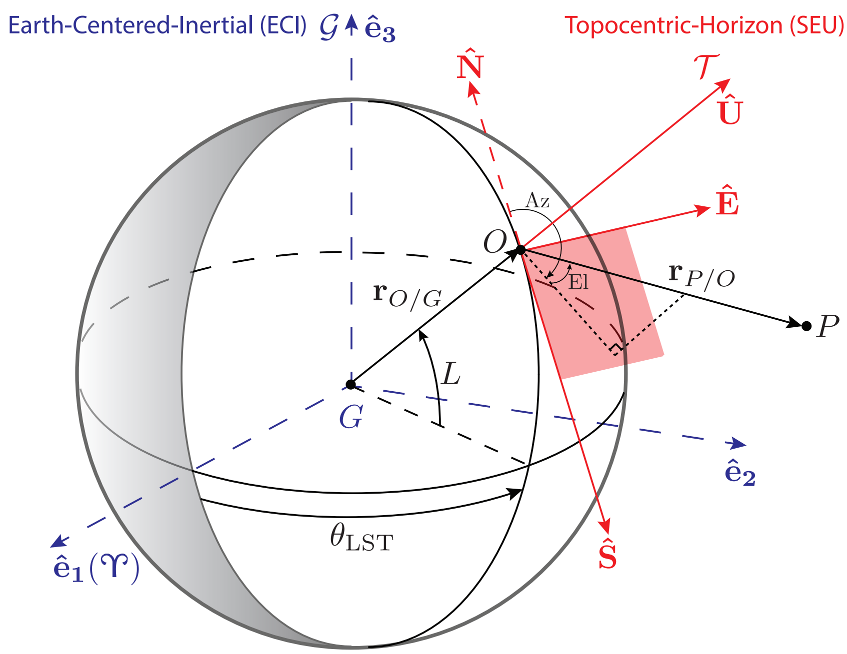
| Name | Origin | Reference Plane | Prime Direction | Azimuth Angle | Elevation Angle |
|---|---|---|---|---|---|
| Geographic | Geocentric | Equator | Prime Meridian | Longitude ($\lambda$) | Latitude ($\varphi$ or L) |
| Horizontal (Topocentric) | Observer Location | Horizon | North | Azimuth (Az) | Altitude/Elevation (Alt/El) |
| Equatorial | Geocentric or Heliocentric | Celestial Equator | Vernal Equinox | Right Ascension ($\alpha$) | Declination ($\delta$) |
| Ecliptic | Geocentric or Heliocentric | Ecliptic | Vernal Equinox | Ecliptic Longitude ($\lambda$) | Ecliptic Latitude ($\beta$) |
| Galactic | Heliocentric | Galactic Plane | Galactic Center | Galactic Longitude ($l$) | Galactic Latitude ($b$) |

# ICRF3 4,356 Extragalactic Sources, 303 defining




http://hpiers.obspm.fr/icrs-pc/newwww/icrf/index.php

# Topocentric-Horizon Coordinate System



# The Reference Geoid

**World Geodetic System (WGS) 84**

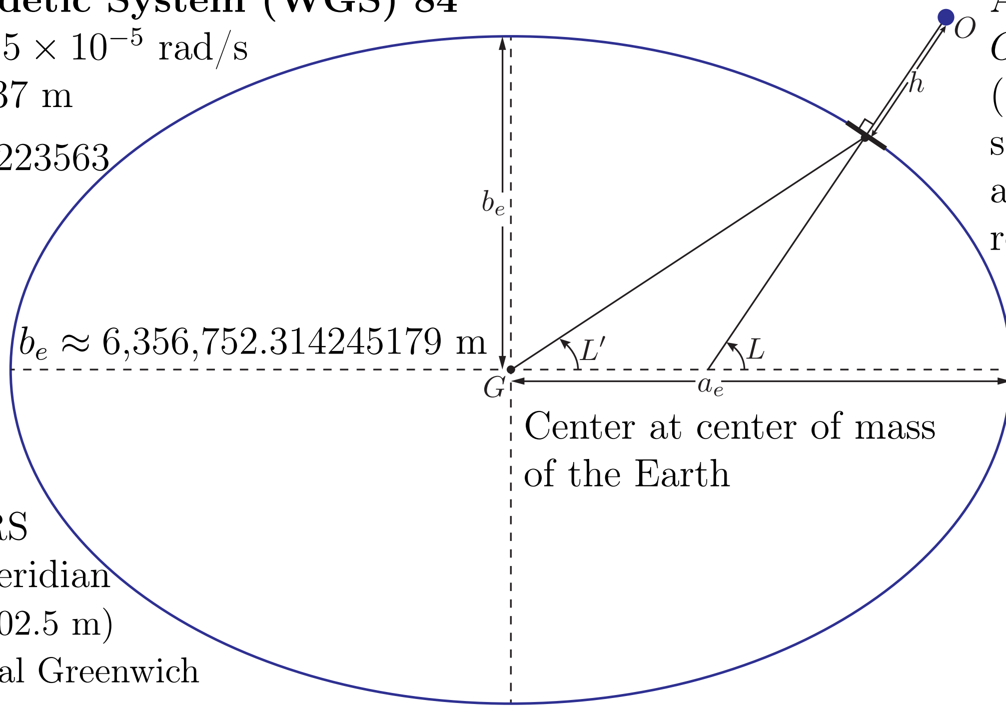
$\omega_e = 7.292115 \times 10^{-5}$ rad/s

$a_e = 6{,}378{,}137$ m

$\frac{1}{f} = 298.257223563$

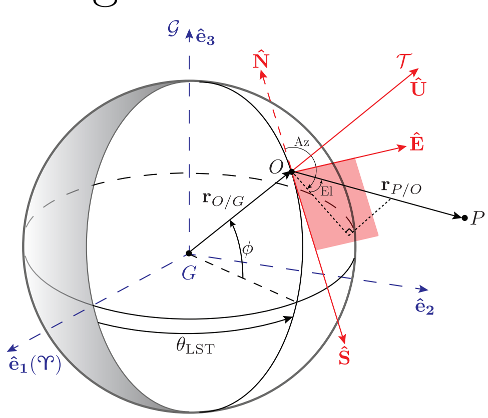
$f \triangleq \frac{a-b}{a}$

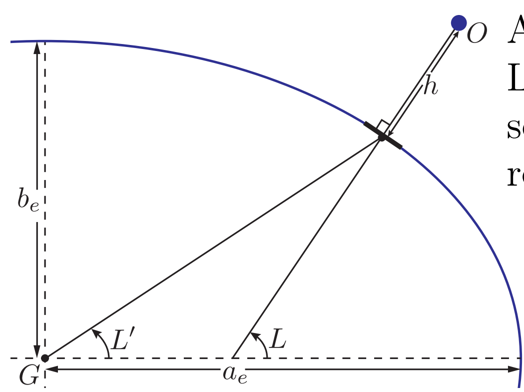
$\lambda = 0$ at IERS Reference Meridian ~5.3 arcsec(102.5 m) East of original Greenwich




A surface point $O$ at Lon/Lat $(\lambda, L)$ is at some height $h$ above the reference geoid

# Finding Where You Are




A surface point $O$ at Lon/Lat $(\lambda, L)$ is at some height $h$ above the reference geoid

Geoid described by $a_e$ and $e_e$ where: $e_e^2 = 2f - f^2$

$$[\mathbf{r}_{O/G}]_{\mathcal{G}} = \begin{bmatrix} x\cos\theta_{\mathrm{LST}} \\ x\sin\theta_{\mathrm{LST}} \\ z \end{bmatrix}_{\mathcal{G}}$$

$$x = \left(\frac{a_e}{\sqrt{1 - e_e^2 \sin^2\phi}} + h\right)\cos\phi$$

$$y = \left(\frac{a_e(1 - e_e^2)}{\sqrt{1 - e_e^2 \sin^2\phi}} + h\right)\sin\phi$$