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sghiya-2

$$u = \begin{bmatrix} \delta \\ F \end{bmatrix}, \quad s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix}, \quad s_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{s}_1 = A_1 s_1 + B_1 u$$

$$\& \quad \dot{s}_2 = A_2 s_2 + B_2 u$$

Let, $s_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ $\quad \& \quad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} F \\ \delta \end{bmatrix}$

$$\therefore \quad \dot{s}_1 = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} x_2 \\ \ddot{y} \\ x_4 \\ \ddot{\psi} \end{bmatrix}$$

We have,

$$\ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}\left[\left(\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right)\cos\delta - \frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right]$$

$$\& \quad \ddot{\psi} = \frac{2l_f C_\alpha}{I_z}\left[\delta - \frac{\dot{y} + l_f\dot{\psi}}{\dot{x}}\right] - \frac{2l_r C_\alpha}{I_z}\left[-\frac{\dot{y} - l_r\dot{\psi}}{\dot{x}}\right]$$

$$\therefore \quad \cancel{\ddot{y} = -\dot{\psi}\dot{x} + \frac{2C_\alpha}{m}}$$

$$\therefore \quad \ddot{y} = -\dot{x}x_4 + \frac{2C_\alpha}{m}\left[\left(\delta - \frac{x_2 + l_f \cdot x_4}{\dot{x}}\right)\cos\delta - \frac{x_2 - l_r x_4}{\dot{x}}\right]$$

$$\ddot{\psi} = \frac{2l_f C_\alpha}{I_z}\left[\delta - \frac{x_2 + l_f \cdot x_4}{\dot{x}}\right] - \frac{2l_r C_\alpha}{I_z}\left[-\frac{x_2 - l_r x_4}{\dot{x}}\right]$$

$$\therefore \quad \dot{S}_1 = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\dot{x}x_4 + \frac{2C_\alpha}{m}\left[\left(\delta - \frac{x_2 + l_f x_4}{\dot{x}}\right)\cos\delta - \frac{x_2 - l_r x_4}{\dot{x}}\right] \\ x_4 \\ \frac{2l_f C_\alpha}{I_z}\cdot\left[\delta - \frac{x_2 + l_f x_4}{\dot{x}}\right] - \frac{2l_r C_\alpha}{I_z}\left[-\frac{x_2 - l_r x_4}{\dot{x}}\right] \end{bmatrix}$$

$\Rightarrow$ finding equilibrium point $\Rightarrow \dot{S}_1 = 0$

$\therefore \; x_2 = 0, \; x_4 = 0 \;\; \& \; \dot{x}_2 = \dot{x}_4 = 0$

$$\Rightarrow -\cancelto{0}{\dot{x}x_4} + \frac{2C_\alpha}{m}\left[\left(\delta - \cancelto{0}{\frac{x_2 + l_f x_4}{\dot{x}}}\right)\cos\delta - \cancelto{0}{\frac{x_2 - l_r x_4}{\dot{x}}}\right] = 0$$

$\Rightarrow \quad \frac{2C_\alpha}{m}.\delta.\cos\delta = 0 \quad \Rightarrow \boxed{\delta = 0} \rightarrow$ (for equilibrium)

for linearizing the system,

$$\therefore A_1 = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} & \frac{\partial f_3}{\partial x_4} \\ \frac{\partial f_4}{\partial x_1} & \frac{\partial f_4}{\partial x_2} & \frac{\partial f_4}{\partial x_3} & \frac{\partial f_4}{\partial x_4} \end{bmatrix} \quad \& \quad B_1 = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \frac{\partial f_1}{\partial u_2} \\ \frac{\partial f_2}{\partial u_1} & \frac{\partial f_2}{\partial u_2} \\ \frac{\partial f_3}{\partial u_1} & \frac{\partial f_3}{\partial u_2} \\ \frac{\partial f_4}{\partial u_1} & \frac{\partial f_4}{\partial u_2} \end{bmatrix}$$

Considering $f_1 = x_2$.

$\therefore \frac{\partial f_1}{\partial x_1} = 0 \qquad \frac{\partial f_2}{\partial x_2} = 1 \qquad \frac{\partial f_1}{\partial x_3} = 0 \qquad \frac{\partial f_1}{\partial x_4} = 0$

$\frac{\partial f_1}{\partial u_1} = 0 \qquad \frac{\partial f_1}{\partial u_2} = 0$

Considering $f_2 = -\dot{x}x_4 + \frac{2C_\alpha}{m}\left[\left(\delta - \frac{x_2 + l_f x_4}{\dot{x}}\right)\cos\delta - \frac{x_2 - l_r x_4}{\dot{x}}\right]$

$\frac{\partial f_2}{\partial x_1} = 0$, $\frac{\partial f_2}{\partial x_2} = \frac{2C_\alpha}{m}\left[\frac{-\cos\delta}{\dot{x}} - \frac{1}{\dot{x}}\right]$, $\frac{\partial f_3}{\partial x_3} = 0$, $\frac{\partial f_4}{\partial x_4} = -\dot{x} + \frac{2C_\alpha}{m}\left[\frac{-l_f\cos\delta}{\dot{x}} + \frac{l_r}{\dot{x}}\right]$

$\frac{\partial f_2}{\partial u_1} = \frac{2C_\alpha}{m}\left[\delta.(-\sin\delta) + \cos\delta\right]$, $\frac{\partial f_2}{\partial u_2} = 0$

Considering $f_3 = x_4$

$\frac{\partial f_3}{\partial x_1} = 0$, $\frac{\partial f_3}{\partial x_2} = 0$, $\frac{\partial f_3}{\partial x_3} = 0$, $\frac{\partial f_3}{\partial x_4} = 1$

$\frac{\partial f_3}{\partial u_1} = 0$, $\frac{\partial f_3}{\partial u_2} = 0$.

Considering $f_4 = \frac{2l_f C_\alpha}{I_z}\cdot\left[\delta - \frac{x_2 + l_f x_4}{\dot{x}}\right] - \frac{2l_r C_\alpha}{I_z}\left[-\frac{x_2 - l_r x_4}{\dot{x}}\right]$

$\frac{\partial f_4}{\partial x_1} = 0$, $\frac{\partial f_4}{\partial x_2} = -\frac{2C_\alpha}{I_z\dot{x}}\left[l_f - l_r\right]$, $\frac{\partial f_4}{\partial x_3} = 0$, $\frac{\partial f_4}{\partial x_4} = -\frac{2C_\alpha}{I_z\dot{x}}\left[l_f^2 + l_r^2\right]$

$\frac{\partial f_4}{\partial u_1} = \frac{2l_f C_\alpha}{I_z}$, $\frac{\partial f_4}{\partial u_2} = 0$

put in $\delta = 0$

$$\therefore \quad A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-4C_\alpha}{m\dot{x}} & 0 & -\dot{x} + \dfrac{2C_\alpha}{m\dot{x}}.(l_r - l_f) \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-2C_\alpha.(l_f - l_r)}{I_z \dot{x}} & 0 & \dfrac{-2C_\alpha}{I_z \dot{x}}.(l_f^2 + l_r^2) \end{bmatrix}$$

$$\& \quad B_1 = \begin{bmatrix} 0 & 0 \\ \dfrac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \dfrac{2l_f C_\alpha}{I_z} & 0 \end{bmatrix}$$

putting in values, $C_\alpha = 15000$, $m = 1000$, $l_r = 1.7$, $l_f = 1.1$, $I_z = 3344$

For $S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\therefore \dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} x_2 \\ \dot{x}_2 \end{bmatrix}$$

$$\therefore \dot{S}_2 = \begin{bmatrix} x_2 \\ \dot{\psi}\dot{y} + \frac{1}{m}\cdot(F - fmg) \end{bmatrix}$$

$$\frac{\partial f_1}{\partial x_1} = 0 \quad , \quad \frac{\partial f_1}{\partial x_2} = 1$$

$$\frac{\partial f_2}{\partial x_1} = 0 \quad , \quad \frac{\partial f_2}{\partial x_2} = 0$$

$$\frac{\partial f_1}{\partial u_1} = 0 \qquad \frac{\partial f_1}{\partial u_2} = 0$$

$$\frac{\partial f_2}{\partial u_1} = 0 \qquad \frac{\partial f_2}{\partial u_2} = 1/m$$

$$\therefore \boxed{A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \& \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1/m \end{bmatrix}}$$