6 - Time, Time Systems, and Canonical Units

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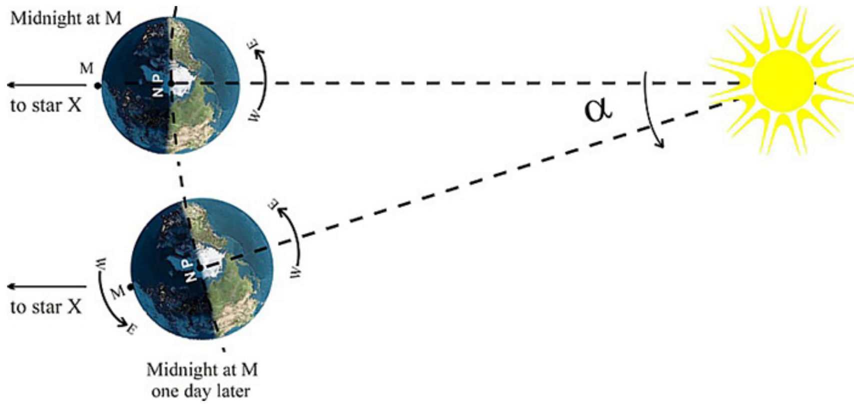
SI Seconds

- The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the Cesium-133 atom.
  –NIST (http://physics.nist.gov/cuu/Units/second.html)
- This definition refers to a cesium atom at rest at a temperature of 0 K.
  –BIPM
  (http://www.bipm.org/en/publications/si-brochure/second.html)

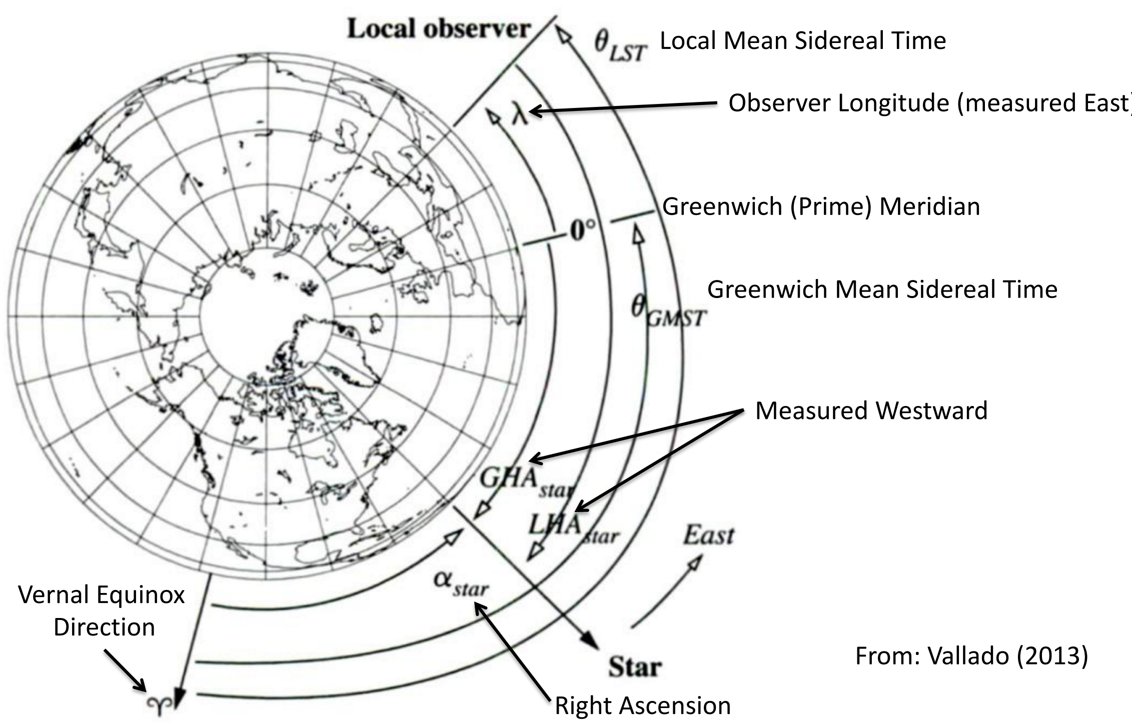
Solar vs Sidereal Time



From: https://dept.astro.lsa.umich.edu/resources/ugactivities/Labs/Detroit/index.html

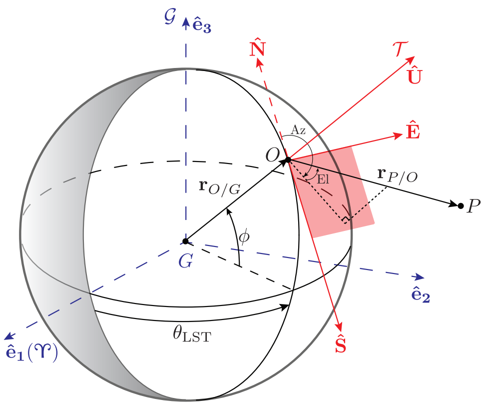
Mean Solar Day (d)	24 SI hours = 86400 SI seconds
Solar (Tropical) Year	365.242190402 d
Mean Sidereal Day	23h56m4.09054s
Sidereal Year	365.256363004 d

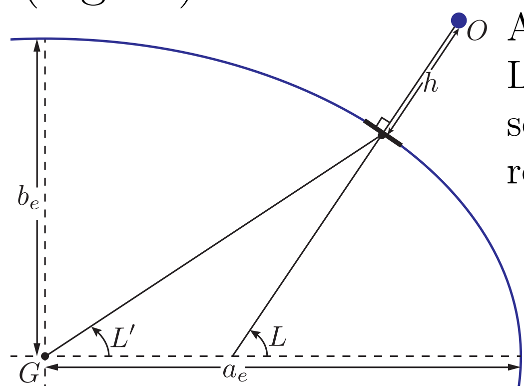
Hour Angle



From: Vallado (2013)

Finding Where You Are (Again)





A surface point O at Lon/Lat (λ, L) is at some height h above the reference geoid

Geoid described by a_e and e_e where: $e_e^2 = 2f - f^2$

$$[\mathbf{r}_{O/G}]_{\mathcal{G}} = \begin{bmatrix} x\cos\theta_{\mathrm{LST}} \\ x\sin\theta_{\mathrm{LST}} \\ z \end{bmatrix}_{\mathcal{G}}$$

$$x = \left(\frac{a_e}{\sqrt{1 - e_e^2\sin^2\phi}} + h\right)\cos\phi$$

$$y = \left(\frac{a_e(1-e_e^2)}{\sqrt{1 - e_e^2\sin^2\phi}} + h\right)\sin\phi$$

Finding When You Are

$$\theta_{\mathrm{LST}} = \theta_{g0} + \omega_e(t - t_0) + \lambda_E$$

Reference Value at Epoch

OR

MEAN SIDEREAL TIME, 2019

Greenwich mean sidereal time at 0^{h} UT

	h		h		h		h
Jan. 0	6·6250	Apr. 0	12·5389	July 0	18·5185	Oct. 0	0·5638
Feb. 0	8·6620	May 0	14·5102	Aug. 0	20·5555	Nov. 0	2·6008
Mar. 0	10·5019	June 0	16·5472	Sept. 0	22·5925	Dec. 0	4·5721

Greenwich mean sidereal time (GMST) on day d of month at hour t UT

= GMST at 0^{h} UT on day 0 + $0^{\mathrm{h}}065\,71\ d + 1^{\mathrm{h}}002\,74\ t$

Local mean sidereal time = GMST $\begin{smallmatrix}+\ \text{east}\\ -\ \text{west}\end{smallmatrix}$ longitude

https://aa.usno.navy.mil/publications/reports/ap19_for_web.pdf

$\omega_e = 7.292115 \times 10^{-5}$ rad/s (WGS84)

$\theta_{g0} = 100.4606184° + 36,000.77005361 T_{\mathrm{UT1}} + 0.00038793 T_{\mathrm{UT1}}^2 - 2.6 \times 10^{-8} T_{\mathrm{UT1}}^3$
T_{UT1} = number of Julian centuries from J2000.0

Julian Date

- Days since January 1, 4713 BCE, 12^{h} UT

$$\begin{aligned}\mathrm{JD} =& 367\mathrm{Y} - \mathrm{int}\left(\frac{7\left(\mathrm{Y} + \mathrm{int}\left(\frac{\mathrm{M}+9}{12}\right)\right)}{4}\right) + \mathrm{int}\left(\frac{275\mathrm{M}}{9}\right) + \mathrm{D} + 1721013.5 + \frac{\mathrm{UT}}{24} \\ & - \frac{1}{2}\,\mathrm{sgn}\left(100\mathrm{Y} + \mathrm{M} - 190002.5\right) + \frac{1}{2}\end{aligned}$$

$$\mathrm{int}(x) = \begin{cases} \lfloor x \rfloor & x \geq 0 \\ \lceil x \rceil & x < 0 \end{cases} \qquad \mathrm{sgn}(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

- 1 Julian year is <u>exactly</u> 365.25 days, 1 Julian century is 100 Julian years
- Define: MJD = JD -2,400,000.5

IEEE 754: Standard for Floating-Point Arithmetic

- A floating point number is represented by two values:
  1. s: The significand (mantissa, coefficient)—fixed length (p) digit string in base b
  2. e: The exponent—a signed integer

$$f \approx \frac{s}{b^{p-1}} b^e$$

- The IEEE 754 double precision (binary64, default in MATLAB) data type has: $b = 2$, $p = 52$, and 11 exponent bits ($e \in [-1022, 1023]$).