Algorithms: Assignment Two

Deadline: Nov 1, 2020

1. You work for an advertising company that wishes to place billboards along a stretch of the highway. The company has identified n sites at distances $d_1 < d_2 < \ldots < d_n$ (measured in km) from the start of the highway; placing a billboard on the ith site will give a revenue/profit of r_i. However government regulations insist that any two billboards must be at least 5km apart. Your goal is to identify a subset of sites that satisfy the regulations and give maximum revenue.

   Example input, given as pairs (d_i, r_i): (6,5), (7,6), (11,5), (14,2). The optimal solution is to choose the first and third site for a max revenue of 10.

2. There is a railway route with n stations labeled in order as $1, 2, \ldots, n$; the train starts at station 1 and goes to station 2, then to station 3, and so on till station n. For each pair $i < j$, the cost of a ticket for boarding the train at station i and getting off at station j is equal to $C_{i,j}$. Interestingly, the costs are not well-chosen in the sense that it may be possible to travel via several hops at a lower cost. For example, suppose that $n = 3$ and we have $C_{1,3} = 100, C_{1,2} = 50, C_{2,3} = 40$. Then to travel from station 1 to station 3, it is less expensive to travel via station 2 than to travel directly. Your goal is to to travel from station 1 to station n with minimum cost. Describe an efficient algorithm for this, given the various $C_{i,j}$s as input.

3. Let $A[1...m]$ and $B[1...n]$ be two strings (character arrays). A common supersequence of A and B is a string that contains both A and B as subsequences. For example, the string $MATCHES$ is a supersequence of the two strings $MATE$ and $ACHES$. Describe an efficient algorithm to find a common supersequence of A and B of minimum length.

4. You have an unlimited supply of coins of denominations $c_1, c_2, \ldots, c_n$, which are positive integers. You wish to make change for a value of V (also a pos-

itive integer), using minimum number of coins. You may assume $c_1 = 1$, so that it is always possible to make change for any positive integer value. Describe and analyze an efficient algorithm for this problem.